International J.Math. Combin. Vol.3(2013), 96-103

Ratio by Using Coefficients of Fibonacci Sequence

Megha Garg and Pertik Garg

(Punjab Technical University, Jalanahar, Punjab)

Ravinder Kumar

(Bhai Gurdas Group of Institution, Sangrur, Punjab)

E-mail: garg3032@gmail.com, waytogarg@gmail.com, r.k.vohra49@gmail.com

Abstract: In this paper, ratio by using coefficients of Fibonacci sequence has been discussed in detail. The Fibonacci series is made from $F_{n+2} = F_n + F_{n+1}$. New sequences from the formula $F_{n+2} = aF_n + bF_{n+1}$ by using a and b, where a and b are consecutive coefficients of Fibonacci sequence are formed. These all new sequences have their own ratios. When find the ratio of these ratios, it always becomes 1.6, which is known as golden ratio in Fibonacci series.

Key Words: Fibonacci series, Fibonacci in nature, golden ratio, ratios of new sequences, ratio of all new ratios.

AMS(2010): 05C78

§1. Introduction

The Fibonacci numbers were first discovered by a man named Leonardo Pisano. He was known by his nickname, Fibonacci. The Fibonacci sequence is a sequence in which each term is the sum of the 2 numbers preceding it. The first 10 Fibonacci numbers are: $1, 1, 2, 3, 5, 8, 13, 21, 34, 55$ and 89. These numbers are obviously recursive. Leonardo Pisano Bogollo, (c.1170 - c.1250) known as Leonardo of Pisa, Fibonacci was an Italian mathematician (Anderson, Frazier, & Popendorf, 1999). He is considered as the most talented mathematician of the middle ages (Eves, 1990). Fibonacci was first introduced to the number system we currently use with symbols from 0 to 9 along with the Fibonacci sequence by Indian merchants when he was in northern Africa (Anderson, Frazier, & Popendorf, 1999). He then introduced the Fibonacci sequence and the number system we currently use to the western Europe In his book Liber Abaci in 1202 (Singh, Acharya Hemachandra and the (so called) Fibonacci Numbers, 1986) (Singh, The so-called Fibonacci numbers in ancient and medieval India, 1985). Fibonacci was died around 1240 in Italy. He played an important role in reviving ancient mathematics and made significant contributions of his own. Fibonacci numbers are important to perform a run-time analysis of Euclid's algorithm to Find the greatest common divisor (GCD) of two integers. A pair of two consecutive Fibonacci numbers makes a worst case input for this algorithm (Knuth, Art of Computer Programming, Volume 1: Fundamental Algorithms, 1997). Fibonacci numbers

[1]Received May 31, 2013, Accepted September 5, 2013.

have their application in the Polyphone version of the Merge Sort algorithm. This algorithm divides an unsorted list in two Lists such that the length of lists corresponds to two sequential Fibonacci numbers.

If we take the ratio of two successive numbers in Fibonacci series, $(1, 1, 2, 3, 5, 8, 13,\ cdots)$ we find

$$1/1 = 1,\ 2/1 = 2,\ 3/2 = 1.5,\ 5/3 = 1.666...; 8/5 = 1.6; 13/8 = 1.625.$$

Greeks called the golden ratio and has the value 1.61803. It has some interesting properties, for instance, to square it, you just add 1. To take its reciprocal, you just subtract 1. This means all its powers are just whole multiples of itself plus another whole integer (and guess what these whole integers are? Yes! The Fibonacci numbers again!) Fibonacci numbers are a big factor in Math.

1.1 **Fibonacci Credited Two Things**

1. Introducing the Hindu-Arabic place-valued decimal system and the use of Arabic numerals into Europe. (Can you imagine us trying to multiply numbers using Roman numerals?)
2. Developing a sequence of numbers (later called the Fibonacci sequence) in which the first two numbers are one, then they are added to get 2, 2 is added to the prior number of 1 to get 3, 3 is added to the prior number of 2 to get 5, 5 is added to the prior number of 3 to get 8, etc. Hence, the sequence begins as $1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144$, etc Allows users to distribute parallelized workloads to a shared pool of resources to automatically find and use the best available resource. The ability to have pieces of work run in parallel on different nodes in the grid allows the over all job to complete much more quickly than if all the pieces were run in sequence.

1.2 **List of Fibonacci Numbers**

The first 21 Fibonacci numbers F_n for $n = 0, 1, 2, \cdots, ..., 20$ are respectively

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765.$$

The Fibonacci sequence can be also extended to negative index n using the re-arranged recurrence relation

$$F_{n-2} = F_n - F_{n-1}.$$

This yields the sequence of *negafibonacci* numbers satisfying

$$F_{-n} = (-1)^{n+1} F_n.$$

Thus the bidirectional sequence is

F_{-8}	F_{-7}	F_{-6}	F_{-5}	F_{-4}	F_{-3}	F_{-2}	F_{-1}	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8
−21	13	−8	5	−3	2	−1	1	0	1	1	2	3	5	8	13	21

§2. Fibonacci Sequence in Nature

2.1 Sunflower

The Fibonacci numbers have also been observed in the family tree of honeybees. The Fibonacci sequence is a pattern of numbers starting with 0 and 1 and adding each number in sequence to the next $\cdots$, $0+1=1, 1+1=2$ so the first few numbers are $0, 1, 1, 2, 3, 5, 8, \cdots$ and so on and so on infinitely.

Fig.1.1 Sunflower head displaying florets in spirals of 34 and 55 around the outside

One of the most common experiments dealing with the Fibonacci sequence is his experiment with rabbits. Fibonacci put one male and one female rabbit in a field. Fibonacci supposed that the rabbits lived infinitely and every month a new pair of one male and one female was produced. Fibonacci asked how many would be formed in a year. Following the Fibonacci sequence perfectly the rabbit's reproduction was determined 144 rabbits. Though unrealistic, the rabbit sequence allows people to attach a highly evolved series of complex numbers to an everyday, logical, comprehendible thought.

Fibonacci can be found in nature not only in the famous rabbit experiment, but also in beautiful flowers. On the head of a sunflower and the seeds are packed in a certain way so that they follow the pattern of the Fibonacci sequence. This spiral prevents the seed of the sunflower from crowding themselves out, thus helping them with survival. The petals of flowers and other plants may also be related to the Fibonacci sequence in the way that they create new petals.

2.2 Petals on Flowers

Probably most of us have never taken the time to examine very carefully the number or arrangement of petals on a flower. If we were to do so, we would find that the number of petals on a flower that still has all of its petals intact and has not lost any, for many flowers is a Fibonacci number:

(1) 3 petals: lily, iris;
(2) 5 petals: buttercup, wild rose, larkspur, columbine (aquilegia);
(3) 8 petals: delphiniums;
(4) 13 petals: ragwort, corn marigold, cineraria;
(5) 21 petals: aster, black-eyed susan, chicory;
(6) 34 petals: plantain, pyrethrum;

(7) 55, 89 petals: michaelmas daisies, the asteraceae family.

2.3 **Fibonacci Numbers in Vegetables and Fruits**

Romanesque Brocolli/Cauliflower (or Romanesco) looks and tastes like a cross between brocolli and cauliflower. Each floret is peaked and is an identical but smaller version of the whole thing and this makes the spirals easy to see.

Fig.1.2 Brocolli/Cauliflower

2.4 **Human Hand**

Every human has two hands, each one of these has five fingers, each finger has three parts which are separated by two knuckles. All of these numbers fit into the sequence. However keep in mind, this could simply be coincidence.

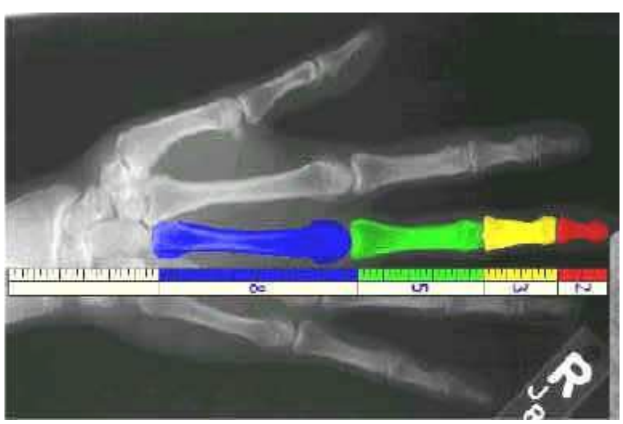

Fig.1.3 Human hand

Subject: The Fibonacci series is a sequence of numbers first created by Leonardo Fibonacci in 1202. The first two numbers of the series are 1 and 1 and each subsequent number is sum of the previous two. Fibonacci numbers are used in computer algorithms. The Fibonacci sequence first appears in the book Liber Abaci by Leonardo of Pisa known as Fibonacci. Fibonacci considers the growth of an idealized rabbit population, assuming that a newly born pair of rabbits, one male, one female and do the study on it. The Fibonacci series become $1, 1, 2, 3, 5, 8, 13, 21 \cdots$.

§3. Ratio by Using Coefficients of Fibonacci Sequence

3.1 Ratio By Using $1, 2$ as Coefficients

Apply the formula by using the next two coefficients of Fibonacci series i.e. 1 for F_{n+1} and 2 for F_n. So the series that becomes from this formula is $F_{n+2} = 2F_n + F_{n+1}$, $F_1 = 1$, $F_2 = 1$, $F_3 = 3$, $5, 11, 21, 43, 85, 171, 341, 683, 1365, \cdots$. K

From this sequence, find the ratio by dividing two consecutive numbers.

$$\frac{F_2}{F_1} = \frac{1}{1} = 1$$
$$\frac{F_3}{F_2} = \frac{3}{1} = 3$$
$$\frac{F_4}{F_3} = \frac{5}{3} = 1.66$$
$$\frac{F_5}{F_4} = \frac{11}{5} = 2.2$$
$$\frac{F_6}{F_5} = \frac{21}{11} = 1.9$$
$$\frac{F_7}{F_6} = \frac{43}{21} = 2.0$$
$$\frac{F_8}{F_7} = \frac{85}{43} = 1.9$$
$$\frac{F_9}{F_8} = \frac{171}{85} = 2.0$$

From here the conclusion is that the ratio (in integer) of this series is 2.

3.2 Ratio by Using 2, 3 as Coefficients

The series that becomes by using $2, 3$ as coefficients is $F_{n+2} = 3F_n + 2F_{n+1}$, i.e., $F_1 = 1, F_2 = 1, F_3 = 5$, $13, 41, 121, 365, 1093, 3281, 9841, \cdots$.

From this sequence, find the ratio by dividing two consecutive numbers.

$$\frac{F_2}{F_1} = \frac{1}{1} = 1$$
$$\frac{F_3}{F_2} = \frac{5}{1} = 5$$
$$\frac{F_4}{F_3} = \frac{13}{5} = 2.6$$
$$\frac{F_5}{F_4} = \frac{41}{13} = 3.15$$

$$\frac{F_6}{F_5} = \frac{121}{41} = 3.15$$
$$\frac{F_7}{F_6} = \frac{365}{121} = 3.01$$
$$\frac{F_8}{F_7} = \frac{1093}{365} = 2.99$$
$$\frac{F_9}{F_8} = \frac{3281}{1093} = 3.01$$

From here the conclusion is that the ratio (in integer) of this series is 3.

3.3 Ratio by Using $3, 5$ as Coefficients

The series that becomes by using $3, 5$ as coefficients is $F_{n+2} = 5F_n + 3F_{n+1}$, i.e., $F_1 = 1, F_2 = 1, F_3 = 8,\ 29, 127, 526, 2213, 9269, 38872, 162961, \cdots$.

From this sequence, find the ratio by dividing two consecutive numbers.

$$\frac{F_2}{F_1} = \frac{1}{1} = 1$$
$$\frac{F_3}{F_2} = \frac{8}{1} = 8$$
$$\frac{F_4}{F_3} = \frac{29}{8} = 3.6$$
$$\frac{F_5}{F_4} = \frac{127}{29} = 4.3$$
$$\frac{F_6}{F_5} = \frac{526}{127} = 4.14$$
$$\frac{F_7}{F_6} = \frac{2213}{526} = 4.20$$
$$\frac{F_8}{F_7} = \frac{9269}{2213} = 4.18$$
$$\frac{F_9}{F_8} = \frac{38872}{9269} = 4.19$$

From here the conclusion is that the ratio (in integer) of this series is 4.

3.4 Ratio by Using $5, 8$ as Coefficients

The series that becomes by using $5, 8$ as coefficients is $F_{n+2} = 8F_n + 5F_{n+1}$, i.e., $F_1 = 1, F_2 = 1, F_3 = 13,\ 73, 469, 2929, 18397, 115417, 724229, \cdots$.

From this sequence , find the ratio by dividing two consecutive numbers.

$$\frac{F_2}{F_1} = \frac{1}{1} = 1$$
$$\frac{F_3}{F_2} = \frac{13}{1} = 13$$
$$\frac{F_4}{F_3} = \frac{73}{13} = 5.6$$

$$\frac{F_5}{F_4} = \frac{469}{73} = 6.4$$
$$\frac{F_6}{F_5} = \frac{2929}{469} = 6.24$$
$$\frac{F_7}{F_6} = \frac{18397}{2929} = 6.28$$
$$\frac{F_8}{F_7} = \frac{115417}{18397} = 6.27$$
$$\frac{F_9}{F_8} = \frac{724229}{115417} = 6.27$$

From here the conclusion is that the ratio (in integer) of this series is 6.

Continuing in this way, find that the ratio of

$F_{n+2} = 13F_n + 8F_{n+1}$ is 9 (in integer);
$F_{n+2} = 21F_n + 13F_{n+1}$ is 14 (in integer);
$F_{n+2} = 34F_n + 21F_{n+1}$ is 22 (in integer);
. . . .

§4. Conclusion

Therefore the sequence becomes from all the ratios by using the consecutive numbers as the coefficients of Fibonacci sequence is:

$$2, 3, 4, 6, 9, 14, 22, 35, 56, 90, 145, 234, 378, \cdots$$

Now find the ratio that on dividing consecutive integers, of this sequence is:

$$3/2 = 1.5, 4/3 = 1.33, 6/4 = 1.5, 14/9 = 1.6, 22/14 = 1.6, 35/22 = 1.6$$

and $$56/35 = 1.6, 90/1.6, 145/90 = 1.6, 234/145 = 1.6 \cdots$$

It always become 1.6, yes it is again the golden ratio of Fibonacci sequence. So the conclusion is that the ratio of these ratios is always become golden ratio in Fibonacci series.

References

[1] Huylebrouck Dirk, Gyllenberg Mats and Sigmund Karl (2000), *The Fibonacci Chimney*, The Mathematical Intelligencer
http://www.mathsisfun.com/numbers/fibonaccisequence.html
http://www.maths.surrey.ac.uk/hostedsites/ R.Knott/Fibonacci/fib.html
http://www.maths.surrey.ac.uk/hostedsites/ R.Knott/Fibonacci/fibmaths.html

[2] The Engineer, *Eden Project Gets Into Flower Power*,
http://www.theengineer.co.uk/news/eden-project-gets-into-flower-power

[3] Di Carlo, Christopher (2001), *Interview with Maynard James Keenan.*

[4] Ingmar Lehman, *Fibonacci-numbers in visual arts and literature*,
http://en.wikipedia.org/wiki/Fibonacci_numbers_in_popular_culture

[5] Munjal Patel (2010), *Fibonacci Number*,
http://munjal.munpat.com/papers/fibonacci-numbers.pdf
[6] Dan Kalman and Robert Mena (2003), *The Fibonacci Numbers-Exposed*,
http://www.math.hmc.edu/ benjamin/papers/exposed.pdf