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Traffic Congestion – A Challenging Problem in the World

Mushtaq Ahmad Shah[1], Mridula Purohit[2] and M.H.Gulzar[3]

1. Department of Mathematics, Vivekananda Global University, Jaipur, India

2. Department of Mathematics, VIT East Jaipur, India

1. Department of Mathematics, University of Kashmir, India

E-mail: shahmushtaq81@gmail.com

Abstract: The purpose of this paper is to describe the problems which involves in the reduction of traffic congestion. In particular we use graph theoretical approach which is quite appropriate. We use crossing number technique to reduce traffic congestion. The minimum number of crossing points in a complete graph is given by $Cr(Kn) \leq \frac{1}{4}\left[\frac{n}{2}\right]\left[\frac{n-1}{2}\right]\left[\frac{n-2}{2}\right]\left[\frac{n-3}{2}\right]$ where[ ] represents greatest integer function. And we illustrate the result with counter examples.

Key Words: crossing number, complete graph, traffic control, edge connectivity, vertex connectivity.

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§1. Introduction

The crossing number (sometimes denoted as $C(G)$) of a graph G is the smallest number of pair wise crossings of edges among all drawings of G in the plane. In the last decade, there has been significant progress on a true theory of crossing numbers. There are now many theorems on the crossing number of a general graph and the structure of crossing critical graphs, whereas in the past, most results were about the crossing numbers of either individual graphs or the members of special families of graphs. The study of crossing numbers began during the Second World War with Paul Turan. In [1], he tells the story of working in a brickyard and wondering about how to design an efficient rail system from the kilns to the storage yards. For each kiln and each storage yard, there was a track directly connecting them. The problem he Consider was how to lay the rails to reduce the number of crossings, where the cars tended to fall off the tracks, requiring the workers to reload the bricks onto the cars. This is the problem of finding the crossing number of the complete bipartite graph. It is also natural to try to compute the crossing number of the complete graph. To date, there are only conjectures for the crossing numbers of these graphs Called Guys conjecture which suggest that crossing number of complete

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graph Kn is given by $V(K_n) = Z(n)[2][3]$ where [ ] represents greatest integer function.

$$z(n) = \frac{1}{4}\left[\frac{n}{2}\right]\left[\frac{n-1}{2}\right]\left[\frac{n-2}{2}\right]\left[\frac{n-3}{2}\right]$$

which can also be written as

$$z(n) = \begin{cases} \frac{1}{64}n(n-2)^2(n-4) & n \text{ even} \\ \frac{1}{64}(n-1)^2(n-3)^2 & n \text{ odd} \end{cases}$$

Guy prove it for $n \leq 10$ in 1972 in 2007 Richter prove it for $n \leq 12$ For any graph G, we say that the crossing number $c(G)$ is the minimum number of crossings with which it is possible to draw G in the plane. We note that the edges of G need not be straight line segments, and also that the result is the same whether G is drawn in the plane or on the surface of a sphere. Another invariant of G is the rectilinear crossing number, $c(G)$, which is the minimum number of crossings when G is drawn in the plane in such a way that every edge is a straight line segment. We will find by an example that this is not the same number obtained by drawing G on a sphere with the edges as arcs of great circles. In drawing G in the plane, we may locate its vertices wherever it is most convenient. A plane graph is one which is already drawn in the plane in such a way that no two of its edges intersect. A planar graph is one which can be drawn as a plane graph [6]. In terms of the notation introduced above, a graph G is planar if and only if $c(G) = 0$. The earliest result concerning the drawing of graphs in the plane is due to Fary [4] [7], who showed that any planar graph (without loops or multiple edges) can be drawn in the plane in such a way that every Edge is straight. Thus Farys result may be rephrased: if $c(G) = 0$, then $c(G) = 0$. In a drawing, the nodes of the graph are mapped into points of a plane, and the arcs into continuous curves of the plane, no three having a point in common. A minimal drawing does not contain an arc which crosses itself, nor two arcs with more than one point in common, [5],[8]. In general for a set of n line segments, there can be up to $\bigcirc(n^2)$ intersection points, since if every segment intersects every other segment, there would be

$$\frac{n(n-1)}{2} = \bigcirc(n^2)$$

Crossing points to compute them all we require $\bigcirc(n^2)$ algorithm.

The *traffic theory* is a physical phenomenon that aims at understanding and improving automobile traffic, and the problem associated with it such as traffic congestion [9]. The traffic control problem is to minimize the waiting time of the public transportation while maintaining the individual traffic flow optimally [10]. Significant development of traffic control systems using traffic lights have been achieved since the first traffic controller was installed in London in 1868. The first green wave was realized in Salt Lake City (U.K.) in 1918, and the first area traffic controller was introduced in Toronto in 1960. At the beginning, electromechanical devices were used to perform traffic control. Then Intelligent Transportation System (ITS) is used extensively in urban areas to control traffic at an intersection [11]. The traffic data in a particular region can be used to direct the traffic flow to improve traffic output without adding new roads. In order to collect accurate traffic data semi conductor-based controllers known as sensors were placed in different places to collect traffic information are used in traffic control system [11], [12], [13]. Nowadays, microprocessor based controller are used in Traffic Control Systems. The combinatorial approach to the optimal traffic control problem was founded by Stoffers [14] in 1968 by introducing the Compatibility Graph of traffic streams. One of the main uses of traffic theory is the development of traffic models which can be used for estimation, prediction, and control related tasks for the automobile traffic process. The term Intelligent Transportation System (ITS) refers to information and communication technology applied to transport infrastructure and vehicles, that improves transport outcomes such as transport safety, transport productivity, transport reliability, informed traveler choice, environmental performance etc. [15] , [16]. ITS mainly comes from the problems caused by traffic congestion and synergy of new information technology for simulation, real time control and communication networks. Traffic congestion has been increased world wide as a result of increased motorization, urbanization, population growth and changes in population density. Congestion reduces efficiency of transportation infrastructure and increases travel time, air pollution and fuel consumption. At the beginning of 1920, in United States large increase in both motorization and urbanization led to the migration of the population from sparsely populated rural areas and densely packed urban areas into suburbs (sub urban areas). Intelligent Transport Systems vary in technologies applied, from basic management system such as car navigation; traffic signal control systems; container management system; variable message sign; automatic number plate recognition or speed cameras to monitor applications; such as security CCTV systems; and to more advanced applications that integrate live data and feedback from a number of other sources, such as parking guidance and information systems; weather information etc. Additional predictive techniques are being developed to allow advanced modeling and comparison with historical data. The traffic flow predictions will be delivered to the drivers via different channels such as roadside billboards, radio stations, internet, and on vehicle GPS (Global Positioning Systems) systems. One of the components of an ITS is the live traffic data collection. To collect accurate traffic data sensors have to be placed on the roads and streets to measure the flow of traffic. Some of the constituent technologies implemented in ITS are namely, Wireless Communication, Computational technologies, Sensing technologies, Video Vehicle Detection etc. Urban traffic congestion is a significant and growing problem in many parts of the world. Moreover, as congestion continues to increase, the conventional approach of "building more roads" doesn't always work for a variety of political, financial, and environmental reasons. In fact, building new roads can actually compound congestion, in some cases, by inducing greater demands for vehicle travel that quickly eat away the additional capacity? Against this backdrop of serious existing and growing congestion traffic Control techniques and information systems are needed that can substantially increase capacity and Improve traffic flow efficiency. Application of ITS technologies in areas such as road user information and navigation systems, improved traffic control systems and vehicle guidance and control systems has significant potential for relieving traffic congestions.

Theorem 1.1 *The edge connectivity of a graph G cannot exceed the degree of the vertex with the smallest degree in G.*

Theorem 1.2 *The vertex connectivity of any graph G can never exceed the edge connectivity of G.*

Theorem 1.3 *The maximum vertex connectivity one can achieve in a graph of n vertices and e edges is $e \geq n-1$ Thus we conclude that* $\text{vertexconnectivity} \leq \text{edgeconnectivity} \leq \frac{2e}{n}$.

Definition 1.4 *A graph $G(v,e)$ where v is the set of vertices and e the set of edges is said to be complete if degree of each vertex is $v-1$.*

Definition 1.5 *The number of edges incident on a vertex is said to be degree of the vertex.*

Proposed Solution 1.6 As congestion continues to increase, the conventional approach of *building more roads* doesn't always work for a variety of political, financial, and environmental reasons. In fact, building new roads can actually compound congestion. There is no particular technique which reduces traffic congestion. Numbers of techniques are simultaneously required to curb this problem. Traffic congestion is one of the challenging problem in the world the aim of this research paper is that how to curb this problem. Before giving the solution to the problem we would like to introduce you the graph theoretical approach of the problem, using underlying graphs. We represent various cities by vertices and roads connected them by edges. Since every city must be connected with all other cities in particular geographical area so first of all we are dealing with complete graphs then we shall remove all the edges in the graph in such a way that maximum crossing pointes will be removed and there is no effect in the connectivity. Following are the techniques require curbing traffic congestion.

Definition 1.7 *Let $G(v,e)$ be a complete graph v the set of vertices and e the set of edges. the crossing number $Cr(G)$ of a complete graph $G(v,e)$ is the least number of crossings, common points of two arcs other than a vertex, in any drawing of graph in a plane (or on a sphere) in a drawing the vertex of the graph are mapped into points of a plane and the arcs into continue curves of the plane no three having a point in common, unless it be an end point (vertex) of the arc. A drawing which exhibits a crossing number is called minimal a minimal drawing does not contain an arc which crosses itself nor two arcs with more than one point in common. For any complete graph K_n it has been shown that the minimum number of crossing points is given by $Cr(Kn) \leq \frac{1}{4}\left[\frac{n}{2}\right]\left[\frac{n-1}{2}\right]\left[\frac{n-2}{2}\right]\left[\frac{n-3}{2}\right]$, where[ ] represents greatest integer function.*

Since minimum number of crossing points means minimum number of interruptions on roads which minimize the waiting time of the traffic participants so traffic congestion is reduces. After drawing the graph with minimum crossings we remove all those edges in a graph which does not effect the connectivity of the graph but reduce more crossings so the graph becomes more efficient as shown in Figure 1 here red dots represent vertices and yellow dots represent crossing points.

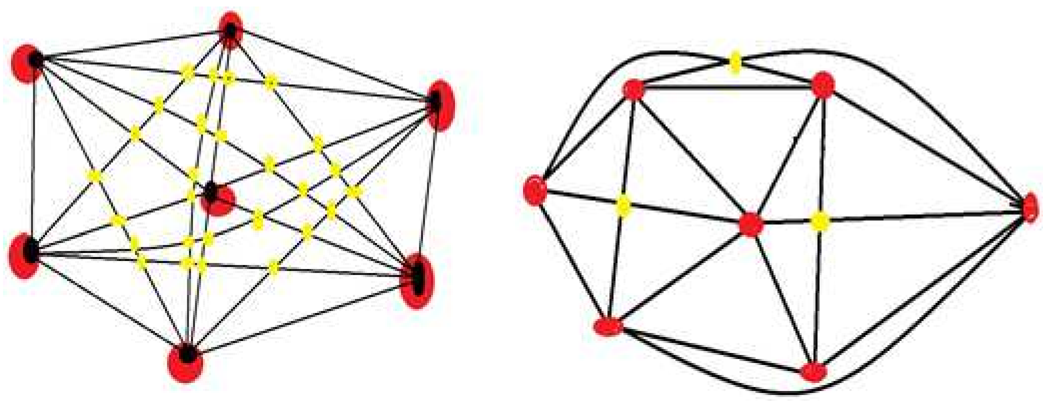

Figure 1

We shall keep this point in mind that no three edges has a point in common if it is necessary then we have to keep other crossing point at least one kilometer away from one another as shown in Figure 2

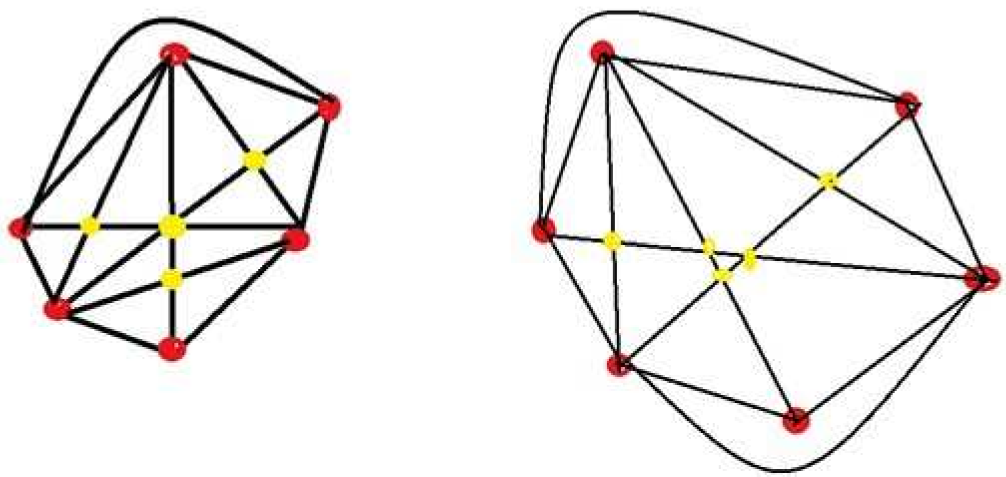

Figure 2

Because more than one crossing at point will increase interruption on traffic flow, first of all we have to try our best to reduce the intensity of crossing points, if it is not possible then flyovers should be constructed at every crossing so that there is no interruption on traffic flow, in this case only slow moments are possible not traffic jam. The above crossing point technique will reduce traffic congestion in a large extent. if there are maximum crossing points on the roads then maximum traffic congestion is possible so minimum crossing means minimum traffic congestion so we have to reduce the crossing points then traffic congestion is reduced. The aim to reduce traffic congestion is to reduce the crossing points. There are certain crossing points where more than two roads cross each other and traffic lights are imposed to allow the traffic flow alternatively one by one. so traffic congestion is increased because more and more vehicles have to be stopped on the roads First of all we shall try over best to reduce those crossing points where more than two crossing points exist which is quite possible, if it is necessary then, on these intersection pointes where more than two crossings points exist we have to make flyovers (as shown in fig 1.3 below these flyovers have already designed in certain parts of the world for this purpose) in such a way that there should not be any crossing point and traffic flow should be in continuous manner may be some times there are slow moments but still it will reduce traffic congestion.

Figure 3

§3. Methods

Method 3.1 There should be exclusive lanes for public transport so that private transport system does not affect the moment of the public transport.

Method 3.2 A turning restriction is imposed on the traffic, and vehicles are allow to turn on certain places turning points are at least one kilometer away from the crossing points if the turning points are at crossing points it will definitely increase traffic congestion, and proper fly-overs system is imposed as shown in Figure 4, which will automatically reduce traffic congestion in a large extent.

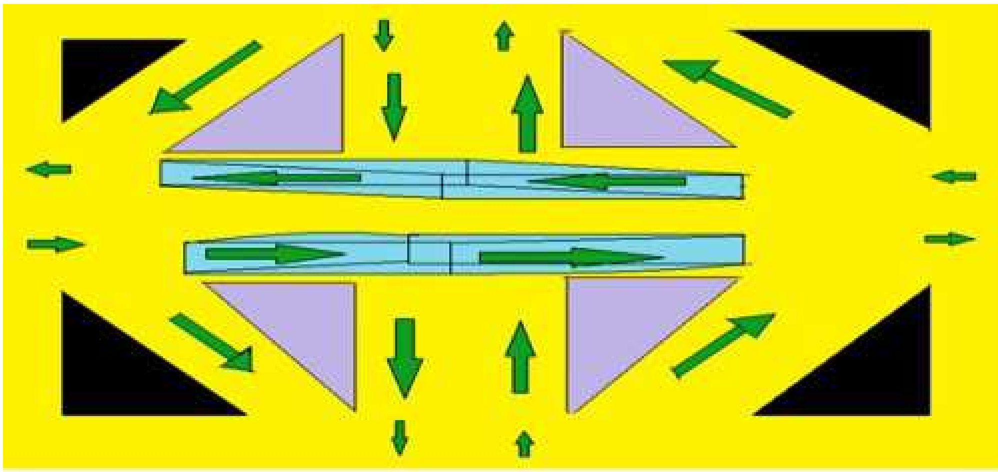

Figure 4

Method 3.3 A double win method has already imposed in certain cities of the world and has been proved to be very happy one. A congestion charge is essentially an economic method of regulating traffic by imposing fees on vehicle users that travels a city more crowed roads, but charge vary by city to city depending up on crowed on the city.

Method 3.4 The Parking restrictions on road side should be banned to reduce congestion.

Method 3.5 Remove some link roads at high efficiency points. Then we have to connect them other side so that minimum crossings are possible.

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