

Smarandache Seminormal Subgroupoids

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Abstract: In this paper, we define Smarandache seminormal subgroupoids. We have proved some results for finding the Smarandache seminormal subgroupoids in $Z(n)$ when n is even and n is odd.

Key Words: Groupoids, Smarandache groupoids, Smarandache seminormal subgroupoids

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§1. Introduction

In [5] and [6], W.B.Kandasamy defined new classes of Smarandache groupoids using Z_n . In this paper we define and prove some theorems for construction of Smarandache seminormal subgroupoids according as n is even or odd.

Definition 1.1 A non-empty set of elements G is said to form a groupoid if in G is defined a binary operation called the product, denoted by $*$ such that $a * b \in G \quad \forall a, b \in G$. We denote groupoids by $(G, *)$.

Definition 1.2 Let $(G, *)$ be a groupoid. A proper subset $H \subset G$ is a subgroupoid if $(H, *)$ is itself a groupoid.

Definition 1.3 Let S be a non-empty set. S is said to be a semigroup if on S is defined a binary operation $*$ such that

- (1) for all $a, b \in S$ we have $a * b \in S$;
- (2) for all $a, b, c \in S$ we have $a * (b * c) = (a * b) * c$.

$(S, *)$ is a semi-group.

Definition 1.4 A Smarandache groupoid G is a groupoid which has a proper subset S such that S under the operation of G is a semigroup.

Definition 1.5 Let $(G, *)$ be a Smarandache groupoid. A non-empty subgroupoid H of G is said

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to be a Smarandache subgroupoid if H contains a proper subset K such that K is a semigroup under the operation $*$.

Definition 1.6 Let G be a Smarandache groupoid. V be a Smarandache subgroupoid of G . We say V is a Smarandache seminormal subgroupoid if $aV = V$ for all $a \in G$ or $Va = V$ for all $a \in G$.

For example, let $(G, *)$ be groupoid given by the following table:

$*$	a_0	a_1	a_2	a_3	a_4	a_5
a_0	a_0	a_3	a_0	a_3	a_0	a_3
a_1	a_2	a_5	a_2	a_5	a_2	a_5
a_2	a_4	a_1	a_4	a_1	a_4	a_1
a_3	a_0	a_3	a_0	a_3	a_0	a_3
a_4	a_2	a_5	a_2	a_5	a_2	a_5
a_5	a_4	a_1	a_4	a_1	a_4	a_1

It is a Smarandache groupoid as $\{a_3\}$ is a semigroup. $V = \{a_1, a_3, a_5\}$ is a Smarandache subgroupoid, also $aV = V$. Therefore V is Smarandache seminormal subgroupoid in G .

Definition 1.7 Let $Z_n = \{0, 1, \dots, n-1\}$, $n \geq 3$ and $a, b \in Z_n \setminus \{0\}$. Define a binary operation $*$ on Z_n as follows:

$a * b = ta + ub \pmod{n}$, where t, u are two distinct elements in $Z_n \setminus \{0\}$ and $(t, u) = 1$. Here '+' is the usual addition of two integers and 'ta' means the product of the two integers t and a .

Elements of Z_n form a groupoid with respect to the binary operation $*$. We denote these groupoid by $\{Z_n(t, u), *\}$ or $Z_n(t, u)$ for fixed integer n and varying $t, u \in Z_n \setminus \{0\}$ such that $(t, u) = 1$. Thus we define a collection of groupoids $Z(n)$ as follows
 $Z(n) = \{\{Z_n(t, u), *\} \mid \text{for integers } t, u \in Z_n \setminus \{0\} \text{ such that } (t, u) = 1\}$.

§2. Smarandache Seminormal Subgroupoids When $n \equiv 0 \pmod{2}$

When n is even we are interested in finding Smarandache seminormal subgroupoid in $Z_n(t, t+1)$.

Theorem 2.1 Let $Z_n(t, t+1) \in Z(n)$, n is even, $n > 3$ and $t = 1, \dots, n-2$. Then $Z_n(t, t+1)$ is Smarandache groupoid.

Proof Let $x = \frac{n}{2}$. Then

$$\begin{aligned} x * x &= xt + x(t+1) = 2xt + x \\ &= (2t+1)x \equiv x \pmod{n} \end{aligned}$$

Consequently, $\{x\}$ is a semigroup in $Z_n(t, t+1)$. Thus $Z_n(t, t+1)$ is a Smarandache groupoid when n is even. $\square$

Remark In the above theorem we can also show that beside $\{n/2\}$ the other semigroup is $\{0, n/2\}$ in $Z_n(t, t+1) \in Z(n)$.

Proof If t is even, $0*t + \frac{n}{2}*(t+1) \equiv \frac{n}{2} \pmod{n}$, $\frac{n}{2}*t + 0*(t+1) \equiv 0 \pmod{n}$, $\frac{n}{2}*t + \frac{n}{2}*(t+1) \equiv \frac{n}{2} \pmod{n}$ and $0*t + 0*(t+1) \equiv 0 \pmod{n}$. So $\{0, \frac{n}{2}\}$ is semigroup in $Z_n(t, t+1)$. If t is odd, $0*t + \frac{n}{2}*(t+1) \equiv 0 \pmod{n}$, $\frac{n}{2}*t + 0*(t+1) \equiv \frac{n}{2} \pmod{n}$, $\frac{n}{2}*t + \frac{n}{2}*(t+1) \equiv \frac{n}{2} \pmod{n}$ and $0*t + 0*(t+1) \equiv 0 \pmod{n}$. So $\{0, \frac{n}{2}\}$ is a semigroup in $Z_n(t, t+1)$. $\square$

Theorem 2.2 Let $n > 3$ be even and $t = 1, \dots, n-2$,

- (1) If $\frac{n}{2}$ is even then $A_0 = \{0, 2, \dots, n-2\} \subseteq Z_n$ is Smarandache subgroupoid in $Z_n(t, t+1) \in Z(n)$.
- (2) If $\frac{n}{2}$ is odd then $A_1 = \{1, 3, \dots, n-1\} \subseteq Z_n$ is Smarandache subgroupoid in $Z_n(t, t+1) \in Z(n)$.

Proof (1) Let $\frac{n}{2}$ is even. $\Rightarrow \frac{n}{2} \in A_0$. We will show that A_0 is subgroupoid. Let $x_i, x_j \in A_0$ and $x_i \neq x_j$. Then

$$\begin{aligned} x_i * x_j &= x_i t + x_j(t+1) \\ &= (x_i + x_j)t + x_j \equiv x_k \pmod{n} \end{aligned}$$

for some $x_k \in A_0$ as $(x_i + x_j)t + x_j$ is even. So $x_i * x_j \in A_0$. Thus A_0 is subgroupoid in $Z_n(t, t+1)$.

Let $x = \frac{n}{2}$. Then

$$\begin{aligned} x * x &= xt + x(t+1) \\ &= (2t+1)x \equiv x \pmod{n}. \end{aligned}$$

Therefore, $\{x\}$ is a semigroup in A_0 . Thus A_0 is a subgroupoid in $Z_n(t, t+1)$.

- (2) Let $\frac{n}{2}$ is odd. $\Rightarrow \frac{n}{2} \in A_1$. We show that A_1 is subgroupoid.

Let $x_i, x_j \in A_1$ and $x_i \neq x_j$. Then

$$\begin{aligned} x_i * x_j &= x_i t + x_j(t+1) \\ &= (x_i + x_j)t + x_j \equiv x_k \pmod{n} \end{aligned}$$

for some $x_k \in A_1$ as $(x_i + x_j)t + x_j$ is odd. Therefore, $x_i * x_j \in A_1$. Thus A_1 is subgroupoid in $Z_n(t, t+1)$.

Let $x = \frac{n}{2}$. Then

$$\begin{aligned} x * x &= xt + x(t+1) \\ &= (2t+1)x \equiv x \pmod{n}. \end{aligned}$$

So $\{x\}$ is a semigroup in A_1 . Thus A_1 is a Smarandache subgroupoid in $Z_n(t, t+1)$. $\square$

Theorem 2.3 Let $n > 3$ be even and $t = 1, \dots, n-2$,

(1) If $\frac{n}{2}$ is even then $A_0 = \{0, 2, \dots, n-2\} \subseteq Z_n$ is Smarandache seminormal subgroupoid of $Z_n(t, t+1) \in Z(n)$.

(2) If $\frac{n}{2}$ is odd then $A_1 = \{1, 3, \dots, n-1\} \subseteq Z_n$ is Smarandache seminormal subgroupoid of $Z_n(t, t+1) \in Z(n)$.

Proof By Theorem 2.1, $Z_n(t, t+1)$ is a Smarandache groupoid.

(1) Let $\frac{n}{2}$ is even. Then by Theorem 2.2, $A_0 = \{0, 2, \dots, n-2\}$ is Smarandache subgroupoid of $Z_n(t, t+1)$. Now we show that either $aA_0 = A_0$ or $A_0a = A_0 \forall a \in Z_n = \{0, 1, 2, \dots, n-1\}$.

Case 1 t is even.

Let $a_i \in A_0$ and $a \in Z_n = \{0, 1, 2, \dots, n-1\}$. Then

$$\begin{aligned} a * a_i &= at + a_i(t+1) \\ &\equiv a_j \pmod{n} \end{aligned}$$

for some $a_j \in A_0$ as $at + a_i(t+1)$ is even. Therefore, $a * a_i \in A_0 \forall a_i \in A_0$, $aA_0 = A_0$. Thus, A_0 is a Smarandache seminormal subgroupoid in $Z_n(t, t+1)$.

Case 2 t is odd.

Let $a_i \in A_0$ and $a \in Z_n = \{0, 1, 2, \dots, n-1\}$. Then

$$\begin{aligned} a_i * a &= a_it + a(t+1) \\ &\equiv a_j \pmod{n} \end{aligned}$$

for some $a_j \in A_0$ as $a_it + a(t+1)$ is even. Therefore, $a_i * a \in A_0 \forall a_i \in A_0$, $A_0a = A_0$. Thus A_0 is a Smarandache seminormal subgroupoid in $Z_n(t, t+1)$.

(2) Let $\frac{n}{2}$ is odd. Then by Theorem 2.2, $A_1 = \{1, 3, 5, \dots, n-1\}$ is Smarandache subgroupoid of $Z_n(t, t+1)$. Now we show that either $aA_1 = A_1$ or $A_1a = A_1 \forall a \in Z_n = \{0, 1, 2, \dots, n-1\}$.

Case 1 t is even.

Let $a_i \in A_1$ and $a \in Z_n = \{0, 1, 2, \dots, n-1\}$. Then

$$\begin{aligned} a * a_i &= at + a_i(t+1) \\ &= (a + a_i)t + a_i \\ &\equiv a_j \pmod{n} \end{aligned}$$

for some $a_j \in A_1$ as $(a + a_i)t + a_i$ is odd. Therefore, $a * a_i \in A_1 \forall a_i \in A_1$, $\therefore aA_1 = A_1$. Thus A_1 is Smarandache seminormal subgroupoid in $Z_n(t, t+1)$.

Case 2 t is odd.

Let $a_i \in A_1$ and $a \in Z_n = \{0, 1, 2, \dots, n-1\}$. Then

$$\begin{aligned} a_i * a &= a_it + a(t+1) \\ &\equiv a_j \pmod{n} \end{aligned}$$

for some $a_j \in A_1$ as $a_i t + a(t+1)$ is odd. Therefore, $a_i * a \in A_1 \forall a_i \in A_1, A_1 a = A_1$.
Thus A_1 is Smarandache seminormal subgroupoid in $Z_n(t, t+1)$. $\square$

By the above theorem we can determine the Smarandache seminormal subgroupoid in $Z_n(t, t+1)$ of $Z(n)$ when n is even and $n > 3$.

n	$n/2$	t	$Z_n(t, t+1)$	Smarandache seminormal subgroupoid in $Z_n(t, t+1)$
4	2	1	$Z_4(1, 2)$	{0, 2}
		2	$Z_4(2, 3)$	
6	3	1	$Z_6(1, 2)$	{1, 3, 5}
		2	$Z_6(2, 3)$	
		3	$Z_6(3, 4)$	
		4	$Z_6(4, 5)$	
8	4	1	$Z_8(1, 2)$	{0, 2, 4, 6}
		2	$Z_8(2, 3)$	
		3	$Z_8(3, 4)$	
		4	$Z_8(4, 5)$	
		5	$Z_8(5, 6)$	
		6	$Z_8(6, 7)$	
10	5	1	$Z_{10}(1, 2)$	{1, 3, 5, 7, 9}
		2	$Z_{10}(2, 3)$	
		3	$Z_{10}(3, 4)$	
		4	$Z_{10}(4, 5)$	
		5	$Z_{10}(5, 6)$	
		6	$Z_{10}(6, 7)$	
		7	$Z_{10}(7, 8)$	
		8	$Z_{10}(8, 9)$	
12	6	1	$Z_{12}(1, 2)$	{0, 2, 4, 6, 8}
		2	$Z_{12}(2, 3)$	
		3	$Z_{12}(3, 4)$	
		4	$Z_{12}(4, 5)$	
		5	$Z_{12}(5, 6)$	
		6	$Z_{12}(6, 7)$	
		7	$Z_{12}(7, 8)$	
		8	$Z_{12}(8, 9)$	
		9	$Z_{12}(9, 10)$	
		10	$Z_{12}(10, 11)$	

§3. Smarandache Seminormal Subgroupoids Depend on t, u when $n \equiv 0 \pmod{2}$

When n is even we are interested in finding Smarandache seminormal subgroupoid in $Z_n(t, u) \in Z(n)$ when t is even and u is odd or when t is odd and u is even.

Theorem 3.1 *Let $Z_n(t, u) \in Z(n)$, if n is even, $n > 3$ and for each $t, u \in Z_n$, if one is even and other is odd then $Z_n(t, u)$ is Smarandache groupoid.*

Proof Let $x = \frac{n}{2}$. Then

$$\begin{aligned} x * x &= xt + xu \\ &= (t + u)x \equiv x \pmod{n}. \end{aligned}$$

So $\{x\}$ is a semigroup in $Z_n(t, u)$. Thus $Z_n(t, u)$ is a Smarandache groupoid when n is even. $\square$

Remark In the above theorem we can also show that beside $\{n/2\}$ the other semigroup is $\{0, n/2\}$ in $Z_n(t, u) \in Z(n)$.

Proof If t is even and u is odd, $0 * t + \frac{n}{2} * u \equiv \frac{n}{2} \pmod{n}$, $\frac{n}{2} * t + 0 * u \equiv 0 \pmod{n}$, $\frac{n}{2} * t + \frac{n}{2} * u \equiv \frac{n}{2} \pmod{n}$ and $0 * t + 0 * u \equiv 0 \pmod{n}$. So $\{0, \frac{n}{2}\}$ is semigroup in $Z_n(t, u)$. If t is odd and u is even, $0 * t + \frac{n}{2} * u \equiv 0 \pmod{n}$, $\frac{n}{2} * t + 0 * u \equiv \frac{n}{2} \pmod{n}$, $\frac{n}{2} * t + \frac{n}{2} * u \equiv \frac{n}{2} \pmod{n}$ and $0 * t + 0 * u \equiv 0 \pmod{n}$. So $\{0, \frac{n}{2}\}$ is semigroup in $Z_n(t, u)$. $\square$

Theorem 3.2 *Let $n > 3$ be even and $t, u \in Z_n$.*

(1) *If $\frac{n}{2}$ is even then $A_0 = \{0, 2, \dots, n-2\} \subseteq Z_n$ is Smarandache subgroupoid of $Z_n(t, u) \in Z(n)$ when one of t and u is odd and other is even.*

(2) *If $\frac{n}{2}$ is odd then $A_1 = \{1, 3, \dots, n-1\} \subseteq Z_n$ is Smarandache subgroupoid of $Z_n(t, u) \in Z(n)$ when one of t and u is odd and other is even.*

Proof (1) Let $\frac{n}{2}$ be even. $\Rightarrow \frac{n}{2} \in A_0$. We show that A_0 is subgroupoid.

Let $x_i, x_j \in A_0$ and $x_i \neq x_j$. Then

$$x_i * x_j = x_i t + x_j u \equiv x_k \pmod{n}$$

for some $x_k \in A_0$ as $x_i t + x_j u$ is even. So $x_i * x_j \in A_0$. Thus A_0 is a subgroupoid in $Z_n(t, u)$.

Let $x = \frac{n}{2}$. Then

$$\begin{aligned} x * x &= xt + xu \\ &= x(t + u) \equiv x \pmod{n}. \end{aligned}$$

Whence, $\{x\}$ is a semigroup in A_0 . Thus, A_0 is a Smarandache subgroupoid in $Z_n(t, u)$.

(2) Let $\frac{n}{2}$ be odd. $\Rightarrow \frac{n}{2} \in A_1$. We show that A_1 is subgroupoid.

Let $x_i, x_j \in A_1$ and $x_i \neq x_j$. Then

$$x_i * x_j = x_i t + x_j u \equiv x_k \pmod{n}$$

for some $x_k \in A_1$ as $x_i + x_j u$ is odd. So $x_i * x_j \in A_1$. Consequently, A_1 is subgroupoid in $Z_n(t, u)$.

Let $x = \frac{n}{2}$. Then

$$\begin{aligned} x * x &= xt + xu \\ &= x(t + u) \equiv x \pmod{n}. \end{aligned}$$

So $\{x\}$ is a semigroup in A_1 . Thus A_1 is a Smarandache subgroupoid in $Z_n(t, u)$. $\square$

Theorem 3.3 *Let $n > 3$ be even and $t = 1, \dots, n - 2$.*

- (1) *If $\frac{n}{2}$ is even then $A_0 = \{0, 2, \dots, n - 2\} \subseteq Z_n$ is Smarandache seminormal subgroupoid of $Z_n(t, u) \in Z(n)$ when one of t and u is odd and other is even;*
- (2) *If $\frac{n}{2}$ is odd then $A_1 = \{1, 3, \dots, n - 1\} \subseteq Z_n$ is Smarandache seminormal subgroupoid of $Z_n(t, u) \in Z(n)$ when one of t and u is odd and other is even.*

Proof By Theorem 3.1, $Z_n(t, u)$ is a Smarandache groupoid.

(1) Let $\frac{n}{2}$ is even. Then by Theorem 3.2, $A_0 = \{0, 2, \dots, n - 2\}$ is Smarandache subgroupoid of $Z_n(t, u)$. Now we show that either $aA_0 = A_0$ or $A_0a = A_0 \forall a \in Z_n = \{0, 1, 2, \dots, n - 1\}$.

Case 1 t is even and u is odd.

Let $a_i \in A_0$ and $a \in Z_n = \{0, 1, 2, \dots, n - 1\}$. Then

$$\begin{aligned} a * a_i &= at + a_i u \\ &\equiv a_j \pmod{n} \end{aligned}$$

for some $a_j \in A_0$ as $at + a_i u$ is even. Whence, $a * a_i \in A_0 \forall a_i \in A_0$, $aA_0 = A_0$. Thus, A_0 is a Smarandache seminormal subgroupoid in $Z_n(t, u)$.

Case 2 t is odd and u is even.

Let $a_i \in A_0$ and $a \in Z_n = \{0, 1, 2, \dots, n - 1\}$. Then

$$\begin{aligned} a_i * a &= a_i t + au \\ &\equiv a_j \pmod{n} \end{aligned}$$

for some $a_j \in A_0$ as $a_i t + au$ is even. Therefore, $a_i * a \in A_0 \forall a_i \in A_0$, $A_0 a = A_0$. Thus, A_0 is Smarandache seminormal subgroupoid in $Z_n(t, u)$.

(2) Let $\frac{n}{2}$ is odd then by Theorem 3.2 is $A_1 = \{1, 3, 5, \dots, n - 1\}$ is Smarandache subgroupoid of $Z_n(t, u)$. We show that either $aA_1 = A_1$ or $A_1a = A_1 \forall a \in Z_n = \{0, 1, 2, \dots, n - 1\}$.

Case 1 t is even and u is odd.

Let $a_i \in A_1$ and $a \in Z_n = \{0, 1, 2, \dots, n - 1\}$. Then

$$\begin{aligned} a * a_i &= at + a_i u \\ &\equiv a_j \pmod{n} \end{aligned}$$

for some $a_j \in A_1$ as $at + a_i u$ is odd. So, $a * a_i \in A_1 \forall a_i \in A_1, \therefore aA_1 = A_1$. Thus, A_1 is a Smarandache seminormal subgroupoid in $Z_n(t, u)$.

Case 2 t is odd and u is even.

Let $a_i \in A_1$ and $a \in Z_n = \{0, 1, 2, \dots, n-1\}$.

$$\begin{aligned} a_i * a &= a_i t + au \\ &\equiv a_j \pmod{n} \end{aligned}$$

for some $a_j \in A_1$ as $a_i t + au$ is odd. Therefore, $a_i * a \in A_1 \forall a_i \in A_1, A_1 a = A_1$. Thus, A_1 is a Smarandache seminormal subgroupoid in $Z_n(t, u)$. $\square$

By the above theorem we can determine Smarandache seminormal subgroupoid in $Z_n(t, u) \in Z(n)$ for $n > 3$, when n is even and when one of t and u is odd and other is even.

n	$n/2$	t	$Z_n(t, u)$	Smarandache seminormal subgroupoid
4	2	1	$Z_4(1, 2)$	{0, 2}
		2	$Z_4(2, 3)$	
6	3	1	$Z_6(1, 2), Z_6(1, 4)$	{1, 3, 5}
		2	$Z_6(2, 1), Z_6(2, 3), Z_6(2, 5)$	
		3	$Z_6(3, 2), Z_6(3, 4)$	
		4	$Z_6(4, 1), Z_6(4, 3), Z_6(4, 5)$	
		5	$Z_6(5, 2), Z_6(5, 4)$	
8	4	1	$Z_8(1, 2), Z_8(1, 4), Z_8(1, 6)$	{0, 2, 4, 6}
		2	$Z_8(2, 1), Z_8(2, 3), Z_8(2, 5),$ $Z_8(2, 7)$	
		3	$Z_8(3, 2), Z_8(3, 4)$	
		4	$Z_8(4, 1), Z_8(4, 3), Z_8(4, 5),$ $Z_8(4, 7)$	
		5	$Z_8(5, 2), Z_8(5, 4), Z_8(5, 6)$	
		6	$Z_8(6, 1), Z_8(6, 5), Z_8(6, 7),$	
		7	$Z_8(7, 2), Z_8(7, 4), Z_8(7, 6),$	

n	$n/2$	t	$Z_n(t, u)$	Smarandache seminormal subgroupoid
10	5	1	$Z_{10}(1, 2), Z_{10}(1, 4), Z_{10}(1, 6),$ $Z_{10}(1, 8)$	$\{1, 3, 5, 7, 9\}$
		2	$Z_{10}(2, 1), Z_{10}(2, 3), Z_{10}(2, 5),$ $Z_{10}(2, 7), Z_{10}(2, 9)$	
		3	$Z_{10}(3, 2), Z_{10}(3, 4), Z_{10}(3, 8),$	
		4	$Z_{10}(4, 1), Z_{10}(4, 3), Z_{10}(4, 5),$ $Z_{10}(4, 7), Z_{10}(4, 9)$	
		5	$Z_{10}(5, 2), Z_{10}(5, 4), Z_{10}(5, 6),$ $Z_{10}(5, 8)$	
		6	$Z_{10}(6, 1), Z_{10}(6, 5), Z_{10}(6, 7),$	
		7	$Z_{10}(7, 2), Z_{10}(7, 4), Z_{10}(7, 6),$ $Z_{10}(7, 8)$	
		8	$Z_{10}(8, 1), Z_{10}(8, 3), Z_{10}(8, 5),$ $Z_{10}(8, 7), Z_{10}(8, 9)$	
		9	$Z_{10}(9, 2), Z_{10}(9, 4), Z_{10}(9, 8)$	
12	6	1	$Z_{12}(1, 2), Z_{12}(1, 4), Z_{12}(1, 6),$ $Z_{12}(1, 8), Z_{12}(1, 10)$	$\{0, 2, 4, 6, 8, 10\}$
		2	$Z_{12}(2, 1), Z_{12}(2, 3), Z_{12}(2, 5),$ $Z_{12}(2, 7), Z_{12}(2, 9), Z_{12}(2, 11)$	
		3	$Z_{12}(3, 2), Z_{12}(3, 4), Z_{12}(3, 8),$ $Z_{12}(3, 10)$	
		4	$Z_{12}(4, 1), Z_{12}(4, 3), Z_{12}(4, 5),$ $Z_{12}(4, 7), Z_{12}(4, 9), Z_{12}(4, 11)$	
		5	$Z_{12}(5, 2), Z_{12}(5, 4), Z_{12}(5, 6),$ $Z_{12}(5, 8)$	
		6	$Z_{12}(6, 1), Z_{12}(6, 3), Z_{12}(6, 5),$ $Z_{12}(6, 7), Z_{12}(6, 11)$	
		7	$Z_{12}(7, 2), Z_{12}(7, 4), Z_{12}(7, 6),$ $Z_{12}(7, 8), Z_{12}(7, 10)$	
		8	$Z_{12}(8, 1), Z_{12}(8, 3), Z_{12}(8, 5),$ $Z_{12}(8, 7), Z_{12}(8, 9), Z_{12}(8, 11)$	
		9	$Z_{12}(9, 2), Z_{12}(9, 4), Z_{12}(9, 8),$ $Z_{12}(9, 10)$	
		10	$Z_{12}(10, 1), Z_{12}(10, 3), Z_{12}(10, 7),$ $Z_{12}(10, 9), Z_{12}(10, 11)$	
		11	$Z_{12}(11, 2), Z_{12}(11, 4), Z_{12}(11, 6),$ $Z_{12}(11, 8), Z_{12}(11, 10)$	

§4. Smarandache Seminormal Subgroupoids When $n \equiv 1 \pmod{2}$

When n is odd we are interested in finding Smarandache seminormal subgroupoid in $Z_n(t, u) \in Z(n)$. We have proved the similiar result in [4].

Theorem 4.1 *Let $Z_n(t, u) \in Z(n)$. If n is odd, $n > 4$ and for each $t = 2, \dots, \frac{n-1}{2}$ and $u = n - (t-1)(t, u) = 1$, then $Z_n(t, u)$ is a Smarandache groupoid.*

Proof Let $x \in \{0, \dots, n-1\}$. Then

$$x * x = xt + xu = (n+1)x \equiv x \pmod{n}.$$

So $\{x\}$ is semigroup in Z_n . Thus $Z_n(t, u)$ is a Smarandache groupoid in $Z(n)$. $\square$

Remark We note that all $\{x\}$ where $x \in \{1, \dots, n-1\}$ are proper subsets which are semigroups in $Z_n(t, u)$.

Theorem 4.2 *Let $n > 4$ be odd and $t = 2, \dots, \frac{n-1}{2}$ and $u = n - (t-1)$ such that $(t, u) = 1$ if $s = (n, t)$ or $s = (n, u)$ then $A_k = \{k, k+s, \dots, k+(r-1)s\}$ for $k = 0, 1, \dots, s-1$ where $r = \frac{n}{s}$ is a Smarandache subgroupoid in $Z_n(t, u) \in Z(n)$.*

Proof Let $x_p, x_q \in A_k$. Then

$$x_p \neq x_q \Rightarrow \left. \begin{array}{l} x_p = k + ps \\ x_q = k + qs \end{array} \right\} p, q \in \{0, 1, \dots, r-1\}.$$

Also,

$$\begin{aligned} x_p * x_q &= x_p t + x_q u \\ &= (k + ps)t + (k + qs)(n - (t-1)) \\ &= k(n+1) + ((p-q)t + q(n+1))s \\ &\equiv (k + ls) \pmod{n} \\ &\equiv x_l \pmod{n} \end{aligned}$$

$x_l \in A_k$ as $x_l = k + ls$ for some $l \in \{0, 1, \dots, r-1\}$. Whence, $x_p * x_q \in A_k$. Consequently, A_k is a subgroupoid in $Z_n(t, u)$. By the above remark all singleton sets are semigroup. Thus, A_k is a Smarandache subgroupoid. $\square$

Theorem 4.3 *Let $n > 4$ be odd and $t = 2, \dots, \frac{n-1}{2}$ and $u = n - (t-1)$ such that $(t, u) = 1$ if $s = (n, t)$ or $s = (n, u)$ then $A_k = \{k, k+s, \dots, k+(r-1)s\}$ for $k = 0, 1, \dots, s-1$ where $r = \frac{n}{s}$ is a Smarandache seminormal subgroupoid in $Z_n(t, u) \in Z(n)$.*

Proof By Theorem 4.1, $Z_n(t, u)$ is a Smarandache groupoid. Also by Theorem 4.2, $A_k = \{k, k+s, \dots, k+(r-1)s\}$ for $k = 0, 1, \dots, s-1$ is Smarandache subgroupoid of $Z_n(t, u)$.

If $s = (n, t)$, let $x_p \in A_k$ and $a \in Z_n = \{0, 1, 2, \dots, n-1\}$. Then

$$\begin{aligned}
 a * x_p &= at + x_p u \\
 &= at + (k + ps)(n - t + 1) \\
 &= k(n + 1) + [(a - k)v_1 + (pn - pt + p)]s \text{ where } t = v_1 s \\
 &\equiv k + ls \pmod{n}
 \end{aligned}$$

$x_l \in A_k$ as $x_l = k + ls$ for some $l \in \{0, 1, \dots, r-1\}$. So, $a * x_p \in A_k$, $a * A_k = A_k$. Thus, A_k is a Smarandache seminormal subgroupoid in $Z_n(t, u)$.

If $s = (n, u)$, let $x_p \in A_k$ and $a \in Z_n = \{0, 1, 2, \dots, n-1\}$. Then

$$\begin{aligned}
 x_p * a &= x_p t + au \\
 &= (k + ps)(n - u + 1) + au \\
 &= k(n + 1) + [(a - k)v_2 + (pn - pu + p)]s \text{ where } t = v_2 s \\
 &\equiv (k + ls) \pmod{n}
 \end{aligned}$$

$x_l \in A_k$ as $x_l = k + ls$ for some $l \in \{0, 1, \dots, r-1\}$. Therefore, $a * x_p \in A_k$, $a * A_k = A_k$. Thus A_k is a Smarandache seminormal subgroupoid in $Z_n(t, u)$. $\square$

By the above theorem we can determine Smarandache seminormal subgroupoid in $Z_n(t, u)$ when n is odd and $n > 4$.

n	t	u	$Z_n(t, u)$	$s = (n, u)$ or $s = (n, t)$	$r = n/s$	Smarandache seminormal subgroupoid in $Z_n(t, u)$
9	3	7	$Z_9(3, 7)$	$3 = (9, 3)$	3	$A_0 = \{0, 3, 6\}$
						$A_1 = \{1, 4, 7\}$
						$A_2 = \{2, 5, 8\}$
15	3	13	$Z_{15}(3, 13)$	$3 = (15, 3)$	5	$A_0 = \{0, 3, 6, 9, 12\}$
						$A_1 = \{1, 4, 7, 10, 13\}$
						$A_2 = \{2, 5, 8, 11, 14\}$
	5	11	$Z_{15}(5, 11)$	$5 = (15, 5)$	3	$A_0 = \{0, 5, 10\}$
						$A_1 = \{1, 6, 11\}$
						$A_2 = \{2, 7, 12\}$
						$A_3 = \{3, 8, 13\}$
						$A_4 = \{4, 9, 14\}$
	7	9	$Z_{15}(7, 9)$	$3 = (15, 9)$	5	$A_0 = \{0, 3, 6, 9, 12\}$
						$A_1 = \{1, 4, 7, 10, 13\}$
						$A_2 = \{2, 5, 8, 11, 14\}$

n	t	u	$Z_n(t, u)$	$s = (n, u)$ or $s = (n, t)$	$r = n/s$	Smarandache seminormal subgroupoid in $Z_n(t, u)$
21	3	19	$Z_{21}(3, 19)$	$3 = (21, 3)$	7	$A_0 = \{0, 3, 6, 9, 12, 15, 18\}$
						$A_1 = \{1, 4, 7, 10, 13, 16, 19\}$
						$A_2 = \{2, 5, 8, 11, 14, 17, 20\}$
	7	15	$Z_{21}(7, 15)$	$7 = (21, 7)$	3	$A_0 = \{0, 7, 14\}$
						$A_1 = \{1, 8, 15\}$
						$A_2 = \{2, 9, 16\}$
						$A_3 = \{3, 10, 17\}$
						$A_4 = \{4, 11, 18\}$
						$A_5 = \{5, 12, 19\}$
						$A_6 = \{6, 13, 14\}$
				$3 = (21, 15)$	7	$A_0 = \{0, 3, 6, 9, 12, 15, 18\}$
						$A_1 = \{1, 4, 7, 10, 13, 16, 19\}$
						$A_2 = \{2, 5, 8, 11, 14, 17, 20\}$
	9	13	$Z_{21}(9, 13)$	$3 = (21, 9)$	7	$A_0 = \{0, 3, 6, 9, 12, 15, 18\}$
						$A_1 = \{1, 4, 7, 10, 13, 16, 19\}$
						$A_2 = \{2, 5, 8, 11, 14, 17, 20\}$

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