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On Dynamical Chaotic Weyl Representations of the Vacuüm C Metric and Their Retractions

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Abstract: In this article we will introduce the dynamical chaotic vacuüm C metric when $m = 0$. The relations between the dynamical chaotic vacuüm and its deformation retract ar obtained. Many types of dynamical chaotic vacuüm are deduced. The end limits of n-dimensional chaotic vacuüm are presented.

Key Words: Chaotic vacuüm, Retraction, deformation retract.

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§1. Introduction

Chaos Theory is the qualitative study of unstable aperiodic behavior in deterministic nonlinear dynamical systems. Aperiodic behavior is observed when there is no variable, describing the state of the system, that undergoes a regular repetition of values. Unstable aperiodic behavior is highly complex it never repeats and it continues to manifest the effects of any small perturbation. As per the current mathematical theory a chaotic system is defined as showing sensitivity to initial conditions. In other words, to predict the future state of a system with certainty, you need to know the initial conditions with infinite accuracy, since errors increase rapidly with even the slightest inaccuracy. This is why the weather is so difficult to forecast. The theory also has been applied to business cycles, and dynamics of animal populations, as well as in fluid motion, planetary orbits, electrical currents in semi-conductors, medical conditions like epileptic seizures, and the modeling of arms races.

During the 1960s Edward Lorenz, a meteorologist at MIT, worked on a project to simulate weather patterns on a computer. He accidentally stumbled upon the butterfly effect after deviations in calculations off by thousandths greatly changed the simulations. The Butterfly Effect reflects how changes on the small-scale, can influence things on the large-scale. It is the classic example of chaos, where small changes may cause large changes. A butterfly, flapping its wings in Hong Kong, may change tornado patterns in Texas.

Chaos Theory regards organizations businesses as complex, dynamic, non-linear, co-creative and far-from-equilibrium systems. Their future performance cannot be predicted by past and

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present events and actions. In a state of chaos, organizations behave in ways which are simultaneously both unpredictable chaotic and patterned orderly [6,10,11].

The vacuüm C-metric was first discovered by Levi-Civita within a class of degenerate static vacuüm metrics. However, over the years it has been rediscovered many times: by Newman and Tamburino, by Robinson and Trautman and again by Ehlers and Kundt who called it the C-metric in 1962. The charged C-metric has been studied in detail by Kinnersley and Walker. In general the space-time represented by the C-metric contains one or, via an extension, two uniformly accelerated particles as explained in. A description of the geometric properties of various extensions of the C-metric as well as a more complete list of references is contained in . The main property of the C-metric is the existence of two hypersurface-orthogonal Killing vectors, one of which is time like (showing the static property of the metric) in the space-time region of interest in this work. The C-metric is a vacuüm solution of the Einstein equations of the Petrov type D. Kinnersley and Walker showed that it represents black holes uniformly accelerated by nodal singularities in opposite directions along the axis of the axial symmetry [5,7,9].

Many types of dynamical manifolds And systems are discussed in [1-4,11]. A dynamical system in the space X is a function $q = f(p,t)$ which assigns to each point p of the space X and to each real number t, $\infty < t < \infty$ a definite point $q \in X$ and possesses the following three properties:

a – Initial condition: $f(p,0) = p$ for any point $p \in X$;

b – Property of continuity in both arguments simultaneously:

$$\lim_{\substack{p\to p_0\\ t\to t_0}} f(p,t) = f(p_0,t_0).$$

c – Group property $f(f(p,t_1),t_2) = f(p,t_1+t_2)$ [11].

A subset A of a topological space X is called a retract of X if there exists a continuous map $r : X \to A$ (called a retraction) such that $r(a) = a$, $\forall a \in A$ [8]. A subset A of a topological space X is a deformation retract of X if there exists a retraction $r : X \to A$ and a homotopy $f : X \times I \to X$ such that $f(x,0) = x, f(x,1) = r(x)$, $\forall x \in X$ and $f(a,t) = a$, $\forall a \in A, t \in [0,1]$ [8].

§2. Main Results

In this paper we will discuss some types of retractions and deformations retracts in Weyl representation of the space-time of the vacuüm C metric when $m = 0$.
The chaotic vacuüm C metric when $m = 0$ is defined as

$$ds^2 = \frac{1}{A^2(x(t)+y(t))^2}\begin{bmatrix} -k^2A^2(-1+y^2(t))du^2(t) + \frac{1}{1-x^2(t)}dx^2(t)+ \\ \frac{1}{-1+y^2(t)}dy^2(t) + \frac{1-x^2(t)}{k^2}dw^2(t)\end{bmatrix} \tag{1}$$

where $x(t), y(t), u(t), w(t)$ are functions of time. The chaotic Weyl coordinates system are

$$\begin{aligned} z(t) &= \frac{1+x(t)y(t)}{A(x(t)+y(t))^2} \\ r(t) &= \frac{(1-x^2(t))^{\frac{1}{2}}(y^2(t)-1)^{\frac{1}{2}}}{A(x(t)+y(t))^2} \\ w(t) &= w(t) \\ u(t) &= u(t) \end{aligned}$$

Now we will use the following Lagrangian equations:

$$\frac{d}{ds}\left(\frac{\partial T}{\partial \Psi_i'}\right) - \frac{\partial T}{\partial \Psi_i} = 0,\ i = 1, 2, 3, 4. \tag{2}$$

To deduce a chaotic geodesic which is a retraction of $\overset{ch}{C_0}$ by using Lagrangian equations, where $\overset{ch}{C_0}$ is the chaotic vacuüm C metric when $m = 0$. Since, $T = \frac{1}{2}ds^2$ it follows that

$$T = \frac{1}{2}\left(\frac{1}{A^2(x(t)+y(t))^2}\left[\begin{array}{c} -k^2A^2(-1+y^2(t))du^2(t) + \frac{1}{1-x^2(t)}dx^2(t) + \\ \frac{1}{-1+y^2(t)}dy^2(t) + \frac{1-x^2(t)}{k^2}dw^2(t) \end{array}\right]\right). \tag{3}$$

Then the Lagrangian equations of chaotic vacuüm $\overset{ch}{C_0}$ are

$$\frac{d}{ds}\left[-\frac{k^2(-1+y^2(t)}{(x(t)+y(t))^2})u'(t)\right] = 0 \tag{4}$$

$$\frac{d}{ds}\left[\frac{1-x^2(t)}{k^2A^2(x(t)+y(t))^2}w'(t)\right] = 0 \tag{5}$$

$$\begin{aligned} &\frac{d}{ds}\left[\frac{x'(t)}{A^2(x(t)+y(t))^2(1-x^2(t))}\right] - \frac{1}{A^2(x(t)+y(t))^2}\times \\ &\qquad \left[\frac{x(t)}{1-x^2(t)}(x'(t))^2 + \frac{-x(t)}{k^2}(w'(t))^2\right] + \\ &\left[\begin{array}{c} -k^2A^2(-1+y^2(t))(u'(t))^2 + \frac{1}{1-x^2(t)}(x'(t))^2 + \\ \frac{1}{-1+y^2(t)}(y'(t))^2 + \frac{1-x^2(t)}{k^2}(w'(t))^2 \end{array}\right]\left[\frac{1}{A^2(x(t)+y(t))^3}\right] = 0 \end{aligned} \tag{6}$$

$$\begin{aligned} &\frac{d}{ds}\left[\frac{y'(t)}{A^2(x(t)+y(t))^2(-1+y^2(t))}\right] - \frac{1}{A^2(x(t)+y(t))^2}\times \\ &\qquad \left[-k^2A^2y(t)(u'(t))^2 - \frac{y(t)}{(-1+y^2(t))^2}(y'(t))^2\right] + \\ &\left[\begin{array}{c} -k^2A^2(-1+y^2(t))(u'(t))^2 + \frac{1}{1-x^2(t)}(x'(t))^2 + \\ \frac{1}{-1+y^2(t)}(y'(t))^2 + \frac{1-x^2(t)}{k^2}(w'(t))^2 \end{array}\right]\left[\frac{1}{A^2(x(t)+y(t))^3}\right] = 0 \end{aligned} \tag{7}$$

From Eq.(2.4), we obtain $\frac{k^2(-1+y^2(t)}{(x(t)+y(t))^2})u'(t)$ =constant ,say λ,if $\lambda=0$ then $u'(t)=0$, and so $u(t)$ =constant α, if $\alpha=0$ we have the following retraction

$$\begin{aligned} z(t) &= \frac{1+x(t)y(t)}{A(x(t)+y(t))^2} \\ r(t) &= \frac{(1-x^2(t))^{\frac{1}{2}}(y^2(t)-1)^{\frac{1}{2}}}{A(x(t)+y(t))^2} \\ w(t) &= w(t) \\ u(t) &= 0 \end{aligned}$$

which is the chaotic retraction $\overset{ch}{C}_{01}$ in the chaotic vacuüm $\overset{ch}{C}_{0}$. Also, from Eq.(2.5), we get $\frac{1-x^2(t)}{k^2A^2(x(t)+y(t))^2}w'(t)$ =constant ,say ν , if $\nu=0$ then $w'(t)=0$, and so $w(t)$ =constant δ, if $\delta=0$ we have the following retraction

$$\begin{aligned} z(t) &= \frac{1+x(t)y(t)}{A(x(t)+y(t))^2} \\ r(t) &= \frac{(1-x^2(t))^{\frac{1}{2}}(y^2(t)-1)^{\frac{1}{2}}}{A(x(t)+y(t))^2} \\ w(t) &= 0 \\ u(t) &= u(t) \end{aligned}$$

which is the chaotic retraction $\overset{ch}{C}_{02}$ in the chaotic vacuüm $\overset{ch}{C}_{0}$. Moreover from Eq.(2.6), we have $\frac{d}{ds}\left[\frac{x'(t)}{A^2(x(t)+y(t))^2(1-x^2(t))}\right]$ =constant , say ϖ , if $\varpi=0$ then $x'(t)=0$, and so $x(t)$ =constant β, if $\beta=0$ we have the following retraction

$$\begin{aligned} z(t) &= \frac{1}{Ay^2(t)} \\ r(t) &= \frac{(y^2(t)-1)^{\frac{1}{2}}}{Ay^2(t)} \\ w(t) &= w(t) \\ u(t) &= u(t) \end{aligned}$$

which is chaotic geodesic $\overset{ch}{C}_{03}$ in chaotic hyper affine subspace of chaotic vacuüm $\overset{ch}{C}_{0}$. Now, from Eq.(2.7), we have $\frac{d}{ds}\left[\frac{y'(t)}{A^2(x(t)+y(t))^2(-1+y^2(t))}\right]$ =constant ,say γ, if $\gamma=0$ then $y'(t)=0$, and so $y(t)$ =constant ρ, if $\rho=0$ we have the following retraction

$$\begin{aligned} z(t) &= \frac{1}{Ax^2(t)} \\ r(t) &= \frac{i(1-x^2(t))^{\frac{1}{2}}}{Ax^2(t)} \\ w(t) &= w(t) \\ u(t) &= u(t) \end{aligned}$$

which is chaotic geodesic $\overset{ch}{C}_{04}$ in chaotic hyper affine subspace of chaotic vacuüm $\overset{ch}{C}_0$

From the above discussion we can formulate the following theorem.

Theorem 2.1 *The geodesic of the chaotic vacuüm $\overset{ch}{C}_0$ by using Lagrangian equations is a type of retraction which is chaotic hyper affine subspace of $\overset{ch}{C}_0$.*

Now we will discuss the relations between the deformation retracts of chaotic vacuüm and their geodesics. The deformation retract of the chaotic vacuüm $\overset{ch}{C}_0$ is defined as $\Psi : \overset{ch}{C}_0 \times I \to \overset{ch}{C}_0$,where I is the closed interval $[0, 1]$. The retraction of the chaotic vacuüm $\overset{ch}{C}_0$ is defined as $r : \overset{ch}{C}_0 \to \overset{ch}{C}_{01}, \overset{ch}{C}_{02}, \overset{ch}{C}_{03}$ and $\overset{ch}{C}_{04}$. The deformation retract of the chaotic vacuüm $\overset{ch}{C}_0$ into a geodesic $\overset{ch}{C}_{01} \subseteq \overset{ch}{C}_0$ is given by

$$\begin{aligned}\Psi(m,s) &= \cos(\frac{\pi s}{2})\left(\frac{1+x(t)y(t)}{A(x(t)+y(t))^2}, \frac{(1-x^2(t))^{\frac{1}{2}}(y^2(t)-1)^{\frac{1}{2}}}{A(x(t)+y(t))^2}, w(t), u(t)\right)\\ &\quad + \sin(\frac{\pi s}{2})\left(\frac{1+x(t)y(t)}{A(x(t)+y(t))^2}, \frac{(1-x^2(t))^{\frac{1}{2}}(y^2(t)-1)^{\frac{1}{2}}}{A(x(t)+y(t))^2}, w(t), 0\right)\end{aligned}$$

and so $\Psi(m, 0) = \overset{ch}{C}_0$, $\Psi(m, 1) = \overset{ch}{C}_{01}$. The deformation retract of the chaotic vacuüm $\overset{ch}{C}_0$ into a geodesic $\overset{ch}{C}_{02} \subseteq \overset{ch}{C}_0$ is given by

$$\begin{aligned}\Psi(m,s) &= \cos(\frac{\pi s}{2})\left(\frac{1+x(t)y(t)}{A(x(t)+y(t))^2}, \frac{(1-x^2(t))^{\frac{1}{2}}(y^2(t)-1)^{\frac{1}{2}}}{A(x(t)+y(t))^2}, w(t), u(t)\right)\\ &\quad + \sin(\frac{\pi s}{2})\left(\frac{1+x(t)y(t)}{A(x(t)+y(t))^2}, \frac{(1-x^2(t))^{\frac{1}{2}}(y^2(t)-1)^{\frac{1}{2}}}{A(x(t)+y(t))^2}, 0, u(t)\right).\end{aligned}$$

Thus $\Psi(m, 0) = \overset{ch}{C}_0$, $\Psi(m, 1) = \overset{ch}{C}_{02}$. The deformation retract of the chaotic vacuüm $\overset{ch}{C}_0$ into a geodesic $\overset{ch}{C}_{03} \subseteq \overset{ch}{C}_0$ is given by

$$\begin{aligned}\Psi(m,s) &= \cos(\frac{\pi s}{2})\left(\frac{1+x(t)y(t)}{A(x(t)+y(t))^2}, \frac{(1-x^2(t))^{\frac{1}{2}}(y^2(t)-1)^{\frac{1}{2}}}{A(x(t)+y(t))^2}, w(t), u(t)\right)\\ &\quad + \sin(\frac{\pi s}{2})\left(\frac{1}{Ay^2(t)}, \frac{(y^2(t)-1)^{\frac{1}{2}}}{Ay^2(t)}, w(t), u(t)\right).\end{aligned}$$

So $\Psi(m, 0) = \overset{ch}{C}_0$, $\Psi(m, 1) = \overset{ch}{C}_{03}$. The deformation retract of the chaotic vacuüm $\overset{ch}{C}_0$ into a geodesic $\overset{ch}{C}_{04} \subseteq \overset{ch}{C}_0$ isgiven by

$$\begin{aligned}\Psi(m,s) &= \cos(\frac{\pi s}{2})\left(\frac{1+x(t)y(t)}{A(x(t)+y(t))^2}, \frac{(1-x^2(t))^{\frac{1}{2}}(y^2(t)-1)^{\frac{1}{2}}}{A(x(t)+y(t))^2}, w(t), u(t)\right)\\ &\quad + \sin(\frac{\pi s}{2})\left(\frac{1}{Ax^2(t)}, \frac{i(1-x^2(t))^{\frac{1}{2}}}{Ax^2(t)}, w(t), u(t)\right),\end{aligned}$$

and so $\Psi(m,0) = \overset{ch}{C_0}$, $\Psi(m,1) = \overset{ch}{C}_{04}$.

Theorem 2.2 *The end limit of dynamical chaotic n-dimensional vacuüm* $\overset{ch}{V_n}$ *is zero-dimensional chaotic vacuüm* $\overset{ch}{V_0}$.

Proof Let D_i be the dynamical chaotic n-dimensional vacuüm $\overset{ch}{V_n}$ .Then we get the following chains:

$$\overset{ch}{V_n} \overset{D_1^1}{\to} \overset{ch}{V_n^1} \overset{D_2^1}{\to} \overset{ch}{V_n^2} \to \cdots \to \overset{ch}{V_n^{m-1}} \text{ such that } \lim_{m\to\infty} D_m^1(\overset{ch}{V_n^{m-1}}) = \overset{ch}{V}_{n-1};$$

$$\overset{ch}{V}_{n-1} \overset{D_1^1}{\to} \overset{ch}{V_{n-1}^1} \overset{D_2^1}{\to} \overset{ch}{V_{n-1}^2} \to \cdots \to \overset{ch}{V_{n-1}^{m-1}} \text{ such that } \lim_{m\to\infty} D_m^1(\overset{ch}{V_{n-1}^{m-1}}) = \overset{ch}{V}_{n-2},$$

$$\vdots$$

$$\overset{ch}{V_1} \overset{D_1^1}{\to} \overset{ch}{V_1^1} \overset{D_2^1}{\to} \overset{ch}{V_1^2} \to \cdots \to \overset{ch}{V_1^{m-1}} \text{ such that } \lim_{m\to\infty} D_m^1(\overset{ch}{V_1^{m-1}}) = \overset{ch}{V_0}.$$

Therefore, from the last chain the end limits of the dynamical chaotic n-dimensional vacuüm $\overset{ch}{V_n}$ is zero-dimensional chaotic vacuüm. □

Now we are going to discuss some types of dynamical chaotic vacuüm $\overset{ch}{C_0}$. Let $D : \overset{ch}{C_0} \to \overset{ch}{C_0}$ be the dynamical chaotic vacuüm on $\overset{ch}{C_0}$ which preserve the isometry of chaotic vacuüm $\overset{ch}{C_0}$ into itself such that $D(x_1, x_2, x_3, x_4) = (|x_1|, x_2, x_3, x_4)$. So we can define D as

$$D : \left(\frac{1 + x(t)y(t)}{A(x(t) + y(t))^2}, \frac{(1 - x^2(t))^{\frac{1}{2}}(y^2(t) - 1)^{\frac{1}{2}}}{A(x(t) + y(t))^2}, w(t), u(t)\right)$$
$$\longrightarrow \left(\left|\frac{1 + x(t)y(t)}{A(x(t) + y(t))^2}\right|, \frac{(1 - x^2(t))^{\frac{1}{2}}(y^2(t) - 1)^{\frac{1}{2}}}{A(x(t) + y(t))^2}, w(t), u(t)\right).$$

The deformation retracts of the dynamical chaotic vacuüm $\overset{ch}{C_0}$ into the dynamical chaotic retraction $\overset{ch}{C}_{01} \subseteq \overset{ch}{C_0}$ is given by

$$\Psi_D : \left(\left|\frac{1 + x(t)y(t)}{A(x(t) + y(t))^2}\right|, \frac{(1 - x^2(t))^{\frac{1}{2}}(y^2(t) - 1)^{\frac{1}{2}}}{A(x(t) + y(t))^2}, w(t), u(t)\right) \times I$$
$$\longrightarrow \left(\left|\frac{1 + x(t)y(t)}{A(x(t) + y(t))^2}\right|, \frac{(1 - x^2(t))^{\frac{1}{2}}(y^2(t) - 1)^{\frac{1}{2}}}{A(x(t) + y(t))^2}, w(t), 0\right)$$

with

$$\Psi_D\ (m,s) = \cos(\frac{\pi s}{2})\left(\left|\frac{1 + x(t)y(t)}{A(x(t) + y(t))^2}\right|, \frac{(1 - x^2(t))^{\frac{1}{2}}(y^2(t) - 1)^{\frac{1}{2}}}{A(x(t) + y(t))^2}, w(t), u(t)\right)$$
$$+ \sin(\frac{\pi s}{2})\left(\left|\frac{1 + x(t)y(t)}{A(x(t) + y(t))^2}\right|, \frac{(1 - x^2(t))^{\frac{1}{2}}(y^2(t) - 1)^{\frac{1}{2}}}{A(x(t) + y(t))^2}, w(t), 0\right).$$

The deformation retracts of the dynamical chaotic vacuüm $\overset{ch}{C_0}$ into the dynamical chaotic retraction $\overset{ch}{C}_{02} \subseteq \overset{ch}{C}_0$ is given by

$$\Psi_D : \left(\left| \frac{1+x(t)y(t)}{A(x(t)+y(t))^2} \right|, \frac{(1-x^2(t))^{\frac{1}{2}}(y^2(t)-1)^{\frac{1}{2}}}{A(x(t)+y(t))^2}, w(t), u(t) \right) \times I \longrightarrow \left(\left| \frac{1+x(t)y(t)}{A(x(t)+y(t))^2} \right|, \frac{(1-x^2(t))^{\frac{1}{2}}(y^2(t)-1)^{\frac{1}{2}}}{A(x(t)+y(t))^2}, 0, u(t) \right)$$

with

$$\Psi_D\ (m,s) = \cos(\frac{\pi s}{2}) \left(\left| \frac{1+x(t)y(t)}{A(x(t)+y(t))^2} \right|, \frac{(1-x^2(t))^{\frac{1}{2}}(y^2(t)-1)^{\frac{1}{2}}}{A(x(t)+y(t))^2}, w(t), u(t) \right) + \sin(\frac{\pi s}{2}) \left(\left| \frac{1+x(t)y(t)}{A(x(t)+y(t))^2} \right|, \frac{(1-x^2(t))^{\frac{1}{2}}(y^2(t)-1)^{\frac{1}{2}}}{A(x(t)+y(t))^2}, 0, u(t) \right).$$

The deformation retracts of the dynamical chaotic vacuüm $\overset{ch}{C}_0$ into the dynamical chaotic geodesic $\overset{ch}{C}_{03} \subseteq \overset{ch}{C}_0$ is given by

$$\Psi_D : \left(\left| \frac{1+x(t)y(t)}{A(x(t)+y(t))^2} \right|, \frac{(1-x^2(t))^{\frac{1}{2}}(y^2(t)-1)^{\frac{1}{2}}}{A(x(t)+y(t))^2}, w(t), u(t) \right) \times I \longrightarrow \left(\left| \frac{1}{Ay^2(t)} \right|, \frac{(y^2(t)-1)^{\frac{1}{2}}}{Ay^2(t)}, w(t), u(t) \right)$$

with

$$\Psi_D\ (m,s) = \cos(\frac{\pi s}{2}) \left(\left| \frac{1+x(t)y(t)}{A(x(t)+y(t))^2} \right|, \frac{(1-x^2(t))^{\frac{1}{2}}(y^2(t)-1)^{\frac{1}{2}}}{A(x(t)+y(t))^2}, w(t), u(t) \right) + \sin(\frac{\pi s}{2}) \left(\left| \frac{1}{Ay^2(t)} \right|, \frac{(y^2(t)-1)^{\frac{1}{2}}}{Ay^2(t)}, w(t), u(t) \right).$$

The deformation retracts of the dynamical chaotic vacuüm $\overset{ch}{C}_0$ into the dynamical chaotic geodesic $\overset{ch}{C}_{04} \subseteq \overset{ch}{C}_0$ is given by

$$\Psi_D : \left(\left| \frac{1+x(t)y(t)}{A(x(t)+y(t))^2} \right|, \frac{(1-x^2(t))^{\frac{1}{2}}(y^2(t)-1)^{\frac{1}{2}}}{A(x(t)+y(t))^2}, w(t), u(t) \right) \times I \longrightarrow \left(\left| \frac{1}{Ax^2(t)} \right|, \frac{i(1-x^2(t))^{\frac{1}{2}}}{Ax^2(t)}, w(t), u(t) \right)$$

with

$$\Psi_D\ (m,s) = \cos(\frac{\pi s}{2}) \left(\left| \frac{1+x(t)y(t)}{A(x(t)+y(t))^2} \right|, \frac{(1-x^2(t))^{\frac{1}{2}}(y^2(t)-1)^{\frac{1}{2}}}{A(x(t)+y(t))^2}, w(t), u(t) \right) + \sin(\frac{\pi s}{2}) \left(\left| \frac{1}{Ax^2(t)} \right|, \frac{i(1-x^2(t))^{\frac{1}{2}}}{Ax^2(t)}, w(t), u(t) \right).$$

Then the following theorem has been proved.

Theorem 2.3 *The deformation retracts of the dynamical chaotic vacuüm* $\overset{ch}{C_0}$ *into chaotic geodesic is different from the deformation retracts of the chaotic vacuüm* $\overset{ch}{C_0}$ *into the chaotic geodesic.*

References

[1] M. Abu-Saleem, Folding on the chaotic Cartesian product of manifolds and their fundamental group, *Tamkang Journal of Mathematics*, Vol.39 (2008), 353-361.

[2] M. Abu-Saleem, Folding on the wedge sum of graphs and their fundamental group, *Applied Sciences*, Vol.12 (2010),14-19.

[3] M. Abu-Saleem, Dynamical manifold and their fundamental group, *Adv.Stud.Contemp.Math*, Vol.20, No.1 (2010),125-131.

[4] M.Abu-Saleem, Dynamical Knot and their fundamental group, *International J. Mathematical Combinatorics*, Vol.1, (2010) ,80-86.

[5] D.Bini, C.Cherubini and B.Mashhoon, Vacuüm C-metric and the gravitational stark effect, *Phys.Rev.*, D70 (2004) 044020.

[6] G.D. Birkhoof: Dynamical systems, *Bulletin of the American Mathematics Society*, Vol.37, No.1 (2000) 88-121.

[7] W. Kinnersley and M. Walker, Uniformly accelerating charged mass in general relativity, *Phys. Rev.*, D 2, 1359 (1970).

[8] W. S. Massey, *Algebraic Topology: An Introduction*, Harcourt Brace and world, New York (1967).

[9] V. Pravda, A. Pravdova, Co-accelerated particles in the C-metric, *Classical and Quantum Gravity*, 18 (2001), 1205-1216

[10] K. S. Sibirsky, *Introduction to Topological Dynamics*, Noordhoff Int. Pub. Leyden, The Netherlands (1975).

[11] H. Stephani, D. Kramer, M.A.H. MacCallum, C. Hoenselaers and E. Herlt, *Exact Solutions of Einstein's Theory*, Cambridge Univ. Press, Cambridge, second edition (2003).

[12] S. Wiggins, *Introduction to Applied Non-Linear Dynamical System and Chaos*, New York, Heidelberg and Berlin, Spring Verlag, 1997.