

A Note On Jump Symmetric n -Sigraph

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Abstract: A *Smarandachely k -signed graph* (*Smarandachely k -marked graph*) is an ordered pair $S = (G, \sigma)$ ($S = (G, \mu)$) where $G = (V, E)$ is a graph called *underlying graph of S* and $\sigma : E \rightarrow (\bar{e}_1, \bar{e}_2, \dots, \bar{e}_k)$ ($\mu : V \rightarrow (\bar{e}_1, \bar{e}_2, \dots, \bar{e}_k)$) is a function, where each $\bar{e}_i \in \{+, -\}$. Particularly, a Smarandachely 2-signed graph or Smarandachely 2-marked graph is called abbreviated a *signed graph* or a *marked graph*. In this note, we obtain a structural characterization of jump symmetric n -sigraphs. The notion of jump symmetric n -sigraphs was introduced by E. Sampathkumar, P. Siva Kota Reddy and M. S. Subramanya [Proceedings of the Jangjeon Math. Soc., 11(1) (2008), 89-95].

Key Words: Smarandachely symmetric n -sigraph, Smarandachely symmetric n -marked graph, Balance, Jump symmetric n -sigraph.

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§1. Introduction

For standard terminology and notion in graph theory we refer the reader to West [6]; the non-standard will be given in this paper as and when required. We treat only finite simple graphs without self loops and isolates.

Let $n \geq 1$ be an integer. An n -tuple $(a_1, a_2, \dots, a_n)$ is *symmetric*, if $a_k = a_{n-k+1}$, $1 \leq k \leq n$. Let $H_n = \{(a_1, a_2, \dots, a_n) : a_k \in \{+, -\}, a_k = a_{n-k+1}, 1 \leq k \leq n\}$ be the set of all symmetric n -tuples. Note that H_n is a group under coordinate wise multiplication, and the order of H_n is 2^m , where $m = \lceil \frac{n}{2} \rceil$.

A *Smarandachely symmetric n -sigraph* (*Smarandachely symmetric n -marked graph*) is an ordered pair $S_n = (G, \sigma)$ ($S_n = (G, \mu)$), where $G = (V, E)$ is a graph called the *underlying graph of S_n* and $\sigma : E \rightarrow H_n$ ($\mu : V \rightarrow H_n$) is a function.

A *sigraph* (*marked graph*) is an ordered pair $S = (G, \sigma)$ ($S = (G, \mu)$), where $G = (V, E)$ is a graph called the *underlying graph of S* and $\sigma : E \rightarrow \{+, -\}$ ($\mu : V \rightarrow \{+, -\}$) is a function. Thus a Smarandachely symmetric 1-sigraph (Smarandachely symmetric 1-marked graph) is a sigraph (marked graph).

The *line graph* $L(G)$ of graph G has the edges of G as the vertices and two vertices of $L(G)$

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are adjacent if the corresponding edges of G are adjacent.

The *jump graph* $J(G)$ of a graph $G = (V, E)$ is $\overline{L(G)}$, the complement of the line graph $L(G)$ of G (See [1] and [2]).

In this paper by an *n-tuple/n-sigraph/n-marked graph* we always mean a symmetric *n-tuple/Smarandachely symmetric n-sigraph/Smarandachely symmetric n-marked graph*.

An *n-tuple* $(a_1, a_2, \dots, a_n)$ is the *identity n-tuple*, if $a_k = +$, for $1 \leq k \leq n$, otherwise it is a *non-identity n-tuple*. In an *n-sigraph* $S_n = (G, \sigma)$ an edge labelled with the identity *n-tuple* is called an *identity edge*, otherwise it is a *non-identity edge*.

Further, in an *n-sigraph* $S_n = (G, \sigma)$, for any $A \subseteq E(G)$ the *n-tuple* $\sigma(A)$ is the product of the *n-tuples* on the edges of A .

In [4], the authors defined two notions of balance in *n-sigraph* $S_n = (G, \sigma)$ as follows (See also R. Rangarajan and P. Siva Kota Reddy [3]):

Definition 1.1 Let $S_n = (G, \sigma)$ be an *n-sigraph*. Then,

- (i) S_n is identity balanced (or *i-balanced*), if product of *n-tuples* on each cycle of S_n is the identity *n-tuple*, and
- (ii) S_n is balanced, if every cycle in S_n contains an even number of non-identity edges.

Note An *i-balanced n-sigraph* need not be balanced and conversely.

The following characterization of *i-balanced n-sigraphs* is obtained in [4].

Proposition 1.1(E. Sampathkumar et al. [4]) An *n-sigraph* $S_n = (G, \sigma)$ is *i-balanced* if, and only if, it is possible to assign *n-tuples* to its vertices such that the *n-tuple* of each edge uv is equal to the product of the *n-tuples* of u and v .

The *line n-sigraph* $L(S_n)$ of an *n-sigraph* $S_n = (G, \sigma)$ is defined as follows (See [5]): $L(S_n) = (L(G), \sigma')$, where for any edge ee' in $L(G)$, $\sigma'(ee') = \sigma(e)\sigma(e')$.

The *jump n-sigraph* of an *n-sigraph* $S_n = (G, \sigma)$ is an *n-sigraph* $J(S_n) = (J(G), \sigma')$, where for any edge ee' in $J(S_n)$, $\sigma'(ee') = \sigma(e)\sigma(e')$. This concept was introduced by E. Sampathkumar et al. [4]. This notion is analogous to the line *n-sigraph* defined above. Further, an *n-sigraph* $S_n = (G, \sigma)$ is called *jump n-sigraph*, if $S_n \cong J(S'_n)$ for some signed graph S' . In the following section, we shall present a characterization of jump *n-sigraphs*. The following result indicates the limitations of the notion of jump *n-sigraphs* defined above, since the entire class of *i-unbalanced n-sigraphs* is forbidden to be jump *n-sigraphs*.

Proposition 1.2(E. Sampathkumar et al. [4]) For any *n-sigraph* $S_n = (G, \sigma)$, its *jump n-sigraph* $J(S_n)$ is *i-balanced*.

§2. Characterization of Jump *n-Sigraphs*

The following result characterize *n-sigraphs* which are jump *n-sigraphs*.

Proposition 2.1 An *n-sigraph* $S_n = (G, \sigma)$ is a *jump n-sigraph* if, and only if, S_n is *i-balanced*

n -sigraph and its underlying graph G is a jump graph.

Proof Suppose that S_n is i -balanced and G is a jump graph. Then there exists a graph H such that $J(H) \cong G$. Since S_n is i -balanced, by Proposition 1.1, there exists a marking μ of G such that each edge uv in S_n satisfies $\sigma(uv) = \mu(u)\mu(v)$. Now consider the n -sigraph $S'_n = (H, \sigma')$, where for any edge e in H , $\sigma'(e)$ is the marking of the corresponding vertex in G . Then clearly, $J(S'_n) \cong S_n$. Hence S_n is a jump n -sigraph.

Conversely, suppose that $S_n = (G, \sigma)$ is a jump n -sigraph. Then there exists a n -sigraph $S'_n = (H, \sigma')$ such that $J(S'_n) \cong S_n$. Hence G is the jump graph of H and by Proposition 1.2, S_n is i -balanced. $\square$

References

- [1] G. T. Chartrand, H. Hevia, E. B. Jaretre, and M. Schutz, Subgraph distance in graphs defined by edge transfers, *Discrete mathematics*, 170 (1997), 63-79.
- [2] F. Harary, *Graph Theory*, Addison-Wesley Publishing Co., 1969.
- [3] R. Rangarajan and P. Siva Kota Reddy, Notions of balance in symmetric n -sigraphs, *Proceedings of the Jangjeon Math. Soc.*, 11(2) (2008), 145-151.
- [4] E. Sampathkumar, P. Siva Kota Reddy, and M. S. Subramanya, Jump symmetric n -sigraph, *Proceedings of the Jangjeon Math. Soc.* 11(1) (2008), 89-95.
- [5] E. Sampathkumar, P. Siva Kota Reddy, and M. S. Subramanya, The Line n -sigraph of a symmetric n -sigraph, *Southeast Asian Bull. Math.*(Springer-Verlag), to appear.
- [6] D. B. West, *Introduction to Graph Theory*, Prentice-Hall of India Pvt. Ltd., 1996.