

Super Mean Labeling of Some Classes of Graphs

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Abstract: Let G be a (p, q) graph and $f : V(G) \rightarrow \{1, 2, 3, \dots, p + q\}$ be an injection. For each edge $e = uv$, let $f^*(e) = (f(u) + f(v))/2$ if $f(u) + f(v)$ is even and $f^*(e) = (f(u) + f(v) + 1)/2$ if $f(u) + f(v)$ is odd. Then f is called a super mean labeling if $f(V) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, 3, \dots, p + q\}$. A graph that admits a super mean labeling is called a super mean graph. In this paper we prove that $S(P_n \odot K_1)$, $S(P_2 \times P_4)$, $S(B_{n,n})$, $\langle B_{n,n} : P_m \rangle$, $C_n \odot \overline{K_2}$, $n \geq 3$, generalized antiprism $\mathcal{A}_n^m$ and the double triangular snake $D(T_n)$ are super mean graphs.

Key Words: Smarandachely super m -mean labeling, Smarandachely super m -mean graph, super mean labeling, super mean graph.

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§1. Introduction

By a graph we mean a finite, simple and undirected one. The vertex set and the edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. The disjoint union of two graphs G_1 and G_2 is the graph $G_1 \cup G_2$ with $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$. The disjoint union of m copies of the graph G is denoted by mG . The corona $G_1 \odot G_2$ of the graphs G_1 and G_2 is obtained by taking one copy of G_1 (with p vertices) and p copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2 . Armed crown $C_n \Theta P_m$ is a graph obtained from a cycle C_n by identifying the pendent vertex of a path P_m at each vertex of the cycle. Bi-armed crown is a graph obtained from a cycle C_n by identifying the pendant vertices of two vertex disjoint paths of equal length $m - 1$ at each vertex of the cycle. We denote a bi-armed crown by $C_n \Theta 2P_m$, where P_m is a path of length $m - 1$. The double triangular snake $D(T_n)$ is the graph obtained from the path $v_1, v_2, v_3, \dots, v_n$ by joining v_i and v_{i+1} with two new vertices i_i and w_i for $1 \leq i \leq n - 1$. The bistar $B_{m,n}$ is a graph obtained from

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K_2 by joining m pendant edges to one end of K_2 and n pendant edges to the other end of K_2 . The generalized prism graph $C_n \times P_m$ has the vertex set $V = \{v_i^j : 1 \leq i \leq n \text{ and } 1 \leq j \leq m\}$ and the edge set $E = \{v_i^j v_{i+1}^j, v_n^j v_1^j : 1 \leq i \leq n-1 \text{ and } 1 \leq j \leq m\} \cup \{v_i^j v_{i-1}^{j+1}, v_1^j v_n^{j+1} : 2 \leq i \leq n \text{ and } 1 \leq j \leq m-1\}$. The generalized antiprism $\mathcal{A}_n^m$ is obtained by completing the generalized prism $C_n \times P_m$ by adding the edges $v_i^j v_i^{j+1}$ for $1 \leq i \leq n$ and $1 \leq j \leq m-1$. Terms and notations not defined here are used in the sense of Harary [1].

§2. Preliminary Results

Let G be a graph and $f : V(G) \rightarrow \{1, 2, 3, \dots, |V| + |E(G)|\}$ be an injection. For each edge $e = uv$ and an integer $m \geq 2$, the induced Smarandachely edge m -labeling f_S^* is defined by

$$f_S^*(e) = \left\lceil \frac{f(u) + f(v)}{m} \right\rceil.$$

Then f is called a Smarandachely super m -mean labeling if $f(V(G)) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, 3, \dots, |V| + |E(G)|\}$. A graph that admits a Smarandachely super mean m -labeling is called Smarandachely super m -mean graph. Particularly, if $m = 2$, we know that

$$f^*(e) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u) + f(v) \text{ is even;} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u) + f(v) \text{ is odd.} \end{cases}$$

Such a labeling f is called a super mean labeling of G if $f(V(G)) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, 3, \dots, p + q\}$. A graph that admits a super mean labeling is called a super mean graph. The concept of super mean labeling was introduced in [7] and further discussed in [2-6].

We use the following results in the subsequent theorems.

Theorem 2.1([7]) *The bistar $B_{m,n}$ is a super mean graph for $m = n$ or $n + 1$.*

Theorem 2.2([2]) *The graph $\langle B_{n,n} : w \rangle$, obtained by the subdivision of the central edge of $B_{n,n}$ with a vertex w , is a super mean graph.*

Theorem 2.3([2]) *The bi-armed crown $C_n \Theta 2P_m$ is a super mean graph for odd $n \geq 3$ and $m \geq 2$.*

Theorem 2.4([7]) *Let $G_1 = (p_1, q_1)$ and $G_2 = (p_2, q_2)$ be two super mean graphs with super mean labeling f and g respectively. Let $f(u) = p_1 + q_1$ and $g(v) = 1$. Then the graph $(G_1)_{f^*}(G_2)_g$ obtained from G_1 and G_2 by identifying the vertices u and v is also a super mean graph.*

§3. Super Mean Graphs

If G is a graph, then $S(G)$ is a graph obtained by subdividing each edge of G by a vertex.

Theorem 3.1 *The graph $S(P_n \odot K_1)$ is a super mean graph.*

Proof Let $V(P_n \odot K_1) = \{u_i, v_i : 1 \leq i \leq n\}$. Let $x_i (1 \leq i \leq n)$ be the vertex which divides the edge $u_i v_i (1 \leq i \leq n)$ and $y_i (1 \leq i \leq n-1)$ be the vertex which divides the edge $u_i u_{i+1} (1 \leq i \leq n-1)$. Then $V(S(P_n \odot K_1)) = \{u_i, v_i, x_i, y_j : 1 \leq i \leq n, 1 \leq j \leq n-1\}$.

Define $f : V(S(P_n \odot K_1)) \rightarrow \{1, 2, 3, \dots, p+q = 8n-3\}$ by

$$f(v_1) = 1; f(v_2) = 14; f(v_{2+i}) = 14 + 8i \text{ for } 1 \leq i \leq n-4;$$

$$f(v_{n-1}) = 8n-11; f(v_n) = 8n-10; f(x_1) = 3;$$

$$f(x_{1+i}) = 3 + 8i \text{ for } 1 \leq i \leq n-2; f(x_n) = 8n-7;$$

$$f(u_1) = 5; f(u_2) = 9; f(u_{2+i}) = 9 + 8i \text{ for } 1 \leq i \leq n-3;$$

$$f(u_n) = 8n-5; f(y_i) = 8i-1 \text{ for } 1 \leq i \leq n-2; f(y_{n-1}) = 8n-3.$$

It can be verified that f is a super mean labeling of $S(P_n \odot K_1)$. Hence $S(P_n \odot K_1)$ is a super mean graph. $\square$

Example 3.2 The super mean labeling of $S(P_5 \odot K_1)$ is given in Fig.1.

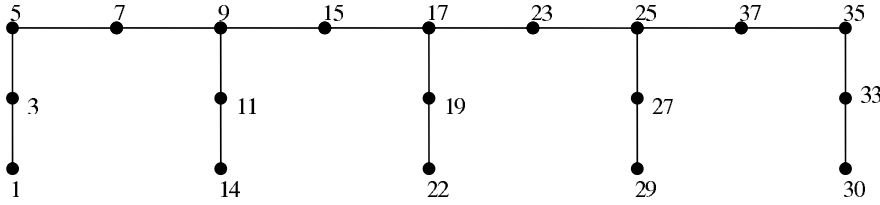


Fig.1

Theorem 3.2 The graph $S(P_2 \times P_n)$ is a super mean graph.

Proof Let $V(P_2 \times P_n) = \{u_i, v_i : 1 \leq i \leq n\}$. Let $u_i^1, v_i^1 (1 \leq i \leq n-1)$ be the vertices which divide the edges $u_i u_{i+1}, v_i v_{i+1} (1 \leq i \leq n-1)$ respectively. Let $w_i (1 \leq i \leq n)$ be the vertex which divides the edge $u_i v_i$. That is $V(S(P_2 \times P_n)) = \{u_i, v_i, w_i : 1 \leq i \leq n\} \cup \{u_i^1, v_i^1 : 1 \leq i \leq n-1\}$.

Define $f : V(S(P_2 \times P_n)) \rightarrow \{1, 2, 3, \dots, p+q = 11n-6\}$ by

$$f(u_1) = 1; f(u_2) = 9; f(u_3) = 27;$$

$$f(u_i) = f(u_{i-1}) + 5 \text{ for } 4 \leq i \leq n \text{ and } i \text{ is even}$$

$$f(u_i) = f(u_{i-1}) + 17 \text{ for } 4 \leq i \leq n \text{ and } i \text{ is odd}$$

$$f(v_1) = 7; f(v_2) = 16;$$

$$f(v_i) = f(v_{i-1}) + 5 \text{ for } 3 \leq i \leq n \text{ and } i \text{ is odd}$$

$$f(v_i) = f(v_{i-1}) + 17 \text{ for } 3 \leq i \leq n \text{ and } i \text{ is even}$$

$$f(w_1) = 3; f(w_2) = 12;$$

$$f(w_{2+i}) = 12 + 11i \text{ for } 1 \leq i \leq n-2;$$

$$f(u_1^1) = 6; f(u_2^1) = 24;$$

$$\begin{aligned}
f(u_i^1) &= f(u_{i-1}^1) + 6 \text{ for } 3 \leq i \leq n-1 \text{ and } i \text{ is odd} \\
f(u_i^1) &= f(u_{i-1}^1) + 16 \text{ for } 3 \leq i \leq n-1 \text{ and } i \text{ is even} \\
f(v_1^1) &= 13; f(v_i^1) = f(v_{i-1}^1) + 6 \text{ for } 2 \leq i \leq n-1 \text{ and } i \text{ is even} \\
f(v_i^1) &= f(v_{i-1}^1) + 16 \text{ for } 2 \leq i \leq n-1 \text{ and } i \text{ is odd}.
\end{aligned}$$

It is easy to check that f is a super mean labeling of $S(P_2 \times P_n)$. Hence $S(P_2 \times P_n)$ is a super mean graph. $\square$

Example 3.4 The super mean labeling of $S(P_2 \times P_6)$ is given in Fig.2.

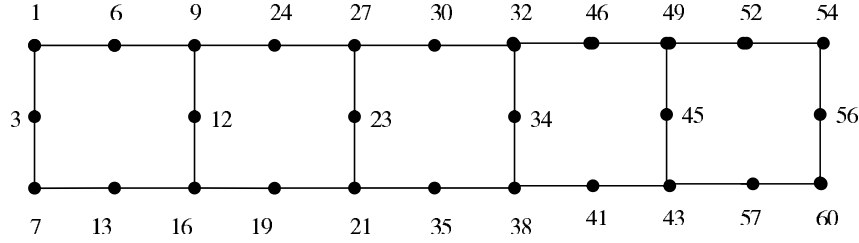


Fig.2

Theorem 3.5 The graph $S(B_{n,n})$ is a super mean graph.

Proof Let $V(B_{n,n}) = \{u, u_i, v, v_i : 1 \leq i \leq n\}$ and $E(B_{n,n}) = \{uu_i, vv_i, uv : 1 \leq i \leq n\}$. Let $w, x_i, y_i, (1 \leq i \leq n)$ be the vertices which divide the edges $uv, uu_i, vv_i (1 \leq i \leq n)$ respectively. Then $V(S(B_{n,n})) = \{u, u_i, v, v_i, x_i, y_i, w : 1 \leq i \leq n\}$ and $E(S(B_{n,n})) = \{ux_i, x_iu_i, uw, wv, vy_i, y_iv_i : 1 \leq i \leq n\}$.

Define $f : V(S(B_{n,n})) \rightarrow \{1, 2, 3, \dots, p+q = 8n+5\}$ by

$f(u) = 1; f(x_i) = 8i - 5$ for $1 \leq i \leq n; f(u_i) = 8i - 3$ for $1 \leq i \leq n; f(w) = 8n + 3; f(v) = 8n + 5; f(y_i) = 8i - 1$ for $1 \leq i \leq n; f(v_i) = 8i + 1$ for $1 \leq i \leq n$. It can be verified that f is a super mean labeling of $S(B_{n,n})$. Hence $S(B_{n,n})$ is a super mean graph. $\square$

Example 3.6 The super mean labeling of $S(B_{n,n})$ is given in Fig.3.

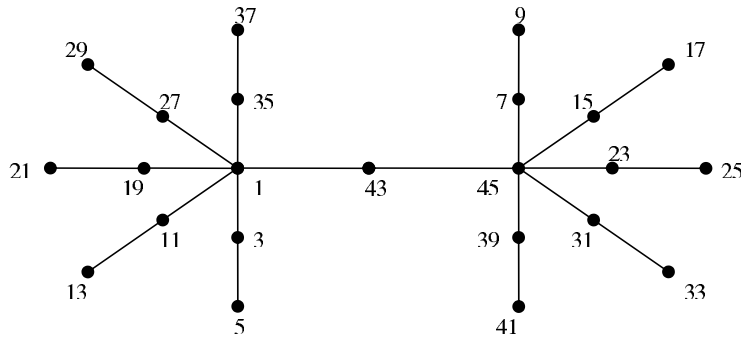


Fig.3

Next we prove that the graph $\langle B_{n,n} : P_m \rangle$ is a super mean graph. $\langle B_{m,n} : P_k \rangle$ is a graph obtained by joining the central vertices of the stars $K_{1,m}$ and $K_{1,n}$ by a path P_k of length $k - 1$.

Theorem 3.7 *The graph $\langle B_{n,n} : P_m \rangle$ is a super mean graph for all $n \geq 1$ and $m > 1$.*

Proof Let $V(\langle B_{n,n} : P_m \rangle) = \{u_i, v_i, u, v, w_j : 1 \leq i \leq n, 1 \leq j \leq m \text{ with } u = w_1, v = w_m\}$ and $E(\langle B_{n,n} : P_m \rangle) = \{uu_i, vv_i, w_j w_{j+1} : 1 \leq i \leq n, 1 \leq j \leq m - 1\}$.

Case 1 n is even.

Subcase 1 m is odd.

By Theorem 2.2, $\langle B_{n,n} : P_3 \rangle$ is a super mean graph. For $m > 3$, define $f : V(\langle B_{n,n} : P_m \rangle) \rightarrow \{1, 2, 3, \dots, p + q = 4n + 2m - 1\}$ by

$$\begin{aligned} f(u) &= 1; f(u_i) = 4i - 1 \text{ for } 1 \leq i \leq n \text{ and for } i \neq \frac{n}{2} + 1; \\ f(u_{\frac{n}{2}+1}) &= 2n + 2; f(v_i) = 4i + 1 \text{ for } 1 \leq i \leq n; f(v) = 4n + 3; \\ f(w_2) &= 4n + 4; f(w_3) = 4n + 9; \\ f(w_{3+i}) &= 4n + 9 + 4i \text{ for } 1 \leq i \leq \frac{m-5}{2}; f(w_{\frac{m+3}{2}}) = 4n + 2m - 4 \\ f(w_{\frac{m+3}{2}+i}) &= 4n + 2m - 4 - 4i \text{ for } 1 \leq i \leq \frac{m-5}{2}. \end{aligned}$$

It can be verified that f is a super mean labeling of $\langle B_{n,n} : P_m \rangle$.

Subcase 2 m is even.

By Theorem 2.1, $\langle B_{n,n} : P_2 \rangle$ is a super mean graph. For $m > 2$, define $f : V(\langle B_{n,n} : P_m \rangle) \rightarrow \{1, 2, 3, \dots, p + q = 4n + 2m - 1\}$ by

$$\begin{aligned} f(u) &= 1; f(u_i) = 4i - 1 \text{ for } 1 \leq i \leq n \text{ and for } i \neq \frac{n}{2} + 1; \\ f(u_{\frac{n}{2}+1}) &= 2n + 2; f(v_i) = 4i + 1 \text{ for } 1 \leq i \leq n; f(v) = 4n + 3; \\ f(w_2) &= 4n + 4; f(w_{2+i}) = 4n + 4 + 2i \text{ for } 1 \leq i \leq \frac{m-4}{2}; \\ f(w_{\frac{m+2}{2}}) &= 4n + m + 3; \\ f(w_{\frac{m+2}{2}+i}) &= 4n + m + 3 + 2i \text{ for } 1 \leq i \leq \frac{m-4}{2}. \end{aligned}$$

It can be verified that f is a super mean labeling of $\langle B_{n,n} : P_m \rangle$.

Case 2 n is odd.

Subcase 1 m is odd.

By Theorem 2.1, $\langle B_{n,n} : P_2 \rangle$ is a super mean graph. For $m > 2$, define $f : V(\langle B_{n,n} : P_m \rangle) \rightarrow$

$\{1, 2, 3, \dots, p + q = 4n + 2m - 1\}$ by

$$\begin{aligned} f(u) &= 1; f(v) = 4n + 3; f(u_i) = 4i - 1 \text{ for } 1 \leq i \leq n; \\ f(v_i) &= 4i + 1 \text{ for } 1 \leq i \leq n \text{ and for } i \neq \frac{n+1}{2}; \\ f(v_{\frac{n+1}{2}}) &= 2n + 2; f(w_2) = 4n + 4; \\ f(w_{2+i}) &= 4n + 4 + 2i \text{ for } 1 \leq i \leq \frac{m-4}{2}; f(w_{\frac{m+2}{2}}) = 4n + m + 3; \\ f(w_{\frac{m+2}{2}+i}) &= 4n + m + 3 + 2i \text{ for } 1 \leq i \leq \frac{m-4}{2}. \end{aligned}$$

It can be verified that f is a super mean labeling of $\langle B_{n,n} : P_m \rangle$.

Subcase 2 m is even.

By Theorem 2.2, $\langle B_{n,n} : P_3 \rangle$ is a super mean graph. For $m > 3$, define $f : V(\langle B_{n,n} : P_m \rangle) \rightarrow \{1, 2, 3, \dots, p + q = 4n + 2m - 1\}$ by

$$\begin{aligned} f(u) &= 1; f(v) = 4n + 3; f(u_i) = 4i - 1 \text{ for } 1 \leq i \leq n; \\ f(v_i) &= 4i + 1 \text{ for } 1 \leq i \leq n \text{ and for } i \neq \frac{n+1}{2}; f(v_{\frac{n+1}{2}}) = 2n + 2; \\ f(w_2) &= 4n + 4; f(w_3) = 4n + 9; \\ f(w_{3+i}) &= 4n + 9 + 4i \text{ for } 1 \leq i \leq \frac{m-5}{2}; f(w_{\frac{m+3}{2}}) = 4n + 2m - 4; \\ f(w_{\frac{m+3}{2}+i}) &= 4n + 2m - 4 - 2i \text{ for } 1 \leq i \leq \frac{m-5}{2}. \end{aligned}$$

It can be verified that f is a super mean labeling of $\langle B_{n,n} : P_m \rangle$. Hence $\langle B_{n,n} : P_m \rangle$ is a super mean graph for all $n \geq 1$ and $m > 1$. $\square$

Example 3.8 The super mean labeling of $\langle B_{4,4} : P_5 \rangle$ is given in Fig.4.

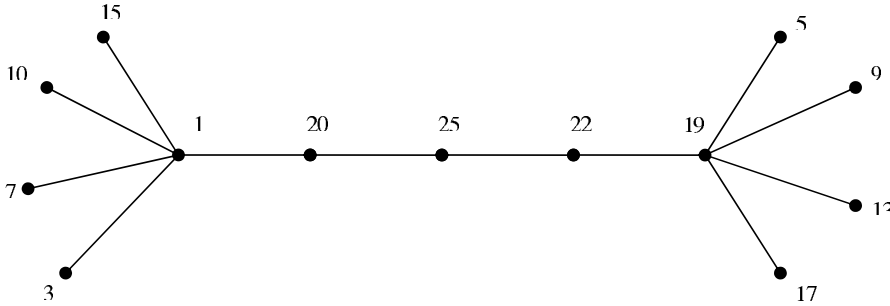


Fig.4

Theorem 3.9 The corona graph $C_n \odot \overline{K_2}$ is a super mean graph for all $n \geq 3$.

Proof Let $V(C_n) = \{u_1, u_2, \dots, u_n\}$ and $V(C_n \odot \overline{K_2}) = \{u_i, v_i, w_i : 1 \leq i \leq n\}$. Then $E(C_n \odot \overline{K_2}) = \{u_i u_{i+1}, u_n u_1, u_j v_j, u_j w_j : 1 \leq i \leq n-1 \text{ and } 1 \leq j \leq n\}$.

Case 1 n is odd.

The proof follows from Theorem 2.3 by taking $m = 2$.

Case 2 n is even.

Take $n = 2k$ for some k . Define $f : V(C_n \odot \overline{K_2}) \rightarrow \{1, 2, 3, \dots, p + q = 6n\}$ by

$$\begin{aligned} f(u_i) &= 6i - 3 \text{ for } 1 \leq i \leq k - 1; f(u_k) = 6k - 2; \\ f(u_{k+i}) &= 6k - 2 + 6i \text{ for } 1 \leq i \leq k - 2; f(u_{2k-1}) = 12k - 2; \\ f(u_{2k}) &= 12k - 9; f(v_i) = 6i - 5 \text{ for } 1 \leq i \leq k - 1; f(v_k) = 6k - 6; \\ f(v_{k+1}) &= 6k + 2; f(v_{k+1+i}) = 6k + 2 + 6i \text{ for } 1 \leq i \leq k - 3; f(v_{2k-1}) = 12k; \\ f(v_{2k}) &= 12k - 6; f(w_i) = 6i - 1 \text{ for } 1 \leq i \leq k - 1; f(w_k) = 6k; \\ f(w_{k+i}) &= 6k + 6i \text{ for } 1 \leq i \leq k - 2; f(w_{2k-1}) = 12k - 4; \\ f(w_{2k}) &= 12k - 11. \end{aligned}$$

It can be verified that $f(V) \cup (f^*(e) : e \in E) = \{1, 2, 3, \dots, 6n\}$. Hence $C_n \odot \overline{K_2}$ is a super mean graph. $\square$

Example 3.10 The super mean labeling of $C_8 \odot \overline{K_2}$ is given in Fig.5.

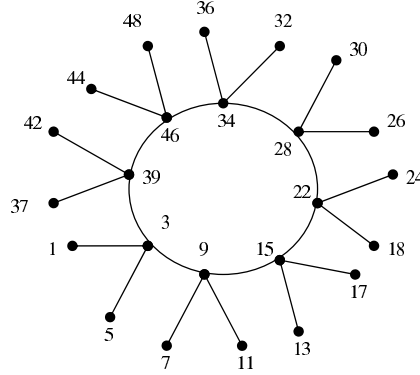


Fig.5

Theorem 3.11 The double triangular snake $D(T_n)$ is a super mean graph.

Proof We prove this result by induction on n . A super mean labeling of $G_1 = D(T_2)$ is given in Fig.6.

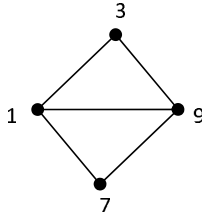


Fig.6

Therefore the result is true for $n = 2$. Let f be the super mean labeling of G_1 as in the above figure. Now $D(T_3) = (G_1)_f * (G_1)_f$, by Theorem 2.4, $D(T_3)$ is a super mean graph. Therefore the result is true for $n = 3$. Assume that $D(T_{n-1})$ is a super mean graph with the super mean labeling g . Now by Theorem 2.4, $(D(T_{n-1}))_g * (G_1)_f = D(T_n)$ is a super mean graph. Therefore the result is true for n . Hence by induction principle the result is true for all n . Thus $D(T_n)$ is a super mean graph. $\square$

Example 3.12 The super mean labeling of $D(T_6)$ is given in Fig.7.

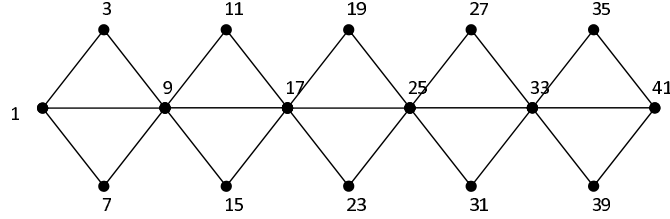


Fig.7

Theorem 3.13 The generalized antiprism $\mathcal{A}_n^m$ is a super mean graph for all $m \geq 2, n \geq 3$ except for $n = 4$.

Proof Let $V(\mathcal{A}_n^m) = \{v_i^j : 1 \leq i \leq n, 1 \leq j \leq m\}$ and $E(\mathcal{A}_n^m) = \{v_i^j v_{i+1}^j, v_n^j v_1^j : 1 \leq i \leq n-1, 1 \leq j \leq m\} \cup \{v_i^j v_{i-1}^{j+1}, v_1^j v_n^{j+1} : 2 \leq i \leq n, 1 \leq j \leq m-1\} \cup \{v_i^j v_i^{j+1} : 1 \leq i \leq n \text{ and } 1 \leq j \leq m-1\}$.

Case 1 n is odd.

Define $f : V(\mathcal{A}_n^m) \rightarrow \{1, 2, 3, \dots, p+q = 4mn - 2n\}$ by

$$f(v_i^j) = 4(j-1)n + 2i - 1 \text{ for } 1 \leq i \leq \frac{n+1}{2} \text{ and } 1 \leq j \leq m;$$

$$f(v_{\frac{n+3}{2}}^j) = 4(j-1)n + n + 3 \text{ for } 1 \leq j \leq m;$$

$$f(v_{\frac{n+3}{2}+i}^j) = 4(j-1)n + n + 3 + 2i \text{ for } 1 \leq i \leq \frac{n-3}{2} \text{ and } 1 \leq j \leq m.$$

Then f is a super mean labeling of $\mathcal{A}_n^m$. Hence $\mathcal{A}_n^m$ is a super mean graph.

Case 2 n is even and $n \neq 4$.

Define $f : V(\mathcal{A}_n^m) \rightarrow \{1, 2, 3, \dots, p+q = 4mn - 2n\}$ by

$$f(v_1^j) = 4(j-1)n + 1 \text{ for } 1 \leq j \leq m; f(v_2^j) = 4(j-1)n + 3 \text{ for } 1 \leq j \leq m;$$

$$f(v_3^j) = 4(j-1)n + 7 \text{ for } 1 \leq j \leq m; f(v_4^j) = 4(j-1)n + 12 \text{ for } 1 \leq j \leq m;$$

$$f(v_{4+i}^j) = 4(j-1)n + 12 + 4i \text{ for } 1 \leq j \leq m \text{ and } 1 \leq i \leq \frac{n-6}{2};$$

$$f(v_{\frac{n+2}{2}+i}^j) = 4(j-1)n + 2n + 1 - 4i \text{ for } 1 \leq j \leq m \text{ and } 1 \leq i \leq \frac{n-6}{2};$$

$$f(v_{n-1}^j) = 4(j-1)n + 9 \text{ for } 1 \leq j \leq m; f(v_n^j) = 4(j-1)n + 6 \text{ for } 1 \leq j \leq m.$$

Then f is a super mean labeling of $\mathcal{A}_n^m$. Hence $\mathcal{A}_n^m$ is a super mean graph. $\square$

Example 3.14 The super mean labeling of $\mathcal{A}_6^3$ is given in Fig.8.

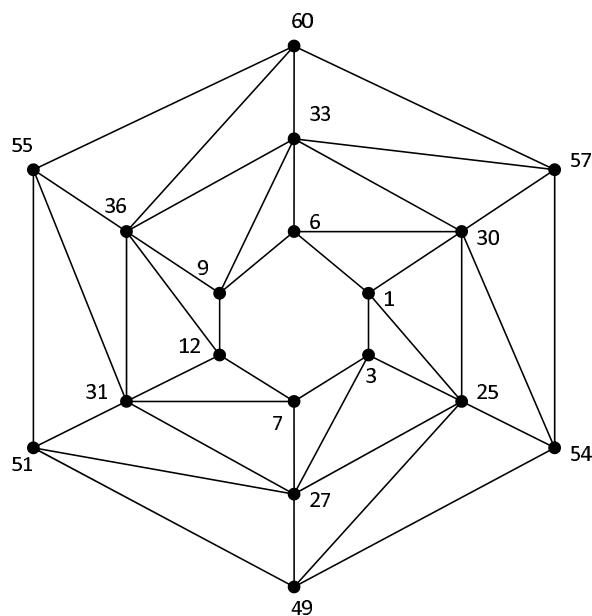


Fig.8

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