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On (r, m, k)-Regular Fuzzy Graphs

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Abstract: In this paper, (r, m, k)- regular fuzzy graph and totally (r, m, k)- regular fuzzy graph are defined and compared through various examples. A necessary and sufficient condition under which they are equivalent is provided. Also (r, m, k)-regularity on some fuzzy graphs whose underlying crisp graph is a cycle is studied with some specific membership functions.

Key Words: Degree of a vertex in fuzzy graph, regular fuzzy graph, total degree, totally regular fuzzy graph, d_m- degree of a vertex in graph, semiregular graphs, (m, k)-regular fuzzy graphs, totally (m, k)-regular fuzzy graphs.

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§1. Introduction

Azriel Rosenfeld introduced fuzzy graphs in 1975 [12]. It has been growing fast and has numerous applications in various fields. A.Nagoor Gani and K.Radha [11] introduced regular fuzzy graphs, total degree and totally regular fuzzy graphs. Alison Northup introduced Semiregular graphs that we call it as $(2, k)$-regular graphs and studied some properties on $(2, k)$-regular graphs [2].

N.R.Santhi Maheswari and C. Sekar introduced d_2-degree of a vertex in fuzzy graphs, total d_2-degree of a vertex in fuzzy graphs, $(2, k)$-regular fuzzy graphs and totally $(2, k)$-regular fuzzy graphs [14]. Also they introduced $(r, 2, k)$-regular fuzzy graphs and totally $(r, 2, k)$-regular fuzzy graphs [15].

Also they introduced d_m-degree of a vertex in fuzzy graphs, total d_m-degree of a vertex in fuzzy graphs, m-Neighbourly irregular fuzzy graphs and totally m-Neighbourly irregular fuzzy graphs [16]. Also, they introduced (m, k)-regular fuzzy graphs and totally (m, k)-regular fuzzy graphs [17].

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These motivate us to introduce (r, m, k)-regular fuzzy graphs and totally (r, m, k)-regular fuzzy graphs. We make comparative study between (r, m, k)-regular fuzzy graphs and totally (r, m, k)-regular fuzzy graphs. Then we provide a necessary and sufficient condition under which they are equivalent. Also (r, m, k)-regularity on fuzzy graphs whose underlying crisp graph is a cycle is studied with some specific membership functions.

§2. Preliminaries

We present some known definitions and results for ready reference to go through the work presented in this paper.

Definition 2.1([9]) *A Fuzzy graph denoted by $G : (\sigma, \mu)$ on graph $G^* : (V, E)$ is a pair of functions (σ, μ) where $\sigma : V \to [0, 1]$ is a fuzzy subset of a non empty set V and $\mu : V \times V \to [0, 1]$ is a symmetric fuzzy relation on σ such that for all u, v in V the relation $\mu(u, v) = \mu(uv) \le \sigma(u) \wedge \sigma(v)$ is satisfied, where σ and μ are called membership function. A fuzzy graph G is complete if $\mu(u, v) = \mu(uv) = \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$, where uv denotes the edge between u and v. $G^* : (V, E)$ is called the underlying crisp graph of the fuzzy graph $G : (\sigma, \mu)$.*

Definition 2.2([10]) *The strength of connectedness between two vertices u and v is $\mu^\infty(u, v) = sup\{\mu^k(u, v)/k = 1, 2, \dots\}$ where $\mu^k(u, v) = sup\{\mu(uu_1) \wedge \mu(u_1u_2) \wedge \cdots \wedge \mu(u_{k-1}v)/u, u_1, u_2, \cdots, u_{k-1}, v$ is a path connecting u and v of length $k\}$.*

Definition 2.3([11]) *Let $G : (\sigma, \mu)$ be a fuzzy graph. The degree of a vertex u is $d_G(u) = \sum_{u \ne v} \mu(uv)$ for $uv \in E$ and $\mu(uv) = 0$, for uv not in E; this is equivalent to $d_G(u) = \sum_{uv \in E} \mu(uv)$.*

Definition 2.4([11]) *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. If $d(v) = k$ for all $v \in V$, then G is said to be regular fuzzy graph of degree k.*

Definition 2.5([11]) *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. The total degree of a vertex u is defined as $td(u) = \sum \mu(u, v) + \sigma(u) = d(u) + \sigma(u)$, $uv \in E$. If each vertex of G has the same total degree k, then G is said to be totally regular fuzzy graph of degree k or k-totally regular fuzzy graph.*

Definition 2.6([14]) *Let $G : (\sigma, \mu)$ be a fuzzy graph. The d_2-degree of a vertex u in G is $d_2(u) = \sum \mu^2(u, v)$, where $\mu^2(uv) = \sup\{\mu(uu_1) \wedge \mu(u_1v) : u, u_1, v$ is the shortest path connecting u and v of length $2\}$. Also, $\mu(uv) = 0$, for uv not in E.*
The minimum d_2-degree of G is $\delta_2(G) = \wedge\{d_2(v) : v \in V\}$.
The maximum d_2-degree of G is $\Delta_2(G) = \vee\{d_2(v) : v \in V\}$.

Definition 2.7([14]) *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. If $d_2(v) = k$ for all $v \in V$, then G is said to be $(2, k)$-regular fuzzy graph.*

Definition 2.8([14]) *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. The total d_2-degree of a* *vertex* $u \in V$ *is defined as* $td_2(u) = \sum \mu^2(u,v) + \sigma(u) = d_2(u) + \sigma(u)$.

The minimum td_2*-degree of* G *is* $t\delta_2(G) = \wedge\{td_2(v) : v \in V\}$.

The maximum td_2*-degree of* G *is* $t\Delta_2(G) = \vee\{td_2(v) : v \in V\}$.

Definition 2.9([14]) *If each vertex of* G *has the same total* d_2 *- degree* k, *then* G *is said to be totally* $(2,k)$*-regular fuzzy graph.*

Definition 2.10([15]) *If each vertex of* G *has the same degree* r *and same* d_2*-degree* k, *then* G *is said to be* $(r,2,k)$*-regular fuzzy graph.*

Definition 2.11([15]) *If each vertex of* G *has the same total degree* r *and same total* d_2*-degree* k, *then* G *is said to be totally* $(r,2,k)$*-regular fuzzy graph.*

Definition 2.12([16]) *Let* $G : (\sigma,\mu)$ *be a fuzzy graph on* $G^* : (V,E)$.. *The* d_m*-degree of a vertex* u *in* G *is* $d_m(u) = \sum \mu^m(uv)$, *where* $\mu^m(uv) = sup\{\mu(uu_1) \wedge \mu(u_1u_2) \wedge \ldots, \mu(u_{m-1}v) : u, u_1, u_2, \ldots, u_{m-1}, v$ *is the shortest path connecting* u *and* v *of length* $m\}$. *Also,* $\mu(uv) = 0$, *for* uv *not in* E.

The minimum d_m*-degree of* G *is* $\delta_m(G) = \wedge\{d_m(v) : v \in V\}$.

The maximum d_m*-degree of* G *is* $\Delta_m(G) = \vee\{d_m(v) : v \in V\}$.

Definition 2.13([16]) *Let* $G : (\sigma,\mu)$ *be a fuzzy graph on* $G^* : (V,E)$. *The total* d_m*-degree of a vertex* $u \in V$ *is defined as* $td_m(u) = \sum \mu^m(uv) + \sigma(u) = d_m(u) + \sigma(u)$.

The minimum td_m*-degree of* G *is* $t\delta_m(G) = \wedge\{td_m(v) : v \in V\}$.

The maximum td_m*-degree of* G *is* $t\Delta_m(G) = \vee\{td_m(v) : v \in V\}$.

Definition 2.14([17]) *Let* $G : (\sigma,\mu)$ *be a fuzzy graph on* $G^* : (V,E)$. *If* $d_m(v) = k$ *for all* $v \in V$, *then* G *is said to be* (m,k)*-regular fuzzy graph.*

Definition 2.15([17]) *If each vertex of* G *has the same total* d_m *- degree* k, *then* G *is said to be totally* (m,k)*-regular fuzzy graph.*

§3. (r,m,k)-Regular Fuzzy Graphs

In this section, we define (r,m,k)-Regular Fuzzy Graphs and illustrates this with $(r,3,k)$-regular graph.

Definition 3.1 *Let* $G : (\sigma,\mu)$ *be a fuzzy graph on* $G^* : (V,E)$. *If* $d(v) = r$ *and* $d_m(v) = k$, *for all* $v \in V$, *then* G *is said to be* (r,m,k)*-regular fuzzy graph. That is, if each vertex of* G *has the same degree* r *and same* d_m*-degree* k, *then* G *is said to be* (r,m,k)*-regular fuzzy graph.*

Example 3.2 Consider $G^? : (V,E)$, where $V = \{u_1,u_2,u_3,u_4,u_5,u_6,u_7,u_8,u_9,u_{10}\}$ and $E = \{u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_5u_6, u_6u_7, u_7u_8, u_8u_9, u_9u_{10}, u_{10}u_1\}$. Define $G : (\sigma,\mu)$ by $\sigma(u_1) = 0.3$, $\sigma(u_2) = 0.4$, $\sigma(u_3) = 0.5$, $\sigma(u_4) = 0.6$, $\sigma(u_5) = 0.7$, $\sigma(u_6) = 0.6, \sigma(u_7) = 0.5$, $\sigma(u_8) = 0.4, \sigma(u_9) = 0.3$, $\sigma(u_{10}) = 0.2$ and $\mu(u_1u_2) = 0.3$, $\mu(u_2u_3) = 0.4$, $\mu(u_3u_4) = 0.3$, $\mu(u_4u_5) = 0.4$, $\mu(u_5u_6) = 0.3$, $\sigma(u_6u_7) = 0.4$, $\sigma(u_7u_8) = 0.3$, $\sigma(u_8u_9) = 0.4$, $\sigma(u_9u_{10}) = 0.3$, $\sigma(u_{10}u_1) =$

0.4.

$$d_3(u_1) = \{0.3 \wedge 0.4 \wedge 0.3\} + \{0.3 \wedge 0.4 \wedge 0.3\} = 0.3 + 0.3 = 0.6.$$
$$d_3(u_2) = \{0.3 \wedge 0.3 \wedge 0.4\} + \{0.4 \wedge 0.3 \wedge 0.4\} = 0.3 + 0.3 = 0.6.$$
$$d_3(u_3) = \{0.4 \wedge 0.3 \wedge 0.3\} + \{0.3 \wedge 0.4 \wedge 0.3\} = 0.3 + 0.3 = 0.6.$$
$$d_3(u_4) = \{0.3 \wedge 0.4 \wedge 0.3\} + \{0.4 \wedge 0.3 \wedge 0.4\} = 0.3 + 0.3 = 0.6.$$
$$d_3(u_5) = \{0.3 \wedge 0.4 \wedge 0.3\} + \{0.4 \wedge 0.3 \wedge 0.4\} = 0.3 + 0.3 = 0.6.$$
$$d_3(u_6) = \{0.4 \wedge 0.3 \wedge 0.4\} + \{0.3 \wedge 0.4 \wedge 0.3\} = 0.3 + 0.3 = 0.6.$$
$$d_3(u_7) = \{0.3 \wedge 0.4 \wedge 0.3\} + \{0.4 \wedge 0.3 \wedge 0.4\} = 0.3 + 0.3 = 0.6.$$
$$d_3(u_8) = \{0.4 \wedge 0.3 \wedge 0.3\} + \{0.3 \wedge 0.4 \wedge 0.3\} = 0.3 + 0.3 = 0.6.$$
$$d_3(u_9) = \{0.3 \wedge 0.3 \wedge 0.4\} + \{0.4 \wedge 0.3 \wedge 0.4\} = 0.3 + 0.3 = 0.6.$$
$$d(u_i) = \{0.3 + 0.4\} = 0.7 \text{ for i } = 1,2,3,4,5,6,7,8,9,10.$$

It is noted that, each vertex has the same d_3-degree 0.6 and each vertex has the same degree 0.7. Hence G is $(0.7, 3, 0.6)$-regular fuzzy graph.

Example 3.3 Consider $G^* : (V, E)$, where $V = \{u, v, w, x, y, z\}$ and
$E = \{uv, vw, wx, xy, yz, zu\}$.

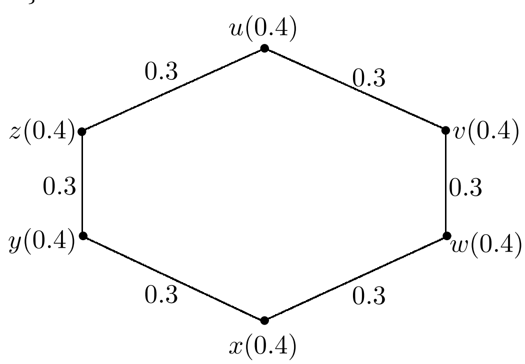

Figure 1

$$d_3(u) = Sup\{0.3 \wedge 0.3 \wedge 0.3, 0.3 \wedge 0.3 \wedge 0.3\} = Sup\{0.3, 0.3\} = 0.3.$$
$$d_3(v) = Sup\{0.3 \wedge 0.3 \wedge 0.3, 0.3 \wedge 0.3 \wedge 0.3\} = Sup\{0.3, 0.3\} = 0.3.$$
$$d_3(w) = Sup\{0.3 \wedge 0.3 \wedge 0.3, 0.3 \wedge 0.3 \wedge 0.3\} = Sup\{0.3, 0.3\} = 0.3.$$
$$d_3(x) = Sup\{0.3 \wedge 0.3 \wedge 0.3, 0.3 \wedge 0.3 \wedge 0.3\} = Sup\{0.3, 0.3\} = 0.3.$$
$$d_3(y) = Sup\{0.3 \wedge 0.3 \wedge 0.3, 0.3 \wedge 0.3 \wedge 0.3\} = Sup\{0.3, 0.3\} = 0.3.$$
$$d_3(z) = Sup\{0.3 \wedge 0.3 \wedge 0.3, 0.3 \wedge 0.3 \wedge 0.3\} = Sup\{0.3, 0.3\} = 0.3.$$

In Figure 1, $d(u) = 0.3 + 0.3 = 0.6,\ d(v) = 0.6,\ d(w) = 0.6, d(x) = 0.6,\ d(y) = 0.6,\ d(z) = 0.6$. Each vertex has the same d_3-degree 0.3 and each vertex has the same degree 0.3. Hence G is a $(0.6, 3, 0.3)$-regular fuzzy graph.

Example 3.4 Non regular fuzzy graphs which is (m, k)-regular

1. Let $G:(\sigma,\mu)$ be a fuzzy graph such that $G^*:(V,E)$, a path on $2m$ vertices. Let all the edges of G have the same membership value c. Then, for $i=1,2,3,4,5,\cdots,m$,

$$\begin{aligned}
d_m(v_i) &= \{\mu(e_i)\wedge\mu(e_{i+1})\wedge\{\mu(e_{i+2})\cdots\wedge\mu(e_{m-1+i})\}\\
&= \{c\wedge c\wedge c\cdots\wedge c\} = c.\\
d_m(v_{m+i}) &= \{\mu(e_i)\wedge\mu(e_{i+1})\}+\{\mu(e_{i+2})\cdots\wedge\mu(e_{m-1+i})\}\\
&= \{c\wedge c\wedge c\cdots\wedge c\} = c.\\
d_m(v) &= c,\ \text{for all } v\in V.
\end{aligned}$$

Hence $G:(\sigma,\mu)$ is (m,c)-regular fuzzy graph.

For $i=2,3,4,5,\cdots,2m-1$,

$$\begin{aligned}
d(v_i) &= \{\mu(e_{i-1})+\mu(e_i) = 2c.\\
d(v_1) &= \{\mu(e_1)\} = c.\\
d(v_{2m}) &= \mu(e_{2m-1}) = c.
\end{aligned}$$

$d(v_1)\neq d(v_i)\neq d(v_{2m})$ for $i=2,4,5,\cdots,2m-1$. Hence G is non- regular fuzzy graph which is (m,c)-regular.

Example 3.5 Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$, a cycle of length $\geq 2m+1$. Let

$$\mu(e_i)=\begin{cases} c_1 & \text{if i is odd}\\ \text{membership value } x\geq c_1 & \text{if } i \text{ is even, where } x \text{ is not constant and}\end{cases}$$

$$d_m(v)=min\{c_1,x\}+min\{x,c_1\}=c_1+c_1=2c_1$$

for all $v\in V$.

Case 1. Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$ an even cycle of length $\leq 2m+2$. Then $d(v_i)=x+c_1$, for $i=1,2,4,5,\cdots,2m+1$. So, $G:(\sigma,\mu)$ is non-regular (m,k)-regular fuzzy graph, since x is not constant.

Case 2 Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$ a odd cycle of length $\leq 2m+1$. Hence $G:(\sigma,\mu)$ is $(m,2c_1)$-regular fuzzy graph and $d(v_1)=2c_1,\ d(v_i)=x+c_1$ for $i=2,4,5,\cdots,2m+1$. So, $G:(\sigma,\mu)$ is non-regular (m,k)-regular fuzzy graph since x is not constant.

§4. Totally (r,m,k)-Regular Fuzzy Graphs

In this section, we introduce totally (r,m,k)-regular fuzzy graph and the necessary and sufficient condition under which (r,m,k)-regular fuzzy graph and totally (r,m,k)-regular fuzzy graph are equivalent is provided.

Definition 4.1 *If each vertex of G has the same total degree r and same total d_m-degree k, then G is said to be totally (r,m,k)-regular fuzzy graph.*

From Figure 1, it is noted that each vertex has the same total d_3-degree 0.7.

$$td_3(u) = d_3(u) + \sigma(u) = 0.3 + 0.4 = 0.7$$
$$td_3(v) = d_3(v) + \sigma(v) = 0.3 + 0.4 = 0.7$$
$$td_3(w) = d_3(w) + \sigma(w) = 0.3 + 0.4 = 0.7$$
$$td_3(x) = d_3(x) + \sigma(x) = 0.3 + 0.4 = 0.7$$
$$td_3(y) = d_3(y) + \sigma(y) = 0.3 + 0.4 = 0.7$$
$$td_3(z) = d_3(z) + \sigma(z) = 0.3 + 0.4 = 0.7$$

$$td(u) = d(u) + \sigma(u) = 0.8 + 0.4 = 1.2$$
$$td(v) = d(v) + \sigma(v) = 0.8 + 0.4 = 1.2$$
$$td(w) = d(w) + \sigma(w) = 0.8 + 0.4 = 1.2$$
$$td(x) = d(x) + \sigma(x) = 0.8 + 0.4 = 1.2$$
$$td(y) = d(y) + \sigma(y) = 0.8 + 0.4 = 1.2$$
$$td(z) = d(z) + \sigma(z) = 0.8 + 0.4 = 1.2$$

In Figure 1, Each vertex has the same total d_3-degree 0.7 and each vertex has the same total degree 1.2. Hence $G : (\sigma, \mu)$ is totally $(1.2, 3, 0.7)$-regular fuzzy graph.

Theorem 4.2 *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. Then σ is constant function iff the following conditions are equivalent:*

(1) *$G : (\sigma, \mu)$ is (r, m, k)-regular fuzzy graph;*
(2) *$G : (\sigma, \mu)$ is totally (r, m, k)-regular fuzzy graph.*

Proof Suppose that σ is constant function. Let $\sigma(u) = c$, constant for all $u \in V$. Assume that $G : (\sigma, \mu)$ is (r, m, k)-regular fuzzy graph. Then $d(u) = r$ and $d_m(u) = k$, for all $u \in V$. So

$$td(u) = d(u) + \sigma(u) \text{ and } td_m(u) = d_m(u) + \sigma(u) \text{ for all } u \in V.$$
$$\Rightarrow td(u) = r + c \text{ and } td_m(u) = k + c \text{ for all } u \in V.$$

Hence $G : (\sigma, \mu)$ is totally $(r + c, m, k + c)$- regular fuzzy graph. Thus (1) $\Rightarrow$ (2) is proved. Now suppose G is totally (r, m, k)-regular fuzzy graph.

$$\Rightarrow td_m(u) = k \text{ and } td(u) = r \text{ for all } u \in V.$$
$$\Rightarrow d_m(u) + \sigma(u) = k \text{ and } d(u) + \sigma(u) = r \text{ for all } u \in V.$$
$$\Rightarrow d_m(u) + c = k \text{ and } d(u) + \sigma(u) = r \text{ for all } u \in V.$$
$$\Rightarrow d_m(u) = k - c \text{ and } d(u) = r - c \text{ for all } u \in V.$$

Hence $G : (\sigma, \mu)$ is $(r - c, m, k - c)$-regular fuzzy graph and (1) and (2) are equivalent.

Conversely assume that (1) and (2) are equivalent. Suppose σ is not constant function. Then $\sigma(u) \neq \sigma(w)$, for at least one pair $u, w \in V$. Let $G : (\sigma, \mu)$ be a (r, m, k)-regular fuzzy graph. Then, $d_m(u) = d_m(w) = k$ and $d(u) = d(w) = r$. So, $td_m(u) = d_m(u) + \sigma(u) = k + \sigma(u)$ and $td_m(w) = d_m(w) + \sigma(w) = k + \sigma(w)$ and $td(u) = d(u) + \sigma(u) = r + \sigma(u)$ and $td(w) = d(w) + \sigma(w) = r + \sigma(w)$. Since $\sigma(u) \neq \sigma(w) \Rightarrow k + \sigma(u) \neq k + \sigma(w)$ and $r + \sigma(u) \neq r + \sigma(w \Rightarrow td_m(u) \neq td_m(w)$ and $td(u) \neq td(w)$. So $G : (\sigma, \mu)$ is not totally (r, m, k)-regular fuzzy graph which is contradiction to our assumption. Let $G : (\sigma, \mu)$ be a totally (r, m, k)-regular fuzzy graph. Then, $td_m(u) = td_m(w)$ and $td(u) = td(w)$.

$$\begin{aligned}
&\Rightarrow d_m(u) + \sigma(u) = d_m(w) + \sigma(w) \text{ and } d(u) + \sigma(u) = d(w) + \sigma(w) \\
&\Rightarrow d_m(u) - d_m(w) = \sigma(w) - \sigma(u) \neq 0 \text{ and } d(u) - d(w) \\
&\qquad\qquad\qquad = \sigma(w) - \sigma(u) \neq 0 \\
&\Rightarrow d_m(u) \neq d_m(w) \text{ and } d(u) \neq d(w).
\end{aligned}$$

So $G : (\sigma, \mu)$ is not (r, m, k)-regular fuzzy graph which is a contradiction to our assumption. Hence σ is constant function. □

Theorem 4.3 *If a fuzzy graph $G : (\sigma, \mu)$ is both (r, m, k)-regular and totally (r, m, k)-regular then σ is constant function.*

Proof Let G be (r_1, m, k_1)-regular and totally (r_2, m, k_2)-regular fuzzy graph. Then $d_m(u) = k_1$ and $td_m(u) = k_2, d(u) = r_1$ and $td(u) = r_2$, for all $u \in V$. Now, $td_m(u) = k_2$ and $td(u) = r_2$, for all $u \in V$.

$$\begin{aligned}
&\Rightarrow d_m(u) + \sigma(u) = k_2 \text{ and } d(u) + \sigma(u) = r_2 \text{ for all } u \in V. \\
&\Rightarrow k_1 + \sigma(u) = k_2 \text{ and } r_1 + \sigma(u) = r_2 \text{ for all } u \in V. \\
&\Rightarrow \sigma(u) = k_2 - k_1 \text{ and } \sigma(u) = r_2 - r_1 \text{ for all } u \in V.
\end{aligned}$$

Hence σ is constant function. □

§5. (r, m, k)- Regular Fuzzy Graph on a Cycle with Some Specific Membership Function.

In this section, (r, m, k)-regularity on a cycle C_{2m}, C_{2m+1} is studied with some specific membership functions.

Theorem 5.1 *For any $m \geq 1$, let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, a cycle of length $\geq 2m$. If μ is constant function, then $G : (\sigma, \mu)$ is (r, m, k)-regular fuzzy graph, where $r = 2\mu(uv)$ and $k = \mu(uv)$.*

Proof If μ is constant function say $\mu(uv) = c$, then $d_m(v) = Sup\{(c \wedge c \cdots \wedge c)), (c \wedge c \cdots \wedge c\}$ = c, for all $v \in V$ and $d(v) = c + c = 2c$. Hence G is $(2c, m, c)$-regular fuzzy graph. □

Remark 5.2 Converse of the above Theorem need not be true.

Theorem 5.3 *For any $m \geq 1$, let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, a cycle of length*

$\geq 2m+1$. *If μ is constant function, then G is (r, m, k)-regular fuzzy graph, where $r = 2\mu(uv)$ and $k = 2\mu(uv)$.*

Proof If μ is constant function say $\mu(uv) = c$, then $d_m(v) = \{c \wedge c \cdots \wedge c\} + \{c \wedge c \cdots \wedge c\} = c + c = 2c$, for all $v \in V$ and $d(v) = c + c = 2c$. Hence G is $(2c, m, 2c)$-regular fuzzy graph. □

Remark 5.4 Converse of the above Theorem need not be true.

Theorem 5.5 *For any $m \geq 1$, let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, an even cycle of length $\geq 2m+2$. If the alternate edges have the same membership values, then $G : (\sigma, \mu)$ is (r, m, k)-regular fuzzy graph.*

Proof If the alternate edges have the same membership values, then

$$\mu(e_i) = \begin{cases} c_1, & \text{if } i \text{ is odd} \\ c_2, & \text{if } i \text{ is even.} \end{cases}$$

If $c_1 = c_2$, then μ is constant function. So, $G : (\sigma, \mu)$ is $(2c_1, m, 2c_1)$-regular fuzzy graph. If $c_1 < c_2$, then $d_m(v) = \{c_1 \wedge c_2 \ldots c_1 \wedge c_2\} + \{c_1 \wedge c_2 \ldots c_1 \wedge c_2\} = c_1 + c_1 = 2c_1$, for all $v \in V$ and $d(v) = c_1 + c_2$. Hence $G : (\sigma, \mu)$ is $(c_1 + c_2, m, 2c_1)$-regular fuzzy graph.

If $c_1 > c_2$, then $d_m(v) = \{c_1 \wedge c_2 \ldots c_1 \wedge c_2\} + \{c_1 \wedge c_2 \ldots c_1 \wedge c_2\} = c_2 + c_2 = 2c_2$, for all $v \in V$ and $d(v) = c_1 + c_2$. Hence $G : (\sigma, \mu)$ is $(c_1 + c_2, m, 2c_2)$-regular fuzzy graph. □

Remark 5.6 Even if the alternate edges of a fuzzy graph whose underlying graph is an even cycle of length $\geq 2m+2$ have the same membership values, then $G : (\sigma, \mu)$ need not be totally (r, m, k)-regular fuzzy graph, since if σ is not constant function then $G : (\sigma, \mu)$ is not totally (r, m, k)-regular fuzzy graph, for any $m \geq 1$.

Theorem 5.7 *For any $m > 1$, let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, a cycle of length $\geq 2m+1$. Let*

$$\mu(e_i) = \begin{cases} c_1, & \textit{if } i \textit{ is odd} \\ c_2 \geq c_1, & \textit{if } i \textit{ is even,} \end{cases}$$

then $G : (\sigma, \mu)$ is a (m, k)-regular fuzzy graph.

Proof Let

$$\mu(e_i) = \begin{cases} c_1, & \text{if } i \text{ is odd} \\ c_2 \geq c_1, & \text{if } i \text{ is even} \end{cases}$$

Case 1. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$ an even cycle of length $\leq 2m+2$. Then by theorem 6.3, G is $(c_1 + c_2, m, 2c_1)$-regular fuzzy graph.

Case 2. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$ an odd cycle of length $\leq 2m+1$. For any $m > 1$, $d_m(v) = 2c_1$, for all $v \in V$. But $d(v_1) = c_1 + c_1 = 2c_1$ and $d(v_i) = c_1 + c_2$, for $i \neq 1$. Hence G is not (r, m, k)-regular fuzzy graph. □

Remark 5.8 Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, an even cycle of length $\geq 2m + 1$. Even if

$$\mu(e_i) = \begin{cases} c_1, & \text{if } i \text{ is odd} \\ c_2 \geq c_1 & \text{if } i \text{ is even,} \end{cases}$$

then G need not be totally (r, m, k)-regular fuzzy graph, since if σ is not constant function then G is not totally (r, m, k)-regular fuzzy graph.

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