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New Families of Odd Mean Graphs

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Abstract: Let $G(V,E)$ be a graph with p vertices and q edges. A graph G is said to have an odd mean labeling if there exists a function $f : V(G) \to \{0,1,2,\ldots,2q-1\}$ satisfying f is $1-1$ and the induced map $f^* : E(G) \to \{1,3,5,\ldots,2q-1\}$ defined by

$$f^*(uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

is a bijection. A graph that admits an odd mean labeling is called an odd mean graph. In this paper, we discuss the construction of two kinds of odd mean graphs. Here we prove that $(P_n;S_1)$ for $n \geq 1$, $(P_{2n};S_m)$ for $m \geq 2, n \geq 1$, $(P_m;C_n)$ for $n \equiv 0(mod\ 4)$ and $m \geq 1$, $(P_m;Q_3)$ for $m \geq 1$, $[P_m;C_n]$ for $n \equiv 0(mod\ 4)$ and $m \geq 1$, $[P_m;Q_3]$ for $m \geq 1$ and $[P_m;C_n^{(2)}]$ for $n \equiv 0(mod\ 4)$ and $m \geq 1$ are odd mean graphs.

Key Words: Labeling, Smarandache m-module labeling, odd mean labeling, odd mean graph.

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§1. Introduction

All graphs considered here are finite, undirected and simple graph. We follow the basic notations and terminologies of graph theory as in [3]. Given a graph G, the symbols $V(G)$ and $E(G)$ denote the vertex set and the edge set of the graph G, respectively. For terminologies and notations, we follow the reference [7] with some of them mentioned in the following.

A path on n vertices is denoted by P_n and a cycle on n vertices is denoted by C_n. The graph $P_2 \times P_2 \times P_2$ is called a cube and is denoted by Q_3.

Let C_n be a cycle with fixed vertex v and $(P_m;C_n)$ the graph obtained from m copies of C_n and the path $P_m : u_1u_2\cdots u_m$ by joining u_i with the vertex v of the i^{th} copy of C_n by means of an edge, for $1 \leq i \leq m$.

Let Q_3 be a cube with fixed vertex v and $(P_m;Q_3)$ the graph obtained from m copies of Q_3 and the path $P_m : u_1u_2\cdots u_m$ by joining u_i with the vertex v of the i^{th} copy of Q_3 by means of an edge, for $1 \leq i \leq m$.

Let S_m be a star with vertices $v_0, v_1, v_2, \cdots, v_m$ and let $(P_{2n};S_m)$ be the graph obtained

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from $2n$ copies of S_m and the path $P_{2n} : u_1u_2\cdots u_{2n}$ by joining u_j with the vertex v_0 of the j^{th} copy of S_m by means of an edge, for $1 \le j \le 2n, (P_n; S_1)$ the graph obtained from n copies of S_1 and the path $P_n : u_1u_2\cdots u_n$ by joining u_j with the vertex v_0 of the j^{th} copy of S_1 by means of an edge, for $1 \le j \le n$.

Suppose $C_n : v_1v_2\cdots v_nv_1$ be a cycle of length n. Let $[P_m; C_n]$ be the graph obtained from m copies of C_n with vertices $v_{1_1}, v_{1_2}, \cdots, v_{1_n}, v_{2_1}, \cdots, v_{2_n}, \cdots, v_{m_1}\ldots, v_{m_n}$ and joining v_{i_j} and $v_{(i+1)_j}$ by means of an edge, for some j and $1 \le i \le m-1$.

Let Q_3 be a cube and $[P_m; Q_3]$ the graph obtained from m copies of Q_3 with vertices $v_{1_1}, v_{1_2}, \cdots, v_{1_8}, v_{2_1}, v_{2_2}, \cdots, v_{2_8}, \cdots, v_{m_1}, v_{m_2}, \cdots, v_{m_8}$ and the path $P_m : u_1u_2\cdots u_m$ by adding the edges $v_{1_1}v_{2_1}, v_{2_1}v_{3_1}, \cdots, v_{m-1_1}v_{m_1} (i.e) v_{i_1}v_{i+1_1}$, $1 \le i \le m-1$.

Let $C_n^{(2)}$ be a friendship graph. Define $\left[P_m; C_n^{(2)}\right]$ to be the graph obtained from m copies of $C_n^{(2)}$ and the path $P_m : u_1, u_2, \cdots, u_m$ by joining u_i with the center vertex of the i^{th} copy of $C_n^{(2)}$ for $1 \le i \le m$.

The graceful labelings of graphs was first introduced by Rosa in 1961 [1] and R.B. Gnanajothi introduced odd graceful graphs [2]. The concept of mean labeling was first introduced and studied by S. Somasundaram and R. Ponraj [8]. Further some more results on mean graphs are discussed in [6, 7, 10, 11]. The concept of odd mean labeling was introduced and studied by K. Manickam and M. Marudai [4]. Some more results on odd mean graphs are discussed in [9, 12, 13].

In [4], R. Vasuki et al. introduced the concept of even vertex odd mean labeling and studied even vertex odd meanness of some standard graphs. In [5], some construction of even vertex odd mean graphs are discussed and proved that $(P_n; S_1)$ for $n \ge 1, (P_{2n}; S_m)$ for $m \ge 2, n \ge 1, (P_m; C_n)$ for $n \equiv 0(mod\ 4), m \ge 1, [P_m; C_n]$ for $n \equiv 0(mod\ 4), m \ge 1$ and $[P_m; C_n^{(2)}]$ for $n \equiv 0(mod\ 4), m \ge 1$ are even vertex odd mean graphs.

A graph G with p vertices and q edges is said to have an even vertex odd mean labeling if there exists an injective function $f : V(G) \to \{0, 2, 4, \cdots, 2q-2, 2q\}$ such that the induced map $f^* : E(G) \to \{1, 3, 5, \cdots, 2q-1\}$ defined by

$$f^*(uv) = \frac{f(u) + f(v)}{2}$$

is a bijection. A graph that admits an even vertex odd mean labeling is called an even vertex odd mean graph. Generally, if there exists an injective function $f : V(G) \to \{0, m, 2m, \cdots, mq - m, mq\}$ such that the induced map $f^* : E(G) \to \{m-1, 2m-1, 3m-1, \cdots, mq-1\}$ defined by

$$f^*(uv) = \frac{f(u) + f(v)}{m}$$

is a bijection, G is said to have a *Smarandache m-module labeling*, where $m \ge 1$ is an integer. Clearly, a Smarandache 2-module labeling is an even vertex odd mean labeling on G.

A graph G is said to have an odd mean labeling if there exists a function $f : V(G) \to \{0, 1, 2, \cdots, 2q-1\}$ satisfying f is $1-1$ and the induced map $f^* : E(G) \to \{1, 3, 5, \cdots, 2q-1\}$

defined by

$$f^*(uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

is a bijection. A graph that admits an odd mean labeling is called an odd mean graph [4]. An odd mean labeling of $K_{2,5}$ is given in Figure 1.

In this paper, we prove that, the graphs $(P_n; S_1)$ for $n \geq 1$, $(P_{2n}; S_m)$ for $m \geq 2, n \geq 1$, $(P_m; C_n)$ for $n \equiv 0(mod\ 4)$ and $m \geq 1$, $(P_m; Q_3)$ for $m \geq 1$, $[P_m; C_n]$ for $n \equiv 0(mod\ 4)$ and $m \geq 1$, $[P_m; Q_3]$ for $m \geq 1$ and $\left[P_m; C_n^{(2)}\right]$ for $n \equiv 0(mod\ 4)$ and $m \geq 1$ are odd mean graphs.

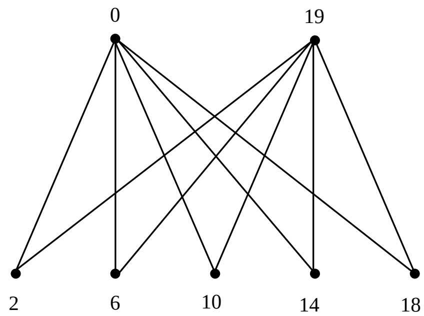


Figure 1

§2. Odd Mean Graphs $(P_m; G)$

Let G be a graph with fixed vertex v and let $(P_m; G)$ be the graph obtained from m copies of G and the path $P_m : u_1u_2 \cdots u_m$ by joining u_i with the vertex v of the i^{th} copy of G by means of an edge, for $1 \leq i \leq m$. For example, $(P_4; C_4)$ is shown in Figure 2.

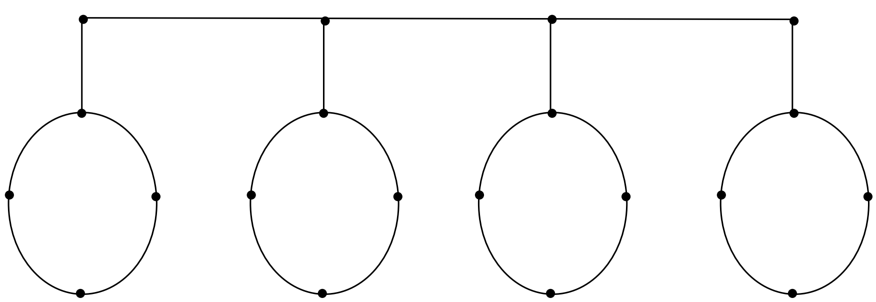


Figure 2

Theorem 2.1 *$(P_{2n}; S_m)$ is an odd mean graph, $m \geq 2, n \geq 1$.*

Proof Let $u_1, u_2, \cdots, u_{2n}$ be the vertices of P_{2n}. Let $v_{0_j}, v_{1_j}, v_{2_j}, \cdots, v_{m_j}$ be the vertices of the j^{th} copy of S_m, where v_{0_j} is the center vertex, $1 \leq j \leq 2n$. Define $f : V(P_{2n}; S_m) \rightarrow$

$\{0, 1, 2, \cdots, 2q-2, 2q-1 = 4n(m+2)-3\}$ as follows:

$$f(u_j) = \begin{cases} (2m+4)(j-1)+2, & 1 \le j \le 2n \text{ and } j \text{ is odd} \\ (2m+4)j-4, & 1 \le j \le 2n \text{ and } j \text{ is even} \end{cases}$$

$$f(v_{0_j}) = \begin{cases} (2m+4)(j-1), & 1 \le j \le 2n \text{ and } j \text{ is odd} \\ (2m+4)j-3, & 1 \le j \le 2n \text{ and } j \text{ is even} \end{cases}$$

$$f(v_{i_j}) = \begin{cases} (2m+4)(j-1)+4i+2, & 1 \le i \le m, 1 \le j \le 2n \text{ and } j \text{ is odd} \\ (2m+4)(j-2)+4i, & 1 \le i \le m, 1 \le j \le 2n \text{ and } j \text{ is even.} \end{cases}$$

For the vertex labeling f, the induced edge labeling f^* is obtained as follows:

$$f^*(u_j u_{j+1}) = (2m+4)j-1, \quad 1 \le j \le 2n-1$$

$$f^*(u_j v_{0_j}) = \begin{cases} (2m+4)(j-1)+1, & 1 \le j \le 2n \text{ and } j \text{ is odd} \\ (2m+4)j-3, & 1 \le j \le 2n \text{ and } j \text{ is even} \end{cases}$$

$$f^*(v_{0_j} v_{i_j}) = \begin{cases} (2m+4)(j-1)+2i+1, & 1 \le i \le m, 1 \le j \le 2n \\ & \text{and } j \text{ is odd} \\ (2m+4)(j-1)+2i-1, & 1 \le i \le m, 1 \le j \le 2n \text{ and } j \text{ is even.} \end{cases}$$

It can be verified that f is an odd mean labeling and hence $(P_{2n}; S_m)$ is an odd mean graph for $n \ge 1$ and $m \ge 2$. For example, an odd mean labeling of $(P_6; S_4)$ is shown in Figure 3. □

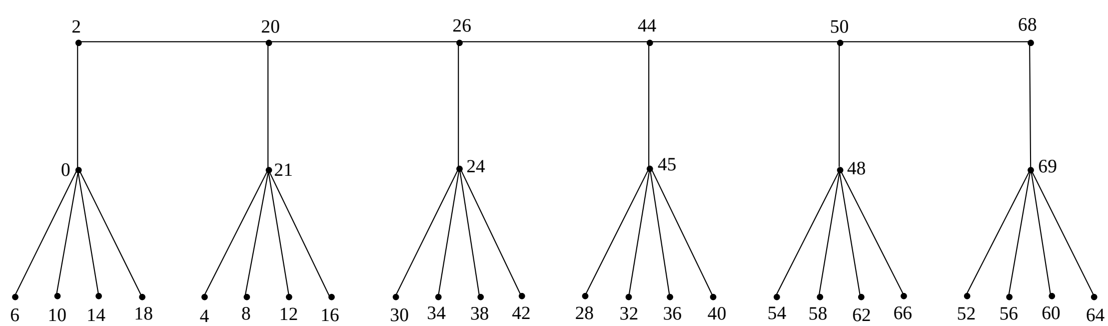


Figure 3

Theorem 2.2 *The graph $(P_n; S_1)$ $n \ge 1$ is an odd mean graph.*

Proof Let $u_1, u_2, \cdots, u_n$ be the vertices of P_n. Let v_{0_j} and v_{1_j} be the vertices. Define

$f : V(P_n; S_1) \to \{0, 1, 2, \cdots, 2q-2, 2q-1 = 6n-3\}$ as follows:

$$f(u_j) = \begin{cases} 6j-6, & 1 \le j \le n \text{ and } j \text{ is odd} \\ 6j-3, & 1 \le j \le n \text{ and } j \text{ is even} \end{cases}$$

$$f(v_{0_j}) = 6j-4, \quad 1 \le j \le n$$

$$f(v_{1_j}) = \begin{cases} 6j-3, & 1 \le j \le n \text{ and } j \text{ is odd} \\ 6j-7, & 1 \le j \le n \text{ and } j \text{ is even.} \end{cases}$$

The induced edge labels are obtained as follows:

$$f^*(u_j u_{j+1}) = 6j-1, \quad 1 \le j \le n-1$$

$$f^*(u_j v_{0_j}) = \begin{cases} 6j-5, & 1 \le j \le n \text{ and } j \text{ is odd} \\ 6j-3, & 1 \le j \le n \text{ and } j \text{ is even} \end{cases}$$

$$f^*(v_{0_j} v_{1_j}) = \begin{cases} 6j-3, & 1 \le j \le n \text{ and } j \text{ is odd} \\ 6j-5, & 1 \le j \le n \text{ and } j \text{ is even.} \end{cases}$$

Thus, f is an odd mean labeling. Hence, $(P_n; S_1)$ is an odd mean graph for any $n \ge 1$. For example, an odd mean labeling of $(P_8; S_1)$ is shown in Figure 4. □

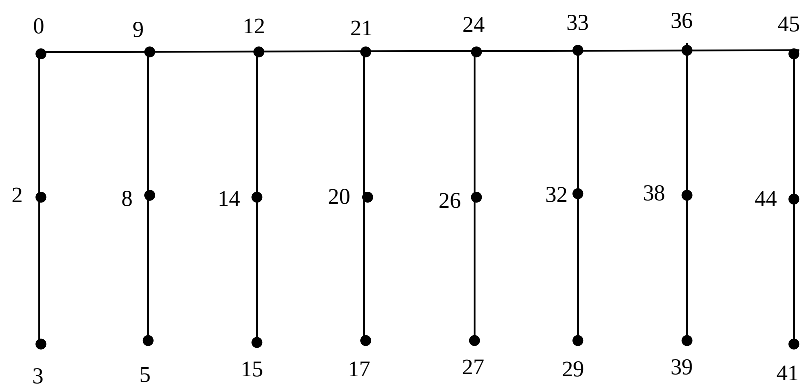


Figure 4

Theorem 2.3 *$(P_m; C_n)$ is an odd mean graph, for $n \equiv 0(mod\ 4)$ and $m \ge 1$.*

Proof Let $v_{i_1}, v_{i_2}, \cdots, v_{i_n}$ be the vertices in the i^{th} copy of C_n, $1 \le i \le m$ and $u_1, u_2, \cdots, u_m$ be the vertices of P_m. In $(P_m; C_n)$, u_i is joined with v_{i_1} by an edge, for each $i, 1 \le i \le m$. Define $f : V(P_m; C_n) \to \{0, 1, 2, \cdots, 2q-1 = (2n+4)m-3\}$ as follows:

$$f(u_i) = \begin{cases} 2(n+2)(i-1) & \text{if } i \text{ is odd and } 1 \le i \le m \\ 2(n+2)i-3 & \text{if } i \text{ is even and } 1 \le i \le m \end{cases}$$

and for $1 \le i \le m$, i is odd,

$$f(v_{i_j}) = \begin{cases} 2(n+2)(i-1)+2j, & 1 \le j \le \frac{n}{2} \\ 2(n+2)(i-1)+2j+3, & \frac{n}{2}+1 \le j \le n \text{ and } j \text{ is odd} \\ 2(n+2)(i-1)+2j, & \frac{n}{2}+2 \le j \le n \text{ and } j \text{ is even} \end{cases}$$

and for $1 \le i \le m$, i is even,

$$f(v_{i_j}) = \begin{cases} 2(n+2)i-2(j+1), & 1 \le j \le \frac{n}{2} \\ 2(n+2)i-2(j+3), & \frac{n}{2}+1 \le j \le n \text{ and } j \text{ is odd} \\ 2(n+2)i-2(j+1), & \frac{n}{2}+2 \le j \le n \text{ and } j \text{ is even.} \end{cases}$$

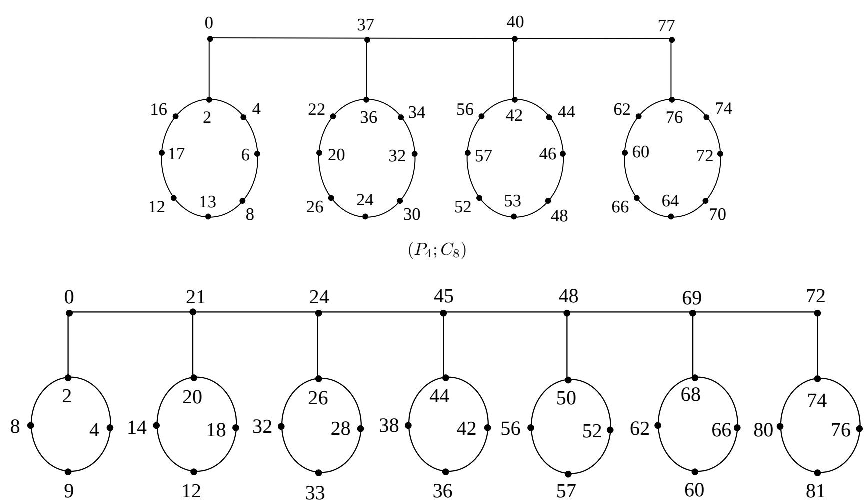
The induced edge labels are obtained by $f^*(u_iu_{i+1}) = 2i(n+2)-1$ for integers $1 \le i \le m-1$, and for $1 \le i \le m$, i is odd, $f^*(v_{i_n}v_{i_1}) = 2(n+2)(i-1)+n+1$, $f^*(u_iv_{i_1}) = 2(n+2)(i-1)+1$,

$$f^*(v_{i_j}v_{i_{(j+1)}}) = \begin{cases} 2(n+2)(i-1)+2j+1, & 1 \le j \le \frac{n}{2}-1 \\ 2(n+2)(i-1)+2j+3, & \frac{n}{2} \le j \le n-1 \end{cases}$$

and for $1 \le i \le m$, i is even, $f^*(v_{i_n}v_{i_1}) = 2(n+2)i-n-3$, $f^*(u_iv_{i_1}) = 2(n+2)i-3$,

$$f^*(v_{i_j}v_{i_{(j+1)}}) = \begin{cases} 2(n+2)i-(2j+3), & 1 \le j \le \frac{n}{2}-1 \\ 2(n+2)i-(2j+5), & \frac{n}{2} \le j \le n-1 \end{cases}$$

Thus, f is an odd mean labeling and hence $(P_m; C_n)$ is an odd mean graph for $n \equiv 0(mod\, 4)$, $m \ge 1$. For example, an odd mean labeling of $(P_4; C_8)$ and $(P_7; C_4)$ are shown in Figure 5. □



$(P_4; C_8)$

$(P_7; C_4)$

Figure 5

Theorem 2.4 $(P_m; Q_3), m \ge 1$ *is an odd mean graph.*

Proof For $1 \leq j \leq 8$, let v_{i_j} be the vertices in the i^{th} copy of $Q_3, 1 \leq i \leq m$ and $u_1, u_2, \cdots, u_m$ be the vertices of P_m.

The vertices of $(P_m; Q_3)$ are denoted as in Figure 6.

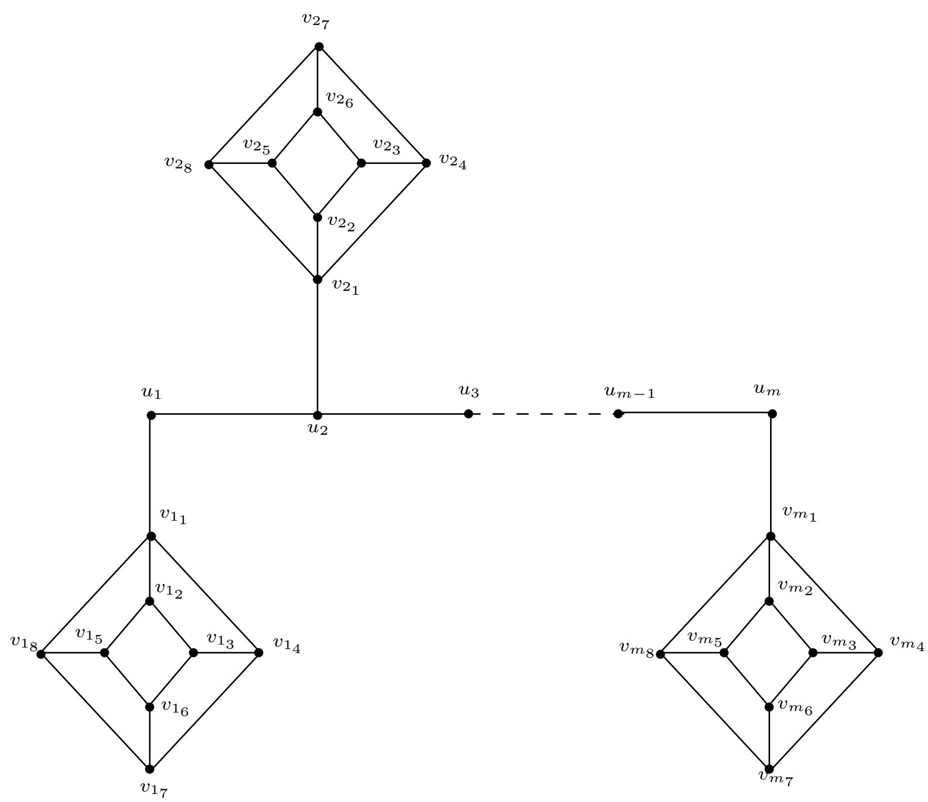


Figure 6

Define $f : V(P_m; Q_3) \to \{0, 1, 2, \cdots, 2q-2, 2q-1 = 28m-3\}$ as follows:

$$f(u_i) = \begin{cases} 28i - 28, & 1 \leq i \leq m \text{ and } i \text{ is odd} \\ 28i - 3, & 1 \leq i \leq m \text{ and } i \text{ is even,} \end{cases}$$

when i is odd,

$$f(v_{i_j}) = (28i - 28) + 2j, \quad 1 \leq i \leq m, \quad j = 1, 2, 4$$
$$f(v_{i_3}) = 28i - 18, \quad 1 \leq i \leq m$$
$$f(v_{i_j}) = 28i - 20 + 2j, \quad 1 \leq i \leq m, \quad j = 5, 6, 8$$
$$f(v_{i_7}) = 28i - 3, \quad 1 \leq i \leq m$$

and when i is even,

$$f(v_{i_j}) = 28i - (2j+2), \quad 2 \le i \le m, \quad 1 \le j \le 3$$
$$f(v_{i_4}) = 28i - 14, \quad 2 \le i \le m$$
$$f(v_{i_j}) = 28i - (2j+10), \quad 2 \le i \le m, \quad 5 \le j \le 7$$
$$f(v_{i_8}) = 28i - 30, \quad 2 \le i \le m.$$

The label of the edge u_iu_{i+1} is $28i-1, 1 \le i \le m-1$ and for $1 \le i \le m$, the label of the edge

$$u_iv_{i_1} = \begin{cases} 28i - 27 & \text{if } i \text{ is odd} \\ 28i - 3 & \text{if } i \text{ is even} \end{cases}$$

The label of the edges of the i^{th} copy of Q_3 are $28i-3, 28i-5, \ldots, 28i-25$ if i is odd and $28i-5, 28i-7, \ldots, 28i-27$ if i is even. Thus, $(P_m; Q_3), m \ge 1$ is an odd mean graph. For example, an odd mean labeling of $(P_5; Q_3)$ is shown in Figure 7. □

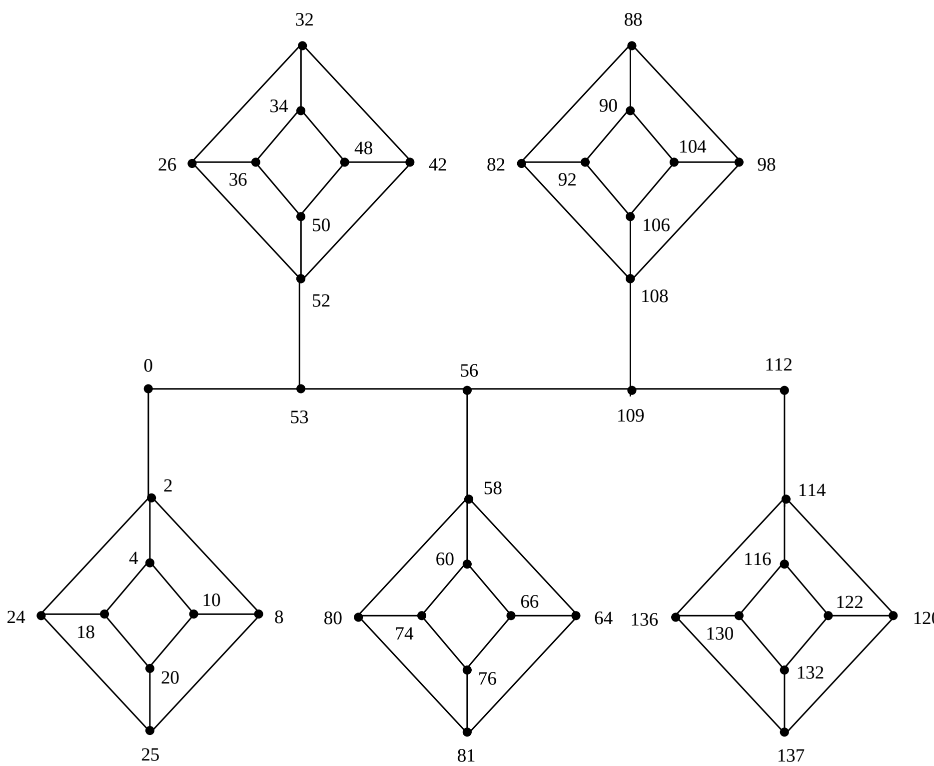


Figure 7

§3. Odd Mean Graphs $[P_m; G]$

Let G be a graph with fixed vertex v and let $[P_m; G]$ be the graph obtained from m copies of G by joining v_{i_j} and $v_{(i+1)_j}$ by means of an edge for some j and $1 \le i \le m-1$. For example, $[P_5; C_4]$ is shown in Figure 8.

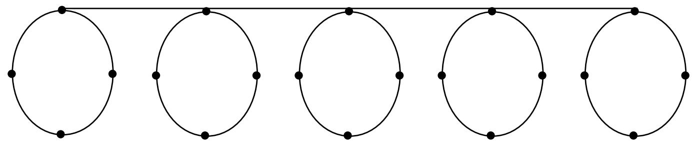


Figure 8

Theorem 3.1 *$[P_m; C_n]$ is an odd mean graph, for $n \equiv 0(mod\ 4)$ and $m \geq 1$.*

Proof Let $u_1, u_2, \cdots, u_m$ be the vertices of P_m. Let $v_{i_1}, v_{i_2}, \cdots, v_{i_n}$ be the vertices of the i^{th} copy of C_n, $1 \leq i \leq m$ and joining $v_{i_j}(= u_i)$ and $v_{(i+1)_j}(= u_{i+1})$ by means of an edge, for some j. Define $f : V([P_m; C_n]) \to \{0, 1, 2, \cdots, 2q-2, 2q-1 = (2n+2)m-3\}$ as follows:

for $1 \leq i \leq m$ and i is odd,

$$f(v_{i_j}) = \begin{cases} 2(n+1)(i-1)+2j-2, & 1 \leq j \leq \frac{n}{2} \\ 2(n+1)(i-1)+2j+1, & \frac{n}{2}+1 \leq j \leq n \text{ and } j \text{ is odd} \\ 2(n+1)(i-1)+2j-2, & \frac{n}{2}+2 \leq j \leq n \text{ and } j \text{ is even} \end{cases}$$

and for $1 \leq i \leq m$ and i is even,

$$f(v_{i_1}) = 2(n+1)i-3$$

$$f(v_{i_j}) = \begin{cases} 2(n+1)i-2j, & 2 \leq j \leq \frac{n}{2} \\ 2(n+1)i-2(j+2), & \frac{n}{2}+1 \leq j \leq n \text{ and } j \text{ is odd} \\ 2(n+1)i-2j, & \frac{n}{2}+2 \leq j \leq n \text{ and } j \text{ is even.} \end{cases}$$

The induced edge labels are obtained $f^*(v_{i_1}v_{(i+1)_1}) = 2(n+1)i-1, 1 \leq i \leq m-1$ and for $1 \leq i \leq m$, i is odd,

$$f^*(v_{i_j}v_{i_{(j+1)}}) = \begin{cases} 2(n+1)(i-1)+2j-1, & 1 \leq j \leq \frac{n}{2}-1 \\ 2(n+1)(i-1)+2j+1, & \frac{n}{2} \leq j \leq n-1 \end{cases}$$

$$f^*(v_{i_n}v_{i_1}) = 2(n+1)i-(n+3)$$

and for $1 \leq i \leq m$, i is even,

$$f^*(v_{i_j}v_{i_{(j+1)}}) = \begin{cases} 2(n+1)i-2j-1, & 1 \leq j \leq \frac{n}{2}-1 \\ 2(n+1)i-2j-3, & \frac{n}{2} \leq j \leq n-1 \end{cases}$$

$$f^*(v_{i_n}v_{i_1}) = 2(n+1)i-(n+1).$$

Thus, f is an odd mean labeling and hence $[P_m; C_n]$ is an odd mean graph for $n \equiv 0(mod\ 4)$ and $m \geq 1$. For example, an odd mean labeling of $[P_5; C_8]$ is shown in Figure 9. □

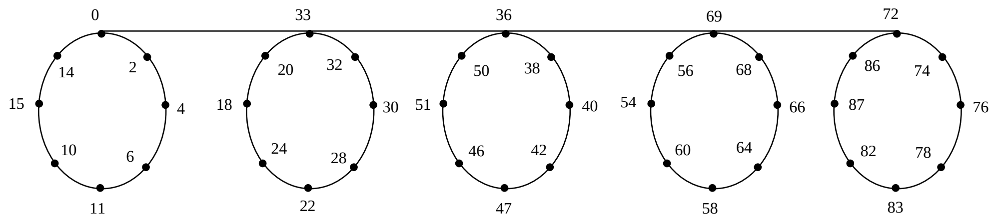


Figure 9

Theorem 3.2 *$[P_m; Q_3]$ is an odd mean graph.*

Proof For $1 \leq j \leq 8$, Let v_{i_j} be the vertices in the i^{th} copy of $Q_3, 1 \leq i \leq m$. Then $f : V([P_m; Q_3]) \rightarrow \{0, 1, 2, \cdots, 2q-2, 2q-1 = 26m-3\}$ as follows:

when i is odd,

$$f(v_{i_j}) = 26i + 2j - 28, \quad 1 \leq i \leq m, \quad j = 1, 2, 4$$
$$f(v_{i_3}) = 26i - 18, \quad 1 \leq i \leq m$$
$$f(v_{i_j}) = 26i + 2j - 20, \quad 1 \leq i \leq m, \quad j = 5, 6, 8$$
$$f(v_{i_7}) = 26i - 3, \quad 1 \leq i \leq m,$$

when i is even,

$$f(v_{i_1}) = 26i - 3, \quad 2 \leq i \leq m$$
$$f(v_{i_j}) = 26i - 2j, \quad 2 \leq i \leq m, \quad 2 \leq j \leq 3$$
$$f(v_{i_4}) = 26i - 12, \quad 2 \leq i \leq m$$
$$f(v_{i_j}) = 26i - 2j - 8, \quad 2 \leq i \leq m, \quad 5 \leq j \leq 7$$
$$f(v_{i_8}) = 26i - 28, \quad 2 \leq i \leq m.$$

The label of the edge $v_{i_1} v_{(i+1)_1}$ is $26i - 1, 1 \leq i \leq m-1$. The label of the edges of the cube are $26i - 3, 26i - 5, \cdots, 26i - 25, 1 \leq i \leq m$. For example, an odd mean labeling of $[P_4; Q_3]$ is shown in Figure 10. □

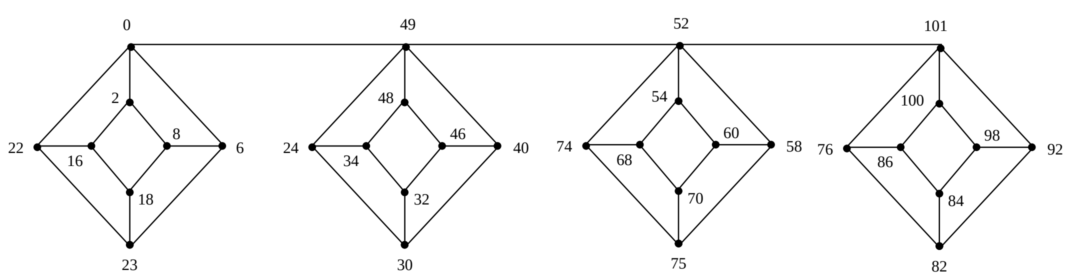


Figure 10

Theorem 3.3 *$\left[P_m; C_n^{(2)}\right]$ is an odd mean graph for $n \equiv 0 (mod\ 4)$ and $m \geq 1$.*

Proof Let $u_1, u_2, \cdots, u_m$ be the vertices of P_m and each vertex $u_i, 1 \le i \le m$ is attached with the common vertex in the i^{th} copy of $C_n^{(2)}$. Let v'_{i_j} and v''_{i_j} for $1 \le j \le n$ be the vertices in the i^{th} copy of $C_n^{(2)}$ in which $v'_{i_1} = v''_{i_1}, 1 \le i \le m$. Define $f : V\left(\left[P_m; C_n^{(2)}\right]\right) \to \{0, 1, 2, \ldots, 2q-2, 2q-1 = (4n+2)m-3\}$ as follows:

for $1 \le i \le m$,

$$f(v'_{i_j}) = \begin{cases} (4n+2)i - 2(n+j), & 1 \le j \le 2 \\ (4n+2)(i-1) + 2j - 6, & 3 \le j \le \frac{n}{2} + 2 \\ (4n+2)(i-1) + 2j - 3, & \frac{n}{2} + 3 \le j \le n \text{ and } j \text{ is odd} \\ (4n+2)(i-1) + 2j - 6, & \frac{n}{2} + 3 \le j \le n \text{ and } j \text{ is even} \end{cases}$$

$$f(v''_{i_j}) = \begin{cases} (4n+2)i - 2(n-j+2), & 2 \le j \le \frac{n}{2} \\ (4n+2)i - 2n + 2j - 1, & \frac{n}{2} + 1 \le j \le n \text{ and } j \text{ is odd} \\ (4n+2)i - 2(n-j+2), & \frac{n}{2} + 2 \le j \le n \text{ and } j \text{ is even.} \end{cases}$$

The induced edge labels are obtained as follows:

for $1 \le i \le m$,

$$f^*(v'_{i_1} v'_{i_2}) = (4n+2)i - (2n+3)$$

$$f^*(v'_{i_2} v'_{i_3}) = (4n+2)i - 3(n+1)$$

$$f^*(v'_{i_j} v'_{i_{(j+1)}}) = \begin{cases} (4n+2)(i-1) + 2j - 5, & 3 \le j \le \frac{n}{2} + 1 \\ (4n+2)(i-1) + 2j - 3, & \frac{n}{2} + 2 \le j \le n \end{cases}$$

$$f^*(v''_{i_j} v''_{i_{(j+1)}}) = \begin{cases} (4n+2)i - 2n + 2j - 5, & 1 \le j \le \frac{n}{2} - 1 \\ (4n+2)i - 2n + 2j - 1, & \frac{n}{2} \le j \le n \end{cases}$$

$$f^*(v''_{i_n} v''_{i_1}) = (4n+2)i - (n+3).$$

Thus, f is an odd mean labeling. Hence, $\left[P_m; C_n^{(2)}\right]$ is an odd mean graph for $n \equiv 0 (mod\, 4)$ and $m \ge 1$. For example, an odd mean labeling of $\left[P_4; C_8^{(2)}\right]$ is shown in Figure 11. □

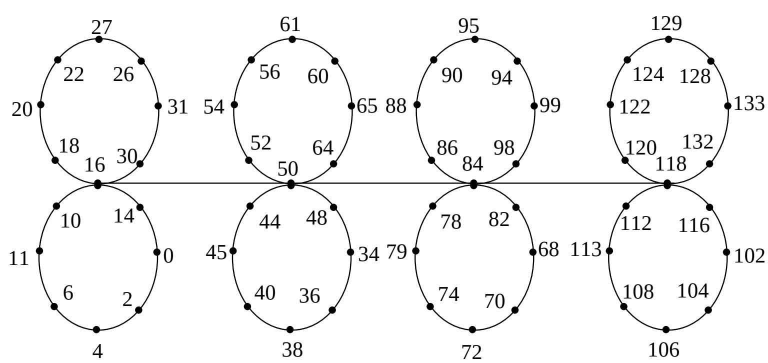


Figure 11

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