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A Note on 3-Remainder Cordial Labeling Graphs

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Abstract: Let G be a (p, q) graph. Let f be a function from $V(G)$ to the set $\{1, 2, \cdots, k\}$ where k is an integer $2 < k \leq |V(G)|$. For each edge uv assign the label r where r is the remainder when $f(u)$ is divided by $f(v)$ (or) $f(v)$ is divided by $f(u)$ according as $f(u) \geq f(v)$ or $f(v) \geq f(u)$. The function f is called a k-remainder cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$, $i, j \in \{1, \cdots, k\}$ where $v_f(x)$ denote the number of vertices labeled with x and $|\eta_e(0) - \eta_o(1)| \leq 1$ where $\eta_e(0)$ and $\eta_o(1)$ respectively denote the number of edges labeled with even integers and number of edges labeled with odd integers. A graph with admits a k-remainder cordial labeling is called a k-remainder cordial graph. In this paper we investigate the 3-remainder cordial labeling behavior of dumbbell graph, butterfly graph, umbrella graph, $C_3 \odot K_{1,n}$.

Key Words: Dumbbell graph, butterfly graph, umbrella graph, $C_3 \odot K_{1,n}$, Smarandache k-remainder cordial labeling.

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§1. Introduction

All graphs considered here are finite and simple. The origin of graph labeling is graceful labeling which was introduced by Rosa (1967). The concept of cordial labeling was introduced by Cahit [1]. Motivated by this Ponraj et al. [4, 6], introduced remainder cordial labeling of graphs and investigate the remainder cordial labeling behavior of several graphs. Also the notion of k-remainder cordial labeling introduced in [5] and investigate the $k-$remainder cordial labeling behavior of grid, subdivision of crown, subdivision of bistar, book, Jelly fish, subdivision of Jelly fish, mongolian tent, flower graph, sunflower graph and subdivision of ladder graph, $L_n \odot K_1$, $L_n \odot 2K_1$, $L_n \odot K_2$. Recently [9, 10] they investigate the 3-remainder cordial labeling behavior of the subdivision of the star, wheel, subdivision of the path, cycle, star, complete graph, comb, crown, wheel, subdivision of the comb, armed crown, fan, square of the path, $K_{1,n} \odot K_2$. In this paper we investigate the 3-remainder cordial labeling behavior of dumbbell graph, butterfly graph, umbrella graph, $C_3 \odot K_{1,n}$, etc. Terms are not defined here follows from Harary [3] and Gallian [2].

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§2. Preliminary Results

Definition 2.1 *The corona of G_1 with G_2, $G_1 \odot G_2$ is the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2.*

Definition 2.2 *The graph obtained by joining two disjoint cycles, $u_1u_2 \cdots u_nu_1$ and $v_1v_2 \ldots v_nv_1$ with an edge u_1v_1 is called dumbbell graph Db_n.*

Definition 2.3 *The butterfly graph $BF_{m,n}$ is a two even cycles of the same order say C_n, sharing a common vertex with m pendant edges attached at the common vertex is called a butterfly graph.*

Definition 2.4 *The umbrella graph $U_{n,m}$ is obtained from a fan $F_n = P_n + K_1$ where $P_n : u_1, u_2, \cdots, u_n$ and $V(K_1) = \{u\}$ by pasting the end vertex of the path $P_m : v_1, v_2, \cdots, v_m$ to the vertex of K_1 of the fan F_n.*

§3. k-Remainder Cordial Labeling

Definition 3.1 *Let G be a (p, q) graph. Let f be a function from $V(G)$ to the set $\{1, 2, \cdots, k\}$ where k is an integer $2 < k \leq |V(G)|$. For each edge uv assign the label r where r is the remainder when $f(u)$ is divided by $f(v)$ (or) $f(v)$ is divided by $f(u)$ according as $f(u) \geq f(v)$ or $f(v) \geq f(u)$. The function f is called a k-remainder cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$, $i, j \in \{1, \cdots, k\}$, otherwise, Smarandachely if $|v_f(i) - v_f(j)| \geq 1$ or $|e_f(0) - e_f(1)| \geq 1$ for integers $i, j \in \{1, \cdots, k\}$, where $v_f(x)$ denote the number of vertices labeled with x and $|\eta_e(0) - \eta_o(1)| \leq 1$ where $\eta_e(0)$ and $\eta_o(1)$ respectively denote the number of edges labeled with even integers and number of edges labeled with odd integers. A graph with a k-remainder cordial labeling is called a k-remainder cordial graph.*

Now, we investigate the $3-$remainder cordial labeling behavior of the dumbbell graph Db_n.

Theorem 3.2 *The dumbbell graph Db_n is 3-remainder cordial for all n.*

Proof Let $C_n : u_1u_2 \cdots u_nu_1$ and $C_n^{'} : v_1v_2 \cdots v_nv_1$ be two disjoint cycles of the same order n. Let $V(Db_n) = V(C_n) \cup V(C_n^{'})$ and $E(Db_n) = E(C_n) \cup E(C_n^{'}) \cup \{u_1v_1\}$. Then the order and size of the dumbbell graph are $2n$ and $2n + 1$ respectively.

Case 1. $n \equiv 0 \pmod 3$.

Assign the labels $2, 3$ and 1 respectively to the vertices u_1, u_2 and u_3. Next assign the labels $1, 2$ and 3 to the vertices u_4, u_5 and u_6 respectively. Then assign the labels $2, 3$ and 1 respectively to the vertices u_7, u_8 and u_9. Then next assign the labels $1, 2$ and 3 to the vertices u_{10}, u_{11} and u_{12} respectively. Proceeding like this until we reach the vertex u_n. If n is odd then assign the labels $2, 3$ and 1 respectively to the vertices u_{n-2}, u_{n-1} and u_n. If n is even then assign the labels $1, 2$ and 3 respectively to the vertices u_{n-2}, u_{n-1} and u_n of C_n. On the other hand assign the labels $3, 2$ and 1 respectively to the vertices v_1, v_2 and v_3. Next assign the labels $1, 3$ and 2 to the vertices v_4, v_5 and v_6 respectively. Then assign the labels $3, 2$ and 1 respectively to the vertices v_7, v_8 and v_9. Then next assign the labels $1, 3$ and 2 to the vertices v_{10}, v_{11} and v_{12} respectively. Continuing like this until we reach the vertex v_n. If n is odd then assign the labels $3, 2$ and 1 respectively to the vertices v_{n-2}, v_{n-1} and v_n. If n is even then assign the labels $1, 3$ and 2 respectively to the vertices v_{n-2}, v_{n-1} and v_n of $C_n^{'}$. Table 1 shows that this vertex labeling is called 3-remainder cordial labeling of the dumbbell graph for $n \equiv 0 \pmod 3$.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	η_e	η_o
n is odd	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$	$n+1$	n
n is even	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$	n	$n+1$

Table 1

Case 2. $n \equiv 1 \pmod 3$.

Subcase 2.1 n is even.

Assign the labels to the vertices $u_i, 1 \le i \le n$ in the following way.

$$f(u_i) = \begin{cases} 2, & \text{if } i = 1, 3, 5, \cdots, i+2 \cdots, n-1, \\ 3, & \text{if } i = 2, 4, 6, \cdots, i+2 \cdots, n. \end{cases}$$

we consider the vertices $v_i, 1 \le i \le n$ of the cycle $C_n^{'}$. Assign the label 1 to the first $\frac{2n+1}{3}$ vertices $v_1, v_2, \cdots, v_{\frac{2n+1}{3}}$. Next assign the label 2 to the vertices $v_{\frac{2n+1}{3}+1}, v_{\frac{2n+1}{3}+2}, \cdots, u_{\frac{5n+4}{6}}$. Finally assign the label 3 to the remaining vertices of the cycle $C_n^{'}$.

Subcase 2.2 n is odd.

Assign the labels to the vertices $u_i, 1 \le i \le n$ in the following ways.

$$f(u_i) = \begin{cases} 3, & \text{if } i = 1, 3, 5, \cdots, i+2 \cdots, n, \\ 2, & \text{if } i = 2, 4, 6, \cdots, i+2 \cdots, n-1. \end{cases}$$

Next assign the labels to the vertices $v_i, 1 \le i \le n$ of the cycle $C_n^{'}$ in the following way. Assign the label 1 to the first $\frac{2n+1}{3}$ vertices $v_1, v_2, \ldots, v_{\frac{2n+1}{3}}$. Next assign the label 2 to the vertices $v_{\frac{2n+1}{3}+1}, v_{\frac{2n+1}{3}+2}, \ldots, u_{\frac{5n+1}{6}}$. Finally assign the label 3 to the remaining vertices of the cycle $C_n^{'}$. Table 2 shows that this vertex labeling is called 3-remainder cordial labeling of the dumbbell graph for $n \equiv 1 \pmod 3$.

Nature of n,$n \equiv 1 \pmod 3$	$v_f(1)$	$v_f(2)$	$v_f(3)$	η_e	η_o
n is odd	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{2n-2}{3}$	$n+1$	n
n is even	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{2n-2}{3}$	n	$n+1$

Table 2

Case 3. $n \equiv 2 \pmod 3$.

Fix the labels in the following pattern : $3, 2, 1, 1$ and 2 to the vertices u_1, u_2, u_3, u_{n-1} and u_n respectively and $2, 3, 2, 1$ and 3 to the vertices v_1, v_2, v_3, v_{n-1} and v_n respectively. Next assign the labels to the remaining vertices u_i, and v_i, $(4 \le i \le n-2)$ in the following two cases.

Subcase 3.1 First assign the labels to the vertices $u_i, 4 \le i \le n-2$. Assign the labels $1, 2$ and 3 to the vertices u_4, u_5 and u_6 respectively. Then assign the labels $2, 3$ and 1 respectively to the vertices u_7, u_8 and u_9. Then next assign the labels $1, 2$ and 3 to the vertices u_{10}, u_{11} and u_{12} respectively. Then assign the labels $2, 3$ and 1 respectively to the vertices u_{13}, u_{14} and u_{15}. Proceeding like this until we reach the vertex u_{n-2}. When n is odd then the vertices u_{n-4}, u_{n-3} and u_{n-2} are receive the labels $2, 3$ and 1 respectively. When n is even then the vertices u_{n-4}, u_{n-3} and u_{n-2} are receive the labels $1, 2$ and 3 respectively.

Subcase 3.2 We consider the vertices $v_i, (4 \leq i \leq n-2)$. Assign the labels to the vertices v_i for $(4 \leq i \leq n-2)$ as in subcase(i). Table 3 shows that this vertex labeling is called 3-remainder cordial labeling of the dumbbell graph for $n \equiv 2 \pmod 3$.

Nature of n,$n \equiv 2 \pmod 3$	$v_f(1)$	$v_f(2)$	$v_f(3)$	η_e	η_o
n is odd	$\frac{2n-1}{3}$	$\frac{2n+2}{3}$	$\frac{2n-1}{3}$	$n+1$	n
n is even	$\frac{2n-1}{3}$	$\frac{2n+2}{3}$	$\frac{2n-1}{3}$	n	$n+1$

Table 3

This completes the proof. □

Theorem 3.3 *The umbrella $U_{n,n}$ is 3-remainder cordial for all n.*

Proof Let $F_n = P_n + K_1$ where $P_n : u_1, u_2, \cdots, u_n$ and $V(K_1) = \{u\}$. Let $P_n' : v_1, v_2, \cdots, v_n$ be another path. Identify v_1 with u. Clearly the umbrella graph has $2n$ vertices and $3n-2$ edges.

Case 1. $n \equiv 0 \pmod 3$.

Subcase 1.1 n is odd.

Assign the labels to the vertices $u_i, (1 \leq i \leq n)$ as follows:

$$f(u_i) = \begin{cases} 2, & \text{if } i = 1, 3, 5, \ldots, i+2 \cdots, n, \\ 3, & \text{if } i = 2, 4, 6, \ldots, i+2 \cdots, n-1. \end{cases}$$

Next assign the labels to the vertices $v_i, 1 \leq i \leq n$. Assign the label 3 to the first $\frac{n+3}{6}$ vertices $v_1, v_2, \cdots, v_{\frac{n+3}{6}}$ and assign the label 1 consecutively to the vertices $v_{\frac{n+3}{6}+1}, v_{\frac{n+3}{6}+2}, \cdots, v_{\frac{5n+3}{6}}$. Next assign the label 2 to the remaining vertices.

Subcase 2. n is even.

Assign the labels to the vertices $u_i, (1 \leq i \leq n)$ as follows:

$$f(u_i) = \begin{cases} 2, & \text{if } i = 1, 3, 5, \cdots, i+2 \ldots, n-1, \\ 3, & \text{if } i = 2, 4, 6, \cdots, i+2 \ldots, n. \end{cases}$$

Next we consider the vertices $v_i, 1 \leq i \leq n$. Assign the label 3 to the first $\frac{n}{6}$ vertices $v_1, v_2, \cdots, v_{\frac{n}{6}}$ and assign the label 1 consecutively to the vertices $v_{\frac{n}{6}+1}, v_{\frac{n}{6}+2}, \cdots, v_{\frac{5n}{6}}$. Next assign the label 2 to the remaining vertices $v_{\frac{5n}{6}+1}, v_{\frac{5n}{6}+2}, \ldots, v_n$. Table 4 shows that this vertex labeling is called 3-remainder cordial labeling of $U_{n,n}$ for $n \equiv 0 \pmod 3$.

Nature of n,$n \equiv 0 \pmod 3$	$v_f(1)$	$v_f(2)$	$v_f(3)$	η_e	η_o
n is odd	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{3n-3}{2}$	$\frac{3n-1}{2}$
n is even	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{3n-2}{2}$	$\frac{3n-2}{2}$

Table 4

Case 2. $n \equiv 1 \pmod 3$.

Subcase 2.2 n is odd.

Assign the labels to the vertices $u_i, (1 \le i \le n)$ as follows:

$$f(u_i) = \begin{cases} 2, & \text{if } i = 1, 3, 5, \cdots, i+2 \cdots, n, \\ 3, & \text{if } i = 2, 4, 6, \cdots, i+2 \cdots, n-1. \end{cases}$$

Next assign the labels to the vertices $v_i, 1 \le i \le n$. Assign the label 3 to the first $\frac{n+5}{6}$ vertices $v_1, v_2, \cdots, v_{\frac{n+5}{6}}$ and assign the label 1 consecutively to the vertices $v_{\frac{n+5}{6}+1}, v_{\frac{n+5}{6}+2}, \cdots, v_{\frac{5n+7}{6}}$. Next assign the label 2 to the remaining vertices.

Subcase 2.2 n is even.

Assign the labels to the vertices $u_i, (1 \le i \le n)$ as follows:

$$f(u_i) = \begin{cases} 2, & \text{if } i = 1, 3, 5, \cdots, i+2 \cdots, n-1, \\ 3, & \text{if } i = 2, 4, 6, \cdots, i+2 \cdots, n. \end{cases}$$

Next we consider the vertices $v_i, 1 \le i \le n$. Assign the label 3 to the first $\frac{n+2}{6}$ vertices $v_1, v_2, \cdots, v_{\frac{n+2}{6}}$ and assign the label 1 to the vertices $v_{\frac{n+2}{6}+1}, v_{\frac{n+2}{6}+2}, \cdots, v_{\frac{5n+4}{6}}$ consecutively. Next assign the label 2 to the remaining vertices $v_{\frac{5n+4}{6}+1}, v_{\frac{5n+4}{6}+2}, \cdots, v_n$. Table 5 shows that this vertex labeling is called 3-remainder cordial labeling of $U_{n,n}$ for $n \equiv 1 \pmod 3$.

Nature of n,$n \equiv 0 \pmod 3$	$v_f(1)$	$v_f(2)$	$v_f(3)$	η_e	η_o
n is odd	$\frac{2n+1}{3}$	$\frac{2n-2}{3}$	$\frac{2n+1}{3}$	$\frac{3n-3}{2}$	$\frac{3n-1}{2}$
n is even	$\frac{2n+1}{3}$	$\frac{2n-2}{3}$	$\frac{2n+1}{3}$	$\frac{3n-2}{2}$	$\frac{3n-2}{2}$

Table 5

Case 3. $n \equiv 2 \pmod 3$.

Subcase 3.1 n is odd.

Assign the labels to the vertices $u_i, (1 \le i \le n)$ as follows:

$$f(u_i) = \begin{cases} 2, & \text{if } i = 1, 3, 5, \cdots, i+2 \cdots, n, \\ 3, & \text{if } i = 2, 4, 6, \cdots, i+2 \cdots, n-1. \end{cases}$$

Next assign the labels to the vertices $v_i, 1 \le i \le n$. Assign the label 3 to the first $\frac{n+1}{6}$ vertices $v_1, v_2, \cdots, v_{\frac{n+1}{6}}$ and assign the label 1 to the vertices $v_{\frac{n+1}{6}+1}, v_{\frac{n+1}{6}+2}, \cdots,$ $v_{\frac{5n+5}{6}}$ consecutively. Next assign the label 2 to the remaining $(\frac{n-5}{6})$ vertices.

Subcase 3.2 n is even.

Assign the labels to the vertices $u_i, (1 \le i \le n)$ as follows:

$$f(u_i) = \begin{cases} 2, & \text{if } i = 1, 3, 5, \cdots, i+2 \cdots, n-1, \\ 3, & \text{if } i = 2, 4, 6, \cdots, i+2 \cdots, n. \end{cases}$$

Next we consider the vertices $v_i, 1 \le i \le n$. Assign the label 3 to the first $\frac{n-2}{6}$ vertices

$v_1, v_2, \cdots, v_{\frac{n-2}{6}}$ and assign the label 1 to the vertices $v_{\frac{n-2}{6}+1}, v_{\frac{n-2}{6}+2}, \cdots, v_{\frac{5n+2}{6}}$ consecutively. Next assign the label 2 to the remaining $\frac{n-2}{6}$ vertices $v_{\frac{5n+2}{6}+1}, v_{\frac{5n+2}{6}+2}, \cdots, v_n$. Table 6 shows that this vertex labeling is called 3-remainder cordial labeling of $U_{n,n}$ for all $n \equiv 2 \pmod 3$.

Nature of n,$n \equiv 2 \pmod 3$	$v_f(1)$	$v_f(2)$	$v_f(3)$	η_e	η_o
n is odd	$\frac{2n+2}{3}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$	$\frac{3n-3}{2}$	$\frac{3n-1}{2}$
n is even	$\frac{2n+2}{3}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$	$\frac{3n-2}{2}$	$\frac{3n-2}{2}$

Table 6

This completes the proof. □

Theorem 3.4 *The butterfly graph $BF_{n,n}$ is 3-remainder cordial for all n.*

Proof Let $C_n : u_1u_2\cdots u_nu_1$ and $C_n^{'} : v_1v_2\cdots v_nv_1$ be two cycles of the same order n. Identify the vertex u_1 with the vertex v_1. Let $w_1, w_2\cdots, w_n$ be the n-pendant vertices adjacent to the vertex u_1. Then the given graph has $3n-1$ vertices and $3n$ edges.

First assign the labels to the vertices $u_i, (1 \le i \le n)$ as follows:

$$f(u_i) = \begin{cases} 2, & \text{if } i = 1, 3, 5, \cdots, i+2\cdots, n-1, \\ 3, & \text{if } i = 2, 4, 6, \cdots, i+2\cdots, n. \end{cases}$$

Next assign the labels to the vertices $v_i, (2 \le i \le n)$. Assign the label 1 to the vertices $v_2, v_3, \ldots, v_n$. Finally assign the labels to the vertices $w_i, (1 \le i \le n)$ as follows:

$$f(w_i) = \begin{cases} 2, & \text{if } i = 1, 3, 5, \cdots, i+2\cdots, \frac{n}{2}, \\ 3, & \text{if } i = \frac{n}{2}+1, \frac{n}{2}+2, \cdots, i+2\cdots, n. \end{cases}$$

Thus $v_f(1) = n-1$, $v_f(2) = v_f(3) = n$ and $\eta_e = \frac{3n}{2} = \eta_o$. Hence this vertex labeling is called 3-remainder cordial labeling of butterfly graph for all n. □

Theorem 3.5 *The graph $C_3 \odot K_{1,n}$ is 3-remainder cordial for all n.*

Proof Let $V(C_3 \odot K_{1,n}) = \{u, v, w, u_i, v_i, w_i : 1 \le i \le n\}$, $E(C_3 \odot K_{1,n}) = \{uv, vw, wu, uu_i, vv_i, ww_i : 1 \le i \le n\}$. Clearly the order and size of the given graph are $3n+3$ and $3n+3$ respectively.

Fix Tables 1, 2 and 3 respectively to the central vertices u, v and w of $C_3 \odot K_{1,n}$ and also fix the label 3 to the vertices $v_1, v_2, v_3, \cdots, v_n$ into the following two cases.

Case 1. n is even.

First we consider the vertices $u_i, (1 \le i \le n)$. Assign the label 1 consecutively to the vertices $u_1, u_2, \cdots, u_{\frac{n+1}{2}}$. Next assign the label 2 to the remaining vertices $u_{\frac{n+1}{2}+1}, u_{\frac{n+1}{2}+2}, \cdots, u_n$.

Next we consider the vertices $w_i, (1 \le i \le n)$. Assign the label 2 consecutively to the vertices $w_1, w_2, \cdots, w_{\frac{n+1}{2}}$. Next assign the label 1 to the remaining vertices $w_{\frac{n+1}{2}+1}, w_{\frac{n+1}{2}+2}, \cdots, w_n$.

Case 2. n is odd.

Assign the label 1 to the first $(\frac{n}{2})$ vertices $u_1, u_2, \cdots, u_{\frac{n}{2}}$ and assign the label 2 to the remaining $(\frac{n}{2})$ vertices $u_{\frac{n}{2}+1}, u_{\frac{n}{2}+2}, \ldots, u_n$. Next we consider the vertices $w_i, (1 \le i \le n)$. Assign the label 2 consecutively to the vertices $w_1, w_2, \cdots, w_{\frac{n}{2}}$ and assign the label 1 to the next $(\frac{n}{2})$ vertices

$w_{\frac{n}{2}+1}, w_{\frac{n}{2}+2}, \cdots, w_n$. Table 7 shows that this vertex labeling is called 3-remainder cordial labeling of $C_3 \odot K_{1,n}$ for all n.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	η_e	η_o
n is odd	$n+1$	$n+1$	$n+1$	$\frac{3n+3}{2}$	$\frac{3n+3}{2}$
n is even	$n+1$	$n+1$	$n+1$	$\frac{3n+4}{2}$	$\frac{3n+2}{2}$

Table 7

This completes the proof. □

Example 3.6 A 3-remainder cordial labeling of $C_3 \odot K_{1,9}$ is shown in Figure 1.

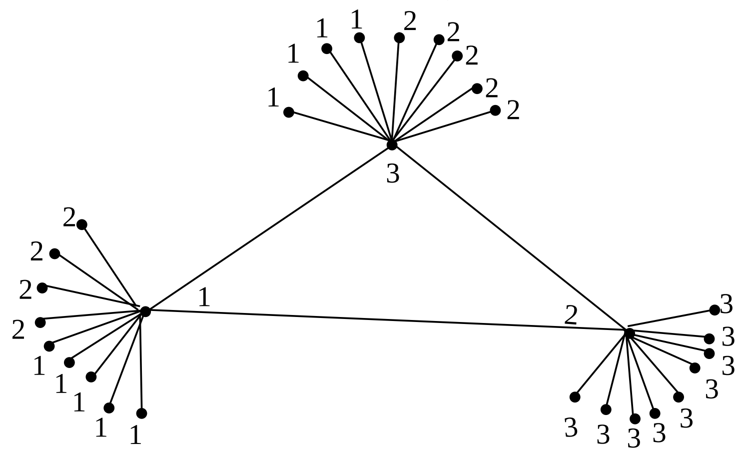

Figure 1

References

[1] Cahit I., Cordial graphs: A weaker version of graceful and harmonious graphs, *Ars Combin.*, 23 (1987), 201–207.

[2] Gallian J.A., A Dynamic survey of graph labeling, *The Electronic Journal of Combinatorics.*, 19, (2017).

[3] Harary F., *Graph Theory*, Addision Wesley, New Delhi, 1969.

[4] Ponraj R., Annathurai K. and Kala R., Remainder cordial labeling of graphs, *Journal of Algorithms and Computation*, Vol.49(1), (2017), 17-30.

[5] Ponraj R., Annathurai K. and Kala R., *k*-Remaider cordial graphs, *Journal of Algorithms and Computation*, Vol.49(2), (2017), 41-52.

[6] Ponraj R., Annathurai K. and Kala R., Remaider cordiality of some graphs, *Palestin Journal of Mathematics* (Accepted).

[7] Ponraj R., Annathurai K. and Kala R., 4−Remainder cordial labeling of some special graphs, *International Journal of Pure and Applied Mathematics*, Vol. 118, No.6 (2018), 399 - 405.

[8] Ponraj R., Annathurai K. and Kala R., 4−Remainder cordial labeling of some graphs, *International Journal of Mathematical Combinatorics*, Vol.1(2018), 138 - 145.

[9] Annathurai K., Ponraj R. and Kala R., Some 3- Remainder cordial graphs, *Global Journal of Engineering Science and Researches*,Vol.5(6)(2018), 169 - 175.

[10] Annathurai K., Ponraj R. and Kala R., Further Results On 3- Remainder cordial labeling of graphs, *International Journal of Management, IT and Engineering*, Vol.8(8)(2018), 120 - 129.

[11] Seoud M.A. and Shakir M. Salman, On difference cordial graphs, *Mathematica Aeterna*, Vol.5, No.1(2015), 105–124.

[12] Seoud M.A. and Shakir M. Salman, Some results and examples On difference cordial graphs, *Turkish Journal of Mathematics*, 40,(2016), 417 – 427.