International J.Math. Combin. Vol.2(2017), 105-115

Some More 4-Prime Cordial Graphs

R.Ponraj

(Department of Mathematics, Sri Paramakalyani College, Alwarkurichi-627412, India)

Rajpal Singh and R.Kala

(Department of Mathematics, Manonmaniam Sundaranar University, Tirunelveli-627012, India)

E-mail: ponrajmaths@gmail.com, rajpalsingh@outlook.com; karthipyi91@yahoo.co.in

Abstract: Let G be a (p,q) graph, $H \prec G$ and $f : V(G) \rightarrow \{1,2,\cdots,k\}$ be a map. For each edge uv, assign the label $\gcd(f(u),f(v))$. Then, f is called Smarandachely k-prime cordial labeling on G to H if $\left|v_f^H(i) - v_f^H(j)\right| \leq 1$, $i,j \in \{1,2,\cdots,k\}$ and $\left|e_f^H(0) - e_f^H(1)\right| \leq 1$, but there exist integers $0 \leq i \neq j \leq k$ such that $\left|v_f^{G\setminus H}(i) - v_f^{G\setminus H}(j)\right| \geq 2$, or $\left|e_f^{G\setminus H}(0) - e_f^{G\setminus H}(1)\right| \geq 2$, where $v_f^H(x)$, $v_f^{G\setminus H}(x)$ respectively denotes the numbers of vertices of H, $G\setminus H$ labeled with x, $e_f^H(1)$, $e_f^H(0)$ and $e_f^{G\setminus H}(1)$, $e_f^{G\setminus H}(0)$ respectively denote the number of edges labeled with 1 and not labeled with 1 in H, $G\setminus H$. Particularly, a Smarandachely k-prime cordial labeling on G to G is called k-prime cordial labeling with $v_f(x)$, $e_f(1)$ and $e_f(0)$ replacing notations $v_f^H(x)$, $e_f^H(1)$ and $e_f^H(0)$ for abbreviation. A graph with a k-prime cordial labeling is called a k-prime cordial graph. In this paper we investigate 4-prime cordial labeling behavior of lotus inside a circle, sunflower graph, $S(K_2 + mK_1)$, $S(P_n \odot K_1)$, dodecahedron, and some more graphs.

Key Words: Smarandachely k-prime labelling, k-prime labelling, cycle, wheel, join, sunflower graph, lotus inside a circle.

AMS(2010): 05C78.

§1. Introduction

In this paper graphs are finite, simple and undirected. Let G be a (p,q) graph where p refers the number of vertices of G and q refers the number of edge of G. The number of vertices of a graph G is called order of G, and the number of edges is called size of G. In 1987, Cahit introduced the concept of cordial labeling of graphs [1]. Sundaram, Ponraj, Somasundaram [5] have introduced the notion of prime cordial labeling of graphs. Also they discussed the prime cordial labeling behavior of various graphs. Recently Ponraj et al. [7], introduced k-prime cordial labeling of graphs. They have studied 3-prime cordiality of several graphs in [7, 8]. In [9, 10] Ponraj et al. studied the 4-prime cordial labeling behavior of complete graph,

[1]Received September 8, 2016, Accepted May 26, 2017.

book, flower, mC_n, wheel, gear, double cone, helm, closed helm, butterfly graph, and friendship graph and some more graphs. In this paper we have studied about the 4-prime cordiality of lotus inside a circle, sunflower graph and some more graphs. Let x be any real number. Then $\lfloor x \rfloor$ stands for the largest integer less than or equal to x and $\lceil x \rceil$ stands for smallest integer greater than or equal to x. Terms not defined here follow from Harary [3] and Gallian [2].

§2. Preliminaries

Remark 2.1([6]) A 2-prime cordial labeling is a product cordial labeling.

Remark 2.2([5]) A p-prime cordial labeling is a prime cordial labeling.

Definition 2.3 *The join of two graphs $G_1 + G_2$ is obtained from G_1 and G_2 and whose vertex set is $V(G_1 + G_2) = V(G_1) \cup V(G_2)$ and edge set $E(G_1 + G_2) = E(G_1) \cup E(G_2) \cup \{uv : u \in V(G_1), v \in V(G_2)\}$.*

Definition 2.4 *The graph $C_n + K_1$ is called a wheel. In a wheel, the vertex of degree n is called the central vertex and the vertices on the cycle C_n are called rim vertices.*

Definition 2.5 *The sunflower graph SF_n is obtained by taking a wheel with central vertex u and the cycle $C_n : u_1u_2 \cdots u_nu_1$ and new vertices $v_1, v_2, \ldots, v_n$ where v_i is joined by vertices $u_i, u_{i+1 \pmod n}$.*

Definition 2.6 *The lotus inside a circle LC_n is a graph obtained from the cycle $C_n : v_1v_2 \cdots v_nv_1$ and a star $K_{1,n}$ with central vertex u and the end vertices $u_1, u_2, \cdots, u_n$ by joining each u_i to v_i and $v_{i+1 \pmod n}$.*

Definition 2.7 *The subdivision graph $S(G)$ of a graph G is obtained by replacing each edge uv by a path uwv.*

Definition 2.8 *The graph P_n^2 is obtained from the path P_n by adding edges that joins all vertices u and v with $d(u, v) = 2$.*

Definition 2.9 *Let G_1, G_2 respectively be (p_1, q_1), (p_2, q_2) graphs. The corona of G_1 with G_2, $G_1 \odot G_2$ is the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2.*

Definition 2.10 *The one-point union of t copies of the cycle C_3 is called a friendship graph $C_3^{(t)}$.*

Definition 2.11 *The DH_n is a graph with vertex set $V(DH_n) = \{u_i, v_i, x_i, y_i : 1 \leq i \leq n\}$ and the edge set $E(DH_n) = \{uu_{i+1}, y_iv_{i+1}, x_ix_{i+1} : 1 \leq i \leq n-1\} \bigcup \{u_iv_i, v_iy_i, x_iy_i : 1 \leq i \leq n\} \bigcup \{u_1u_n, y_nv_1, x_1x_n\}$. DH_5 is called a Tetrahedron.*

Definition 2.12 *The Cartesian product graph $G_1 \Box G_2$ is defined as follows: Consider any two points $u = (u_1, u_2)$ and $v = (v_1, v_2)$ in $V = V_1 \times V_2$. Then u and v are adjacent in $G_1 \Box G_2$*

whenever $[u_1 = v_1$ *and* $u_2v_2 \in E(G_2)]$ *or* $[u_2 = v_2$ *and* $u_1v_1 \in E(G_1)]$.

Definition 2.13 *A ladder* L_n *is the graph* $P_n \times P_2$. *Let* $V(L_n) = \{u_i, v_i : 1 \le i \le n\}$ *and* $E(L_n) = \{u_iu_{i+1}, v_iv_{i+1} : 1 \le i \le n-1\} \bigcup \{u_iv_i : 1 \le i \le n\} \bigcup \{u_1u_n, v_1v_n\}$.

The graph GL_n *is obtained from the ladder* L_n *with* $V(GL_n) = V(L_n)$ *and* $E(GL_n) = E(L_n) \cup \{u_iv_{i+1} : 1 \le i \le n-1\}$.

Theorem 2.14([7]) *The cycle* C_n, $n \ne 3$ *is* k*-prime cordial where* k *is even.*

§3. Main Results

3.1 Cycle Related Graphs

Theorem 3.1 *The lotus inside a circle* LC_n *is* 4*-prime cordial if and only if* $n > 4$.

Proof Note that the order and size of LC_n are $2n+1$ and $4n$ respectively. Suppose $n = 3$ or 4 then one can easily check that there does not exists a 4-prime cordial labeling and so we assume $n > 4$. Here we divide the proof into four cases.

Case 1. $n \equiv 0 \pmod 4$.

We construct a labeling f as follows: assign the label 4 to the vertices $v_1, v_2, \cdots v_{\frac{n}{2}}$ then put the label 3 to the vertex $v_{\frac{n}{2}+1}$. The remaining vertices of the cycle, namely, $v_{\frac{n}{2}+2}, v_{\frac{n}{2}+3}, \cdots, v_n$ are labeled by 1. Now we consider the center of the star u. The vertex u is labeled by 2. For the vertices $u_1, u_2, \cdots, u_n$, first we consider the vertices u_{n-1} and u_n. Assign the labels $1, 2$ respectively to the vertices u_{n-1} and u_n. Consider the vertices $u_1, u_2, \cdots u_{\frac{n}{2}-1}$. Fix the label 2 to this vertices. Then the vertices $u_{\frac{n}{2}}, u_{\frac{n}{2}+1}, \cdots, u_{n-2}$ are labeled by 3.

Case 2. $n \equiv 1 \pmod 4$.

Assign the labels to the vertices u, u_i, $1 \le i \le n-3$, v_j, $1 \le j \le n-1$ as in case 1. The assign the labels $3, 1, 2$ to the vertices u_{n-2}, u_{n-1}, u_n respectively. Finally we assign the label 4 to the vertex v_n.

Case 3. $n \equiv 2 \pmod 4$.

Assign the label 4 to the vertices $v_1, v_2, \cdots v_{\frac{n}{2}}$. For the vertex $v_{\frac{n}{2}+1}$ we assign the number 3. The remaining vertices of the cycle from $v_{\frac{n}{2}+2}, v_{\frac{n}{2}+3}, \cdots, v_n$ receives the label 1. Put the label 2 to the vertex u. Now we consider the vertices u_i, $1 \le i \le n$. Assign the label 2 to the vertices u_n, u_i where $1 \le i \le \frac{n}{2} - 1$, then assign the integer 3 to the vertices $u_{\frac{n}{2}}, u_{\frac{n}{2}+1}, \cdots, u_{n-2}$. Finally we assign the label 1 to the vertex u_{n-1}.

Case 4. $n \equiv 3 \pmod 4$.

First we consider the vertices of the cycle. The label 4 is used to the vertices v_i, $1 \le i \le \frac{n+1}{2}$. Put the label 3 to the vertex $v_{\frac{n+3}{2}}$. The unlabeled vertices of the cycle are now labeled by 1. Then put the number 2 to the vertex u. If we consider the vertices u_i, $1 \le i \le n$, we assign the label 2 to the vertices u_j where $1 \le j \le \frac{n-1}{2}$ then assign 3 to the vertices $u_{\frac{n+1}{2}}, \cdots, u_{n-1}$.

Finally put the number 1 to the vertex u_n. □

The following Table 1 establish that the above mentioned labeling f is a 4-prime cordial labeling.

Values of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$v_f(4)$	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 4$	$\lfloor\frac{2n+1}{4}\rfloor$	$\lceil\frac{2n+1}{4}\rceil$	$\lfloor\frac{2n+1}{4}\rfloor$	$\lfloor\frac{2n+1}{4}\rfloor$	$2n$	$2n$
$n \equiv 1 \pmod 4$	$\lfloor\frac{2n+1}{4}\rfloor$	$\lceil\frac{2n+1}{4}\rceil$	$\lceil\frac{2n+1}{4}\rceil$	$\lceil\frac{2n+1}{4}\rceil$	$2n$	$2n$
$n \equiv 2 \pmod 4$	$\lfloor\frac{2n+1}{4}\rfloor$	$\lceil\frac{2n+1}{4}\rceil$	$\lfloor\frac{2n+1}{4}\rfloor$	$\lfloor\frac{2n+1}{4}\rfloor$	$2n$	$2n$
$n \equiv 3 \pmod 4$	$\lfloor\frac{2n+1}{4}\rfloor$	$\lceil\frac{2n+1}{4}\rceil$	$\lceil\frac{2n+1}{4}\rceil$	$\lceil\frac{2n+1}{4}\rceil$	$2n$	$2n$

Table 1

Theorem 3.2 *The sunflower graph SF_n is 4-prime cordial for all n.*

Proof First we observe that the order and size of SF_n are $2n+1$ and $4n$ respectively. We consider the following cases.

Case 1. $n \equiv 0 \pmod 4$.

Assign the label 2 to the vertices $u_1, u_2, \cdots u_{\frac{n}{4}}$. Then put the number 4 to the next consecutive vertices $u_{\frac{n}{4}+1}, \cdots, u_{\frac{n}{2}+1}$. The next vertex $u_{\frac{n}{2}+2}$ is labeled by 3. Then the remaining vertices of the cycle, namely, $u_{\frac{n}{2}+3}, \cdots, u_n$ are labeled by 1. For the central vertex u, we use the label 2. We now move to the vertices v_i, $1 \le i \le n$. Assign the label 2 to the vertices $v_1, v_2, \cdots v_{\frac{n}{4}-1}$. Then assign the label 4 to the vertices $v_{\frac{n}{4}}, v_{\frac{n}{4}+1}, \cdots, v_{\frac{n}{2}-1}$. The next three vertices $v_{\frac{n}{2}}, v_{\frac{n}{2}+1}, v_{\frac{n}{4}+2}$ are labeled by $1, 3, 1$ respectively. Finally the remaining unlabeled vertices received the integer 3.

Case 2. $n \equiv 1 \pmod 4$.

First we consider the vertices of the cycle C_n. For the vertices $u_1, u_2, \cdots u_{\frac{n-5}{4}}$, we assign the label 2. The successive vertices $u_{\frac{n-1}{4}}, \cdots, u_{\frac{n-1}{2}}$ are labeled by 4. Put the label 3 to the vertex $u_{\frac{n+1}{2}}$. The vertices $u_{\frac{n+3}{2}}, \cdots, u_{n-1}$ are labeled by 1. Put the label 2 to the vertex u_n. Then assign the label 2 to the central vertex u. We now move to the vertices v_i, $1 \le i \le n$. Assign the label 2 to the vertices $v_1, v_2, \cdots v_{\frac{n-5}{4}}$. Then put the number 4 to the vertices $v_{\frac{n-1}{4}}, \cdots, v_{\frac{n-3}{2}}$. Put the labels $33, 1$ respectively to the vertices $v_{\frac{n-1}{2}}, v_{\frac{n+1}{2}}$. Assign the label 3 to the vertices $v_{\frac{n+3}{2}}, \cdots, v_{n-1}$. Finally we assign the number 2 to v_n.

Case 3. $n \equiv 2 \pmod 4$.

We first consider the vertex u. Label it by 2. Then we consider the vertices of the cycle C_n. Assign the label 2 to the vertices $u_1, u_2, \cdots u_{\frac{n+2}{4}}$. Then put the integer 4 to the vertices $u_{\frac{n+6}{4}}, \cdots, u_{\frac{n+2}{2}}$. The next vertex $u_{\frac{n+4}{2}}$ is labeled by 3. The remaining vertices of the cycle are labeled by 1. Then consider the vertices v_i, $1 \le i \le n$. Assign the label 2 to the vertices v_i where $1 \le i \le \frac{n-2}{4}$ then the vertices $v_{\frac{n-2}{4}+i}$ where $1 \le i \le \frac{n-2}{4}$ are labeled with 4. Put the labels $1, 3, 1$ to the next consecutive vertices $v_{\frac{n}{2}}, v_{\frac{n+2}{2}}, v_{\frac{n+4}{2}}$. Finally put the number 3 for the unlabeled vertices.

Case 4. $n \equiv 3 \pmod 4$.

Assign the label 2 to the vertices u_i, where $1 \leq i \leq \frac{n+1}{4}$, then assign 4 to the vertices $u_{\frac{n+1}{4}+i}$ where $1 \leq i \leq \frac{n+1}{4}$. The next vertex $u_{\frac{n+3}{2}}$ received the label 3. Then assign the label 1 to the remaining vertices of the cycle. Put the integer 2 to the vertex u. Then we consider the vertices v_i, $1 \leq i \leq n$. Assign the number 2 to the vertices $v_1, v_2, \cdots v_{\frac{n-3}{4}}$. Then assign 4 to the vertices $v_{\frac{n+1}{4}}, \cdots, v_{\frac{n-1}{2}}$. The next two consecutive vertices $v_{\frac{n+1}{2}}, v_{\frac{n+3}{2}}$ are labeled by $3, 1$ respectively. The rest of the unlabeled vertices are labeled by 3.

The Table 2 shows that the above labeling f is a required 4-prime cordial labeling. □

Values of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$v_f(4)$	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 4$	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n}{2}+1$	$2n$	$2n$
$n \equiv 1 \pmod 4$	$\frac{n-1}{2}$	$\frac{n+1}{2}$	$\frac{n+1}{2}$	$\frac{n+1}{2}$	$2n$	$2n$
$n \equiv 2 \pmod 4$	$\frac{n}{2}$	$\frac{n}{2}+1$	$\frac{n}{2}$	$\frac{n}{2}$	$2n$	$2n$
$n \equiv 3 \pmod 4$	$\frac{n-1}{2}$	$\frac{n+1}{2}$	$\frac{n+1}{2}$	$\frac{n+1}{2}$	$2n$	$2n$

Table 2

A 4-prime cordial labeling of SF_8 is given in Figure 1.

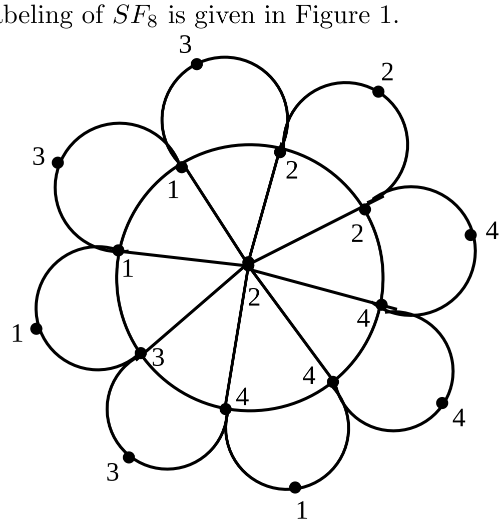

Figure 1

Theorem 3.3 *DH_n is 4-prime cordial.*

Proof Clearly DH_n consists of $4n$ vertices and $6n$ edges. We now give the label to the vertices of DH_n as follows: Assign the label 2 to the vertices u_i, $1 \leq i \leq n$ and assign the label 4 to the vertices v_i, $1 \leq i \leq n$. Now we move to the vertices y_i. Assign the label 1 to the vertices $y_1, y_2, \cdots, y_n$. Finally assign the label 3 to the vertices $x_1, x_2, \cdots, x_n$. This vertex labeling f is obviously a 4-prime cordial labeling of DH_n. Since, $v_f(1) = v_f(2) = v_f(3) = v_f(4) = n$ and $e_f(0) = e_f(1) = 3n$. □

Theorem 3.4 *Let C_n be the cycle $u_1u_2 \cdots u_nu_1$. Let G_n be the graph with $V(G_n) = V(C_n \bigcup \{v_i :$*

$1 \leq i \leq n\}$ *and* $E(G_n) = E(C_n) \bigcup \{u_iv_i : 1 \leq i \leq n\} \cup \{u_{i+1}v_i : 1 \leq i \leq n-1\}$. *Then* G_n *is* 4*-prime cordial for all* $n \neq 4$.

Proof Clearly for any vertex labeling of G_4, the maximum possible edges with label 0 is 4. Hence G_4 is not 4-prime cordial.

Case 1. $n \equiv 0 \pmod 4$, $n \neq 4$.

Let $n = 4t$. Assign the label 2 to the vertices $u_1, u_2, \cdots, u_{2t}$ and 4 to the vertices $v_1, v_2, \cdots, v_{2t}$. Next assign the label 3 to the vertices u_{2t+1}, u_{2t+2} and u_{2t+3}. Next assign the label 1 to the cycle vertices $u_{2t+4}, u_{2t+6}, \cdots, u_{4t}$ and 3 to the vertices $v_{2t+4}, v_{2t+5}, \cdots, v_{4t-1}, v_{4t}$. Finally assign the remaining non labeled vertices by 1.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4t+1$. In this case, assign the label 2 to the vertices $u_1, u_2, \cdots, u_{2t+1}$ and 4 to the vertices $v_1, v_2, \cdots, v_{2t+1}$. Next assign the label 1 to the cycle vertices $u_{2t+2}, u_{2t+3}, \cdots$ and u_{4t+1}. Finally assign the label 3 to the non labeled vertices $v_{2t+2}, v_{2t+3}, \cdots, v_{4t+1}$.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4t+2$. In this case we assign the label 2 to the vertices u_i $(1 \leq i \leq 2t+1)$ and 4 to the vertices v_i $(1 \leq i \leq 2t+1)$. Next assign the label 3 to the cycle vertices u_{2t+2}, u_{2t+3} and u_{2t+4}. Now we assign the label 1 to the vertices $u_{2t+5}, u_{2t+6}, \cdots, u_{4t+2}$. Next assign the label 3 to the vertices $v_{4t+2}, v_{4t+1}, \cdots, v_{2t+4}$. Finally assign the label to the non labeled vertices by 1.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4t+3$. Assign the label 2 to the vertices $u_1, u_2, \cdots, u_{2t+2}$ and 4 to the vertices $v_1, v_2, \cdots, v_{2t+2}$. Next assign the label 1 to the vertices u_{2t+3}, u_{2t+4}, $\cdots$, u_{4t+3}. Finally assign the label to the vertices $v_{2t+3}, v_{2t+4}, \cdots, v_{4t+3}$.

Hence the vertex labeling given above is obviously a 4-prime cordial labeling of G_n. □

3.2 Subdivided Graphs

Theorem 3.5 $S(K_2 + mK_1)$ *is* 4*-prime cordial.*

Proof Let $V(S(K_2 + mK_1)) = \{u, v, w, u_i, v_i, w_i : 1 \leq i \leq m\}$ and $E(S(K_2 + mK_1)) = \{uv, vw, uu_i, u_iw_i, w_iv_i, v_iu : 1 \leq i \leq m\}$. Note that $S(K_2 + mK_1)$ has $3m+3$ vertices and $4m+2$ edges. The proof is divided into four cases depending upon the nature of n.

Case 1. $n = 4t$.

Assign the label 2 to the vertex u. Next assign the label 2 to the vertices $u_1, u_2, \cdots u_{3t}$. We now assign the label 4 to the vertices $u_{3t+1}, u_{3t+2}, \cdots, u_{4t}$. Next we move to the vertices w_i. Assign the label 4 to the vertices $w_1, w_2, \cdots, w_{2t}$. Next assign 3 to the vertices $w_{3t+1}, w_{3t+2}, \cdots, w_{4t}$. Next assign the label 3 to the vertices $v_{4t}, v_{4-1}, \cdots, v_{3t+1}$. We now assign the labels 4, 3 respectively to the vertices w, v. Finally, assign 1 to all the remaining non labeled vertices.

Case 2. $n = 4t + 1$.

Assign the labels to the vertices u_i, v_i, w_i, $1 \le i \le 4t+1$, u, v, w as in case 1. Finally assign $2, 4$ and 1 respectively to the vertices u_{4t+1}, w_{4t+1} and v_{4t+1}.

Case 3. $n = 4t + 2$.

As in Case 2, assign the labels to the vertices u_i, v_i, w_i, $1 \le i \le 4t+1$, u, v, w. Next assign the labels $2, 1, 3$ to the vertices u_{4t+2}, w_{4t+2} and v_{4t+2} respectively.

Case 4. $n = 4t + 3$.

Assign the labels to the vertices u_i, v_i, w_i, $1 \le i \le 4t + 2$, u, v, w as in case 3. Finally assign $4, 1, 3$ respectively to the vertices u_{4t+3}, w_{4t+3} and v_{4t+3}.

Clearly, the above labeling is a 4-prime cordial labeling of $S(K_2 + mK_1)$. □

Theorem 3.6 *$S(P_n \odot K_1)$ is 4-prime cordial.*

Proof Let P_n be the path $u_1u_2 \ldots _n$ and $V(P_n \odot K_1) = V(P_n) \bigcup \{v_i : 1 \le i \le n\}$, $E(P_n \odot K_1) = E(P_n) \bigcup \{u_iv_i : 1 \le i \le n\}$. The graph $S(P_n \odot K_1)$ is obtained by subdividing the edge u_iu_{i+1} with w_i and the edge u_iv_i with x_i. Note that $S(P_n \odot K_1)$ has $4n - 1$ vertices and $4n - 2$ edges. The proof is divided into four cases.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4t$. Assign the label 2 to the vertices $u_1, u_2, \cdots, u_{4t}$ and 4 to the vertices $w_1, w_2, \cdots, w_{4t-1}$ and x_1. We now assign the label 1 to the vertices $x_2, x_3, \cdots, x_{4t}$. Finally assign the label 3 to the pendent vertices $v_1, v_2, \cdots, v_{4t}$.

Case 2. $n \equiv 1 \pmod 4$.

Assign the label to the vertices u_i, v_i, x_i $(1 \le i \le n - 1)$ and w_i $(1 \le i \le n - 2)$ as in Case 1. Finally assign the labels $2, 4, 3$ and 1 respectively to the vertices w_{n-1}, u_n, x_n, v_n.

Case 3. $n \equiv 2 \pmod 4$.

As in Case 2, assign the label to the vertices u_i, v_i, x_i $(1 \le i \le n-1)$ and w_i $(1 \le i \le n-2)$. Next assign the labels $2, 4, 3$ and 1 to the vertices w_{n-1}, u_n, x_n, v_n respectively.

Case 4. $n \equiv 3 \pmod 4$.

In this case also assign labels to the vertices except w_{n-1}, u_n, x_n, v_n as in case 3. Then assign the labels $2, 4, 3$ and 1 to the vertices w_{n-1}, u_n, x_n, v_n respectively.

Clearly, the vertex labeling given in all cases is a 4-prime cordial labeling of $S(P_n \odot K_1)$.□

Theorem 3.7 *$S(C_3^{(t)})$ is 4-prime cordial.*

Proof Note that $S(C_3^{(t)}) \equiv C_6^{(t)}$. Let the i^{th} copy of the C_6 be $u_1^iu_2^iu_3^iu_4^iu_5^iu_6^i$, where $1 \le i \le t$ and $u_1^1 = u_1^2 = u_1^3 = u_1^4 = u_1^5 = u_1^6$.

Case 1. t is even, $t \ge 4$.

Assign the label 2 to the central vertex.

Subcase 1.1 $t \equiv 0 \pmod 4$.

Assign the label 2 to all the vertices of first $\frac{t}{4}$ copies of the cycle C_5. Next we move to the $(\frac{t}{4}+1)^{th}$ copy. Assign the label 4 to all the vertices of the $(\frac{t}{4}+1)^{th}$, $(\frac{t}{4}+2)^{th}$, $\ldots$, $(\frac{t}{2})^{th}$ copies of the cycle C_5. We now consider the $(\frac{t}{2}+1)^{th}$ copy. Assign the label 1 and 3 alternatively to the vertices of the $(\frac{t}{2}+1)^{th}$ copy of the cycle. In a similar fashion assign the label 1 and 3 alternatively to the vertices of the $(\frac{t}{2}+2)^{th}$, $\cdots$, t^{th} copy of the cycle C_5.

Subcase 1.2 $t \equiv 2 \pmod 4$.

In this case assign the label 2 to all the vertices of first $\frac{t-2}{4}$ copies of the cycle C_5. Now consider the $(\frac{t+2}{4})^{th}$ copy. Assign the label 4 to all the vertices of the cycle $(\frac{t+2}{4})^{th}$ copy. Similarly assign the label 4 to the $(\frac{t+6}{4})^{th}$, $\ldots$, $(\frac{t-2}{2})^{th}$ copies of the cycle C_5. We now move to the $(\frac{t}{2})^{th}$ copy. In this copy, assign the label 2 to the vertices u_2^t, u_3^t and 4 to the vertices u_4^t, u_5^t, u_6^t. Next assign 1, 3 alternatively to the vertices of the $(\frac{t}{2}+1)^{th}$, $\cdots$, t^{th} copies of the cycle.

Case 2. t is odd.

Subcase 2.1 $t \equiv 1 \pmod 4$.

As in subcase 1a, assign the label to the vertices of all the i^{th}, $1 \leq i \leq t-1$ copies of C_5. In the last copy, assign the labels $2, 4, 4, 1$ and 3 respectively to the vertices $u_2^t, u_3^t, u_4^t, u_5^t$ and u_6^t.

Subcase 2.2 $t \equiv 3 \pmod 4$.

Assign the label to the vertices of $(t-1)$ copies of the cycle as in subcase 1b. Finally, assign the labels $2, 4, 3, 3$ and 1 to the vertices $u_2^t, u_3^t, u_4^t, u_5^t$ and u_6^t respectively.

The Table 3 establish that the above vertex labeling f is a 4-prime cordial labeling of $S(C_3^{(t)})$, $t \geq 4$.

Nature of t	$v_f(1)$	$v_f(2)$	$v_f(3)$	$v_f(4)$	$e_f(0)$	$e_f(1)$
$t \equiv 0 \pmod 4$	$\frac{5t}{4}$	$\frac{5t}{4}+1$	$\frac{5t}{4}$	$\frac{5t}{4}$	$3t$	$3t$
$t \equiv 1 \pmod 4$	$\frac{5t-1}{4}$	$\frac{5t+3}{4}$	$\frac{5t-1}{4}$	$\frac{5t+3}{4}$	$3t$	$3t$
$t \equiv 2 \pmod 4$	$\frac{5t+2}{4}$	$\frac{5t+2}{4}$	$\frac{5t-2}{4}$	$\frac{5t+2}{4}$	$3t$	$3t$
$t \equiv 3 \pmod 4$	$\frac{5t+1}{4}$	$\frac{5t+1}{4}$	$\frac{5t+1}{4}$	$\frac{5t+1}{4}$	$3t$	$3t$

Table 3

$S(C_3^{(1)}) \cong C_6$ is 4-prime cordial follows from Theorem 2.14. The 4-prime cordial labelings of $S(C_3^{(2)})$ and $S(C_3^{(3)})$ are given in Figure 2. □

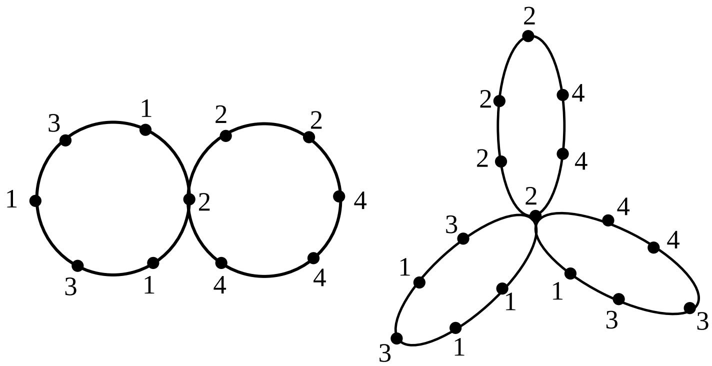

Figure 2

3.3 Miscellaneous Graphs

Theorem 3.8 *P_n^2 is 4-prime cordial if and only if $n \neq 4$.*

Proof Let P_n be the path $u_1, u_2, \cdots, u_n$. Clearly, the order and size of P_n^2 are n and $2n-3$ respectively. It is easy to verify that P_4^2 does not admits a 4-rime cordial labeling. Let us assume that $n \neq 4$.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4t$. Assign the label 1 to the vertices $u_1, u_3, \cdots, u_{2t-1}$. Next assign the label 3 to the vertices $u_2, u_4, \cdots, u_{2t}$. Assign the label 2 to the next t vertices $u_{2t+1}, \cdots, u_{3t}$. Finally, assign the label 4 to the next t non labeled vertices $u_{3t+1}, \cdots, u_{4t}$.

Case 2. $n \equiv 1 \pmod 4$.

Subcase 2.1 $n \equiv 1 \pmod 8$.

Assign the labels to the vertices of u_i, $1 \leq i \leq n-1$ as in Case 1. Finally, assign the label 2 to the vertex u_n.

Subcase 2.1 $n \equiv 5 \pmod 8$.

As in Case 1, assign the labels to the vertices u_i, $1 \leq i \leq n-1$. Then assign the label 1 to the vertex u_n.

Case 3. $n \equiv 2 \pmod 4$.

Subcase 3.1 $n \equiv 2 \pmod 8$.

Assign the labels to the vertices of u_i, $1 \leq i \leq n-1$ as in Subcase 2.1. Then assign the label 1 to the vertex u_n.

Subcase 3.2 $n \equiv 6 \pmod 8$.

As in Case 1, assign the labels to the vertices u_i, $1 \leq i \leq n-2$. Then assign the labels 2 and 1 respectively to the vertices u_{n-1} and u_n.

Case 4. $n \equiv 3 \pmod 4$.

Subcase 4.1 $n \equiv 3 \pmod 8$.

As in Subcase 3.1, assign the labels to the vertices of u_i, $1 \leq i \leq n-1$. Finally, assign the label 4 to the last vertex u_n.

Subcase 4b. $n \equiv 7 \pmod 8$.

Assign the labels to the vertices u_i, $1 \leq i \leq n-1$ as in Subcase 3.2. Finally, assign the label 4 to the vertex u_n.

It is easy to verify that the above vertex labeling pattern is 4-prime cordial labeling. □

The following Figure 3 is an example of a 4-prime cordial labeling of P_{10}^2.

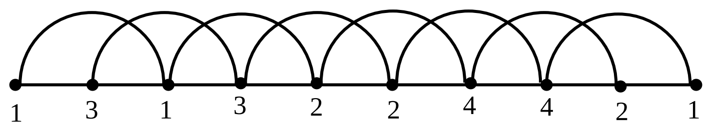

Figure 3

Theorem 3.9 *The graph GL_n is 4-prime cordial for $n > 2$.*

Proof Here we consider the following cases.

Case 1. $n \equiv 0 \pmod 4$.

Clearly GL_n has $2n$ vertices and $4n-3$ edges. Let $n = 4t$. Assign the label 2 to the vertices $u_1, u_2, \cdots, u_{2t}$ and assign the label 4 to the vertices $v_2, v_3, \cdots, v_{2t+1}$. Next assign the label the labels 3, 1 alternatively to the vertices to the vertices $u_{2t+1}, u_{2t+2}, \cdots, u_{4t}$. Assign the label 1 to the vertices $v_{4t}, v_{4t-2}, v_{4t-4}, \cdots, v_{2t+5}$ and assign the label 3 to the vertices v_{4t-1}, v_{4t-3}, $v_{4t-5}, \cdots, v_{2t+4}$. Finally assign the labels 1, 1, 3 and 3 respectively to the vertices $v_{2t+3}, v_{2t+2}, v_{2t+1}$ and v_1.

Case 2. $n \equiv 1 \pmod 4$.

Assign the label to the vertices u_i, v_i, $1 \leq i \leq n-1$ as in case 1. Next assign the labels 2, 4 to the vertices u_n, v_n respectively.

Case 3. $n \equiv 2 \pmod 4$.

As in Case 2, assign the label to the vertices u_i, v_i, $1 \leq i \leq n-1$. Next assign the labels 1, 3 respectively to the vertices u_n, v_n. Finally interchange the labels of u_{2t+2} and u_{2t+3}, that is the label of u_{2t+2} is 3 and the label of u_{2t+3} is 1.

Case 4. $n \equiv 3 \pmod 4$.

As in Case 3, assign the label to the vertices u_i, v_i, $1 \leq i \leq n-1$. Then assign the labels 2, 4 to the vertices u_n, v_n respectively. Clearly the above vertex labeling is a 4 prime cordial labeling of GL_n for all $n \geq 4$ and $n \neq 2$. A 4-prime cordial labeling of GL_3 is shown in the following Figure 4. □

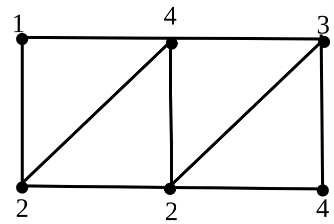

Figure 4

References

[1] I.Cahit, Cordial Graphs: A weaker version of graceful and harmonious graphs, *Ars combin.,* 23 (1987) 201-207.

[2] J.A.Gallian, A dynamic survey of graph labeling, *The Electronic Journal of Combinatorics*, 17 (2015) #Ds6.

[3] F.Harary, *Graph Theory*, Addision Wesley, New Delhi (1969).

[4] M.A.Seoud and M.A.Salim, Two upper bounds of prime cordial graphs, *JCMCC*, 75(2010) 95-103.

[5] M.Sundaram, R.Ponraj and S.Somasundaram, Prime cordial labeling of graphs, *J. Indian Acad. Math.,* 27(2005) 373-390.

[6] M. Sundaram, R. Ponraj and S. Somasundaram, Product cordial labeling of graphs, *Bull. Pure and Appl. Sci. (Math. & Stat.)*, 23E (2004) 155-163.

[7] R.Ponraj, Rajpal singh, R.Kala and S. Sathish Narayanan, k-prime cordial graphs, *J. Appl. Math. & Informatics*, 34 (3-4) (2016) 227 - 237.

[8] R.Ponraj, Rajpal Singh, and S.Sathish Narayanan, A note on 3-Prime cordial graphs, *Journal of Algorithms and Computation*, **48(1)**(2016), 45- 55.

[9] R.Ponraj, Rajpal singh and S. Sathish Narayanan, 4-prime cordiality of some classes of graphs, *Journal of algorithms and computation*, 48(1)(2016), 69- 79.

[10] R.Ponraj, Rajpal singh and S. Sathish Narayanan, 4-prime cordiality of some cycle related graphs, *Applications and Applied Mathematics: an international journal*, 12(1)(2017), 230-240.