

Smarandache Half-Groups

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Abstract: In this paper we introduce the concept of *half-groups*. This is a totally new concept and demands considerable attention. R.H.Bruck [1] has defined a half groupoid. We have imposed a group structure on a half groupoid wherein we have an identity element and each element has a unique inverse. Further, we have defined a new structure called Smarandache half-group. We have derived some important properties of Smarandache half-groups. Some suitable examples are also given.

Key Words: half-group, subhalf-group, Smarandache half-group, Smarandache subhalf-Group, Smarandache hyper subhalf-group.

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§1. Introduction

Definition 1.1 Let $(S, *)$ be a half groupoid (a partially closed set with respect to $*$) such that

(1) There exists an element $e \in S$ such that $a * e = e * a = a, \forall a \in S$. e is called identity element of S ;

(2) For every $a \in S$ there exists $b \in S$ such that $a * b = b * a = e$ (identity) b is called the inverse of a .

Then $(S, *)$ is called a half-group.

Remark It is easy to verify that

- (a) identity element in S is unique;
- (b) each element in S has a unique inverse;
- (c) associativity does not hold in S as there is at least one product that is not defined in S .

Note In all composition tables in the following examples the blank entries show that the corresponding products are not defined.

Example 1.1 Let $S = \{1, -i, i\}$. Then S is a half-group w.r.t. multiplication. We write this multiplication table in the following.

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*	1	-i	i
1	1	-i	i
-i	-i		1
i	i	1	

Example 1.2 Let $S = \{e, a, b, c\}$. Then $(S, *)$ is a half subgroup defined by

*	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b	c	e	a
c	c	e	a	

Here the product $c * c$ is not defined.

Definition 1.2 Let $(S, *)$ be a half-group and H a subset of S . If H itself is a half-group w.r.t. $*$, then H is called a subhalf-group of S .

Example 1.3 Let $S = \{e, a, b, c, d\}$ be a half-group defined by the following table.

*	e	a	b	c	d
e	e	a	b	c	d
a	a	c	e	b	a
b	b	e	c	a	d
c	c	d	a	e	b
d	d	b	c		e

Then, $H = \{e, a, b\}$ is a subhalf-group of S .

Definition 1.3 A half-group $(S, *)$ is called a Smarandache half-group if S contains a proper subset G such that G is a nontrivial group w.r.t. $*$.

Definition 1.4 If S is Smarandache half-group such that every group contained properly in S is commutative, then S is called Smarandache commutative half-group.

Definition 1.5 If S is a Smarandache half-group such that every group contained properly in S is cyclic, then S is called a Smarandache cyclic half-group.

Example 1.4 Let S be a half-group defined by the following table.

*	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	e		e

Then $G = \{e, a\}$ is a nontrivial group contained in S . So, S is a Smarandache half-group. Also, $\{e, a, b\}$ is a Smarandache half-group. S is also a Smarandache commutative half-group. Also S is a Smarandache cyclic half-group.

Example 1.5 $S = \{1, -i, i\}$ is not a Smarandache half-group.

Example 1.6 Let L be the Half-Group given by the following table.

*	e	f	g	h	i	j	k	l
e	e	f	g	h	i	j	k	l
f	f	e	j	g	k	h	l	i
g	g	j	e	k	h	l	i	f
h	h	g	k	e	l	i	f	j
i	i	k	h	l	e	f	j	g
j	j	h	l	i	f	e	g	k
k	k	l	i	f	j	g	e	
l	l	i	f	j	g	k		e

Then L is a half-group which contains a group $G = \{e, g\}$. So, L is a Smarandache Half-Group.

There are many Smarandache half-groups in this structure. Results following are obtained immediately by definition

(1) *The smallest half-group is of order 3.*

This follows from the very definition of half-groups.

(2) *The smallest Smarandache half-group is of order 3.*

As a nontrivial group has order at least 2, the half-group which will contain this group properly will have order at least 3.

§2. Substructures of Smarandache Half-Groups

In this section we introduce Smarandache substructure.

Definition 2.1 Let S be a half-group w.r.t. $*$. A nonempty subset T of S is said to be Smarandache subhalf-group of S if T contains a proper subset G such that G is a nontrivial group under the operation of S .

Theorem 2.1 *If S is a half-group and T is a Smarandache subhalf-group of S then S is a Smarandache half-group.*

Proof As T is a Smarandache subhalf-group of S , S contains T properly. Also, T properly contains a non trivial group. As a result S is a hlf-group which properly contains a nontrivial group. Therefore S is a Smarandache half-group. $\square$

We also note facts following on Smarandache half-groups.

(1) If R is a Smarandache half-group then every subhalf-group of R need not be a Smarandache subhalf-group.

We give an example to justify our claim.

Example 2.1 Consider a half-group S defined by the following table.

*	e	f	g	h	i	j
e	e	f	g	h	i	j
f	f	h	e	g	j	i
g	g	e	h	f	i	i
h	h	g	f	e	e	j
i	i	j	i	j	e	
j	j	i	f	i		e

Then $S \supset H = \{e, f, g, h\}$ and H is a group. Therefore S is a Smarandache half-group. Consider a half-group $R = \{e, f, g\}$. Then R is not a Smarandache subhalf-group of S as there does not exist a non trivial group contained in R .

We give a typical example of a half-group following whose subhalf-groups are Smarandache subhalf-group.

Example 2.2 Consider the following table.

*	e	f	g	h	i	j	k	l
e	e	f	g	h	i	j	k	l
f	f	e	j	g	k	h	l	i
g	g	j	e	k	h	l	i	f
h	h	g	k	e	l	i	f	j
i	i	k	h	l	e	f	j	g
j	j	h	l	i	f	e	g	k
k	k	l	i	f	j	g	e	
l	l	i	f	j	g	k		e

One can easily verify that every subhalf-group is a Smarandache subhalf-group.

Definition 2.2 *If S is a Smarandache half-group such that a subhalf-group A of S contains the largest group in S then A is called a Smarandache hyper subhalf-group.*

In the example above, the largest non-trivial group in S is of order 2 and every Smarandache subhalf-Group of S contains the largest group in S . Thus, every Smarandache subhalf-Group in S is a Smarandache hyper subhalf-Group.

References

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