International J.Math. Combin. Vol.2(2017), 129-134

Non-Existence of Skolem Mean Labeling for Five Star

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Abstract: In this paper, we prove if $\ell \leq m < n$, the five star $G = K_{1,\ell} \bigcup K_{1,\ell} \bigcup K_{1,\ell} \bigcup K_{1,m} \bigcup K_{1,n}$ is not a skolem mean graph if $|m-n| > 4+3\ell$ for $\ell = 2,3,\cdots$ and $m = 2,3,\cdots$.

Key Words: Labeling, Smarandachely edge m-labeling f_S^*, skolem mean labeling, skolem mean graph, star.

AMS(2010): 05C78.

§1. Introduction

Let G be a simple graph. A vertex labeling of G is an assignment $f : V(G) \to \{1,2,3,\cdots,p+q\}$ be an injection. For a vertex labeling f, the induced Smarandachely edge m-labeling f_S^* for an edge $e = uv$, an integer $m \geq 2$ is defined by $f_S^*(e) = \left\lceil \frac{f(u)+f(v)}{m} \right\rceil$. Then f is called a Smarandachely super m-mean labeling if $f(V(G)) \cup \{f^*(e) : e \in E(G)\} = \{1,2,3,\ldots,p+q\}$. Particularly, in the case of $m = 2$, we know that

$$f^*(e) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even;} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd.} \end{cases}$$

Such a labeling is usually called a mean labeling. A graph that admits a Smarandachely super mean m-labeling is called a Smarandachely super m-mean graph, particularly, a skolem mean graph if $m = 2$

In [2], we proved the following theorems to study the existence of skolem mean graphs. We proved the three star $K_{1,\ell} \bigcup K_{1,m} \bigcup K_{1,n}$ is a skolem mean graph if $|m-n| = 4+\ell$ for $\ell = 1,2,3,\cdots$; $m = 1,2,3,\cdots$ and $\ell \leq m < n$. The three star $K_{1,\ell} \bigcup K_{1,m} \bigcup K_{1,n}$ is not a skolem mean graph if $|m-n| > 4+\ell$ for $\ell = 1,2,3,\cdots$; $m = 1,2,3,\cdots$ and $\ell \leq m < n$. The four star $K_{1,\ell} \bigcup K_{1,\ell} \bigcup K_{1,m} \bigcup K_{1,n}$ is a skolem mean graph if $|m-n| = 4+2\ell$ for $\ell = 2,3,\cdots$; $m = 2,3,\cdots$ and $\ell \leq m < n$. The four star $K_{1,\ell} \bigcup K_{1,\ell} \bigcup K_{1,m} \bigcup K_{1,n}$ is not a skolem mean graph if $|m-n| > 4+2\ell$ for $\ell = 2,3,\cdots$; $m = 2,3,\cdots$ and $\ell \leq m < n$. In [3]. The five star $K_{1,\ell} \bigcup K_{1,\ell} \bigcup K_{1,\ell} \bigcup K_{1,m} \bigcup K_{1,n}$ is a skolem mean graph if $|m-n| = 4+3\ell$ for $\ell = 2,3,\cdots$;

[1]Received August 23, 2016, Accepted May 30, 2017.

$m = 2, 3, \cdots$ and $\ell \leq m < n$. Further, we prove the four star $K_{1,1} \bigcup K_{1,1} \bigcup K_{1,m} \bigcup K_{1,n}$ is a skolem mean graph if $|m - n| = 7$ for $m = 1, 2, 3, \cdots$ and $1 \leq m < n$; The four star $K_{1,1} \bigcup K_{1,1} \bigcup K_{1,m} \bigcup K_{1,n}$ is not a skolem mean graph if $|m - n| > 7$ for $m = 1, 2, 3, \cdots$ and $1 \leq m < n$; The five star $K_{1,1} \bigcup K_{1,1} \bigcup K_{1,1} \bigcup K_{1,m} \bigcup K_{1,n}$ is a skolem mean graph if $|m - n| = 8$ for $m = 1, 2, 3, \cdots$ and $1 \leq m < n$.

Definition 1.1 *The five star is the disjoint union of* $K_{1,a}, K_{1,b}, K_{1,c}, K_{1,d}$ *and* $K_{1,e}$ *and is denoted by* $K_{1,a} \bigcup K_{1,b} \bigcup K_{1,c} \bigcup K_{1,d} \bigcup K_{1,e}$.

§2. Main Result

Theorem 2.1 *The five star* $G = K_{1,\ell} \bigcup K_{1,\ell} \bigcup K_{1,\ell} \bigcup K_{1,m} \bigcup K_{1,n}$ *is not a skolem mean graph if* $|m - n| > 4 + 3\ell$ *for* $\ell = 2, 3, \cdots$ *and* $m = 2, 3, \cdots$.

Proof Let $G = 4K_{1,2} \bigcup K_{1,13}$, where $V(G) = \{v_{i,j} : 1 \leq i \leq 4; 0 \leq j \leq 2\} \bigcup \{v_{5,j} : 0 \leq j \leq 13\}$ and $E(G) = \{v_{i,0} : v_{i,j} : 1 \leq i \leq 4; 1 \leq j \leq 2\} \bigcup \{v_{5,0}v_{5,j} : 1 \leq j \leq 13\}$. Then, $p = 26$ and $q = 21$. Suppose G is a skolem mean graph. Then there exists a function f from the vertex set of G to $\{1, 2, 3, \cdots p\}$ such that the induced map f^* from the edge set of G to $\{2, 3, 4, \cdots p\}$ defined by

$$f^*(e = uv) = \begin{cases} \dfrac{f(u) + f(v)}{2} & \text{if} f(u) + f(v) \text{is even} \\ \dfrac{f(u) + f(v) + 1}{2} & \text{if} f(u) + f(v) \text{is odd} \end{cases}$$

then the resulting edges get distinct labels from the set $\{2, 3, \cdots p\}$.

Let $t_{i,j}$ be the label given to the vertex $v_{i,j}$ for $1 \leq i \leq 4$; $0 \leq j \leq 2$ and $v_{5,j}$ for $0 \leq j \leq 13$ and $X_{i,j}$ be the corresponding edge label of the edge $v_{i,0}v_{i,j}$ for $1 \leq i \leq 5$; $0 \leq j \leq 2$ and $v_{5,0}v_{5,j}$ for $1 \leq j \leq 13$.

Let us first consider the case that $t_{5,0} = 26$. If $v_{5,j} = 2n$ and $t_{5,k} = 2n + 1$ for some n and for some j and k then $f^*(v_{5,0}v_{5,j}) = \dfrac{26 + 2n}{2} = 13 + n = f^*(v_{5,0}v_{5,k})$. This is not possible as f^* is a bijection.

Therefore the thirteen vertices $t_{5,j}$ for $1 \leq j \leq 13$ are among the 13 numbers (1 or 2), (3 or 4), (5 or 6), (7 or 8), (9 or 10), (11 or 12), (13 or 14), (15 or 16), (17 or 18), (19 or 20), (21 or 22), (23 or 24) and 25.

Primarily, $t_{5,2}$ is either of 23 or 24. We first consider the case that $t_{5,2} = 23$.

Case 1. $t_{5,2} = 23$.

We have $t_{5,0} = 26$; $t_{5,1} = 25$; $t_{5,2} = 23$; $t_{1,0} = 24$. Now 24 is a label of either $t_{i,0}$ for $1 \leq i \leq 4$ or $t_{i,j}$ for $1 \leq i \leq 4; 1 \leq j \leq 2$. That is 24 is a label of pendent or non pendent vertex in a $k_{1,2}$ component of G. Let us assume that $t_{1,0} = 24$.

Subcase 1.1 $t_{1,0} = 24$.

We have $t_{5,0} = 26$; $t_{5,1} = 25$; $t_{5,2} = 23$; $t_{1,0} = 24$. If $t_{1,0} = 24$ then $t_{1,1}$ take the values 1 or 2. As $t_{1,1} \geq 3$ would imply that $X_{1,1} \geq 14$ this is not possible . The corresponding edge labels are $X_{1,1} = 13$.

Next $t_{5,3}$ is either 21 or 22. If $t_{5,3} = 21$ then $t_{2,0} = 22$. If $t_{2,1} = 3$ or 4 then $X_{2,1} = \dfrac{22 + 3 \text{ or } 4}{2} = 13$ this is not possible.

Similarly, if $t_{5,3} = 22$ then $t_{2,0} = 21$. Then $t_{2,1}$ take the value 3 or 4. The corresponding edge labels are $X_{2,1} = 12$, $X_{1,1} = 13$.

If $t_{2,2} \geq 5$ then $X_{2,2} \geq 14$ this is not possible. Hence it is not possible that $t_{1,0} = 24$. That is 24 is not a label of a non-pendent vertex in $k_{1,2}$ component of G. Next we consider the case that 24 is a label of a pendent vertex in a $k_{1,2}$ component of G. Let us assume that $t_{1,1} = 24$.

Subcase 1.2 $t_{1,1} = 24$.

We have $t_{5,0} = 26;\ t_{5,1} = 25;\ t_{5,2} = 23;\ t_{1,1} = 24$. If $t_{1,0} \geq 3$ then $X_{1,1} \geq 14$. This is not possible. Hence the value of $t_{1,0}$ is 1 or 2.

First, $t_{1,0} = 1$ or 2. We have $t_{5,0} = 26;\ t_{5,1} = 25;\ t_{5,2} = 23;\ t_{1,1} = 24$. Then $X_{1,1} = 13$. Now $t_{5,3}$ is either of 21 or 22.

Next case let, $t_{5,3} = 21$ and hence $t_{2,1} = 22$. If $t_{2,0} \geq 5$ then $X_{2,1} \geq 14$. This is not possible. If $t_{2,0} = 3 or 4$ then $X_{2,1} = \left(\dfrac{26 + 3 or 4}{2}\right) = 15$ this is not possible.

Suppose $t_{5,3} = 22$ and hence $t_{2,1} = 21$. We have $t_{5,0} = 26;\ t_{5,1} = 25;\ t_{5,2} = 23;\ t_{1,1} = 24;\ t_{1,0} = 1$ or $2; t_{2,1} = 21; t_{2,0} = 3$. Then $X_{1,1} = 13, X_{2,1} = 12$. Now $t_{5,4}$ is either of 19 or 20.

Consider the case that $t_{5,4} = 19$ hence $t_{3,1} = 20$. We have $t_{5,0} = 26;\ t_{5,1} = 25;\ t_{5,2} = 23;\ t_{1,1} = 24;\ t_{5,3} = 22;\ t_{2,1} = 21;\ t_{2,0} = 3$. Here the value $t_{3,0} \geq 4$ then $X_{3,1} \geq 13$ this is not possible. If $t_{5,4} = 20$, then $t_{3,1} = 19$. Notice that $t_{5,0} = 26;\ t_{5,1} = 25;\ t_{5,2} = 23;\ t_{1,1} = 24;\ t_{5,3} = 22;\ t_{2,1} = 21;\ t_{2,0} = 3$. Here the value $t_{3,0} \geq 4$ then $X_{3,1} \geq 12$. This is not possible. Hence $t_{5,4} \neq 19$.

Similarly $t_{5,4} \neq 20$; $t_{5,3} \neq 22$; $t_{5,3} \neq 21$. Hence $t_{1,0} \neq 1 or 2$ therefore $t_{1,1} \neq 24$;$t_{5,2} \neq 23$.

Case 2. $t_{5,2} = 24$.

Now 23 is a label of either $t_{i,0}$ for $1 \leq i \leq 4$ or $t_{i,j}$ for $1 \leq i \leq 4; 1 \leq j \leq 2$; that is 23 is a label of pendent or non-pendent vertex in a $K_{1,2}$ component of G.

Subcase 2.1 $t_{1,0} = 23$.

We have $t_{5,0} = 26;\ t_{5,1} = 25;\ t_{5,2} = 24;\ t_{1,0} = 23)$. If $t_{1,0} = 23$ then $t_{1,1}$ and $t_{1,2}$ take the values of 1 and 2 or 3 as $t_{1,1} \geq 4$ would imply that $X_{1,1} \geq 14$ is not possible. The corresponding edge labels are $X_{1,1} = 12;\ X_{1,2} = 13$.

Now $t_{5,3}$ is either of 21 or 22. If $t_{5,3} = 21$ then $t_{2,0} = 22$ then $X_{5,3} = \dfrac{26 + 21}{2} = 24$ and $t_{2,j} \geq 4$ and this is not possible. As $t_{2,j} \geq 4$ would imply that $X_{2,j} \geq 13$ and this not possible.

Similarly $t_{5,3} = 22$ then $X_{5,3} = \dfrac{26 + 22}{2} = 24;\ t_{2,0} = 21$ and also $t_{2,j} \geq 4$ this is not possible. As $t_{2,j} \geq 4$ would imply that $X_{2,j} \geq 13$ and this not possible.

Hence, it is not possible that $t_{1,0} = 23$ that is 23 is not a label of non-pendent vertex in $K_{1,2}$ component of G.

Next we consider the case that 23 is a label of a pendent vertex in a $K_{1,2}$ component of G. Let us assume that $t_{1,1} = 23$.

Subcase 2.2 $t_{1,1} = 23$.

We have $t_{5,0} = 26;\ t_{5,1} = 25;\ t_{5,2} = 24;\ t_{1,1} = 23$. If $t_{1,0} \geq 4$ then $X_{1,1} \geq 14$ and this is not possible. Hence the value of $t_{1,0}$ can either be 1 or 2 or 3. There exist two cases, i.e., $t_{1,0} = 1$ and $t_{1,0} = 2$ or 3.

Subcase 2.2.1 $t_{1,0} = 1$.

We have $t_{5,0} = 26;\ t_{5,1} = 25;\ t_{5,2} = 24;\ t_{1,0} = 1;\ t_{1,1} = 23$. Then $X_{1,1} = 12$. Now $t_{5,3}$ is either of 21 or 22.

Let $t_{5,3} = 21$ hence $t_{2,1} = 22$. If $t_{2,0} \geq 5$ then $X_{2,1} \geq 14$ and is not possible. If $t_{2,0} = 2$ then $X_{2,1} = \dfrac{26+2}{2} = 12$ and this is not possible. Hence $t_{2,0}$ is either of 3 or 4. We have $t_{5,0} = 26; t_{5,1} = 25; t_{5,2} = 24;\ t_{1,0} = 1;\ t_{5,3} = 21;\ t_{2,1} = 22;\ t_{2,0} = 3$ or 4 then $X_{1,1} = 12;\ X_{2,1} = 13$.

Now $t_{5,4}$ is either 19 or 20. Assume $t_{5,4} = 19$ Hence $t_{3,1} = 20$. If $t_{3,0} \geq 5$ then $X_{3,1} \geq 13$ and is not possible. Hence $t_{3,0}$ is 2. Notice that $t_{5,0} = 26;\ t_{5,1} = 25;\ t_{5,2} = 24;\ t_{5,3} = 21;\ t_{1,0} = 1;\ t_{2,1} = 22;\ t_{2,0} = 3$ or 4; $t_{3,1} = 20;\ t_{3,0} = 2$ then $X_{1,1} = 12;\ X_{2,1} = 13;\ X_{3,1} = 11$.

Now $t_{5,5}$ is either 17 or 18. Consider $t_{5,5} = 17$. Hence $t_{4,1} = 18$. We have $t_{5,0} = 26;\ t_{5,1} = 25;\ t_{5,2} = 24;\ t_{5,3} = 21;\ t_{5,4} = 19;\ t_{1,0} = 1;\ t_{1,1} = 23;\ t_{2,1} = 22;\ t_{2,0} = 3$ or 4; $t_{3,0} = 2;\ t_{3,1} = 20;\ t_{4,1} = 18$. Here the value $t_{4,0} \geq 5$ then $X_{4,1} \geq 12$, which is not possible.

Let $t_{5,5} = 18$ and hence $t_{4,1} = 17$. We have $t_{5,0} = 26;\ t_{5,1} = 25;\ t_{5,2} = 24;\ t_{5,3} = 21;\ t_{5,4} = 19;\ t_{5,5} = 18;\ t_{1,0} = 1;\ t_{1,1} = 23;\ t_{2,1} = 22;\ t_{2,0} = 3$ or 4; $t_{3,0} = 2;\ t_{3,1} = 20;\ t_{4,1} = 17$.

If the value $t_{4,0} \geq 5$ then $X_{4,1} \geq 11$, which is not possible. Hence $t_{5,4} \neq 19$. Similarly we can prove $t_{5,4} \neq 20$ and therefore $t_{5,3} \neq 21$.

Consider the case that $t_{5,3} = 22$ and hence $t_{2,1} = 21$. If $t_{2,0} \geq 6$ then $X_{2,1} \geq 14$ and is not possible. Hence the value of $t_{2,0}$ can either of 4 or 5.

First we consider $t_{2,0} = 4$ or 5. We have $t_{5,0} = 26;\ t_{5,1} = 25;\ t_{5,2} = 24;\ t_{5,3} = 22;\ t_{1,1} = 23;\ t_{1,0} = 1;\ t_{2,1} = 21;\ t_{2,0} = 4$ or 5, then $X_{1,1} = 12$ and $X_{2,1} = 13$.

Now $t_{5,4}$ is either 19 or 20. Considering, $t_{5,4} = 19$ and $t_{3,1} = 20$. If $t_{3,0} \geq 7$ then $X_{3,1} \geq 14$ and is not possible. Hence the value of $t_{3,0}$ can either be 2 or 6.

If $t_{3,0} = 6$ then $X_{3,1} = \dfrac{20+6}{2} = 13$, which is not possible. Hence $t_{3,0}$ is 2. Notice that $t_{5,0} = 26;\ t_{5,1} = 25;\ t_{5,2} = 24;\ t_{5,3} = 22;\ t_{1,1} = 23;\ t_{1,0} = 1;\ t_{2,1} = 21;\ t_{2,0} = 4$ or 5; $t_{3,1} = 20;\ t_{3,0} = 2$.

Now $t_{5,5}$ is either 17 or 18. Let us consider $t_{5,5} = 17$ and $t_{4,1} = 18$. Notice that $t_{5,0} = 26;\ t_{5,1} = 25;\ t_{5,2} = 24;\ t_{5,3} = 22;\ t_{1,1} = 23;\ t_{1,0} = 1;\ t_{2,1} = 21;\ t_{2,0} = 4$ or 5; $t_{3,1} = 20;\ t_{3,0} = 2;\ t_{4,1} = 18$.

Here the value $t_{4,0} = 3$ then $X_{4,1} = \dfrac{18+3}{2} = 11$, which is not possible.

Now $t_{5,5}$ is either 18 or 17. Let $t_{5,5} = 18$ and $t_{4,1} = 17$. Notice that $t_{5,0} = 26;\ t_{5,1} = 25;\ t_{5,2} = 24;\ t_{5,3} = 22;\ t_{1,1} = 23;\ t_{1,0} = 1;\ t_{2,1} = 21;\ t_{2,0} = 4$ or 5; $t_{3,1} = 20;\ t_{3,0} = 2;\ t_{4,1} = 17;\ t_{4,0} = 3$.

Now $t_{5,6}$ is either 15 or 16. If $t_{5,6} = 15$ and $t_{5,1} = 16$, we have $t_{5,0} = 26; t_{5,1} = 25;\ t_{5,2} = 24;\ t_{5,3} = 22;\ t_{1,1} = 23;\ t_{1,0} = 1;\ t_{2,1} = 21;\ t_{2,0} = 4$ or 5; $t_{3,1} = 20;\ t_{3,0} = 2;\ t_{4,1} = 17;\ t_{4,0} = 3;\ t_{5,1} = 16$. Here the value of $t_{5,0} \geq 6$. This is not possible.

If $t_{5,6} = 16$ and $t_{5,1} = 15$, we have $t_{5,0} = 26;\ t_{5,1} = 25;\ t_{5,2} = 24;\ t_{5,3} = 22;\ t_{1,1} = 23;\ t_{1,0} = 1;\ t_{2,1} = 21;\ t_{2,0} = 4$or 5; $t_{3,1} = 20;\ t_{3,0} = 2;\ t_{4,1} = 17;\ t_{4,0} = 3;\ t_{5,1} = 15$. Here the value of $t_{5,0} \geq 6$. This is not possible. Hence $t_{5,4} \neq 19$.

Similarly $t_{5,4} \neq 20$ and $t_{2,0} \neq 4 or 5$. Therefore $t_{5,3} \neq 18$. Hence $t_{1,0} \neq 1$.

Subcase 2.2.2 $t_{1,0} = 2$ or 3.

In this case, we have $t_{5,0} = 26;\ t_{5,1} = 25;\ t_{5,2} = 24;\ t_{1,1} = 23$. Then $X_{1,1} = 13$.

Now $t_{5,3}$ is either 21 or 22. If $t_{5,3} = 21$ and $t_{2,1} = 22$. If $t_{2,0} \geq 4$ then $X_{2,1} \geq 13$. This is not possible. Hence $t_{2,0}$ is 1. Notice that $t_{5,0} = 26;\ t_{5,1} = 25;\ t_{5,2} = 24;\ t_{5,3} = 21;\ t_{1,1} = 23;\ t_{1,0} = 2$ or 3; $t_{2,0} = 1$.

Now $t_{5,4}$ is either 19 or 20. Suppose $t_{5,4} = 19$ and $t_{3,1} = 20$. Notice that $t_{5,0} = 26;\ t_{5,1} = 25;\ t_{5,2} = 24;\ t_{5,3} = 21;\ t_{1,1} = 23;\ t_{1,0} = 2$ or 3; $t_{2,0} = 1;\ t_{5,4} = 19;\ t_{3,1} = 20$. Here the value of $t_{3,0} \geq 4$, which is not possible.

Let $t_{5,4} = 20$ and $t_{3,1} = 19$. Notice that $t_{5,0} = 26;\ t_{5,1} = 25;\ t_{5,2} = 24;\ t_{5,3} = 21;\ t_{1,1} = 23;\ t_{1,0} = 2$ or 3; $t_{2,0} = 1;\ t_{5,4} = 19;\ t_{3,1} = 20$. Here the value of $t_{3,0} \geq 4$, which is not possible. Hence $t_{5,3} \neq 21$.

Similarly $t_{5,3} \neq 22$ and $t_{5,4} \neq 19; t_{5,4} \neq 20$ therefore $t_{1,0} \neq 2$ or 3. Hence $t_{5,2} \neq 24$. Therefore $t_{5,0} \neq 26$ and hence $t_{5,1} \neq 25$. We have proved that if $t_{5,0} = 26$ the five star $G = 4K_{1,2} \bigcup K_{1,13}$ does not admit a skolem mean labelling .

Similarly, we can prove the result for other values of $t_{5,0}$. Hence the five star

$$\begin{aligned} G &= K_{1,\ell} \bigcup K_{1,\ell} \bigcup K_{1,\ell} \bigcup K_{1,m} \bigcup K_{1,n} \\ &= K_{1,2} \bigcup K_{1,2} \bigcup K_{1,2} \bigcup K_{1,2} \bigcup K_{1,13} \\ &= 4K_{1,2} \bigcup K_{1,13} \end{aligned}$$

is not a skolem mean graph. That is G is not a skolem mean graph if $|m - n| = 5 + 3\ell$.

In a similar way, we can prove that $G = 4K_{1,2} \bigcup K_{1,14}$ is not a skolem mean graph if $|m - n| = 6 + 3\ell$. Hence on generalizing, $G = K_{1,\ell} \bigcup K_{1,\ell} \bigcup K_{1,\ell} \bigcup K_{1,m} \bigcup K_{1,n}$ is not a skolem mean graph if $|m - n| > 4 + 3\ell$. □

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