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Semi-invariant Sub-manifolds of Generalized Sasakian-Space-Forms

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Abstract: In this paper, we study the decomposition of basic equation of generalized Sasakian space-forms is taken out into horizontal and vertical projections and also we discuss the integrability of distributions D, $D \oplus [\xi]$ and $D^{\perp}$ totally geodesic of semi-invariant sub-manifolds of generalized Sasakian-space-forms.

Key Words: Sub-manifold, semi-invariant sub-manifold, generalized Sasakian-space-forms, totally umbilical(geodesic), integrability condition of distribution.

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§1. Introduction

The notion of semi-invariant sub-manifold is a generalization of invariant and anti-variant sub-manifolds of almost contact metric manifolds. Many authors [6, 8, 9, 20] have obtained the decomposition of basic equations of Kenmotsu, LP-Sasakian, (k, μ)-contact, LP-Cosymplectic manifolds into horizontal and vertical components and also they have studied the integrability of horizontal and vertical distributions. Further, the analysis of totally umbilical and totally geodesics of sub-manifolds of (k, μ)-contact manifolds is done by the author [6]. In [10, 19], the authors studied totally geodesics of sub-manifolds of (ϵ, δ)-trans-Sasakian manifolds. As a generalization of Sasakian space-form, Alegre et al. [1] introduced and studied the notion of generalized Sasakian-space-form with the existence of such notions with various examples.

§2. Preliminaries

An n-dimensional generalized Sasakian-space-forms $\overline{M}$ is a smooth connected manifold with a metric g, that is, $\overline{M}$ admits a smooth symmetric tensor field g of type $(0, 2)$ such that for

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each point the tensor $g_p : T_P\overline{M} \times T_P\overline{M} \to R$ is a non-degenerate bilinear form of signature $(-, +, \cdots, +)$, where $T_P\overline{M}$ denotes the tangent vector space of $\overline{M}$ at p and R is the real number space, which satisfies

$$\phi^2(X_1) = -X_1 + \eta(X_1)\xi, \;\; \phi\xi = 0, \;\; \eta(\phi X_1) = 0, \tag{2.1}$$

$$g(\phi X_1, \phi Y_1) = g(X_1, Y_1) - \eta(X_1)\eta(Y_1), \;\; g(X_1, \xi) = \eta(X_1). \tag{2.2}$$

for any $X_1, Y_1 \in T\overline{M}$ denotes the Lie algebra of vector fields on $\overline{M}$. An almost contact metric manifold is called a generalized Sasakian-space-form if

$$(\overline{\nabla}_{X_1}\phi)(Y_1) = (f_1 - f_3)(g(X_1, Y_1)\xi - \eta(Y_1)X_1), \tag{2.3}$$

$$\overline{\nabla}_{X_1}\xi = -(f_1 - f_3)\phi X_1, \tag{2.4}$$

$$(\overline{\nabla}_{X_1}\eta)(Y_1) = g(\overline{\nabla}_{X_1}\xi, Y_1), \tag{2.5}$$

$$g(X_1, \phi Y_1) = -g(\phi X_1, Y_1), \tag{2.6}$$

where $\overline{\nabla}$ denotes the Levi-Civita connection on $\overline{M}$.

The sub-manifold M of the generalized Sasakian-space-form $\overline{M}$ is said to be semi-invariant if it is endowed with the pair of orthogonal distribution $(D, D^{\perp})$ satisfying the conditions

(*i*) $TM = D \oplus D^{\perp} \oplus [\xi]$;

(*ii*) the distribution D is invariant under ϕ, that is, $\phi D_x = D_x$, for each $x \in M$;

(*iii*) the distribution $D^{\perp}$ is anti-invariant under ϕ, that is, $\phi D_x^{\perp} \subset T_x M^{\perp}$ for each $x \in M$,

where D and $D^{\perp}$ are the horizontal and vertical distribution respectively. A semi-invariant sub-manifold M is said to be invariant if we have $D_x^{\perp} = 0$ and is said to be anti-invariant if $D_x = 0$ for each $x \in M$. We denote the projection morphisms of TM to D and $D^{\perp}$ by P and Q respectively. For any $X_1 \in \Gamma(TM)$ and $N \in \Gamma(TM^{\perp})$, we have

$$X_1 = PX_1 + QX_1 + \eta(X_1)\xi, \tag{2.7}$$

$$\phi N = BN + CN, \tag{2.8}$$

where BN and CN denotes the tangential and normal components of ϕN.

The equations of Gauss and Weingarten for the immersion of M in $\overline{M}$ are given by

$$\overline{\nabla}_{X_1}Y_1 = \nabla_{X_1}Y_1 + h(X_1, Y_1), \tag{2.9}$$

$$\overline{\nabla}_{X_1}N = -A_N X_1 + \nabla^{\perp}_{X_1}N, \tag{2.10}$$

for any $X_1, Y_1 \in \Gamma(TM)$ and $N \in TM^{\perp}$, where ∇ is the Levi-Civita connection on M, $\nabla^{\perp}$ is the linear connection induced by $\overline{\nabla}$ on the normal bundle $TM^{\perp}$, h is the second fundamental form of M and A_N is the fundamental tensor of Weingarten with respect to the normal section

N. Also, we have

$$g(h(X_1,Y_1),N)=g(A_N,Y_1), \tag{2.11}$$

for any $X_1,Y_1\in\Gamma(TM), N\in\Gamma(TM^{\perp})$.

For readers unfamiliar with terminology, notations, recent overviews and introductions, we suggest the authors to refer the papers [2, 3, 4, 5, 7, 11, 11, 12, 13, 14, 15, 16, 17, 18].

§3. Decomposition of Basic Equations

For $X_1,Y_1\in\Gamma(TM)$, we take

$$u(X_1,Y_1)=\nabla_{X_1}\phi PY_1-A_{\phi QY_1}X_1. \tag{3.1}$$

Lemma 3.1 *Let M be a semi-invariant sub-manifold of generalized Sasakian-space-form $\overline{M}$. Then, we have*

$$P(u(X_1,Y_1))=(f_1-f_3)g(X_1,Y_1)P\xi-(f_1-f_3)\eta(Y_1)PX_1+\phi P(\nabla_{X_1}Y_1), \tag{3.2}$$

$$Q(u(X_1,Y_1))=(f_1-f_3)g(X_1,Y_1)Q\xi-(f_1-f_3)\eta(Y_1)QX_1+Bh(X_1,Y_1), \tag{3.3}$$

$$h(X_1,\phi PY_1))=-\nabla^{\perp}_{X_1}\phi QY_1+\phi Q(\nabla_{X_1}Y_1)+Ch(X_1,Y_1), \tag{3.4}$$

$$\eta(u(X_1,Y_1))=(f_1-f_3)g(\phi X_1,\phi Y_1). \tag{3.5}$$

for all $X_1,Y_1\in TM$.

Proof In the view of (2.3) and (2.7), we have

$$\begin{aligned}(\overline{\nabla}_{X_1}\phi)(Y_1)=(f_1-f_3)[&g(X_1,Y_1)P\xi+g(X_1,Y_1)Q\xi+g(X_1,Y_1)\xi\\&-\eta(Y_1)PX_1-\eta(Y_1)QX_1-\eta(X_1)\eta(Y_1)\xi].\end{aligned} \tag{3.6}$$

Now, decompose the LHS of (2.3) and by using (2.8), (2.9), (2.10), we get:

$$\begin{aligned}(\overline{\nabla}_{X_1}\phi)Y_1&=\overline{\nabla}_{X_1}\phi PY_1+\overline{\nabla}_{X_1}\phi QY_1-\phi(\nabla_{X_1}Y_1)-\phi h(X_1,Y_1)\\&=\nabla_{X_1}\phi PY_1+h(X_1,\phi PY_1)-A_{\phi QY_1}+\nabla^{\perp}_{X_1}\phi QY_1-\phi P(\nabla_{X_1}Y_1)\\&\quad-\phi Q(\nabla_{X_1}Y_1)-Bh(X_1,Y_1)-Ch(X_1,Y_1).\end{aligned} \tag{3.7}$$

Now using (3.1) in above equation, we get

$$\begin{aligned}(\overline{\nabla}_{X_1}\phi)Y_1=&\,u(X_1,Y_1)+h(X_1,\phi PY_1)+\nabla^{\perp}_{X_1}\phi QY_1\\&-\phi P(\nabla_{X_1}Y_1)-\phi Q(\nabla_{X_1}Y_1)-Bh(X_1,Y_1)-Ch(X_1,Y_1).\end{aligned} \tag{3.8}$$

Again using (2.7) in above equation, we have

$$(\overline{\nabla}_{X_1}\phi)Y_1 = Pu(X_1,Y_1) + Qu(X_1,Y_1) + \eta(u(X_1,Y_1)\xi) + h(X_1,\phi PY_1) + \nabla^{\perp}_{X_1}\phi QY_1 - \phi P(\nabla_{X_1}Y_1) - \phi Q(\nabla_{X_1}Y_1) - Bh(X_1,Y_1) - Ch(X_1,Y_1). \tag{3.9}$$

Now on comparing (3.6) and (3.9) and equating the horizontal and vertical components, we obtain (3.2), (3.3), (3.4) and (3.5), respectively. □

Lemma 3.2 *Let M be a semi-invariant sub-manifold of generalized Sasakian-space-form M. Then we have*

$$\nabla_{X_1}\xi = -(f_1 - f_3)\phi X_1, \quad h(X_1,\xi) = 0, \quad for\ any\ \ X_1 \in \Gamma(D); \tag{3.10}$$

$$\nabla_{Y_1}\xi = 0 \quad h(Y_1,\xi) = -(f_1 - f_3)\phi QY_1, \quad for\ any\ Y_1 \in \Gamma(D^{\perp}); \tag{3.11}$$

$$\nabla_{\xi}\xi = 0 \quad h(\xi,\xi) = 0. \tag{3.12}$$

Proof In consequence of (2.4) and (2.9), we get

$$-(f_1 - f_3)\phi X_1 = \nabla_{X_1}\xi + h(X_1,\xi). \tag{3.13}$$

Using (2.7) in the above equation, we have

$$\nabla_{X_1}\xi + h(X_1,\xi) = -(f_1 - f_3)(\phi PX_1 + \phi QY_1). \tag{3.14}$$

After equating tangential and normal parts, we get (3.10), (3.11) and (3.12). □

Lemma 3.3 *Let M be a semi-invariant sub-manifold of generalized Sasakian-space-forms $\overline{M}$, then we find:*

$$\begin{aligned} &\nabla_{\xi}X_2 \in \Gamma(D); \quad for\ any\ \ X_2 \in \Gamma(D),\\ &\nabla_{\xi}Y_2 \in \Gamma(D^{\perp}); \quad for\ any\ Y_2 \in \Gamma(D^{\perp}). \end{aligned} \tag{3.15}$$

Proof The above follow from $g(\xi,X_2) = 0, g(\xi,Y_2) = 0$ and (3.12) and covariant differentiation. □

Lemma 3.4 *Let M be a semi-invariant sub-manifold of generalized Sasakian-space-form $\overline{M}$, then we have*

$$[X_1,\xi] \in \Gamma(D) \ \ for\ any\ X_1 \in \Gamma(D), \tag{3.16}$$

$$[Y_1,\xi] \in \Gamma(D^{\perp}) \ \ for\ any\ Y_1 \in \Gamma(D^{\perp}). \tag{3.17}$$

Proof The proof follows from Lemma 3.3. □

§4. Integrability of Invariant and Anti-Invariant Sub-Manifolds

In this section, we study the integrability of $D, D \oplus [\xi]$ and $D^{\perp}$ of semi-invariant sub-manifolds of generalized Sasakian-space-forms.

Proposition *Let M be a semi-invariant sub-manifold such that ξ is tangent to $\overline{M}$. Then the invariant distribution D is integrable provided $f_1 = f_3$.*

Proof We have for $X_1, Y_1 \in D$ and $\xi \in [\xi]$

$$g([X_1, Y_1], \xi) = g(\nabla_{X_1} Y_1 - \nabla_{Y_1} X_1, \xi) \tag{4.1}$$

using (2.9) in above equation, we have

$$\begin{aligned} g([X_1, Y_1], \xi) = g(\overline{\nabla}_{X_1} Y_1 - h(X_1, Y_1) - \overline{\nabla}_{Y_1} X_1 + h(Y_1, X_1), \xi) \\ + g(\overline{\nabla}_{X_1} Y_1, \xi) - g(\overline{\nabla}_{Y_1} X_1, \xi). \end{aligned} \tag{4.2}$$

Taking the covariant differentiation for the above equation, we get

$$\begin{aligned} g([X_1, Y_1], \xi) = \overline{\nabla}_{X_1} g(Y_1, \xi) - g(Y_1, \overline{\nabla}_{X_1} \xi) \\ - \overline{\nabla}_{Y_1} g(X_1, \xi) + g(X_1, \overline{\nabla}_{Y_1} \xi). \end{aligned} \tag{4.3}$$

Now by the definition of semi-invariant sub-manifold, we have

$$g([X_1, Y_1], \xi) = -g(Y_1, \overline{\nabla}_{X_1} \xi) + g(X_1, \overline{\nabla}_{Y_1} \xi). \tag{4.4}$$

Now by taking (2.4) in the above equation, we get

$$g([X_1, Y_1], \xi) = (f_1 - f_3) g(Y_1, \phi X_1) - (f_1 - f_3) g(X_1, \phi Y_1). \tag{4.5}$$

Now with reference to (2.6), we have

$$g([X_1, Y_1], \xi) = 2(f_1 - f_3) g(Y_1, \phi X_1). \tag{4.6}$$

Thus, if $X_1, Y_1 \in D$, then $[X_1, Y_1] \in D$, that is, the invariant distribution D is integrable, provided $f_1 = f_3$. □

Theorem 4.1 *Let M be a semi-invariant sub-manifold in a generalized Sasakian-space-form $\overline{M}$. Then the distribution D is integrable if and only if the second fundamental form h satisfies*

$$h(X_1, \phi Y_1) = h(\phi X_1, Y_1) \quad for\, X_1, Y_1 \in D. \tag{4.7}$$

Proof For $X_1, Y_1 \in D \oplus [\xi]$ and $Y_2 \in T^{\perp}M$ then by the virtue of (2.9), we have

$$\begin{aligned} g(\phi[X_1, Y_1], Y_2) &= g(\phi(\nabla_{X_1} Y_1 - \nabla_{Y_1} X_1), Y_2) \\ &= g(\phi(\overline{\nabla}_{X_1} Y_1) - h(X_1, Y_1) - \overline{\nabla}_{Y_1} X_1) + h(Y_1, X_1), Y_2 \\ &= g(\phi(\overline{\nabla}_{X_1} Y_1), Y_2) - g(\phi(\overline{\nabla}_{Y_1} X_1), Y_2). \end{aligned} \tag{4.8}$$

Now by the covariant differentiation and using (2.3), (2.9), we have

$$\begin{aligned} g(\phi[X_1, Y_1], Y_2) = {}& g(\nabla_{X_1} \phi Y_1, Y_2) + g(h(X_1, \phi Y_1), Y_2) \\ & + (f_1 - f_3)[g(X_1, Y_2)\eta Y_1 - g(Y_1, Y_2)\eta X_1] - g(\nabla_{Y_1} \phi X_1, Y_2) \\ & - g(h(Y_1, \phi X_1), Y_2). \end{aligned} \tag{4.9}$$

By (2.1) and (2.6) in the above equation, we get

$$g(\phi[X_1, Y_1], Y_2) = g(h(X_1, \phi Y_1) - h(Y_1, \phi X_1), Y_2). \tag{4.10}$$

Therefore,

$$\phi[X_1, Y_1] = h(X_1, \phi Y_1) - h(Y_1, \phi X_1). \tag{4.11}$$

Thus, the distribution D is integrable if and only if the second fundamental form h satisfies

$$h(X_1, \phi Y_1) = h(Y_1, \phi X_1). \tag{4.12}$$

This completes the proof. □

Theorem 4.2 *Let M be a semi-invariant sub-manifold of generalized Sasakian-space-form $\overline{M}$ such that ξ is tangent to $\overline{M}$ and $D^{\perp}$ be the anti-invariant subspace of TM. Then the anti-invariant distribution $D^{\perp}$ is always integrable provided $f_1 = f_3$.*

Proof By the definition of covariant differentiation, we have

$$\begin{aligned} g(\phi[Z_1, Z_2], X_1) &= g(\phi(\nabla_{Z_1} Z_2 - \nabla_{Z_2} Z_1), X_1) \\ &= g(\phi(\overline{\nabla}_{Z_1} Z_2 - \phi h(Z_1, Z_2) - \phi(\nabla_{Z_2} Z_1) + \phi h(Z_2, Z_1), X_1). \end{aligned} \tag{4.13}$$

Now using (2.3) and (2.10) in above equation, we have

$$\begin{aligned} g(\phi[Z_1, Z_2], X_1) &= g((\overline{\nabla}_{Z_1} \phi Z_2) - (\overline{\nabla}_{Z_1} \phi) Z_2 - (\overline{\nabla}_{Z_2} \phi Z_1) + (\overline{\nabla}_{Z_2} \phi) Z_1, X_1) \\ &= g(-A_{\phi Z_2} Z_1 + \nabla^{\perp}_{Z_1} \phi Z_2 + A_{\phi Z_1} Z_2 - \nabla^{\perp}_{Z_2} \phi Z_1, X_1) \\ &\quad + (f_1 - f_3) g[\eta Z_2 Z_1 - \eta Z_1 Z_2, X_1] \\ &= g(-A_{\phi Z_2} Z_1 + \nabla^{\perp}_{Z_1} \phi Z_2 + A_{\phi Z_1} Z_2 - \nabla^{\perp}_{Z_2} \phi Z_1, X_1). \end{aligned} \tag{4.14}$$

Since, $A_{\phi Z_1}Z_2 - A_{\phi Z_2}Z_1$ is tangential to M and $\nabla^{\perp}_{Z_1}\phi Z_2 - \nabla^{\perp}_{Z_2}\phi Z_1$ is normal to M.

$$g(\phi[Z_1, Z_2], X_1) = g(-A_{\phi Z_2}Z_1 + A_{\phi Z_1}Z_2, X_1). \tag{4.15}$$

Hence,

$$\phi[Z_1, Z_2] = -A_{\phi Z_2}Z_1 + A_{\phi Z_1}Z_2. \tag{4.16}$$

Therefore, it follows that $[Z_1, Z_2] \in D^{\perp}$ for any $Z_1, Z_2 \in D^{\perp}$ if and only if

$$A_{\phi Z_2}Z_1 = A_{\phi Z_1}Z_2 \quad \text{for any} \;\; Z_1, Z_2 \in D^{\perp} \tag{4.17}$$

and

$$g([Z_1, Z_2], \xi) = 0 \quad \text{for any} \;\; Z_1, Z_2 \in D^{\perp} \;\; \textit{and} \;\; \xi \in [\xi]. \tag{4.18}$$

Conversely, using (2.9) and (2.11) for any $Z_1, Z_2 \in D^{\perp}$ and $X_1 \in TM$, we have

$$\begin{aligned} g(A_{\phi Z_1}Z_2, X_1) &= g(h(Z_2, X_1), \phi Z_1) = g(\overline{\nabla}_{X_1}Z_2, \phi Z_1) \\ &= -g(\phi\overline{\nabla}_{X_1}Z_2, Z_1) \\ &= -g(\overline{\nabla}_{X_1}\phi Z_2 - (\overline{\nabla}_{X_1}\phi)Z_2, Z_1) \\ &= g(-\overline{\nabla}_{X_1}\phi Z_2 + (f_1 - f_3)(g(X_1, Z_2)\xi - \eta(Z_2)X_1, Z_1)) \\ &= -g(-A_{\phi Z_2}X_1 + \nabla^{\perp}_{X_1}\phi Z_2, Z_1) \\ &= g(A_{\phi Z_2}X_1, Z_1) \\ &= g(A_{\phi Z_2}Z_1, X_1). \end{aligned} \tag{4.19}$$

Thus, $A_{\phi Z_1}Z_2 = A_{\phi Z_2}Z_1$ holds.

By using (2.4), we have

$$\begin{aligned} g([Z_1, Z_2], \xi) &= g(\overline{\nabla}_{Z_1}Z_2 - \overline{\nabla}_{Z_2}Z_1, \xi) \\ &= g(Z_2, \overline{\nabla}_{Z_1}\xi) - g(Z_1, \overline{\nabla}_{Z_2}\xi) \\ &= (f_1 - f_3)(g(Z_1, \phi Z_2) - g(Z_2, \phi Z_1)) \\ &= 2(f_1 - f_3)(g(Z_1, \phi Z_2). \end{aligned} \tag{4.20}$$

Hence, (4.17) and (4.18) hold when $f_1 = f_3$ then $g([Z_1, Z_2], \xi) = 0$. □

§5. Totally Umbilical and Totally Geodesic Sub-Manifolds

Here we consider totally umbilical sub-manifolds of generalized Sasakian-space-forms by proving following Lemmas.

Lemma 5.1 *Let D be a distribution on sub-manifold M of a generalized Sasakian-space-form* *such that $\xi \in D$. If M is D-totally umbilical, then M is D-totally geodesic.*

Proof If M is D-totally umbilical, then by $X_1, Y_1 \in D$ we have

$$h(X_1, Y_1) = g(X_1, Y_1)H, \tag{5.1}$$

where H is the mean curvature. With reference to (3.10) and (3.12), we get

$$H = g(\xi, \xi)H = h(\xi, \xi) = 0. \tag{5.2}$$

Hence $H = 0$ and therefore M is D-totally geodesic. □

Lemma 5.1 *Let $D^{\perp}$ be a distribution on sub-manifold M of a generalized Sasakian-space-form such that $\xi \in D^{\perp}$. If M is $D^{\perp}$-totally umbilical, then M is $D^{\perp}$-totally geodesic provided $\phi Q = Q\phi$.*

Proof If M is $D^{\perp}$-totally umbilical, then by $X_1, Y_1 \in D$, we have:

$$h(X_1, Y_1) = g(X_1, Y_1)K, \tag{5.3}$$

where K is the mean curvature. Now with reference to (3.11), we have:

$$K = g(\xi, \xi)K = h(\xi, \xi) = -(f_1 - f_3)\phi Q\xi. \tag{5.4}$$

Suppose $\phi Q = Q\phi$, hence $K = -(f_1 - f_3)Q\phi\xi = 0$. Therefore, M is D-totally geodesic. □

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