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Results on Centralizers of Semiprime Gamma Semirings

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Abstract: Let M be a noncommutative 2-torsion free semiprime Γ-semiring satisfying a certain assumption with centre $Z_\alpha(M)$ and $T : M \to M$ be an additive mapping. We prove results: 1) If T is centralizing on a Jordan Γ-subring J of M, then T is commutating on J; 2) If T is centralizing right centralizer on M, then T is commutating; 3) If T is centralizing right centralizer on M, then T is centralizer and 4) If T is centralizing right centralizer on M, then T satisfies the relation

$$[x, y]_\alpha \beta T(x) = [T(x), y]_\alpha \beta x$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$.

Key Words: Semiprime gamma ring, semiprime gamma semiring, centralizing, left and right centralizers.

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§1. Introduction

The notion of gamma ring was first introduced in [1], which is currently notable as Γ_N-ring. Bernes [2] broadly generalized and extended the concept of Γ_N-ring to Γ-ring and shown that every Γ_N-ring is a Γ-ring. The Γ-ring is more general than the classical ring and it is concluded that Γ-ring need not to be a ring [1, 2]. Later, much theory relevant to the classical rings have been generalized and extended to the theory of Γ-rings, especially, Luh [3] and Kyuno [4] deeply studied on the structure of Γ-rings and explored various generalizations of analogous parts in ring theory.

Over the years, Bell and Martindale [5] and Zalar [6] developed some notable results on centralizing mappings of semiprime rings. Vukman [7-10] presented may remarkable findings via the concept of centralizers on prime and semiprime rings. Recently, the research on centralizers of prime and semiprime rings have been extended to prime and semiprime gamma rings and semiprime gamma semirings in the aspects of Jordan centralizers [12, 13], centralizers [11, 13C15], centralizers on Lie ideals [16, 17], centralizers with involutions [18, 19], Jordan derivations on Lie ideals [16] and generalized derivations on prime and semiprime gamma rings with centralizing and commuting [20, 21] as well.

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H.S. Vandiver introduced the algebraic study of semiring in 1934 [22, 23] and Rao [24] extended such research to the Γ-semirings and established some basic theories on gamma rings as well as on gamma semiring. A number of important features on Γ-semirings are presented in [12, 25, 26]. However, the research on centralizing left/right centralizers on prime and semiprime gamma semiring is still unknown area. Thus the purpose of this article is to study on semiprime gamma semiring via centralizing right centralizers [11, 20]. The study is inspired by the work of [25, 26]. The results presented in this paper through out for right centralizers, which are also true for left centralizers because of left-right symmetry.

§2. Preliminaries

Let M and Γ be additive Abelian groups. If there exists a mapping $(x, \alpha, y) \to x\alpha y$ of $M \times \Gamma \times M \to M$, which satisfies the following conditions:

(a) $x\alpha x \in M$;

(b) $x\alpha(y+z) = x\alpha y + x\alpha z$ and $(x+y)\alpha z = x\alpha z + y\alpha z$;

(c) $x(\alpha+\beta)y = x\alpha y + x\beta y$;

(d) $(x\alpha y)\beta z = x\alpha(y\beta z)$

for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$, then M is called a Γ-ring. Every ring M is a Γ-ring with $M = \Gamma$.

Let M and Γ be two additive commutative semigroups. Then M is called a Γ-semiring if M is itself a Γ-ring. Obviously, every semiring M is a Γ-semiring with $M = \Gamma$. A non-empty subset A of a Γ-semiring M is said to be a sub Γ-semiring of M if $(A, +)$ is a subsemigroup of $(M, +)$ and $x\alpha y \in A$ for all $x, y \in A$ and $\alpha \in \Gamma$. A Γ-semiring M is said to have a zero element if there exists an element $0 \in M$ such that $0 + x = x = x + 0$ and $0\alpha x = 0 = x\alpha 0$ for all $x \in M$ and $\alpha \in \Gamma$. A Γ-semiring M is said to be prime if $x\alpha y = 0$ implies $x = 0$ or $y = 0$ for all $x, y \in S$ and $\alpha \in \Gamma$. A Γ-semiring S is said to be semiprime if $x\alpha x = 0$ implies $x = 0$ for all $x \in S$ and $\alpha \in \Gamma$. A Γ-semiring M is said to be n-torsion free if $nx = 0$ implies $x = 0$ for all $x \in M$. A Γ-semiring M is said to be commutative if $x\alpha y = y\alpha x$ for all $x, y \in M$ and $\alpha \in \Gamma$. Let M be a Γ-semiring. Then the set $Z_\alpha(M) = \{x \in M : x\alpha y = y\alpha x \quad \forall y \in M, \alpha \in \Gamma\}$ is called the centre of the Γ-semiring M. Let M be a Γ-ring. Then $[x, y]_\alpha = x\alpha y - y\alpha x$ is called the commutator of x and y with respect to α, where $x, y \in M$ and $\alpha \in \Gamma$.

We make the basic commutator identities following

$$[a\alpha b, c]_\beta = [a, c]_\beta \alpha b + a[\alpha, \beta]_c b + a\alpha[b, c]_\beta,$$
$$[a, b\alpha c]_\beta = [a, b]_\beta \alpha c + b[\alpha, \beta]_a c + b\alpha[a, c]_\beta$$

for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$. We consider the following assumption [11],

$$a\alpha b\beta c = a\beta b\alpha c \tag{2.1}$$

for all $a, b, c \in M$, and $\alpha, \beta \in \Gamma$, which we extensively used in this paper. According to the assumption(2.1), the above two identities reduce to

$$[a\alpha b, c]_\beta = [a, c]_\beta \alpha b + a\alpha[b, c]_\beta, \tag{2.2}$$

$$[a, b\alpha c]_\beta = [a, b]_\beta \alpha c + b\alpha[a, c]_\beta. \tag{2.3}$$

The identities (2.2) and (2.3) are also used thoroughly in this article.

An additive mapping $T : M \to M$ is called a left (right) centralizer if

$$T(x\alpha y) = T(x)\alpha y (T(x\alpha y) = x\alpha T(y))$$

holds for all $x, y \in M$ and $\alpha \in \Gamma$. An additive mapping $T : M \to M$ is centralizer if it is both a left and a right centralizer. For any fixed $a \in M$ and $\alpha \in \Gamma$, the mapping $T(x) = a\alpha x$ is a left centralizer and $T(x) = x\alpha a$ is a right centralizer. A mapping $T : M \to M$ is called centralizing if $[T(x), x]_\alpha \in Z_\alpha(M)$ for all $x \in M$, $\alpha \in \Gamma$. A mapping T of a Γ-semiring M into itself is said to be commuting if $[T(x), x]_\alpha = 0$. We recall if $T : M \to M$ is commuting, then $[T(x), y]_\alpha = [x, T(y)]_\alpha$ for all $x, y \in M$ and $\alpha \in \Gamma$. Obviously, every commuting mapping $T : M \to M$ is centralizing. If A be a subset of Γ-semiring M and $x\alpha y + y\alpha x \in A$ for all $x, y \in A$ and $\alpha \in \Gamma$, then A is called a Jordan subring of M.

§3. **Main results**

In this section, we obtain the following results with their proofs in sense of 2-torsion free semiprime Γ-semiring with the certain assumption (2.1) and using various commutation identities.

Theorem 3.1 *Suppose M is a 2- torsion free cancellative semiprime Γ-semiring satisfying the assumption (2.1) and J is a Jordan subring of M. If an additive mapping $T : M \to M$ is centralizing on J, then T is commuting on J.*

Proof By the definition of centralizing T on J, we have

$$[T(x), x]_\alpha \in Z_\alpha(M). \tag{3.1}$$

For the linearization, we put $x = x + y$ in (3.1), which yields

$$\begin{aligned} &[T(x), x]_\alpha + [T(x), y]_\alpha + [T(y), x]_\alpha + [T(y), y]_\alpha \in Z_\alpha(M), \\ &\Rightarrow [T(x), y]_\alpha + [T(y), x]_\alpha \in Z_\alpha(M) \end{aligned} \tag{3.2}$$

for all $x, y \in J$ and $\alpha \in \Gamma$. In particular, for $y = x\beta x$, we obtain

$$\begin{aligned} &[T(x), x\beta x]_\alpha + [T(x\beta x), x]_\alpha \in Z_\alpha(M) \quad \text{for all} \quad x, y \in J, \quad \alpha \in \Gamma, \\ &\Rightarrow [T(x), x]_\alpha \beta x + x\beta[T(x), x]_\alpha + [T(x\beta x), x]_\alpha \in Z_\alpha(M). \end{aligned}$$

Using the definition of the centre of Γ-semiring $Z_\alpha(M)$, we have

$$[T(x),x]_\alpha\beta x+[T(x),x]_\alpha\beta x+[T(x\beta x),x]_\alpha\in Z_\alpha(M),$$
$$\Rightarrow 2[T(x),x]_\alpha\beta x+[T(x\beta x),x]_\alpha\in Z_\alpha(M). \tag{3.3}$$

Suppose $x\in J$ is a fixed element with $z=[T(x),x]_\alpha\in Z_\alpha(M)$ and $a=[T(x\beta x),x]_\alpha$. Then (3.3) can rewrite in the following form

$$[T(x),2z\beta x+a]_\alpha=0. \tag{3.4}$$

By the expansion of the commutation identities (3.4), we get

$$[T(x),2z\beta x]_\alpha+[T(x),a]_\alpha=0,$$
$$\Rightarrow 2z\beta[T(x),x]_\alpha+2[T(x),z]_\alpha\beta x+[T(x),a]_\alpha=0,$$
$$\Rightarrow 2z\beta z+[T(x),a]_\alpha=0,$$
$$\Rightarrow [T(x),a]_\alpha=-2z\beta z. \tag{3.5}$$

On the other hand, we have

$$[T(x\beta x),x\beta x]_\alpha\in Z_\alpha(M) \tag{3.6}$$

for all $x\in J$ and $\alpha,\beta\in\Gamma$. This implies

$$[T(x\beta x),x]_\alpha\beta x+x\beta[T(x\beta x),x]_\alpha\in Z_\alpha(M). \tag{3.7}$$

Now using the relation (3.7), we can write $[T(x),a\beta x+x\beta a]_\alpha=0$ and apply (3.4), it takes the following explicit form

$$[T(x),a]_\alpha\beta x+a\beta[T(x),x]_\alpha+[T(x),x]_\alpha\beta a+x\beta[T(x),a]_\alpha=0,$$
$$\Rightarrow -2z\beta z\beta x+a\beta z+z\beta a+x\beta(-2z\beta z)=0,$$
$$\Rightarrow -2z\beta z\beta x+z\beta a+z\beta a-2z\beta z\beta x=0,\quad \text{by using the definition of}\quad Z_\alpha(M),$$
$$\Rightarrow 2z\beta a-4z\beta z\beta x=0\Rightarrow a=2z\beta x. \tag{3.8}$$

Using (3.8) in (3.5), we have

$$[T(x),2z\beta x]_\alpha=-2z\beta z,$$
$$\Rightarrow 2\{[T(x),z]_\alpha\beta x+z\beta[T(x),x]_\alpha\}=-2z\beta z,$$
$$\Rightarrow [T(x),z]_\alpha\beta x+z\beta[T(x),x]_\alpha=-z\beta z,$$
$$\Rightarrow [T(x),z]_\alpha\beta x+z\beta[T(x),x]_\alpha=-z\beta z,$$
$$\Rightarrow z\beta[T(x),x]_\alpha=-z\beta z,$$
$$\Rightarrow z\beta z=-z\beta z\Rightarrow 2z\beta z=0. \tag{3.9}$$

By the 2-torsion free semiprimeness of M, we conclude that $z\beta z = 0$ implies $z = 0$ for all $\beta \in \Gamma$. Therefore $[T(x), x]_\alpha = 0$ for all $x \in J$ and hence T is commutating on J. $\square$

Theorem 3.2 *Suppose that M is a cancellative semiprime Γ-semiring satisfying the assumption* (2.1). *If $T : M \to M$ is a centralizing right centralizer, then T is commuting.*

Proof If we consider M is 2-torsion free cancellative semiprime Γ-semiring satisfying the assumption (2.1), then the theorem is nothing to prove for $J = M$ in account of Theorem 3.1. We now assume that M is not 2-torsion free Γ-semiring. In this case, we consider the following relation

$$2[x, T(x)]_\alpha = 0. \tag{3.10}$$

for all $x \in M$ and $\alpha \in \Gamma$. We now substitute $x + y$ for x in (3.10), which yields

$$\begin{aligned} &2[x + y, T(x + y)]_\alpha = 0, \\ &\Rightarrow 2[x, T(y)]_\alpha + 2[y, T(x)]_\alpha = 0, \\ &\Rightarrow [x, T(y)]_\alpha = -[y, T(x)]_\alpha \end{aligned} \tag{3.11}$$

for all $x, y \in M$ and $\alpha \in \Gamma$. We now again linearise the assumption $[x, T(x)]_\alpha \in Z_\alpha(M)$ by the transformation $x = x + y$, which leads to

$$[x, T(y)]_\alpha + [y, T(x)]_\alpha \in Z_\alpha(M) \tag{3.12}$$

for all $x, y \in M$ and $\alpha \in \Gamma$. By using the definition of $Z_\alpha(M)$ in (3.12), we enable to express as

$$[([x, T(y)]_\alpha + [y, T(x)]_\alpha), x]_\beta = 0 \tag{3.13}$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$. This implies

$$\begin{aligned} &([x, T(y)]_\alpha + [y, T(x)]_\alpha)\beta x - x\beta([x, T(y)]_\alpha + [y, T(x)]_\alpha) = 0, \\ &\Rightarrow [x, T(y)]_\alpha\beta x + [y, T(x)]_\alpha\beta x - x\beta[x, T(y)]_\alpha - x\beta[y, T(x)]_\alpha = 0. \end{aligned} \tag{3.14}$$

Using (3.11) in (3.14), we obtain

$$[x, T(y)]_\alpha\beta x + [y, T(x)]_\alpha\beta x + x\beta[x, T(y)]_\alpha + x\beta[y, T(x)]_\alpha = 0. \tag{3.15}$$

Again from the assumption $[x, T(x)]_\alpha \in Z_\alpha(M)$ and the definition of $Z_\alpha(M)$, we found

$$\begin{aligned} [x, T(x)]_\alpha\beta y &= (x\alpha T(x) - T(x)\alpha x)\beta y \\ &= x\alpha T(x)\beta y - T(x)\alpha x\beta y \\ &= y\beta x\alpha T(x) - y\beta T(x)\alpha x \\ &= y\beta(x\alpha T(x) - T(x)\alpha x) = y\beta[x, T(x)]_\alpha \end{aligned} \tag{3.16}$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$. Applying the result(3.16) in (3.11), we get

$$\begin{aligned}
&2[x, T(x)]_\alpha = 0,\\
&\Rightarrow [x, T(x)]_\alpha + [x, T(x)]_\alpha = 0,\\
&\Rightarrow y\beta[x, T(x)]_\alpha + y\beta[x, T(x)]_\alpha = 0, \quad \text{right multiplying by } y\beta,\\
&\Rightarrow [x, T(x)]_\alpha\beta y + y\beta[x, T(x)]_\alpha = 0, \quad \text{using (3.16)}, \qquad (3.17)
\end{aligned}$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$. Adding the relations (3.15) and (3.17) and simplifying, we have

$$[(x\beta y + y\beta x), T(x)]_\alpha + [x\beta x, T(y)]_\alpha = 0. \qquad (3.18)$$

Now using $x\gamma y$ for y in the relation (3.18), we arrive at

$$[(x\beta x\gamma y + x\gamma y\beta x), T(x)]_\alpha + [x\beta x, T(x\gamma y)]_\alpha = 0 \qquad (3.19)$$

for all $x, y \in M$ and $\alpha, \beta, \gamma \in \Gamma$. By using assumption (2.1) in (3.19), we obtain

$$\begin{aligned}
&[(x\gamma x\beta y + x\gamma y\beta x), T(x)]_\alpha + [x\beta x, T(x\gamma y)]_\alpha = 0,\\
&\Rightarrow [x\gamma(x\beta y + y\beta x), T(x)]_\alpha + [x\beta x, T(x\gamma y)]_\alpha = 0,\\
&\Rightarrow [x, T(x)]_\alpha\gamma(x\beta y + y\beta x) + x\gamma[(x\beta y + y\beta x), T(x)]_\alpha + x\gamma[x\beta x, T(y)]_\alpha = 0. \qquad (3.20)
\end{aligned}$$

Using (3.18) in (3.20), it reduces to

$$\begin{aligned}
&[x, T(x)]_\alpha\gamma(x\beta y + y\beta x) - x\gamma[x\beta x, T(y)]_\alpha + x\gamma[x\beta x, T(y)]_\alpha = 0,\\
&\Rightarrow [x, T(x)]_\alpha\gamma(x\beta y + y\beta x) = 0,\\
&\Rightarrow [x, T(x)]_\alpha\gamma(x\beta y - y\beta x + 2y\beta x) = 0,\\
&\Rightarrow [x, T(x)]_\alpha\gamma(x\beta y - y\beta x) + 2[x, T(x)]_\alpha\gamma y\beta x = 0,\\
&\Rightarrow [x, T(x)]_\alpha\gamma(x\beta y - y\beta x) = 0, \quad \text{using (3.10)},\\
&\Rightarrow [x, T(x)]_\alpha\gamma[x, y]_\beta = 0 \qquad (3.21)
\end{aligned}$$

for all $x, y \in M$ and $\alpha, \beta, \gamma \in \Gamma$. Replacing y by $T(x)$ and $\beta = \alpha$ in (3.21), we obtain

$$[x, T(x)]_\alpha\gamma[x, T(x)]_\alpha = 0 \qquad (3.22)$$

for all $x \in M$ and $\alpha, \gamma \in \Gamma$. For the semiprimeness of Γ-semiring, $[x, T(x)]_\alpha\gamma[x, T(x)]_\alpha = 0$ implies $[x, T(x)]_\alpha = 0$ for all $x \in M$ and $\alpha, \gamma \in \Gamma$. Therefore T is commuting and hence the theorem is proved. □

Theorem 3.3 *Suppose that M is a cancellative semiprime Γ-semiring satisfying the assumption (2.1). If $T : M \to M$ is a centralizing right centralizer on M, then T is centralizer.*

Proof Since T is a centralizing right centralizer on M, we have

$$T(x\alpha y) = x\alpha T(y) \qquad (3.23)$$

for all $x, y \in M$ and $\alpha \in \Gamma$. We aim to prove that $T(x\alpha y) = T(x)\alpha y$ for all $x, y \in M$ and $\alpha \in \Gamma$. According to the statement and Theorem 3.2, T is commuting on M. In this case, we can write $[x, T(x)]_\alpha = 0$. This implies

$$x\alpha T(x) = T(x)\alpha x \tag{3.24}$$

for all $x \in M$ and $\alpha \in \Gamma$. Now linearizing the relation (3.24) by setting $x = x + y$, yields

$$[x, T(y)]_\alpha + [y, T(x)]_\alpha = 0 \tag{3.25}$$

for all $x, y \in M$ and $\alpha \in \Gamma$. Applying $x\beta y$ for x and using the definition, we obtain

$$\begin{aligned}
&[x\beta y, T(y)]_\alpha + [y, T(x\beta y)]_\alpha = 0,\\
&\Rightarrow x\beta[y, T(y)]_\alpha + [x, T(y)]_\alpha \beta y + [y, T(x\beta y)]_\alpha = 0,\\
&\Rightarrow [x, T(y)]_\alpha \beta y + [y, x\beta T(y)]_\alpha = 0,\\
&\Rightarrow x\alpha T(y)\beta y - T(y)\alpha x\beta y + y\alpha x\beta T(y) - x\beta T(y)\alpha y = 0,\\
&\Rightarrow x\beta T(y)\alpha y - x\beta T(y)\alpha y + y\alpha x\beta T(y) - T(y)\alpha x\beta y = 0,\\
&\Rightarrow y\alpha x\beta T(y) - T(y)\alpha x\beta y = 0,\\
&\Rightarrow y\alpha x\beta T(y) = T(y)\alpha x\beta y
\end{aligned} \tag{3.26}$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$. Let $T(y) = y$ in (3.26), then we have $y\alpha x\beta T(y) = y\alpha x\beta y$. This implies $y\beta x\alpha T(y) = y\beta x\alpha y$. By using the cancellation law, it shows that $x\alpha T(y) = x\alpha y$, which implies that $T(x\alpha y) = x\alpha y$ for all $x, y \in M$ and $\alpha \in \Gamma$. Thus for choosing $T(x) = x$, we write $T(x\alpha y) = T(x)\alpha y$. By using the assumption (2.1), this implies

$$x\alpha T(y) = T(x)\alpha y. \tag{3.27}$$

If we consider $z \in M$ and $\beta \in \Gamma$, then we can write $(T(x\alpha y) - x\alpha T(y))\beta z\beta(T(x\alpha y) - x\alpha T(y)) = 0$ for all $x, y \in M$ and $\alpha \in \Gamma$. By the definition of semiprime Γ-semiring M, it leads to $T(x\alpha y) - x\alpha T(y) = 0$. That is,

$$T(x\alpha y) = x\alpha T(y). \tag{3.28}$$

Comparing (3.27) and (3.28), we conclude that

$$T(x\alpha y) = T(x)\alpha y \tag{3.29}$$

for all $x, y \in M$ and $\alpha \in \Gamma$. Therefore, T is a centralizer on M and hence, the theorem is proved. □

The study of the above theorems, we can provide the following remarks.

Remark 3.1 Every centralizer on a cancellative semiprime Γ-semiring is commuting, because of $T(x\alpha x) = T(x)\alpha x = x\alpha T(x)$ for all $x \in M$ and $\alpha \in \Gamma$ and hence is a centralizing right centralizer.

Remark 3.2 An additive mapping T of a cancellative semiprime Γ-semiring M is a centralizer if and only if it is a centralizing right centralizer on M.

Corollary 3.1 *Suppose that T is a commuting right centralizer of a semiprime Γ-semiring M satisfying the assumption (2.1), then T satisfies the relation $[x,y]_\alpha \beta T(x) = [T(x), y]_\alpha \beta x$ for all $x, y \in M$ and $\alpha, \beta \in \Gamma$.*

Proof By the statement, we have $T(x\alpha y) = x\alpha T(y)$ for all $x, y \in M$ and $\alpha \in \Gamma$. Since T is also commuting on M, it is easily seen that $[x, T(x)]_\alpha = 0$. Putting $x = x + y$ for linearization, we arrive at

$$[x, T(y)]_\alpha + [y, T(x)]_\alpha = 0 \tag{3.30}$$

for all $x, y \in M$ and $\alpha \in \Gamma$. Replacing y by $y\beta x$ in (3.30) and using the relation $[x, T(x)]_\alpha = 0$, we obtain

$$\begin{aligned}
&[x, T(y\beta x)]_\alpha + [y\beta x, T(x)]_\alpha = 0,\\
&\Rightarrow [x, y\beta T(x)]_\alpha + [y\beta x, T(x)]_\alpha = 0,\\
&\Rightarrow [x,y]_\alpha \beta T(x) + y\beta [x, T(x)]_\alpha + [y, T(x)]_\alpha \beta x + y\beta [x, T(x)]_\alpha = 0,\\
&\Rightarrow [x,y]_\alpha \beta T(x) + [y, T(x)]_\alpha \beta x = 0,\\
&\Rightarrow [x,y]_\alpha \beta T(x) - [T(x), y]_\alpha \beta x = 0,\\
&\Rightarrow [x,y]_\alpha \beta T(x) = [T(x), y]_\alpha \beta x
\end{aligned} \tag{3.31}$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$. Hence, the theorem is proved. $\square$

Corollary 3.2 *Suppose that M is a prime Γ-semiring and T is a commuting right centralizer on M. If $T(x) \in Z_\alpha(M)$ for all $x \in M$, then $T = 0$ or M is commutative.*

Proof Since $T(x) \in Z_\alpha(M)$, in this case we have $[T(x), y]_\alpha = 0$ for all $x, y \in M$ and $\alpha \in \Gamma$. We also have $[x,y]_\alpha \beta T(x) = [T(x), y]_\alpha \beta x$ for all $x, y \in M$ and $\alpha, \beta \in \Gamma$. Thus

$$[x,y]_\alpha \beta T(x) = 0 \tag{3.32}$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$. Setting $y = y\gamma z$ in (3.32), we have

$$\begin{aligned}
&[x, y\gamma z]_\alpha \beta T(x) = 0,\\
&\Rightarrow \{[x,y]_\alpha \gamma z + y\gamma [x,z]_\alpha\} \beta T(x) = 0,\\
&\Rightarrow [x,y]_\alpha \gamma z \beta T(x) + y\gamma [x,z]_\alpha \beta T(x) = 0,\\
&\Rightarrow [x,y]_\alpha \gamma z \beta T(x) = 0, \quad \text{for all} \quad x, y, z \in M,\\
&\Rightarrow (x\alpha y - y\alpha x)\gamma z \beta T(x) = 0,\\
&\Rightarrow (x\alpha y\gamma z - y\alpha x\gamma z)\beta T(x) = 0.
\end{aligned}$$

For the prime Γ-semiringness of M, we have $(x\alpha y\gamma z - y\alpha x\gamma z) = 0$ or $\beta T(x) = 0$. Thus we have seen that $T = 0$ or M is commutative, and hence, the theorem is proved. $\square$

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