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Topological Efficiency Index of Some Composite Graphs

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Abstract: In this paper, we study the behavior of a new graph invariants ρ for some composite graphs such as splice, link and rooted product of two given graphs.

Key Words: Wiener index, topological efficiency index, composite graph.

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§1. Introduction

Throughout this paper, we consider only simple connected graphs. We use $d_G(v)$ to denote the degree of a vertex v in G. Let $d_G(u,v)$ denote the distance between two vertices u and v in G and let $w_v(G)$ denote the sum of all distance of vertices of G from v, that is, $w_v(G) = \sum\limits_{u\in V(G)} d_G(v,u)$ with $\underline{w}(G) = \min \{w_v(G) : v \in V(G)\}$.

The topological indices (also known as the molecular descriptors) had been received much attention in that past decades, and they have been found to be useful in structure-activity relationships (SAR) and pharmaceutical drug design in organic chemistry see, [2, 3, 7]. Many researchers also were devoted to study their graphical properties. Indeed, the topological index of a graph G can be viewed as a graph invariant under the isomorphism of graphs, that is, for some topological index $TI, TI(G) = TI(H)$ if $G \cong H$.

One of the most thoroughly studied topological indices was the Wiener index which was proposed by Wiener in 1947 [8]. This index has been shown to posses close relation with the graph distance, which is an important concept in pure graph theory. It is also well correlated with many physical and chemical properties of a variety of classes of chemical compounds. For more details, see [1, 4, 5, 6, 9].

The Wiener index of a graph G, denoted by $W(G)$, is defined as

$$W(G) = \frac{1}{2}\sum_{u,v\in V(G)} d_G(u,v) = \frac{1}{2}\sum_{v\in V(G)} w_v(G).$$

Hossein-zadeh et al. [12] proposed a new graph descriptor ρ, called topological efficiency

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index based on minimal vertex contribution $\underline{w}$ defined for a connected graph G as

$$\rho(G) = \frac{2W(G)}{|V(G)|\,\underline{w}(G)}.$$

The topological efficiency index of C_{66} fullerence graph is computed in [13]. In [12], the topological efficiency of some product graphs such as Cartesian product, join, corona product, Hierarchical product, composition are given. In this sequence, here we study the behavior of a new graph invariants ρ for some composite graphs such as splice, link and rooted product of two given graphs are obtained.

§2. Splice Graph

For given vertices $x \in V(G_1)$ and $y \in V(G_2)$ *the splice* of G_1 and G_2 by vertices x and y, which is denoted by $S(G_1, G_2)(x, y)$, is defined by identifying the vertices x and y in the union of G_1 and G_2, see Figure 1. The various topological indices of splice graph are studied in [10, 14].

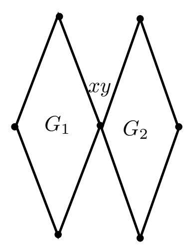

Figure 1 The splice of C_4 and C_4

The proof of the following lemma is easily followed from the structure of splice of graphs G_1 and G_2.

Lemma 2.1 *Let G_1 andG_2 are two connected graphs with $x \in V(G_1)$ and $y \in V(G_2)$. Then*

(i) $|V(S(G_1, G_2)(x, y))| = |V(G_1)| + |V(G_2)| - 1$ *and* $|E(S(G_1, G_2)(x, y))| = |E(G_1)| + |E(G_2)|$;

(ii) If $V(G_1) = \{u_1, u_2, \cdots, u_{n_1}\}$ *and* $V(G_2) = \{v_1, v_2, \cdots, v_{n_2}\}$, *then*

$$\begin{aligned} d_{S(G_1,G_2)(x,y)}(u_i, u_j) &= d_{G_1}(u_i, u_j), \\ d_{S(G_1,G_2)(x,y)}(u_i, v_j) &= d_{G_1}(u_i, x) + d_{G_2}(v_j, y), \\ d_{S(G_2,G_2)(x,y)}(v_i, v_j) &= d_{G_2}(v_i, v_j). \end{aligned}$$

Theorem 2.2 *Let G_1 and G_2 be a connected graph with n_1 and n_2 vertices. For vertices $x \in V(G_1)$ and $y \in V(G_2)$, consider $S(G_1, G_2)(x, y)$. Then*

$$\underline{w}(S(G_1, G_2)(x, y)) = n_1 w_{u_k}(G_2) + n_2 w_{u_k}(G_1).$$

Proof Let $V(G_1) = \{u_1, u_2, \cdots, u_{n_1}\}$ and $V(G_2) = \{v_1, v_2, \cdots, v_{n_2}\}$. For our convenience we denote $S(G_1, G_2)(x, y)$ by G and v_{xy} by the identifying the vertices x and y.

Now we compute the sum of distances between a fixed vertex to all other vertices of G.

Case 1. Let $u_k \neq v_{xy} \in V(G_1)$. Then by Lemma 2.1, for a vertex $u_i \in V(G_1)$, $d_G(u_k, u_i) = d_{G_1}(u_k, u_i)$ and for a vertex $v_j \in V(G_2)$, we have, $d_G(u_k, v_j) = d_{G_1}(u_k, x) + d_{G_2}(y, v_j)$. Thus

$$\begin{aligned} w_{u_k}(G) &= \sum_{i=1}^{n_1-1} d_G(u_k, u_i) + \sum_{j=1}^{n_2} d_G(u_k, v_j) \\ &= w_{u_k}(G_1) + w_y(G_2) + (n_2 - 1)d_{G_1}(u_k, x). \end{aligned}$$

Case 2. Let $v_k \neq v_{xy} \in V(G_2)$. Then by Lemma 2.1, for a vertex $v_j \in V(G_2)$, $d_G(v_k, v_j) = d_{G_2}(v_k, v_j)$ and for a vertex $u_i \in V(G_1)$, we get $d_G(v_k, u_i) = d_{G_2}(v_k, y) + d_{G_1}(x, u_i)$. Thus

$$\begin{aligned} w_{v_k}(G) &= \sum_{j=1}^{n_2-1} d_G(v_k, v_j) + \sum_{i=1}^{n_1} d_G(v_k, u_i) \\ &= w_{v_k}(G_2) + w_x(G_1) + (n_1 - 1)d_{G_2}(v_k, y). \end{aligned}$$

Case 3. Let $v_{xy} \in V(G)$. Then by Lemma 2.1, for a vertices $u_i \in V(G_1)$ and $v_j \in V(G_2)$, $d_G(v_{xy}, u_i) = d_{G_1}(x, u_i)$ and $d_G(v_{xy}, v_j) = d_{G_2}(y, v_j)$ Therefore

$$w_{v_{xy}}(G) = w_x(G_1) + w_y(G_2).$$

From Cases 1 and 3, we know that

$$w_{u_k} - w_{v_{xy}} = w_{u_k}(G_1) + w_y(G_2) + (n_2 - 1)d_{G_1}(u_k, x) - (w_x(G_1) + w_y(G_2)) > 0.$$

From Cases 2 and 3, we get that

$$w_{v_k} - w_{v_{xy}} = w_{v_k}(G_2) + w_x(G_1) + (n_1 - 1)d_{G_2}(v_k, y) - (w_x(G_1) + w_y(G_2)) > 0.$$

Therefore, by the above discussion and the definition of $\underline{w}(G)$, we have that

$$\underline{w}(G) = w_x(G_1) + w_y(G_2).$$ □

From [10] that the Wiener index of the splice graph of G_1 and G_2 is given by the formula

$$W(S(G_1, G_2))(x, y) = W(G_1) + W(G_2) + (|V(G_1)| - 1)w_{v_{xy}}(G_2) + (|V(G_2)| - 1)w_{v_{xy}}(G_1).$$

Using Theorem 2.2 and $W(S(G_1, G_2))(x, y)$, we obtain the ρ value of splice of G_1 and G_2.

Theorem 2.3 *Let G_1 and G_2 be two graphs with n_1 and n_2 vertices. Then*

$$\rho(S(G_1, G_2)(x, y)) = \frac{2\left(W(G_1) + W(G_2) + (n_1 - 1)w_{v_{xy}}(G_2) + (n_2 - 1)w_{v_{xy}}(G_1)\right)}{(n_1 + n_2 - 1)\left(w_x(G_1) + w_y(G_2)\right)}.$$

§3. Link Graph

A *Link* of G_1 and G_2 by the vertices x and y, which is denoted by $L(G_1 \sim G_2)(x, y)$, is defined as the graph obtained by joining x and y by an edge in the union of G_1 and G_2 graph, see Figure2. The various topological indices of link graph are studied in [10].

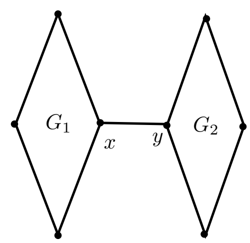

Figure 2 The link of C_4 and C_4

The proof of the following Lemma is easily follows from the structure of link of the graphs G_1 and G_2.

Lemma 3.1 *Let G_1 andG_2 are two connected graphs with $x \in V(G_1)$ and $y \in V(G_2)$. Then*

(i) $|V(L(G_1 \sim G_2)(x, y))| = |V(G_1)| + |V(G_2)|$ *and* $|E(L(G_1 \sim G_2)(x, y))| = |E(G_1)| + |E(G_2)| + 1$;

(ii) If $V(G_1) = \{u_1, u_2, \ldots, u_{n_1}\}$ *and* $V(G_2) = \{v_1, v_2, \ldots, v_{n_2}\}$, *then*

$$\begin{aligned} d_{L(G_1 \sim G_2)(x,y)}(u_i, u_j) &= d_{G_1}(u_i, u_j), \\ d_{L(G_1 \sim G_2)(x,y)}(u_i, v_j) &= d_{G_1}(u_i, x) + d_{G_2}(v_j, y) + 1, \\ d_{L(G_1 \sim G_2)(x,y)}(v_i, v_j) &= d_{G_2}(v_i, v_j). \end{aligned}$$

Theorem 3.2 *Let G_1 and G_2 be a connected graph with n_1 and n_2 vertices. For vertices $x \in V(G_1)$ and $y \in V(G_2)$, consider $L(G_1 \sim G_2)(x, y)$. Then*

$$\underline{w}(G) = \begin{cases} w_x(G_1) + w_y(G_2) + n_2, & \text{if } n_1 > n_2. \\ w_x(G_1) + w_y(G_2) + n_1, & \text{if } n_1 < n_2. \end{cases}$$

Proof Let $V(G_1) = \{u_1, u_2, \cdots, u_{n_1}\}$ and $V(G_2) = \{v_1, v_2, \cdots, v_{n_2}\}$. For our convenience we denote $L(G_1 \sim G_2)(x, y)$ by G. We compute the sum of the distances between a fixed vertex in G to all other vertices of G.

Case 1. Let $u_k \neq x \in V(G_1)$. Then by Lemma 3.1, for a vertex $u_i \in V(G_1)$, $d_G(u_k, u_i) = d_{G_1}(u_k, u_i)$ and for a vertex $v_j \in V(G_2)$,$d_G(u_k, v_j) = d_{G_1}(u_k, x) + d_{G_2}(y, v_j) + 1$. Hence

$$w_{u_k}(G) = \sum_{i=1}^{n_1-1} d_G(u_k, u_i) + \sum_{j=1}^{n_2} d_G(u_k, v_j) = w_{u_k}(G_1) + w_y(G_2) + n_2(d_{G_1}(u_k, x) + 1).$$

Case 2. Let $x \in V(G_1)$. Then by Lemma 3.1, for a vertex $u_i \in V(G_1)$, $d_G(x,u_i) = d_{G_1}(x,u_i)$ and for a vertex $v_j \in V(G_2), d(x,v_j) = d_{G_2}(y,v_j) + 1$. Thus

$$w_x(G) = \sum_{i=1}^{n_1-1} d_G(x,u_i) + \sum_{j=1}^{n_2} d_G(x,v_j) = w_x(G_1) + w_y(G_2) + n_2.$$

Case 3. Let $v_k \neq y \in V(G_2)$. Then by Lemma 3.1, then for a vertex $v_j \in V(G_2)$, $d_G(v_k,v_j) = d_{G_2}(v_k,v_j)$ and for a vertex $u_i \in V(G_1)$,$d_G(v_k,u_i) = d_{G_2}(v_k,y) + d_{G_1}(x,u_i) + 1$. Hence

$$w_{v_k}(G) = w_{v_k}(G_2) + w_x(G_1) + n_1(d_{G_2}(v_k,y) + 1).$$

Case 4. Let $y \in V(G_2)$. Then by Lemma 3.1, for a vertex $v_j \in V(G_2)$, $d_G(y,v_j) = d_{G_2}(y,v_j)$ and for a vertex $u_i \in V(G_1), d(y,u_i) = d_{G_1}(x,u_i) + 1$. Thus

$$w_y(G) = \sum_{j=1}^{n_2-1} d_G(y,v_j) + \sum_{i=1}^{n_1} d_G(y,u_i) = w_y(G_2) + w_x(G_1) + n_1.$$

From Cases 1 and 2, we obtain:

$$w_{u_k}(G) - w_x(G) = w_{u_k}(G_1) + w_y(G_2) + n_2(d_{G_1}(u_k,x) + 1) - (w_x(G_1) + w_y(G_2) + n_2) > 0$$

and

$$w_{v_k}(G) - w_y(G) = w_{v_k}(G_2) + w_x(G_1) + n_1(d_{G_2}(v_k,y) + 1) - (w_x(G_1) + w_y(G_2) + n_1) > 0.$$

From the above discussion and the definition of $\underline{w}(G)$, we have

$$\underline{w}(G) = \begin{cases} w_x(G_1) + w_y(G_2) + n_2, & \text{if } n_1 > n_2. \\ w_x(G_1) + w_y(G_2) + n_1, & \text{if } n_1 < n_2. \end{cases}$$

This completes the proof. □

Recall [10] from that the Wiener index of the link of G_1 and G_2 is given by the formula

$$W(L(G_1 \sim G_2)(x,y)) = W(G_1)+W(G_2)+|V(G_1)|\, w_y(G_2)+|V(G_2)|\, w_x(G_1)+|V(G_1)|\,|V(G_2)|\,.$$

Using Theorem3.2 and $W(L(G_1 \sim G_2))(x,y)$, we obtain the ρ value of the link graph of G_1 and G_2.

Theorem 3.3 *Let G_i be a graph with n_i vertices, $i = 1,2$. Then*

$$\rho(L(G_1 \sim G_2))(x,y) = \begin{cases} \dfrac{2\Big(W(G_1)+W(G_2)+n_1 w_y(G_2)+n_2 w_x(G_1)+n_1 n_2\Big)}{(n_1+n_2)\Big(w_x(G_1)+w_y(G_2)+n_2\Big)}, & \text{if } n_1 > n_2. \\ \dfrac{2\Big(W(G_1)+W(G_2)+n_1 w_y(G_2)+n_2 w_x(G_1)+n_1 n_2\Big)}{(n_1+n_2)\Big(w_x(G_1)+w_y(G_2)+n_1\Big)}, & \text{if } n_1 < n_2. \end{cases}$$

§4. Rooted Product

The *rooted product* $G_1\{G_2\}$, is obtained by taking one copy of $G_1$and $|V(G_1)|$ copies of a rooted graph G_2 and by identifying the root vertex v_i of the i^{th} copy of G_2 with the i^{th} vertex of G_1, $i = 1, 2, \cdots, |V(G_1)|$, one can observe that $|E(G_1\{G_2\})| = |E(G_1)| + |V(G_1)|\,|E(G_2)|$, and $|V(G_1\{G_2\})| = |V(G_1)|\,|V(G_2)|$, see Figure 3 for details. The i^{th} copy of G_2 is denoted by $G_{2,i}$. The various topological indices of the rooted product are studied in [11].

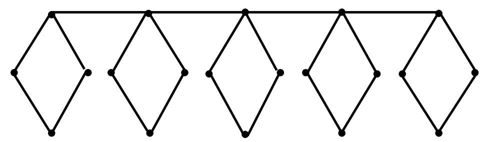

Figure 3 The rooted graph $P_5\{C_4\}$

The proof of the following lemma is easily follows from the structure of the rooted product of the graphs G_1 and G_2.

Lemma 4.1 *Let G_1 be a simple graph and G_2 be a rooted graph with $u_i$as its root. Then for a vertex u_k of $G_1\{G_2\}$ such that $u_k \in V(G_1)$, we have $\delta_{G_1\{G_2\}}(u_k) = \delta_{G_1}(u_k)\delta_{G_2}(u_i)$, and for a vertex v_k of $G_1\{G_2\}$ such that $v_k \notin V(G_1)$ we have $\delta_{(G_1\{G_2\})}(v_k) = \delta_{G_2}(v_0)$ where v_0 is the corresponding vertex in G_2 as v_j of $G_{2,j}$. Moreover*

(i) If $u_k, u_j \in V(G_1)$, then $d_{(G_1\{G_2\})}(u_k, u_j) = d_{G_1}(u_k, u_j)$;

(ii) If $u_k \in V(G)$, $v_{k_i} \in V(G_{2,i})$, where $i = 1, 2, \ldots, |V(G_1)|$, then $d_{(G_1\{G_2\})}(u_k, v_{k_i}) = d_{G_1}(u_k, u_i) + d_{G_{2,i}}(u_i, v_{k_i}) = d_{G_1}(u_k, u_i) + d_{G_2}(u, v_0)$, where u_i is the root of $G_{2,i}$,u is the root of G_2 and v_0 is the corresponding vertex in G_2 as v_{k_i} of $G_{2,i}$;

(iii) If $v_{0_i}, v_{k_i} \in V(G_{2,i})$, where $i = 1, 2, \ldots, |V(G_1)|$, then $d_{(G_1\{G_2\})}(v_{0_i}, v_{k_i}) = d_{G_2}(v_0, v_k)$,where v_0 and v_k are the corresponding vertices in G_2 as v_k and v_{0_i} of $G_{2,i}$;

(iv) If $v_{k_i} \in V(G_{2,i}), v_{k_j} \in V(G_{2,j})$ and $1 \le i < j \le |V(G_1)|$, then $d_{(G_1\{G_2\})}(v_{k_i}, v_{k_j}) = d_{G_{2,i}}(v_{k_i}, u_i) + d_{G_{2,j}}(v_{k_j}, u_j) + d_{G_1}(u_i, u_j) = d_{G_2}(v_0, u) + d_{G_2}(v_n, u) + d_{G_1}(u_i, u_j)$, where u_i is root of $G_{2,i}$ and $u_j$is the root of $G_{2,j}$. Also v_0 and v_n are the corresponding vertices in G_2 as v_{k_i} of $G_{2,i}$ and v_{k_j} of $V(G_{2,j})$, respectively.

Theorem 4.2 *Let G_1 be a graph with n_1 vertices and G_2 be a rooted graph on n_2 vertices with root vertex v_i. Then*

$$\underline{w}(G_1\{G_2\}) = n_2\underline{w}(G_1) + n_1\underline{w}(G_2).$$

Proof Let $V(G_1) = \{u_1, u_2, \cdots, u_{n_1}\}$ and $V(G_2) = \{v_1, v_2, \ldots, v_{n_2}\}$. Let v_i be the root vertex of G_2. We consider the following two cases to find the sum of the distance from a fixed vertex to all the vertices in $G_1\{G_2\}$.

Case 1. Let $u_k \in V(G_1)$. Then by Lemma 4.1, we have$d_{G_1\{G_2\}}(u_k, u_i) = d_{G_1}(u_k, u_i)$, if $u_i \in V(G_1)$ and $d_{G_1\{G_2\}}(u_k, v_k) = d_{G_2}(v_i, v_k)$ if $v_k \in V(G_{2,k})$. Moreover, for $v_k \in V(G_{2,i})$,

$$d_{G_1\{G_2\}}(u_k, v_k) = d_{G_1}(u_k, u_i) + d_{G_2}(v_i, v_k).$$

Hence $w_{u_k}(G) = n_2 w_{u_k}(G_1) + n_1 w_{v_i}(G_2)$.

Case 2. We consider 3 cases for discussion: (1) $v_k \neq v_i \in V(G_2)$. In this case, by Lemma 4.1, for a vertex $v_s \in V(G_2)$, $d_{G_1\{G_2\}}(v_k, v_s) = d_{G_2}(v_k, v_s)$. Thus $w_{v_k}(G_1\{G_2\}) = w_{v_k}(G_2)$; (2) $u_k \in V(G_1)$. In this case, $d_{G_1\{G_2\}}(v_k, u_k) = d_{G_2}(v_k, v_i) + d_{G_1}(u_i, u_k)$. Thus $w_{v_k}(G_1\{G_2\}) = (n_1-1)d_{G_{2,i}}(v_k, v_i) + w_{u_i}(G_1)$; (3) $v_r \in V(G_{2,r})$. In this case, $d_{(G_1\{G_2\})}(v_k, v_r) = d_{G_2}(v_k, v_i) + d_{G_1}(u_i, u_r) + d_{G_2}(v_i, v_r)$. Thus$w_{v_k}(G_1\{G_2\}) = (n_1-1)(n_2-1)d_{G_{2,i}}(v_k, v_i) + (n_1-1)w_{v_i}(G_2) + (n_2-1)w_{u_i}(G_1)$.

The total contribution of $v_k \in V(G_2)$ is

$$w_{v_k}(G_1\{G_2\}) = w_{v_k}(G_2)+w_{v_i}(G_2)n_2+d_{G_{2,i}}(v_k, v_i)\Big[(n_1-1)+(n_1-1)(n_2-1)\Big]+(n_1-1)w_{v_i}(G_2).$$

From Cases 1 and 2 we have

$$w_{v_k}(G_1\{G_2\}) - w_{u_k}(G_1\{G_2\}) > 0.$$

Hence

$$\underline{w}(G_1\{G_2\}) = n_2\underline{w}(G_1) + n_1\underline{w}(G_2). \qquad \square$$

From [11] that the Wiener index of the rooted product of G_1 and G_2 is given by the formula

$$W(G_1\{G_2\}) = |V(G_2)|^2 W(G_1) + |V(G_1)| W(G_2) + (|V(G_1)|^2 - |V(G_2)|) |V(G_2)| w_{v_i}(G_2),$$

where v_i is a root-vertex of G_2. Using Theorem4.2 and $W(G_1\{G_2\})$, we obtain the ρ value of rooted product of G_1 and G_2.

Theorem 4.3 *Let $G_1$and G_2 be two graphs with n_1 and n_2 be a number of vertices in G_1 and G_2 Then*

$$\rho(G_1\{G_2\}) = \frac{2\left(n_2^2 W(G_1) + n_1 W(G_2) + (n_1^2 - n_2)n_2 w_{v_i}(G_2)\right)}{(n_1 n_2)\left(n_1\underline{w}(G_2) + n_2\underline{w}(G_1)\right)}.$$

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