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A Note on Acyclic Coloring of Sunlet Graph Families

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Abstract: In this paper, we find the acyclic chromatic number χ_a for the central graph of sunlet graph $C(S_n)$, line graph of sunlet graph $L(S_n)$, middle graph of sunlet graph $M(S_n)$ and the total graph of sunlet graph $T(S_n)$ for all $n \geqslant 3$.

Key Words: Smarandachely vertex coloring, acyclic coloring, sunlet graph, central graph, line graph, middle graph and total graph.

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§1. Introduction

Let G be a finite graph and let $H \prec G$ be a subgraph of G. A Smarandachely vertex coloring respect to a subgraph $H \prec G$ by colors in $\mathscr{C}$ is a mapping $\varphi_H : \mathscr{C} \rightarrow E(G)$ such that $\varphi_H(e_1) \neq \varphi_H(e_2)$ if e_1 and e_2 are edges of a subgraph isomorphic to H in G. Particularly, let $H = G$. Then, such a Smarandachely vertex coloring is clearly the usual proper vertex coloring (or proper coloring) of G, i.e., a coloring ϕ: $V \rightarrow N^+$ on G such that if v and u are adjacent vertices, then $\phi(v) \neq \phi(u)$. The chromatic number of a graph G is the minimum number of colors required in any proper coloring of G. Generally, The notion of acyclic coloring was introduced by Branko Grünbaum in 1973. An acyclic coloring of a graph G is a proper vertex coloring such that the induced subgraph of any two color classes is acyclic, i.e., disjoint collection of trees. The minimum number of colors needed to acyclically color the vertices of a graph G is called as acyclic chromatic number and is denoted by $\chi_a(G)$.

§2. Preliminaries

A *sunlet graph* on $2n$ vertices is obtained by attaching n pendant edges to the cycle C_n and denoted by S_n.

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For a given graph $G = (V, E)$ we do an operation on G by subdividing each edge exactly once and joining all the non-adjacent vertices of G. The graph obtained by this process is called *central graph* [5] of G denoted by $C(G)$.

A *line graph* [1, 4] of a graph G, denoted by $L(G)$, is a graph whose vertices are the edges of G, and if $u, v \in E(G)$ then $uv \in E(L(G))$ if u and v share a vertex in G.

A *middle graph* [3] of G, is defined with the vertex set $V(G) \cup E(G)$ where two vertices are adjacent iff they are either adjacent edges of G or one is the vertex and the other is an edge incident with it and it is denoted by $M(G)$.

The *total graph* [1 ,3, 4] of G has vertex set $V(G) \cup E(G)$, and edges joining all elements of this vertex set which are adjacent or incident in G.

Additional graph theory terminology used in this paper can be found in [1, 4].

In the following sections we find the acyclic chromatic number for the central graph of sunlet graph $C(S_n)$, line graph of sunlet graph $L(S_n)$, middle graph of sunlet graph $M(S_n)$ and the total graph of sunlet graph $T(S_n)$.

Definition 2.1([2]) *An ayclic coloring of a graph G is a proper coloring such that the union of any two color classes induces a forest.*

§3. Acyclic Coloring on Central Graph of Sunlet Graph

Theorem 3.1 *Let S_n be a sunlet graph with $2n$ vertices, then*

$$\chi_a(C(S_n)) = n, \forall\, n \geq 3.$$

Proof Let $V(S_n) = \{u_1, u_2, \cdots, u_n\} \bigcup \{v_1, v_2, \cdots, v_n\}$ and $E(S_n) = \{e_i : 1 \leq i \leq n\} \bigcup \{e'_i : 1 \leq i \leq n\}$, where e_i is the edge $v_i v_{i+1} (1 \leq i \leq n-1)$, e_n is the edge $v_n v_1$ and e'_i is the edge $v_i u_i (1 \leq i \leq n)$. For $1 \leq i \leq n$, u_i is the pendant vertex and v_i is the adjacent vertex to u_i. By the definition of central graph $V(C(S_n)) = V(S_n) \bigcup E(S_n) = \{u_i : 1 \leq i \leq n\} \bigcup \{v_i : 1 \leq i \leq n\} \bigcup \{v'_i : 1 \leq i \leq n\} \cup \{u'_i : 1 \leq i \leq n\}$, where v'_i and u'_i represents the edge e_i and e'_i, $(1 \leq i \leq n)$ respectively.

Assign the following coloring for $C(S_n)$ as acyclic:

(1) For $1 \leq i \leq n$ assign the color c_i to u_i, v_i;

(2) For $1 \leq i \leq n-1$ assign the color c_{i+1} to u'_i and c_1 to u'_n;

(3) For $1 \leq i \leq n-1$ assign the color c_i to v'_i and c_n to v'_1.

Thus, $\chi_a(C(S_n)) = n$, for if $\chi_a(C(S_n)) < n$, say $n-1$. A contradiction to proper coloring since, $\forall n$, $\{u_i : 1 \leq i \leq n\}$ forms a clique of order n. Hence, $\chi_a(C(S_n)) = n, \forall n \geq 3$. □

§4. Acyclic Coloring on Line Graph of Sunlet Graph

Theorem 4.1 *Let $n \geq 3$ be a positive integer, then $\chi_a(L(S_n)) = 3$.*

Proof Let $V(S_n) = \{u_1, u_2, \cdots, u_n\} \bigcup \{v_1, v_2, \cdots, v_n\}$ and $E(S_n) = \{e_i : 1 \leq i \leq n\} \bigcup \{e'_i : 1 \leq i \leq n\}$, where e_i is the edge $v_iv_{i+1}(1 \leq i \leq n-1)$, e_n is the edge v_nv_1 and e'_i is the edge $v_iu_i(1 \leq i \leq n)$. By the definition of line graph $V(L(S_n)) = E(S_n) = \{u'_i : 1 \leq i \leq n\} \cup \{v'_i : 1 \leq i \leq n-1\} \bigcup \{v'_n\}$ where v'_i and u'_i represents the edge e_i and e'_i, $(1 \leq i \leq n)$ respectively.

Assign the coloring σ as acyclic as follows:

(1) For $1 \leq i \leq 3$, assign the vertices v'_i as $\sigma(v'_i) = c_i$, and for $4 \leq i \leq n$, let $\sigma(v'_i) = c_1$ if $i \equiv 1 \bmod 3$, $\sigma(v'_i) = c_2$ if $i \equiv 2 \bmod 3$, $\sigma(v'_i) = c_3$ if $i \equiv 0 \bmod 3$;

(2) For $1 \leq i \leq n$, assign the vertices u'_i with colors c_1, c_2, c_3 such that $\sigma(u'_i) \neq \sigma(v'_{i-1})$ and $\sigma(u'_i) \neq \sigma(v'_i)$, where $v'_0 = v'_n$.

Thus, $\chi_a(L(S_n)) = 3$, $\forall n \geq 3$.

To the contrary, let $\chi_a(L(S_n)) < 3$, say 2. A contradiction to proper coloring, since, for $\{1 \leq i \leq n-1\}$, $\left\{v'_i, u'_{i+1}, v'_{i+1}\right\}$ is a complete graph K_3. Hence, $\chi_a(L(S_n)) = 3, \forall n \geq 3$. □

§5. Acyclic Coloring on Middle Graph of Sunlet Graph

Theorem 5.1 *Let $n \geq 3$ be a positive integer, then $\chi_a(M(S_n)) = 4$.*

Proof Let $V(S_n) = \{u_1, u_2, \cdots, u_n\} \bigcup \{v_1, v_2, \cdots, v_n\}$ and $E(S_n) = \{e_i : 1 \leq i \leq n\} \bigcup \{e'_i : 1 \leq i \leq n\}$, where e_i is the edge $v_iv_{i+1}(1 \leq i \leq n-1)$, e_n is the edge v_nv_1 and e'_i is the edge $v_iu_i(1 \leq i \leq n)$. By definition of middle graph $V(M(S_n)) = V(S_n) \bigcup E(S_n) = \{u_i : 1 \leq i \leq n\} \bigcup \{v_i : 1 \leq i \leq n\} \bigcup \{v'_i : 1 \leq i \leq n\} \bigcup \{u'_i : 1 \leq i \leq n\}$, where v'_i and u'_i represents the edge e_i and e'_i, $(1 \leq i \leq n)$ respectively.

Define the mapping σ such that $\sigma(V(M(S_n))) \rightarrow c_i$ for $1 \leq i \leq 4$ as follows:

(1) For $1 \leq i \leq n$, assign the vertices v'_i as

$\sigma(v'_i) = \{c_1c_2c_3 \quad c_1c_2c_3 \cdots c_1c_2c_3\}$ if $n \equiv 0 \bmod 3$,
$\sigma(v'_i) = \{c_1c_2c_3 \quad c_1c_2c_3 \cdots c_1c_2c_3 \quad c_2\}$ if $n \equiv 1 \bmod 3$,
$\sigma(v'_i) = \{c_1c_2c_3 \quad c_1c_2c_3 \cdots c_1c_2c_3 \quad c_1c_2\}$ if $n \equiv 2 \bmod 3$;

(2) Assign $\sigma(u_i) = \sigma(v_i) = c_4$ for $1 \leq i \leq n$;

(3) Assign the vertices u'_i with c_1, c_2, c_3 such that $\sigma(u'_i) \neq \sigma(v'_{i-1})$ and $\sigma(u'_i) \neq \sigma(v'_i)$ for $1 \leq i \leq n$ where $v'_0 = v'_n$.

Thus, $\chi_a(M(S_n)) = 4$, $\forall n \geq 3$.

To the contrary, let $\chi_a(M(S_n)) < 4$, say 3. A contradiction to proper coloring, since $\forall n$, $\left\{v'_{i-1}, v'_i, v_i, u'_i\right\}$, where $v'_0 = v'_n$, forms a clique of order 4. Thus σ is a proper acyclic coloring and hence $\chi_a(M(S_n)) = 3, \forall n \geq 3$. □

§6. Acyclic Coloring on Total Graph of Sunlet Graph

Theorem 6.1 *Let S_n be a sunlet graph with $2n$ vertices then for $n \geq 3$, $\chi_a(T(S_n)) = 6$.*

Proof Let $V(S_n) = \{u_1, u_2, \cdots, u_n\} \bigcup \{v_1, v_2, \cdots, v_n\}$ and $E(S_n) = \{e_i : 1 \leq i \leq n\} \bigcup \{e'_i : 1 \leq i \leq n\}$, where e_i is the edge $v_i v_{i+1} (1 \leq i \leq n-1)$, e_n is the edge $v_n v_1$ and e'_i is the edge $v_i u_i (1 \leq i \leq n)$. By the definition of total graph $V(T(S_n)) = V(S_n) \bigcup E(S_n) = \{u_i : 1 \leq i \leq n\} \bigcup \{v_i : 1 \leq i \leq n\} \bigcup \{v'_i : 1 \leq i \leq n\} \bigcup \{u'_i : 1 \leq i \leq n\}$, where v'_i and u'_i represents the edge e_i and e'_i, $(1 \leq i \leq n)$ respectively.

Define the mapping σ such that $\sigma(V(T(S_n))) \to c_i$ for $1 \leq i \leq 6$ as follows:

(1) For $1 \leq i \leq 3$, let $\sigma(v'_i) = c_i$ and for $4 \leq i \leq n$, let

$\sigma(v'_i) = c_1$ if $i \equiv 1 \bmod 3$,
$\sigma(v'_i) = c_2$ if $i \equiv 2 \bmod 3$,
$\sigma(v'_i) = c_3$ if $i \equiv 0 \bmod 3$;

(2) For $1 \leq i \leq 3$, let $\sigma(v_i) = c_{i+3}$ and for $4 \leq i \leq n$, let

$\sigma(v_i) = c_4$ if $i \equiv 1 \bmod 3$,
$\sigma(v_i) = c_5$ if $i \equiv 2 \bmod 3$,
$\sigma(v_i) = c_6$ if $i \equiv 0 \bmod 3$;

(3) Let $\sigma(u'_i) = \sigma(v'_i) + 1$ for $1 \leq i \leq n$;

(4) For $1 \leq i \leq n$, let $\sigma(u_i) = \sigma(v_i) + 1$ and $\sigma(u_i) = c_1$ if $\sigma(v_i) = c_6$.

Thus, $\chi_a(T(S_n)) = 6$ for $n \geq 3$.

For $1 \leq i \leq 6$, the union of any two color classes c_{i-1} and c_i induces subgraph whose components are trees, hence by Definition 1.1, σ is a proper acyclic coloring and $\chi_a(T(S_n)) = 6$ for $n \geq 3$. □

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