International J.Math. Combin. Vol.2(2020), 118-120

Uni-Distance Domination of Square of Paths

Kishori P. Narayankar, Denzil Jason Saldanha

(Department of Mathematics, Mangalore University, Mangalagangothri, Mangalore-574199, India)

John Sherra

(St Aloysius College (Autonomous), Mangalore-575003, India)

E-mail: kishori_pn@yahoo.co.in, denzil53@gmail.com, johnsherra@gmail.com

Abstract: A dominating set D of G which is also a resolving set of G is called a *metro dominating set.* A metro dominating set D of a graph $G(V,E)$ is a *uni-distance dominating set* (in short an *UDD-set*) if $\mid N(v)\cap D\mid = 1$ for each vertex $v\in V-D$ and the minimum of cardinalities of an *UDD-set of G* is the *uni-distance domination number of G* denoted by $\gamma_{\mu\beta}(G)$. In this paper we determine unique distance domination number of P_n^2 graphs.

Key Words: Domination, metric dimension, metro domination, uni-distance domination, Smarandachely distance k dominating set.

AMS(2010): 05C69.

§1. Introduction

All the graphs considered in this paper are simple, connected and undirected. The length of a shortest path between two vertices u and v in a graph G is called the distance between u and v and is denoted by $d(u,v)$. For a vertex v of a graph, $N(v)$ denote the set of all vertices adjacent to v and is called open neighborhood of v. Similarly, the closed neighborhood of v is defined as $N[v]=N(v)\cup\{v\}$.

Let $G(V,E)$ be a graph. For each ordered subset $S=\{v_1,v_2,v_3,\cdots,v_k\}$ of V, each vertex $v\in V$ can be associated with a vector of distances denoted by $\Gamma(v/S)=(d(v_1,v),d(v_2,v),\cdots,$ $d(v_k,v))$. The set S is said to be a *resolving set* of G, if $\Gamma(v/S)\neq\Gamma(u/S)$, for every $u,v\in V-S$. A resolving set of minimum cardinality is a *metric basis* and cardinality of a metric basis is the *metric dimension* of G. The k-tuple, $\Gamma(v/S)$ associated to the vertex $v\in V$ with respect to a metric basis S, is referred as a *code generated by S* for that vertex v. If $\Gamma(v/S)=(c_1,c_2,\cdots,c_k)$, then $c_1,c_2,c_3,\cdots,c_k$ are called components of the code of v generated by S and in particular c_i, $1\le i\le k$, is called i^{th}-component of the code of v generated by S.

A dominating set D of a graph $G(V,E)$ is the subset of V having the property that for each vertex $v\in V-D$, there exists a vertex $u\in D$ such that uv is in E. A dominating set D of G which is also a resolving set of G is called a *metro dominating set.*

A metro dominating set D of a graph $G(V,E)$ is a *uni-distance dominating set* (in short an *UDD-set*) if $\mid N(v)\cap D\mid = 1$ for each vertex $v\in V-D$. Generally, if $\mid N(v)\cap D\mid = k$

[1]Received November 14, 2019, Accepted June 14, 2020.

for each vertex $v \in V - D$, $k \geq 1$, such a metro dominating set D is called a *Smarandachely distance k dominating set* (Smarandachely k DD-sets of G) and the minimum of cardinalities of the Smarandachely DD-sets of G is the number of Smarandachely k UDD-sets of G, denoted by $\gamma^k_{S\mu\beta}(G)$. Particularly, if $k = 1$, i.e., the *uni-distance domination number of G* denoted by $\gamma_{\mu\beta}(G)$. For an integer $n \geq 3$, we determine the uni-distance domination number $\gamma_{\mu\beta}\left(P_n^2\right)$ of P_n^2 in this paper.

§2. Main Results

Consider P_n, $n \geq 3$. Join v_i to v_{i+2} for $1 \leq i \leq n-2$. The resulting graph is denoted by P_n^2.

Lemma 2.1 *For any positive integer n,* $\gamma_{\mu\beta}(P_n^2) \geq \left\lceil \frac{n}{5} \right\rceil$.

Proof A vertex v_i dominates five vertices $v_i, v_{i-1}, v_{i-2}, v_{i+1}, v_{i+2}$. Therefore, if D is minimal dominating set then $|\ D\ | \geq \frac{n}{5}$. Hence we have $\gamma(P_n^2) \geq \left\lceil \frac{n}{5} \right\rceil$.

End vertex v_1 of P_n^2 can dominate only 3 vertices v_1, v_2 and v_3. As we have to minimize $|\ D\ |$, we include v_3 in D, which dominates v_1, v_2, v_3, v_4 and v_5. □

Lemma 2.2 *If* $n = 5k$, $k \in \mathbb{N}$ *then* $\gamma_{\mu\beta}\left(P_n^2\right) = k = \left\lceil \frac{n}{5} \right\rceil$.

Proof When $k = 1$, v_3 dominates all vertices of P_5^2. Hence $\gamma(P_5^2) = 1$.

Let $n = 5k$. Then $D = \{v_3, v_8, v_{13}, \cdots, v_{5k-2}\}$ and $|\ D\ |$=k. When n=5(k+1), take $D' = D \cup \{v_{5k+3}\}$. Observe that $|\ D'\ | = k+1$ and D' dominates all vertices. From Lemma 2.1, we have

$$\gamma(P^2_{5(k+1)}) \geq \left\lceil \frac{5(k+1)}{5} \right\rceil = k+1$$

and $|\ D'\ | = k+1$. Therefore we conclude that $\gamma(P_5(k+1))^2 = k+1$. Thus by induction

$$\gamma\left(P_n^2\right) = k = \left\lceil \frac{n}{5} \right\rceil.$$

In P_n^2, consider any v_j and v_{j+5} in D. Vertex v_j dominates $v_{j-2}, v_{j-1}, v_{j+1}, v_{j+2}$. Vertex v_{j+5} dominates $v_{j+3}, v_{j+4}, v_{j+6}$ and v_{j+7}. These vertices are uniquely dominated by v_j and v_{j+5}. The vertices v_1 and v_2 are uniquely dominated by v_3. The vertex v_{5k} and v_{5k-1} are uniquely dominated by v_{5k-2}.

In P_n^2, we observe that

$$d(v_i, v_j) = d(v_i, v_{j-1}) = \frac{j-i}{2}$$

where i and j are both even and $j \geqslant i$. When i is odd and j is even

$$d(v_i, v_{j+1}) = d(v_i, v_j) = \frac{j-i+1}{2}.$$

We take $D = \{v_3, v_8, v_{13}, \cdots\}$. Note that $d(v_3, v_{j+1}) = d(v_3, v_j)$, $j \geqslant 3$ and j even. Also $d(v_3, v_2) = d(v_3, v_1)$. Now when $j \geqslant 8$ and j is even,

$$d(v_8, v_j) = \frac{j-8}{2} \quad \text{and} \quad d(v_8, v_{j+1}) = \frac{(j+2)-8}{2}.$$

Hence $d(v_8, v_j) \neq d(v_8, v_{j+1})$. Therefore $\{v_3, v_8\}$ resolve all vertices v_j, $j \geqslant 8$, Now $d(v_3, v_1) = 1$ but $d(v_8, v_1) = 4$ and $d(v_3, v_2) = 1$ but $d(v_8, v_2) = 3$. Hence$\{v_3, v_8\}$ resolve v_1 and v_2.

If $3 \leqslant j \leqslant 8$ then $\{v_3, v_8\}$ generate the same code (1,2) for v_4 and v_5. Also $\{v_3, v_8\}$ generate the same code (2,1) for v_6 and v_7. We have $d(v_{13}, v_4) = 5$ and $d(v_{13}, v_5) = 4$. Also $d(v_{13}, v_6) = 4$ and $d(v_{13}, v_7) = 3$. Hence $\{v_3, v_8, v_{13}\}$ resolves all vertices of P_n^2. Therefore to resolve all vertices of P_n^2 we take $n \geq 11$. We observe that $\{v_3, v_8, \cdots, v_{5k-2}\}$ uniquely dominates all vertices in $V - D$. Hence we have the conclusion. □

If $n = 5k + 1$, $n = 5k + 2$, $n = 5k + 3$, $D = \{v_1, v_6, v_{11}, \cdots, v_{5k-4}, v_{5k+1}\}$ is a UDD set. Therefore $\gamma_{\mu\beta}\left(P_n^2\right) = k + 1$. If $n = 5k + 4$, $D = \{v_2, v_7, v_{12}, \cdots, v_{5k-3}, v_{5k+2}\}$ is a UDD set and we have $\gamma_{\mu\beta}\left(P_n^2\right) = k + 1 = \left\lceil \frac{n}{5} \right\rceil$. Thus we obtain $\gamma_{\mu\beta}\left(P_n^2\right) = \left\lceil \frac{n}{5} \right\rceil$ for $\forall n \geq 11$. If $n < 11$, then we observe that $\gamma_{\mu\beta}(P_n^2) = n$. Hence, we have

Theorem 2.3 *For an integer $n \geq 3$,*

$$\gamma_{\mu\beta}(P_n^2) = \begin{cases} \left\lceil \frac{n}{5} \right\rceil, & \text{for } n \geq 11 \\ n, & \text{for } n < 11. \end{cases}$$

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