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F-Root Square Mean Labeling of Some Graphs

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Abstract: A function f is called $F-$ *root square mean labeling* of a graph $G(V,E)$ with p vertices and q edges if $f : V(G) \to \{1, 2, \dots, q+1\}$ is injective and the induced function f^* is defined as $f^*(uv) = \left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$ for all $uv \in E(G)$ is bijective. A graph that admits a $F-$root square mean labeling is called a $F-$*root square mean graph*. In this paper, we study the $F-$root mean square meanness of triangular snake,$A(T_n)$, $D(T_n)$, quadrilateral snake, $A(Q_n)$, $D(Q_n)$.

Key Words: Triangular snake, double triangular snake, quadrilateral snake, double quadrilateral snake, $F-$root square mean labeling, Smarandache m-root mean labeling, $F-$root square mean graph.

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§1. Introduction

Throughout this paper, by a graph we mean a finite, undirected and simple graph. Let $G(V,E)$ be a graph with p vertices and q edges. For notations and terminology, we follow [3]. For a detailed survey on graph labeling, we refer [2].

The concept of root square mean labeling was introduced and studied by S.S.Sandhya et. al [4]. Motivated by the works of so many others in the area of graph labeling, the concept of $F-$root square mean labeling was introduced by S.Arockiaraj et.al. [1].

A function f is called $F-$ *root square mean labeling* of a graph $G(V,E)$ with p vertices and q edges if $f : V(G) \to \{1, 2, \cdots, q+1\}$ is injective and the induced function f^* is defined as

$$f^*(uv) = \left\lfloor \sqrt{\frac{f^2(u)+f^2(v)}{2}} \right\rfloor$$

for all $uv \in E(G)$ is bijective. Generally, if $f : V(G) \to \{m, 2m, \cdots, qm+1\}$ and

$$f^*(uv) = \left\lfloor \sqrt[m]{\frac{f^m(u)+f^m(v)}{m}} \right\rfloor$$

for all $uv \in E(G)$ is bijective, then f is called a *Smarandache m-root mean labeling*, where

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$m \geq 1$ is an integer. Clearly, a Smarandache 2-root mean labeling is nothing else but the $F-$root square mean labeling of G. A graph that admits a $F-$root square mean labeling is called a *$F-$root square mean graph.*

In this paper, we study the $F-$root mean square meanness of triangular snake, $A(T_n)$, $D(T_n)$, quadrilateral snake, $A(Q_n)$ and $D(Q_n)$.

§2. Main Results

Theorem 2.1 *The triangular snake $T_n, n \geq 2$ is a $F-$root square mean graph.*

Proof Let $\{u_i, 1 \leq i \leq n-1, v_i, 1 \leq i \leq n\}$ be the vertices and $\{e_i, 1 \leq i \leq n-1, a_i, 1 \leq i \leq 2(n-1)\}$ the edges of T_n. First we label $f : V(G) \to \{1, 2, \cdots, q+1\}$ on vertices of T_n by $f(u_i) = 3i-2$ if $1 \leq i \leq n-1$ and $f(v_i) = 3i-1$ if $1 \leq i \leq n$. Then the induced edge labels are $f^*(e_i) = 3i-1$ if $1 \leq i \leq n-1$, and if $1 \leq i \leq 2(n-1)$,

$$f^*(a_i) = \begin{cases} \frac{3i-1}{2} & i \text{ is odd} \\ \frac{3i}{2} & i \text{ is even} \end{cases}$$

Hence, f is a $F-$root square mean labeling of the graph T_n. Thus the graph triangular snake $T_n, n \geq 2$ is a $F-$root square mean graph. □

Theorem 2.2 *The alternative triangular snake $A(T_n), n \geq 4$ is a $F-$root square mean graph.*

Proof Let $\{u_i, 1 \leq i \leq \frac{n}{2}, v_i, 1 \leq i \leq n\}$ be the vertices and $\{e_i, 1 \leq i \leq n-1, a_i, 1 \leq i \leq n\}$ the edges of $A(T_n)$. First we label $f : V(G) \to \{1, 2, \cdots, q+1\}$ of vertices of $A(T_n)$ by $f(u_i) = 4i-2$ if $1 \leq i \leq \frac{n}{2}$ and for $1 \leq i \leq n$,

$$f(v_i) = \begin{cases} 2i-1 & i \text{ is odd} \\ 2i & i \text{ is even} \end{cases}$$

Then the induced edge labels are respectively $f^*(e_i) = 2i$ if $1 \leq i \leq n-1$ and $f^*(a_i) = 2i-1$ if $1 \leq i \leq n$. Hence, f is a $F-$root square mean labeling of the graph $A(T_n)$. Thus the graph Alternative triangular snake $A(T_n), n \geq 4$ is a $F-$root square mean graph. □

Theorem 2.3 *The double triangular snake $D(T_n), (n \geq 2)$ is a $F-$root square mean graph.*

Proof Let $\{u_i, 1 \leq i \leq n, v_i^{'}, v_i, 1 \leq i \leq n-1\}$ be the vertices and $\{a_i, 1 \leq i \leq n-1, b_i, c_i, 1 \leq i \leq 2(n-1)$ the edges of $D(T_n)$. First we label $f : V(G) \to \{1, 2, \cdots, q+1\}$ of vertices by $f(u_1) = 1,\ f(u_2) = 4,\ f(u_i) = 5i-4$ if $3 \leq i \leq n$ and $f(v_1) = 6,\ f(v_2) = 10,\ f(v_i) = 5i-1$ if $3 \leq i \leq n-14,\ f(u_1^{'}) = 2, f(u_2^{'}) = 8,\ f(u_i^{'}) = 5i-3$ if $3 \leq i \leq n-1$. Then the induced edge labels are $f^*(a_1) = 2$, $f^*(a_i) = 5i-2$ for $2 \leq i \leq n-1$, $f^*(b_1) = 1, f^*(b_2) = 3$ for $3 \leq i \leq 2n-2$,

$$f^*(b_i) = \begin{cases} \frac{5i-3}{2} & i \text{ is odd} \\ \frac{5i-1}{2} & i \text{ is even} \end{cases}$$

$f^*(c_1) = 4, f^*(c_2) = 5$ and

$$f^*(c_i) = \begin{cases} \frac{5i-1}{2} & i \text{ is odd} \\ \frac{5i}{2} & i \text{ is even} \end{cases}$$

for $3 \leq i \leq 2n-2$. Hence, f is a $F-$root square mean labeling of the graph $D(T_n)$. Thus the graph double triangular snake $D(T_n), (n \geq 2)$ is a $F-$root square mean graph. □

Theorem 2.4 *The quadrilateral snake Q_n is a $F-$root square mean graph.*

Proof Let $\{u_i, 1 \leq i \leq 2(n-1), v_i, 1 \leq i \leq n\}$ be the vertices and $\{a_i, 1 \leq i \leq n-1, b_i, 1 \leq i \leq 2(n-1), c_i, 1 \leq i \leq n-1\}$ the edges of Q_n. First we define $f : V(G) \to \{1, 2, \ldots, q+1\}$ of vertices by $f(u_1) = 1, f(u_2) = 2$,

$$f(u_i) = \begin{cases} 2i & i \text{ is odd} \\ 2i-1 & i \text{ is even} \end{cases}$$

for $3 \leq i \leq 2(n-1)$, $f(v_1) = 3$, $f(v_i) = 4i-3$ for $2 \leq i \leq n$. Then the induced edge labels are respectively $f^*(a_1) = 1$, $f^*(a_i) = 4i-2$ for $2 \leq i \leq n-1$ and $f^*(b_1) = 2, f^*(b_2) = 3$,

$$f^*(b_i) = \begin{cases} 2i-1 & i \text{ is odd} \\ 2i & i \text{ is even} \end{cases}$$

for $3 \leq i \leq 2(n-1)$, $f^*(c_1) = 4$, $f^*(c_i) = 4i-1$ for $2 \leq i \leq n-1$. Hence, f is a $F-$root square mean labeling of the graph Q_n. Thus the graph quadrilateral snake Q_n is a $F-$root square mean graph. □

Theorem 2.5 *The alternative quadrilateral snake $A(Q_n)$ is a $F-$root square mean graph.*

Proof Let $\{u_i, 1 \leq i \leq n, v_i, 1 \leq i \leq n\}$ be the vertices and $\{a_i, 1 \leq i \leq n-1, b_i, 1 \leq i \leq n, c_i, 1 \leq i \leq \frac{n}{2}\}$ the edges of $A(Q_n)$. We define $f : V(G) \to \{1, 2, \cdots, q+1\}$ of vertices by $f(u_1) = 1, f(u_2) = 2$,

$$f(u_i) = \begin{cases} \frac{5i-1}{2} & i \text{ is odd} \\ \frac{5i-4}{2} & i \text{ is even} \end{cases}$$

for $3 \leq i \leq n$ and $f(v_1) = 3$,

$$f(v_i) = \begin{cases} \frac{5i}{2} & i \text{ is odd} \\ \frac{5i-3}{2} & i \text{ is even} \end{cases}$$

for $2 \leq i \leq n$. Then the induced edge labels are respectively $f^*(a_1) = 1$, $f^*(a_i) = 5i-3$ for

$2 \leq i \leq n-1$, $f^*(b_1) = 2, f^*(b_2) = 3$,

$$f^*(b_i) = \begin{cases} \frac{5i-3}{2} & i \text{ is odd} \\ \frac{5i-2}{2} & i \text{ is even} \end{cases}$$

for $3 \leq i \leq 2(n-1)$, $f^*(c_1) = 4$ and $f^*(c_i) = 5i - 2$ for $2 \leq i \leq n-1$. Hence, f is a $F-$root square mean labeling of the graph $A(Q_n)$. Thus the graph alternative quadrilateral snake $A(Q_n)$ is a $F-$root square mean graph. □

Theorem 2.6 *The double quadrilateral snake $D(Q_n), (n \geq 3)$ is a $F-$root square mean graph.*

Proof Let $\{u_i, 1 \leq i \leq 2(n-1), v_i, 1 \leq i \leq n\}$ be the vertices and $\{a_i, 1 \leq i \leq n-1, b_i, 1 \leq i \leq 2(n-1), c_i, 1 \leq i \leq n-1\}$ the edges of $D(Q_n)$. We label define $f : V(G) \to \{1, 2, \cdots, q+1\}$ of vertices by $f(u_1) = 1, f(u_2) = 4$,

$$f(u_i) = \begin{cases} 2i & i \text{ is odd} \\ 2i-1 & i \text{ is even} \end{cases}$$

for $3 \leq i \leq 2n-2$ and $f(v_1) = 3$, $f(v_i) = 4i - 3$ for $2 \leq i \leq n$. Then the induced edge labels are respectively $f^*(a_1) = 1$, $f^*(a_i) = 4i - 2$ for $2 \leq i \leq n-1$ and $f^*(b_1) = 2, f^*(b_2) = 3$,

$$f^*(b_i) = \begin{cases} 2i-1 & i \text{ is odd} \\ 2i & i \text{ is even} \end{cases}$$

for $3 \leq i \leq 2n-2$, $f^*(c_1) = 4, f^*(c_2) = 5$ and $f^*(c_i) = 4i - 1$ for $2 \leq i \leq n-1$. Hence, f is a $F-$root square mean labeling of the graph $D(Q_n)$. Thus the graph double quadrilateral snake $D(Q_n), (n \geq 3)$ is a $F-$root square mean graph. □

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