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b-Chromatic Number of Splitting Graph of Wheel

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Abstract: A proper k-coloring is called a b-coloring if there exits a vertex (b-vertex) that has neighbour(s) in all other $k-1$ color classes. The largest integer k for which G admits a b-coloring is called the b-chromatic number denoted as $\varphi(G)$. If b-coloring exists for every integer k satisfying $\chi(G) \leqslant k \leqslant \varphi(G)$ then G is called b-continuous. The b-spectrum $S_b(G)$ of a graph G is the set of k integers(colors) for which G has a b-coloring. We investigate b-chromatic number of the splitting graph of wheel and also discuss its b-continuity and b-spectrum.

Key Words: b-Coloring, b-continuity, b-spectrum.

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§1. Introduction

A proper k-coloring of a graph $G = (V(G), E(G))$ is a mapping $f : V(G) \to \{1, 2, \cdots, k\}$ such that every two adjacent vertices receives different colors. The chromatic number of a graph G is denoted by $\chi(G)$, is the minimum number for which G has a proper k-coloring. The set of vertices with a specific color is called a color class. A b-coloring of a graph G is a variant of proper k-coloring such that every color class has a vertex which is adjacent to at least one vertex in every other color classes and such a vertex is called a color dominating vertex. If v is a color dominating vertex of color class c then we denote it as $cdv(c) = v$. The b-chromatic number $\varphi(G)$ is the largest integer k such that G admits a b-coloring with k colors. The concept of b-coloring was originated by Irving and Manlove [1] and they also observed that every coloring of a graph G with $\chi(G)$ colors is obviously a b-coloring. In the same paper they have introduced the concepts of b-continuity and b-spectrum. If the b-coloring exists for every integer k satisfying $\chi(G) \leqslant k \leqslant \varphi(G)$ then G is called b-continuous and the b-spectrum $S_b(G)$ of a graph G is the set of k integers(colors) for which G has a b-coloring. Kouider and Maheö [2] have obtained lower and upper bounds for the b-chromatic number of the cartesian products of two graphs while Vaidya and Shukla [3,4,5,6] have investigated b-chromatic numbers for various

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graph families. The concept of b-coloring has been extensively studied by Faik [7], Kratochvil *et al.*[8], Alkhateeb [9] and Balakrishnan *et al.* [10].

Definition 1.1 *The splitting graph $S'(G)$ of a graph G is obtained by adding new vertex v' corresponding to each vertex v of G such that $N(v) = N(v')$, where $N(v)$ and $N(v')$ are the neighborhood sets of v and v' respectively in $S'(G)$.*

Here we investigate b-chromatic number for splitting graph of wheel.

Definition 1.2([1]) *The m-degree of a graph G, denoted by $m(G)$, is the largest integer m such that G has m vertices of degree at least $m-1$.*

Proposition 1.3([1]) *If graph G admits a b-coloring with m-colors, then G must have at least m vertices with degree at least $m-1$.*

Proposition 1.4 *Let $W_n = C_n + K_1$. Then* $\chi(W_n) = \begin{cases} 3, & n \text{ is even} \\ 4, & n \text{ is odd.} \end{cases}$

Proposition 1.5([11]) $\chi(G) \leqslant \varphi(G) \leqslant m(G)$.

Proposition 1.6([12]) *For any graph G, $\chi(G) \geqslant 3$ if and only if G has an odd cycle.*

§2. Main Results

Lemma 2.1 *For a wheel W_n,*

$$\chi[S'(W_n)] = \begin{cases} 4, & n \text{ is odd} \\ 3, & n \text{ is even} \end{cases}$$

Proof Let $v_1, v_2, \ldots, v_n$ be the rim vertices of wheel W_n which are duplicated by the vertices $v_1', v_2', \ldots, v_n'$ respectively and let v denotes the apex vertex of W_n which is duplicated by the vertex v'. Let $e_1, e_2, \ldots, e_n$ be the rim edges of W_n. Then the resultant graph $S'[W_n]$ will have order $2(n+1)$ and size $6n$.

Case 1. n is odd

In this case $S'[W_n]$ contains odd W_n as an induced subgraph. Since $\chi(W_n) = 4 \Rightarrow \chi[S'(W_n)] = 4$.

Case 2. n is even

In this case $S'[W_n]$ contains even W_n as an induced subgraph. Since $\chi(W_n) = 3 \Rightarrow \chi[S'(W_n)] = 3$. □

Theorem 2.2 *For a wheel W_n,*

$$\varphi[S'(W_n)] = \begin{cases} 4, & n=3 \\ 3, & n=4 \\ 5, & n=5,6,8 \\ 6, & n=7 \\ 6, & n\geqslant 9 \end{cases}$$

Proof To prove the result we continue with the terminology and notations used in Lemma 2.1 and consider the following cases.

Case 1. $n=3$

In this case the graph $S'(W_3)$ contains an odd cycle. Then by Proposition 1.6, $\chi[S'(W_3)] \geqslant 3$. As $m[S'(W_3)] = 4$ and by Lemma 2.1, $\chi[S'(W_3)] = 4$. We have $4 \leqslant \varphi[S'(W_3)] \leqslant 4$ by Proposition 1.5. Thus, $\varphi[S'(W_3)] = 4$.

Case 2. $n=4$

In this case the graph $S'(W_4)$ contains an odd cycle. Then by Proposition 1.6, $\chi[S'(W_4)] \geqslant 3$. As $m[S'(W_4)] = 5$ and by Lemma 2.1 $\chi[S'(W_4)] = 3$. Then by Proposition 1.5 we have $3 \leqslant \varphi[S'(W_4)] \leqslant 5$.

If $\varphi[S'(W_4)] = 5$ then by Proposition 1.3, the graph $S'(W_4)$ must have five vertices of degree at least 4 which is possible. But due to the adjacency of vertices of the graph $S'(W_4)$ any proper coloring with five colors have at least one color class which does not have color dominating vertices hence it will not be b-coloring for the graph $S'(W_4)$. Thus, $\varphi[S'(W_4)] \neq 5$.

Suppose $\varphi[S'(W_4)] = 4$. Now consider the color class $c = \{1,2,3,4\}$ and define the color function as $f : V \to \{1,2,3,4\}$ as $f(v) = 4 = f(v')$, $f(v_1) = 1$, $f(v_2) = 2$, $f(v_1') = 1$, $f(v_2') = 2$, $f(v_3') = 3$, $f(v_4') = 3$ which in turn forces to assign $f(v_3) = 1$, $f(v_4) = 2$. This proper coloring gives the color dominating vertices for color classes $1,2$ and 4 but not for 3 which is contradiction to our assumption. Thus, $\varphi[S'(W_4)] \neq 4$. Hence, we can color the graph by three colors. For b-coloring, consider the color class $c = \{1,2,3\}$ and define the color function as $f : V \to \{1,2,3\}$ as $f(v_1) = 1 = f(v_1'), f(v_2) = 2 = f(v_2'), f(v_3) = 1 = f(v_3'), f(v_4) = 2 = f(v_4'), f(v) = 3 = f(v')$. This proper coloring gives the color dominating vertices as $cdv(1) = v_1, cdv(2) = v_2$, $cdv(3) = v$. Thus $\varphi[S'\,(W_4)] = 3$.

Case 3. $n=5,6,8$

Subcase 3.1 $n=5$

In this case the graph $S'(W_5)$ contains an odd cycle. Then by Proposition 1.6, $\chi[S'(W_5)] \geqslant 3$. As $m[S'(W_5)] = 6$ and by Lemma 2.1, $\chi[S'(W_5)] = 4$. Then by Proposition 1.5 we have $4 \leqslant \varphi(S'(W_5) \leqslant 6$.

If $\varphi(S'(W_5) = 6$ then by Proposition 1.3, the graph $S'(W_5)$ must have six vertices of degree at least five which is possible. But due to the adjacency of vertices of the graph $S'(W_5)$ any proper coloring with six colors have at least one color class which does not have color dominating vertices. Hence it will not be b-coloring for the graph $S'(W_5)$. Thus, $\varphi(S'(W_5) \neq 6$.

Suppose $\varphi(S'(W_5) = 5$. Now consider the color class $\underset{\varsigma}{=}\{1,2,3,4,5\}$ and define the color function as $f : V \rightarrow \{1,2,3,4,5\}$ as $f(v) = 5 = f(v'), f(v_1) = 3, f(v_2) = 1, f(v_3) = 2, f(v_4) = 3, f(v_5) = 4, f(v'_1) = 2, f(v'_2) = 4, f(v'_3) = 4, f(v'_4) = 1, f(v'_5) = 1$. This proper coloring gives the color dominating vertices as $cdv(1) = v_2, cdv(2) = v_3, cdv(3) = v_4, cdv(4) = v_5, cdv(5) = v$. Thus, $\varphi(S'(W_5) = 5$.

Subcase 3.2 $n = 6, 8$

In this case the graph $S'(W_n)$ contains an odd cycle. Then by Proposition 1.6, $\chi[S'(W_n)] \geqslant 3$. As $m[S'(W_n)] = 7$ and by Lemma 2.1, $\chi[S'(W_n)] = 3$. Then by Proposition 1.5 we have $3 \leqslant \varphi[S'(W_n)] \leqslant 7$.

If $\varphi[S'(W_n)] = 7$ then by Proposition 1.3, the graph $S'(W_n)$ must have seven vertices of degree at least six which is possible. But due to the adjacency of the vertices of graph $S'(W_n)$ any proper coloring with seven colors have at least one color class which does not have color dominating vertices. Hence it will not be b-coloring for the graph $S'(W_n)$. Thus, $\varphi[S'(W_n)] \neq 7$.

Suppose $\varphi[S'(W_n)] = 6$. Now consider the color class $\underset{\varsigma}{=}\{1,2,3,4,5,6\}$ and define the color function as $f : V \rightarrow \{1,2,3,4,5,6\}$ as $f(v) = 6 = f(v'), f(v_1) = 3, f(v_2) = 1, f(v_3) = 2, f(v_4) = 3, f(v_5) = 4, f(v'_1) = 4, f(v'_2) = 4, f(v'_3) = 5, f(v'_4) = 5, f(v'_5) = 1$ which in turn forces to assign $f(v_6) = 2, f(v'_6) = 1$. This proper coloring gives the color dominating vertices for color classes $1, 2, 3, 4$ and 6 but not for 5 which is contradiction to our assumption. Thus, $\varphi[S'(W_n)] \neq 6$.

Suppose that $S'(W_n)$ has b-coloring with 5 colors. Now consider the color class $\underset{\varsigma}{=}\{1,2,3,4,5\}$ and define the color function as $f : V(G) \rightarrow \{1,2,3,4,5\}$ as $f(v) = 5 = f(v'), f(v_1) = 3, f(v_2) = 1, f(v_3) = 2 = f(v'_3), f(v_4) = 3 = f(v'_4), f(v_5) = 4, f(v_6) = 2, f(v'_1) = 4, f(v'_2) = 4, f(v'_5) = 1, f(v'_6) = 1$. This proper coloring gives the color dominating vertices as $cdv(1) = v_2, cdv(2) = v_3, cdv(3) = v_4, cdv(4) = v_5, cdv(5) = v$. Thus, $\varphi[S'(W_n)] = 5$.

Case 4. $n = 7$

In this case the graph $S'(W_7)$ contains an odd cycle. Then by Proposition 1.6, $\chi[S'(W_7)] \geqslant 3$. As $m[S'(W_7)] = 7$ and by Lemma 2.1, $\chi[S'(W_7)] = 4$. Then by Proposition 1.5 we have $4 \leqslant \varphi[S'(W_7)] \leqslant 7$.

Suppose $\varphi[S'(W_7)] = 7$. Now consider the color class $\underset{\varsigma}{=}\{1,2,3,4,5,6,7\}$ and define the color function as $f : V \rightarrow \{1,2,3,4,5,6,7\}$ as $f(v) = 7, f(v') = 6, f(v_1) = 5, f(v_2) = 1, f(v_3) = 2, f(v_4) = 3, f(v_5) = 1, f(v_6) = 4, f(v'_1) = 1, f(v'_2) = 4, f(v'_3) = 4, f(v'_4) = 5, f(v'_5) = 5, f(v'_6) = 4$ which in turn forces to assign $f(v_7) = 2, f(v'_7) = 3$. This proper coloring gives the color dominating vertices for color classes $1, 2, 3, 4$ and 5 but not for 6 and 7 which is contradiction to our assumption. Thus, $\varphi[S'(W_7)] \neq 7$.

Suppose that $S'(W_7)$ has b-coloring with 6 colors. Now consider the color class $\underset{\varsigma}{=}\{1,2,3,4,5,6\}$ and define the color function $f : V \rightarrow \{1,2,3,4,5,6\}$ as $f(v) = 6 = f(v'), f(v_1) = 3, f(v_2) = 1, f(v_3) = 2, f(v_4) = 3, f(v_5) = 4, f(v_6) = 2, f(v_7) = 5, f(v'_1) = 4, f(v'_2) = 4, f(v'_3) = 5, f(v'_4) = 5, f(v'_5) = 1, f(v'_6) = 1, f(v'_7) = 5$. This proper coloring gives the color dominating vertices as $cdv(1) = v_2, cdv(2) = v_3, cdv(3) = v_4, cdv(4) = v_5, cdv(5) = v_7, cdv(6) = v$. Thus, $\varphi[S'(W_7)] = 6$.

Case 5. $n \geqslant 9$

For $n = 9$, the graph $S'(W_9)$ contains an odd cycle. Then by Proposition 1.6, $\chi[S'(W_9)] \geqslant 3$. As $m[S'(W_9)] = 7$ and by Lemma 2.1, $\chi[S'(W_7)] = 4$. Then by Proposition 1.5 we have $4 \leqslant \varphi[S'(W_7)] \leqslant 7$.

Suppose $\varphi[S'(W_9)] = 7$. Consider the color class $\underset{s}{=}\{1, 2, 3, 4, 5, 6, 7\}$ and define the color function $f : V \to \{1, 2, 3, 4, 5, 6, 7\}$ as $f(v) = 6, f(v') = 7, f(v_1) = 3, f(v_2) = 1, f(v_3) = 2, f(v_4) = 3, f(v_5) = 4, f(v_6) = 1, f(v'_1) = 4, f(v'_2) = 4, f(v'_3) = 5, f(v'_4) = 5, f(v'_5) = 1, f(v'_6) = 2, f(v_7) = 5, f(v'_7) = 5, f(v_8) = 3, f(v'_8) = 4$ which in turn forces to assign $f(v_9) = 2 = f(v'_9)$. This proper coloring gives the color dominating vertices for color classes $1, 2, 3, 4$ and 5 but not for 6 and 7 which is contradiction to our assumption. Thus, $\varphi[S'(W_9)] \neq 7$.

Suppose that $S'(W_9)$ has b-coloring with 6 colors. Consider the color class $\underset{s}{=}\{1, 2, 3, 4, 5, 6\}$ and define the color function $f : V \to \{1, 2, 3, 4, 5, 6\}$ as $f(v) = 6 = f(v'), f(v_1) = 3, f(v_2) = 1, f(v_3) = 2, f(v_4) = 3, f(v_5) = 4, f(v_6) = 2, f(v_7) = 5, f(v_8) = 3, f(v_9) = 1 = f(v'_9), f(v'_1) = 4, f(v'_2) = 4, f(v'_3) = 5, f(v'_4) = 5, f(v'_5) = 1, f(v'_6) = 1, f(v'_7) = 5, f(v'_8) = 4$. This proper coloring gives the color dominating vertices as $cdv(1) = v_2, cdv(2) = v_3, cdv(3) = v_4, cdv(4) = v_5, cdv(5) = v_7, cdv(6) = v$. Thus, $\varphi[S'(W_9)] = 6$.

For $n > 9$, we repeat the colors as in the above graph $S'(W_9)$for the vertices $\{v_1, v_2, \ldots, v_9, v'_1, v'_2, \ldots, v'_9, v, v'\}$ and for the remaining vertices assign the colors as $f(v) = 6 = f(v'), f(v_{3k+7}) = 1 = f(v'_{3k+7}), f(v_{3k+8}) = 2 = f(v'_{3k+8})$ where $k \in N$. Hence, $\varphi[S'(W_9)] = 6$, for all $n \geqslant 9$. □

Theorem 2.3 *Let W_n be a wheel. Then, $S'(W_n)$ is b-continuous.*

Proof To prove this result we continue with the terminology and notations used in Lemma 2.1 and consider the following cases.

Case 1. $n = 3$

In this case the graph $S'(W_3)$ is b-continuous as $\chi[S'(W_3)] = \varphi[S'(W_3)] = 4$.

Case 2. $n = 4$

In this case the graph $S'(W_4)$ is b-continuous as $\chi[S'(W_4)] = \varphi[S'(W_4)] = 3$.

Case 3. $n = 5$

In this case by Lemma 2.1, $\chi[S'(W_5)] = 4$ and by Theorem-2.2, $\varphi[S'(W_5)] = 5$. Hence, b-coloring exists for every integer satisfying $\chi[S'(W_5)] \leqslant k \leqslant \varphi[S'(W_5)]$(Here $k = 4, 5$). Thus, $S'(W_5)$ is b-continuous.

Case 4. $n = 6$

In this case by Lemma 2.1, $\chi[S'(W_6)] = 3$ and by Theorem-2.2, $\varphi[S'(W_6)] = 5$. It is obvious that b-coloring for the graph $S'(W_6)$ is possible using the number of colors $k = 3, 5$. Now for $k = 4$ the b-coloring for the graph $S'(W_6)$ is as follows.

Consider the color class $\underset{s}{=}\{1, 2, 3, 4\}$ and define the color function $f : V \to \{1, 2, 3, 4\}$ as $f(v) = f(v') = 4, f(v_1) = f(v'_1) = 3, f(v_2) = f(v'_2) = 1, f(v_3) = f(v'_3) = 2, f(v_4) = f(v'_4) = 3, f(v_5) = f(v'_5) = 1, f(v_6) = f(v'_6) = 2$. This proper coloring gives the color dominating vertices as $cdv(1) = v_2, cdv(2) = v_3, cdv(3) = v_4, cdv(4) = v$. Thus, $S'(W_6)$ is four colorable. Hence b-coloring exists for every integer k satisfy $\chi[S'(W_6)] \leqslant k \leqslant \varphi[S'(W_6)]$ (Here $k = 3, 4, 5$). Consequently $S'(W_6)$ is b-continuous.

Case 5. $n = 7$

By Lemma 2.1, $\chi[S'(W_7)] = 4$ and by Theorem 2.2, $\varphi[S'(W_7)] = 6$. It is obvious that b-coloring for the graph $S'(W_7)$ is possible using the number of colors $k = 4, 6$. Now for $k = 5$ the b-coloring for the graph $S'(W_7)$ is as follows.

Consider the color class $\overset{=}{_5}\{1, 2, 3, 4, 5\}$ and define the color function $f : V \rightarrow \{1, 2, 3, 4, 5\}$ as $f(v) = f(v') = 5, f(v_1) = 3, f(v_1') = 4, f(v_2) = 1, f(v_2') = 4, f(v_3) = 2, f(v_3') = 2, f(v_4) = 3, f(v_4') = 1, f(v_5) = 4, f(v_5') = 1, f(v_6) = 2, f(v_6') = 2, f(v_7) = 1 = f(v_7')$. This proper coloring gives the color dominating vertices as $cdv(1) = v_2, cdv(2) = v_3, cdv(3) = v_4, cdv(4) = v_5, cdv(5) = v$. Thus, $S'(W_7)$ is five colorable. Hence, b-coloring exists for every integer k satisfy $\chi[S'(W_7)] \leqslant k \leqslant \varphi[S'(W_7)]$(Here $k = 4, 5, 6$). Hence $S'(W_7)$ is b-continuous.

Case 6. $n = 8$

By Lemma 2.1, $\chi[S'(W_8)] = 3$ and by Theorem 2.2,$\varphi[S'(W_8)] = 5$. It is obvious that b-coloring for the graph $S'(W_8)$ is possible using the number of colors $k = 3, 5$. Now for $k = 4$ the b-coloring for the graph $S'(W_8)$ is as follows.

Consider the color class $\overset{=}{_5}\{1, 2, 3, 4\}$ and define the color function as $f : V \rightarrow \{1, 2, 3, 4\}$ as $f(v) = f(v') = 4, f(v_1) = 3 = f(v_1'), f(v_2) = 1 = f(v_2'), f(v_3) = 2 = f(v_3'), f(v_4) = 3 = f(v_4'), f(v_5) = 1 = f(v_5'), f(v_6) = 2 = f(v_6'), f(v_7) = 1 = f(v_7'), f(v_8) = 2 = f(v_8')$. This proper coloring gives the color dominating vertices as $cdv(1) = v_2, cdv(2) = v_3, cdv(3) = v_4, cdv(4) = v$. Thus, $S'(W_8)$ is four colorable. Hence, b-coloring exists for every integer k satisfy $\chi[S'(W_8)] \leqslant k \leqslant \varphi[S'(W_8)]$(Here $k = 3, 4, 5$). Thus, $S'(W_8)$ is b-continuous.

Case 7. $n \geqslant 9$

For $n = 9$, by Lemma 2.1, $\chi[S'(W_9)] = 4$ and by Theorem 2.2,$\varphi[S'(W_9)] = 6$. It is obvious that b-coloring for the graph $S'(W_9)$ is possible using the number of colors $k = 4, 6$. Now for $k = 5$ the b-coloring for the graph $S'(W_9)$ is as follows.

Consider the color class $\overset{=}{_5}\{1, 2, 3, 4, 5\}$ and define the color function as $f : V \rightarrow \{1, 2, 3, 4, 5\}$ as $f(v) = f(v') = 5, f(v_1) = 3, f(v_1') = 4, f(v_2) = 1, f(v_2') = 4, f(v_3) = 2, f(v_3') = 2, f(v_4) = 3, f(v_4') = 1, f(v_5) = 4, f(v_5') = 1, f(v_6) = 2, f(v_6') = 2, f(v_7) = 1 = f(v_7'), f(v_8) = 2, f(v_8') = 2, f(v_9) = f(v_9') = 1$. This proper coloring gives the color dominating vertices as $cdv(1) = v_2, cdv(2) = v_3, cdv(3) = v_4, cdv(4) = v_5, cdv(5) = v$. Thus, $S'(W_9)$ is five colorable. Hence, b-coloring exists for every integer k satisfy $\chi[S'(W_9)] \leqslant k \leqslant \varphi[S'(W_9)]$(Here $k = 4, 5, 6$). Hence, $S'(W_9)$ is b-continuous.

For odd $n \geqslant 9$, we repeat the colors as in $S'(W_9)$ for the vertices $\{v_1,\ v_2,\ v_9, \ldots, v_1',\ v_2', \ldots, v'_9,\ v,\ v'\}$ and for the remaining vertices gives the colors as follows:

When $k = 5,\ f(v') = f(v) = 5,\ f(v_{3k+7}) = f(v'_{3k+7}) = 1,\ f(v_{3k+8}) = f(v'_{3k+8}) = 2,\ k \in N$.

For even $n > 9$, we repeat the color assignment as in case $n = 8$ discussed above for the vertices $\{v,\ v',\ v_1, \ldots, v_8, v_1', v_2', \ldots, v_8'\}$ and for remaining vertices gives the colors as follows:

When $k = 4$, $f(v') = f(v) = 4$, $f(v_{2k+7}) = 1 = f(v'_{2k+7})$, $f(v_{2k+8}) = 2 = f(v'_{2k+8}), k \in N$ and when $k = 5$, $f(v') = f(v) = 5$, $f(v_{2k+8}) = 1 = f(v'_{2k+8})$, $f(v_{2k+9}) = 2 = f(v'_{2k+9}), k \in N$. □

Any coloring with $\chi(G)$ is a b-coloring, we state the following obvious result.

Corollary 2.4 *Let W_n be a wheel. Then*

$$S_b[S'(W_n)] = \begin{cases} \{4\}, & n = 3 \\ \{3\}, & n = 4 \\ \{4,5\} & n = 5 \\ \{3,4,5\}, & n = 6, 8 \\ \{4,5,6\}, & n = 7 \\ \{4,5,6\} & for\ odd\ n \geqslant 9 \\ \{3,4,5\} & for\ even\ n > 9 \end{cases}$$

§3. Concluding Remarks

A discussion about b-coloring of wheel is carried out by Alkhateeb [9] while we investigate b-chromatic number of splitting graph of wheel. We also obtain b-spectrum and show that splitting graph of wheel is b-continuous.

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