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On Mean Cordial Graphs

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Abstract: Let f be a function from the vertex set $V(G)$ to $\{0,1,2\}$. For each edge uv assign the label $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. f is called a mean cordial labeling if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$, $i, j \in \{0,1,2\}$, where $v_f(x)$ and $e_f(x)$ respectively are denote the number of vertices and edges labeled with x $(x = 0,1,2)$. A graph with a mean cordial labeling is called a mean cordial graph. In this paper we investigate mean cordial labeling behavior of union of some graphs, square of paths, subdivision of comb and double comb and some more standard graphs.

Key Words: Path, star, complete graph, comb.

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§1. Introduction

All graphs in this paper are finite, undirected and simple. The vertex set and edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. The union of two graphs G_1 and G_2 is the graph $G_1 \cup G_2$ with $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$. The corona of G with H, $G \odot H$ is the graph obtained by taking one copy of G and p copies of H and joining the i^{th} vertex of G with an edge to every vertex in the i^{th} copy of H. The subdivision graph $S(G)$ of a graph G is obtained by replacing each edge uv by a path uwv. The triangular snake T_n is obtained from the path P_{n+1} by replacing each edge of the path by the triangle C_3. mG denotes the m copies of the graph G. The square G^2 of a graph G has the vertex set $V(G^2) = V(G)$, with u, v adjacent in G^2 whenever $d(u,v) \leq 2$ in G. The powers G^3, $G^4 \ldots$ of G are similarly defined. Ponraj et al. defined the mean cordial labeling of a graph in [4]. Mean cordial labeling behavior of path, cycle, star, complete graph, wheel, comb etc have been investigated in [4]. Here we investigate the mean cordial labeling behavior of some standard graphs. The symbol $\lceil x \rceil$ stands for smallest integer greater than or equal to x. Terms and definitions are not defined here are used in the sense of Harary [3].

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§2. Mean Cordial Labeling

Definition 2.1 *Let f be a function from $V(G)$ to $\{0,1,2\}$. For each edge uv of G assign the label* $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. *$f$ is called a mean cordial labeling if $|v_f(i)-v_f(j)| \leq 1$ and $|e_f(i)-e_f(j)| \leq 1$, $i,j \in \{0,1,2\}$, where $v_f(x)$ and $e_f(x)$ denote the number of vertices and edges labeled with x $(x=0,1,2)$ respectively. A graph with a mean cordial labeling is called a mean cordial graph.*

Theorem 2.2 *If $m \equiv 0 \pmod 3$ then mG is mean cordial for all m.*

Proof Let $m=3t$. Assign the label 0 to all the vertices of first t copies of the graph G. Then assign 1 to the vertices of next t copies of G. Finally assign 2 to remaining vertices of mG. Therefore $v_f(0)=v_f(1)=v_f(2)=pt$, $e_f(0)=e_f(1)=e_f(2)=qt$. □

Theorem 2.3 *If G is mean cordial, then mG, $m \equiv 1 \pmod 3$ is also mean cordial.*

Proof $(m-1)G$ is mean cordial by theorem 2.2. Let g be a mean cordial labeling of $(m-1)G$. Using the mean cordial labeling g of $(m-1)G$ and the mean cordial labeling of G, we get a mean cordial labeling of mG. □

Theorem 2.4 *$P_m \cup P_n$ is mean cordial.*

Proof Let $u_1u_2\ldots u_m$ and $v_1v_2\ldots v_n$ be the paths P_m and P_n respectively. Clearly $P_m \cup P_n$ has $m+n$ vertices and $m+n-2$ edges. Assume $m \geq n$.

Case 1 $m+n \equiv 0 \pmod 3$

Let $m+n=3t$. Define

$$\begin{aligned} f(u_i) &= 2, \quad 1 \leq i \leq t, \\ f(u_{t+i}) &= 1, \quad 1 \leq i \leq m-t, \\ f(v_i) &= 1, \quad 1 \leq i \leq n-t, \\ f(v_{n-t+i}) &= 0, \quad 1 \leq i \leq t. \end{aligned}$$

Clearly $v_f(0)=v_f(1)=v_f(2)=t$ and $e_f(0)=e_f(1)=t-1$, $e_f(2)=t$. Therefore f is a mean cordial labeling.

Case 2 $m+n \equiv 1 \pmod 3$

Similar to Case 1.

Case 3 $m+n \equiv 2 \pmod 3$

Let $m+n=3t+2$. Define

$$\begin{aligned} f(u_i) &= 2, \quad 1 \le i \le t, \\ f(u_{t+i}) &= 1, \quad 1 \le i \le m-t, \\ f(v_i) &= 1, \quad 1 \le i \le n-t-1, \\ f(v_{n-t+i}) &= 0, \quad 1 \le i \le t+1. \end{aligned}$$

Clearly $v_f(0) = v_f(1) = t+1$, $v_f(2) = t$ and $e_f(0) = t-1$, $e_f(1) = e_f(2) = t$. Therefore $P_m \cup P_n$ is mean cordial. □

Theorem 2.5 *$C_n \cup P_m$ is mean cordial if $m \ge n$.*

Proof Let C_n be the cycle $u_1u_2 \ldots u_mu_1$ and P_m be the path $v_1v_2 \ldots v_n$ respectively. Clearly $C_n \cup P_m$ has $m+n$ vertices and $m+n-1$ edges. Assume $m \ge n$.

Case 1 $m+n \equiv 0 \pmod 3$

Let $m+n=3t$. Define

$$\begin{aligned} f(u_i) &= 0, \quad 1 \le i \le t, \\ f(u_{t+i}) &= 1, \quad 1 \le i \le n-,t \\ f(v_i) &= 1, \quad 1 \le i \le m-t, \\ f(v_{m-t+i}) &= 2, \quad 1 \le i \le t. \end{aligned}$$

Clearly $e_f(0) = t-1$, $e_f(1) = e_f(2) = t$.

Case 2 $m+n \equiv 1 \pmod 3$

Let $m+n=3t+1$. Define

$$\begin{aligned} f(u_i) &= 0, \quad 1 \le i \le t+1, \\ f(u_{t+1+i}) &= 1, \quad 1 \le i \le n-t-1, \\ f(v_i) &= 1, \quad 1 \le i \le m-t, \\ f(v_{m-t+i}) &= 2, \quad 1 \le i \le t. \end{aligned}$$

Clearly $e_f(0) = e_f(1) = t-1$, $e_f(2) = t$.

Case 3 $m+n \equiv 2 \pmod 3$

Let $m+n=3t+2$. Define

$$\begin{aligned} f(u_i) &= 0, \quad 1 \le i \le t+1, \\ f(u_{t+1+i}) &= 1, \quad 1 \le i \le n-t-1, \\ f(v_i) &= 1, \quad 1 \le i \le m-t-1, \\ f(v_{m-t-1+i}) &= 2, \quad 1 \le i \le t. \end{aligned}$$

Clearly $e_f(0) = e_f(1) = t$, $e_f(2) = t+1$. Hence $C_n \cup P_m$ is mean cordial. □

Theorem 2.6 *$K_{1,n}\cup P_m$ is mean cordial.*

Proof Let $V(K_{1,n})=\{u,u_i:1\le i\le n\}$ and $E(K_{1,n})=\{uu_i:1\le i\le n\}$. Let P_m be the path $v_1v_2\ldots v_m$ respectively. Clearly $K_{1,n}\cup P_m$ has $m+n+1$ vertices and $m+n-1$ edges.

Case 1 $m+n\equiv 0 \pmod 3$

Let $m+n=3t$. Define $f(u)=1$

$$\begin{aligned} f(u_i) &= 1, \quad 1\le i\le t-1,\\ f(u_{t-1+i}) &= 2, \quad 1\le i\le n-t+1,\\ f(v_i) &= 2, \quad 1\le i\le m-t-1,\\ f(v_{m-t-1+i}) &= 0, \quad 1\le i\le t+1.\end{aligned}$$

Clearly $e_f(0)=e_f(1)=t$, $e_f(2)=t-1$.

Case 2 $m+n\equiv 1 \pmod 3$

Similar to Case 1.

Case 3 $m+n\equiv 2 \pmod 3$

Let $m+n=3t+2$. Assign the labels to the vertices as in case 1 and then $e_f(0)=e_f(2)=t$, $e_f(1)=t+1$. Hence $K_{1,n}\cup P_m$ is mean cordial. □

Example 2.7 A mean cordial labeling of $K_{1,8}\cup P_6$ is given in Figure 1.

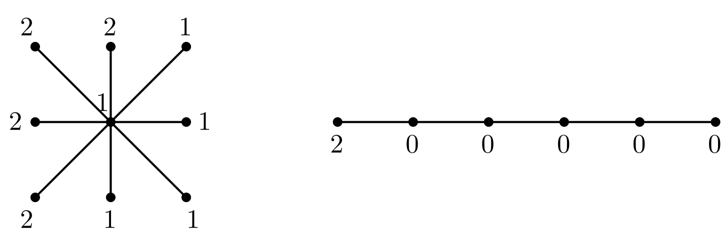

Figure 1

Theorem 2.8 *$S(P_n\odot K_1)$ is mean cordial where $S(G)$ and $G\odot H$ respectively denotes the subdivision of G and corona of G with H.*

Proof Let P_n be the path $u_1u_2\ldots u_n$ and v_i be the pendant vertices adjacent to u_i. Let the edges u_iu_{i+1}, u_iv_i be subdivided by the vertices z_i and w_i respectively.

Case 1 $n\equiv 0 \pmod 3$

Let $n=3t$. Define $f(u_i)=f(v_i)=f(w_i)=2$, $f(u_{t+i})=f(v_{t+i})=f(w_{t+i})=1$, $f(u_{2t+i})=f(v_{2t+i})=f(w_{2t+i})=0$, $1\le i\le t$.

$$\begin{aligned} f(z_i) &= 2, \quad 1\le i\le t,\\ f(z_{t+i}) &= 1, \quad 1\le i\le t-1,\\ f(z_{2t-1+i}) &= 0, \quad 1\le i\le t.\end{aligned}$$

Here $v_f(0) = v_f(2) = 4t$, $v_f(1) = 4t-1$ and $e_f(0) = e_f(1) = 4t-1$, $e_f(2) = 4t$. Hence $S(P_n \odot K_1)$ is mean cordial graph.

Case 2 $n \equiv 1 \pmod 3$

Label the vertices z_i, u_i, v_i $(1 \le i \le n-1)$, w_i $(1 \le i \le n-2)$ as in Case 1. Then assign the labels $0,1,1,2$ to the vertices z_n, u_n, w_{n-1}, v_n respectively. Hence $v_f(0) = v_f(1) = v_f(2) = 4t+1$, $e_f(0) = 4t$, $e_f(1) = e_f(2) = 4t+1$. Hence $S(P_n \odot K_1)$ is mean cordial.

Case 3 $n \equiv 2 \pmod 3$

Label the vertices z_i, u_i, v_i $(1 \le i \le n-2)$, w_i $(1 \le i \le n-3)$ as in case 1. Assign the labels $0,1,2,2,1,1,0,0$ to the vertices u_{n-1}, u_n, v_{n-1}, v_n, w_{n-2}, w_{n-1}, z_{n-1}, z_n respectively. Here $v_f(1) = v_f(2) = 4t+2$, $v_f(0) = 4t+3$, $e_f(0) = e_f(1) = e_f(2) = 4t+2$. Hence $S(P_n \odot K_1)$ is mean cordial. □

Example 2.9 Mean cordial labeling of $S(P_4 \odot K_1)$ is given in Figure 2.

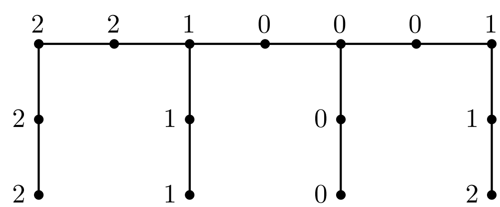

Figure 2

Theorem 2.10 $(P_n \odot 2K_1)$ *is mean cordial.*

Proof Let P_n be the path $u_1u_2\ldots u_n$ and v_i and w_i be the pendant vertices adjacent to u_i $(1 \le i \le n)$. Let the edges u_iu_{i+1}, u_iv_i, u_iw_i be subdivided by the vertices x_i and y_i, z_i respectively.

Case 1 $n \equiv 0 \pmod 3$

Let $n = 3t$. Define $f(u_i) = f(v_i) = f(w_i) = f(y_i) = f(z_i) = 2$, $f(u_{t+i}) = f(v_{t+i}) = f(w_{t+i}) = f(y_{t+i}) = f(z_{t+i}) = 1$, $f(u_{2t+i}) = f(v_{2t+i}) = f(w_{2t+i}) = f(y_{2t+i}) = f(z_{2t+i}) = 0$, $1 \le i \le t$.

$$\begin{aligned} f(x_i) &= 2 \quad 1 \le i \le t \\ f(x_{t+i}) &= 1 \quad 1 \le i \le t-1 \\ f(x_{2t-1+i}) &= 0 \quad 1 \le i \le t. \end{aligned}$$

Here $v_f(0) = v_f(2) = 6t$, $v_f(1) = 6t-1$, $e_f(0) = e_f(1) = 6t-1$, $e_f(2) = 6t$. Hence $S(P_n \odot 2K_1)$ is mean cordial.

Case 2 $n \equiv 1 \pmod 3$

Label the vertices u_i, v_i, w_i, y_i and z_i $(1 \le i \le n-1)$, x_i $(1 \le i \le n-2)$ as in case 1. Assign the labels $0,2,1,0,2,1$ to the vertices u_n, v_n, w_n, x_{n-1}, y_n and z_n respectively. Hence

$v_f(0) = v_f(2) = 6t+2$, $v_f(1) = 6t+1$, $e_f(0) = e_f(2) = 6t+1$, $e_f(1) = 6t+2$. Hence $S(P_n \odot 2K_1)$ is mean cordial.

Case 3 $n \equiv 2 \pmod 3$

Label the vertices u_i, v_i, w_i, y_i and z_i $(1 \le i \le n-2)$, x_i $(1 \le i \le n-3)$ as in case 1. Assign the labels $0, 0, 2, 2, 2, 2, 0, 0, 1, 1, 1, 1$ to the vertices u_{n-1}, u_n, v_{n-1}, v_n, w_{n-1}, w_n, x_{n-2}, x_{n-1}, y_{n-1}, y_n z_{n-1} and z_n respectively. Hence $v_f(0) = v_f(2) = 6t+4$, $v_f(1) = 6t+3$, $e_f(0) = e_f(1) = 6t+3$, $e_f(2) = 6t+4$. Hence $S(P_n \odot 2K_1)$ is mean cordial. □

Theorem 2.11 *P_n^2 is mean cordial iff $n \equiv 1$ (mod 3) and $n \ge 7$.*

Proof Let P_n be the path $u_1u_2 \dots u_n$. Clearly P_n^2 $(n \le 6)$ are not mean cordial. Assume $n \ge 7$. Clearly the order and size of P_n^2 are n and $2n-3$ respectively.

Case 1 $n \equiv 0 \pmod 3$

Let $n = 3t$. In this case $e_f(0) = (t-1) + (t-2) \le 2t-3$. which is a contradiction to the size of P_n^2.

Case 2 $n \equiv 1 \pmod 3$

Let $n = 3t+1$. Define

$$\begin{aligned} f(u_i) &= 0, \quad 1 \le i \le t+1, \\ f(u_{t+1+i}) &= 1, \quad 1 \le i \le t, \\ f(u_{2t+1+i}) &= 2, \quad 1 \le i \le t. \end{aligned}$$

Here $v_f(0) = t+1$, $v_f(1) = v_f(2) = t$, $e_f(0) = 2t-1$, $e_f(1) = e_f(2) = 2t$. Therefore P_n^2 is mean cordial.

Case 3 $n \equiv 2 \pmod 3$

Let $n = 3t+2$. Here $e_f(0) \le 2t-1$, a contradiction to the size of P_n^2. Therefore P_n^2 is not mean cordial. □

Example 2.12 A mean cordial labeling of P_{10}^2 is given in Figure 3.

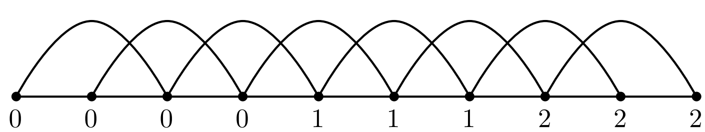

Figure 3

Theorem 2.13 *The triangular snake T_n $(n > 1)$ is mean cordial iff $n \equiv 0$ (mod 3).*

Proof Let $V(T_n) = \{u_i, v_j : 1 \le i \le n+1,\ 1 \le j \le n\}$ and $E(T_n) = \{u_iu_{i+1} : 1 \le i \le n\} \cup \{u_iv_i, v_iu_{i+1} : 1 \le i \le n\}$.

Case 1 $n \equiv 0 \pmod 3$

Let $n = 3t$. Define

$$\begin{aligned} f(u_i) &= 0, \quad 1 \le i \le t+1, \\ f(u_{t+1+i}) &= 1, \quad 1 \le i \le t, \\ f(u_{2t+1+i}) &= 2, \quad 1 \le i \le t, \\ f(v_i) &= 0, \quad 1 \le i \le m-t-1, \\ f(v_{t+i}) &= 1, \quad 1 \le i \le t, \\ f(v_{2t+i}) &= 2, \quad 1 \le i \le t. \end{aligned}$$

Here $v_f(0) = t+1$, $v_f(1) = v_f(2) = t$, $e_f(0) = e_f(1) = e_f(2) = 3t$. Therefore triangular snake T_n is mean cordial.

Case 2 $n \equiv 1 \pmod 3$

Let $n = 3t+1$. Here $v_f(0) = 2t+1$. But $e_f(0) \le 3t$, a contradiction.

Case 3 $n \equiv 2 \pmod 3$

Let $n = 3t+2$. In this case $v_f(0) = 2t+1$ or $2t+2$. But $e_f(0) \le 3t+1$, a contradiction.□

§3. Conclusion

In this paper we have studied the mean cordial behavior of $P_m \cup P_n$, $C_n \cup P_m$, $S(P_n \odot K_1)$, $S(P_n \odot 2K_1)$, P_n^2, T_n. Mean cordial labeling behavior of join and product of given two graphs are the open problems for future research.

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