

Open Packing Number of Triangular Snakes

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Abstract: A set $S \subseteq V(G)$ of vertices in a graph G is called a packing of G if the closed neighborhood of the vertices of S are pairwise disjoint in G . A subset S of $V(G)$ is called an open packing of G if the open neighborhood of the vertices of S are pairwise disjoint in G . We have investigated exact value of these parameters for triangular snakes.

Key Words: Neighborhood, packing, Smarandache k -packing, open packing.

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§1. Introduction

We begin with the finite, connected and undirected graph $G = (V(G), E(G))$ without multiple edges and loops. For a vertex $v \in V(G)$, the open neighborhood $N(v)$ of v is defined as $N(v) = \{u \in V(G) / uv \in E(G)\}$ and the closed neighborhood $N[v] = \{v\} \cup N(v)$. We denote the degree of a vertex $v \in V(G)$ in a graph G by $d_G(v)$. The minimum degree among the vertices of G is denoted by $\delta(G)$ and the maximum degree among the vertices of G is denoted by $\Delta(G)$. For any real number n , $\lfloor n \rfloor$ denotes the greatest integer not greater than that n and $\lceil n \rceil$ denotes the smallest integer not less than that n . For the various graph theoretic notations and terminology, we follow West [8] and Haynes et al. [3].

Definition 1.1 The triangular snake T_n is obtained from the path P_n by replacing every edge of a path by a triangle C_3 .

Definition 1.2 An alternate triangular snake AT_n is obtained from a path P_n with vertices $u_1, u_2, \dots, u_n$ by joining u_i and u_{i+1} (alternately) to a new vertex v_i . That is every alternate edge of a path is replaced by C_3 .

Definition 1.3 The double triangular snake $D(T_n)$ is obtained from a path P_n with vertices $v_1, v_2, \dots, v_n$ by joining v_i and v_{i+1} to a new vertex w_i for $i = 1, 2, \dots, n-1$ and to a new vertex u_i for $i = 1, 2, \dots, n-1$.

Definition 1.4 A double alternate triangular snake $D(AT_n)$ consists of two alternate triangular snakes which have a common path.

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A packing of a graph G is a set of vertices whose closed neighborhoods are pairwise disjoint. Generally, a Smarandache k -packing of a graph G is a set of vertices whose closed neighborhoods intersect just in k vertices, and disjoint if $k = 0$. Equivalently, a packing of a graph G is a set of vertices whose elements are pairwise at distance at least 3 apart in G . The maximum cardinality of a packing set of G is called the packing number and it is denoted by $\rho(G)$. This concept was introduced by Biggs [1].

A subset S of $V(G)$ is an open packing of G if the open neighborhoods of the vertices of S are pairwise disjoint in G . The maximum cardinality of an open packing set is called the open packing number and is denoted by ρ^o . This concept was introduced by Henning and Slater [5]. A brief account of on open packing and its related concepts can be found in [2,4,6,7]. In the present paper, we obtain the packing and open packing number of various snakes.

§2. Main Results

Theorem 2.1 For $n \geq 3$, $\rho(G) = \left\lceil \frac{n}{3} \right\rceil$, where G is triangular snake T_n and double triangular snake $D(T_n)$.

Proof The triangular snake T_n is obtained from a path P_n with vertices $v_1, v_2, \dots, v_n$ by joining v_i and v_{i+1} to a new vertex w_i for $i = 1, 2, 3, \dots, n-1$ while to construct double triangular snake $D(T_n)$ from a path P_n with vertices $v_1, v_2, \dots, v_n$ by joining v_i and v_{i+1} to a new vertex w_i for $i = 1, 2, 3, \dots, n-1$ and to a new vertex u_i for $i = 1, 2, \dots, n-1$.

If S is any packing set of G then it is obvious that v_1 must in S as $d_G(v_1) = 2 = \delta(G)$.

We construct a set S of vertices as follows:

$$S = \left\{ v_{3i+1}/0 \leq i \leq \left\lceil \frac{n}{3} \right\rceil - 1 \right\}$$

Then $|S| = \left\lceil \frac{n}{3} \right\rceil$. Moreover S is a packing set of G as $N[v] \cap N[u] \neq \phi$ for all $v, u \in S$. For any $w \in V(G) - S$, $N[v] \cap N[w] \neq \phi$ and $N[u] \cap N[w] \neq \phi$. Thus, S is a maximal packing set of G . Therefore any superset containing the vertices greater than that of $|S|$ can not be a packing set of G . Hence

$$\rho(G) = \left\lceil \frac{n}{3} \right\rceil. \quad \square$$

Theorem 2.2 For $n \geq 3$, $\rho^o(G) = \left\lceil \frac{n}{3} \right\rceil$, where G is triangular snake T_n and double triangular snake $D(T_n)$.

Proof The triangular snake T_n is obtained from a path P_n with vertices $v_1, v_2, \dots, v_n$ by joining v_i and v_{i+1} to a new vertex w_i for $i = 1, 2, 3, \dots, n-1$ while to construct double triangular snake $D(T_n)$ from a path P_n with vertices $v_1, v_2, \dots, v_n$ by joining v_i and v_{i+1} to a new vertex w_i for $i = 1, 2, 3, \dots, n-1$ and to a new vertex u_i for $i = 1, 2, \dots, n-1$.

If S is any open packing set of G then it is obvious that v_1 must in S as $d_G(v_1) = 2 = \delta(G)$.

We construct a set S of vertices as follows:

$$S = \left\{ v_{3i+1}/0 \leq i \leq \left\lceil \frac{n}{3} \right\rceil - 1 \right\}$$

Then $|S| = \left\lceil \frac{n}{3} \right\rceil$. Moreover S is an open packing set of G as $N(v) \cap N(u) \neq \phi$ for all $v, u \in S$. For any $w \in V(G) - S$, $N(v) \cap N(w) \neq \phi$ and $N(u) \cap N(w) \neq \phi$. Thus, S is a maximal open packing set of G . Therefore any superset containing the vertices greater than that of $|S|$ can not be an open packing set

of G . Hence

$$\rho^o(G) = \left\lceil \frac{n}{3} \right\rceil. \quad \square$$

Illustration 2.3 The graph T_7 and its packing number and open packing number are shown Figure 1 while the graph $D(T_7)$ and its packing number and open packing number are shown in Figure 2.

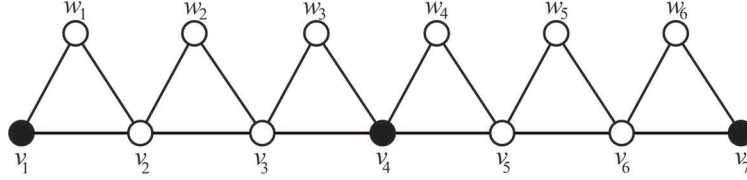


Figure 1 $\rho(T_7) = \rho^o(T_7) = 3$

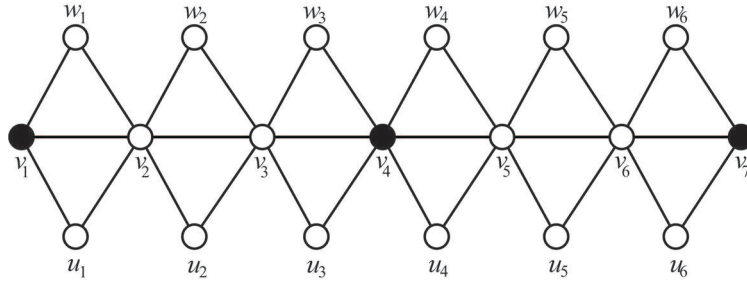


Figure 2 $\rho(D(T_7)) = \rho^o(D(T_7)) = 3$

Theorem 2.4 For $n > 3$, $\rho(G) = \left\lceil \frac{n}{3} \right\rceil$, where G is alternate triangular snake AT_n and double alternate triangular snake $D(AT_n)$.

Proof An alternate triangular snake AT_n is obtained from a path P_n with vertices $v_1, v_2, \dots, v_n$ by joining v_i and v_{i+1} (alternately) to a new vertex w_i , $i = 1, 2, \dots, n-1$ while to construct a double alternate triangular snake $D(AT_n)$ from a path P_n with vertices $v_1, v_2, \dots, v_n$ by joining v_i and v_{i+1} (alternately) to a new vertex w_i , $i = 1, 2, \dots, n-1$ and to a new vertex u_i for $i = 1, 2, \dots, n-1$.

If S is any packing set of G then it is obvious that v_1 must in S as

$$d_G(v_1) = \delta(G) = \begin{cases} 1, & \text{if } n \text{ is odd,} \\ 2, & \text{if } n \text{ is even.} \end{cases}$$

We construct a set S of vertices as follows:

$$S = \left\{ v_{3i+1} / 0 \leq i \leq \left\lceil \frac{n}{3} \right\rceil - 1 \right\}$$

Then $|S| = \left\lceil \frac{n}{3} \right\rceil$. Moreover S is a packing set of G as $N[v] \cap N[u] \neq \phi$ for all $v, u \in S$. For any $w \in V(G) - S$, $N[v] \cap N[w] \neq \phi$ and $N[u] \cap N[w] \neq \phi$. Thus, S is a maximal packing set of G . Therefore any superset containing the vertices greater than that of $|S|$ can not be a packing set of G . Hence

$$\rho(G) = \left\lceil \frac{n}{3} \right\rceil. \quad \square$$

Illustration 2.5 The graph AT_7 and its packing number is shown Figure 3 while the graph $D(AT_7)$

and its packing number is shown in Figure 4.

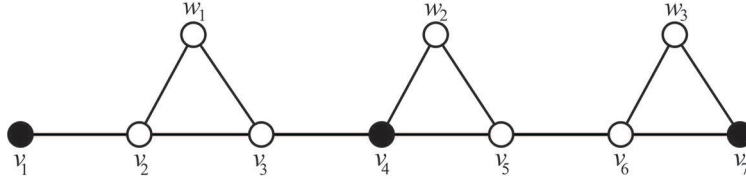


Figure 3 $\rho(AT_7) = 3$

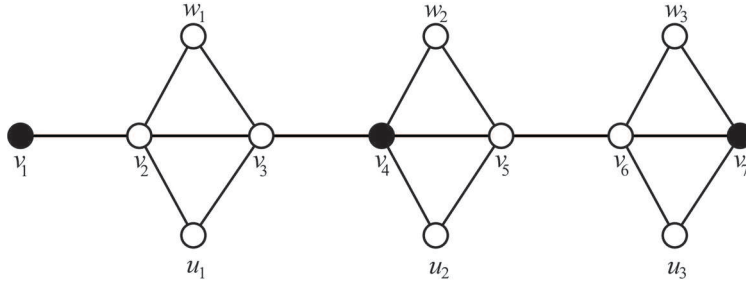


Figure 4 $\rho(D(AT_7)) = 3$

Theorem 2.6 For $n > 3$, $\rho^o(G) = \left\lceil \frac{n}{2} \right\rceil$, where G is alternate triangular snake AT_n and double alternate triangular snake $D(AT_n)$.

Proof An alternate triangular snake AT_n is obtained from a path P_n with vertices $v_1, v_2, \dots, v_n$ by joining v_i and v_{i+1} (alternately) to a new vertex w_i , $i = 1, 2, \dots, n-1$ while to construct a double alternate triangular snake $D(AT_n)$ from a path P_n with vertices $v_1, v_2, \dots, v_n$ by joining v_i and v_{i+1} (alternately) to a new vertex w_i , $i = 1, 2, \dots, n-1$ and to a new vertex u_i for $i = 1, 2, \dots, n-1$.

If S is any open packing set of G then it is obvious that v_1 must in S as

$$d_G(v_1) = \delta(G) = \begin{cases} 1, & \text{if } n \text{ is odd,} \\ 2, & \text{if } n \text{ is even.} \end{cases}$$

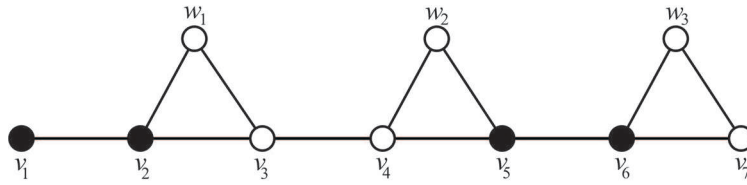
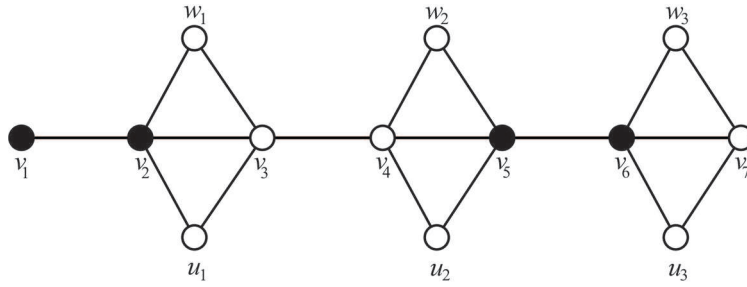
We construct a set S of vertices as follows:

$$S = \begin{cases} \left\{ v_{4i+1}, v_{4i+2} / 0 \leq i \leq \left\lceil \frac{n}{5} \right\rceil \right\} & \text{for } n \text{ is odd} \\ \left\{ v_{4i+2}, v_{4i+3} / 0 \leq i \leq \left\lfloor \frac{n}{5} \right\rfloor \right\} & \text{for } n \text{ is even} \end{cases}$$

Then $|S| = \left\lceil \frac{n}{2} \right\rceil$. Moreover S is an open packing set of G as $N(v) \cap N(u) \neq \phi$ for all $v, u \in S$. For any $w \in V(G) - S$, $N(v) \cap N(w) \neq \phi$ and $N(u) \cap N(w) \neq \phi$. Thus, S is a maximal open packing set of G . Therefore any superset containing the vertices greater than that of $|S|$ can not be an open packing set of G . Hence

$$\rho^o(G) = \left\lceil \frac{n}{2} \right\rceil. \quad \square$$

Illustration 2.7 The graph AT_7 and its open packing number is shown Figure 5 while the graph $D(AT_7)$ and its open packing number is shown in Figure 6.

Figure 5 $\rho^o(AT_7) = 4$ Figure 6 $\rho^o(D(AT_7)) = 4$

§3. Concluding Remarks

The concept of packing number relates three important graph parameters - neighborhood of a vertex, adjacency between two vertices and domination in graphs. We have investigated packing and open packing numbers of triangular snakes.

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