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Skolem Difference Odd Mean Labeling of H-Graphs

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Abstract: A graph G with p vertices and q edges is said to have a skolem difference odd mean labeling if there exists an injective function $f : V(G) \rightarrow \{1, 2, 3, \cdots, 4q-1\}$ such that the induced map $f^* : E(G) \rightarrow \{1, 3, 5, \cdots, 2q-1\}$ defined by $f^*(uv) = \left\lceil \frac{|f(u)-f(v)|}{2} \right\rceil$ is a bijection. A graph that admits skolem difference odd mean labeling is called a skolem difference odd mean graph. In this paper, we investigate skolem difference odd mean labeling of some H-graphs.

Key Words: Skolem difference odd mean labeling, skolem difference odd mean graph.

AMS(2010): 05C78

§1. Introduction

Throughout this paper, by a graph we mean a finite, undirected and simple graph. Let $G(V, E)$ be a graph with p vertices and q edges. For notations and terminology we follow [1].

Path on n vertices is denoted by P_n. $K_{1,m}$ is called a star and is denoted by S_m. The bistar $B_{m,n}$ is the graph obtained from K_2 by identifying the center vertices of $K_{1,m}$ and $K_{1,n}$ at the end vertices of K_2 respectively. The H-graph of a path P_n, denoted by H_n is the graph obtained from two copies of P_n with vertices $v_1, v_2, \cdots, v_n$ and $u_1, u_2, \cdots, u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$ if n is odd and the vertices $v_{\frac{n}{2}+1}$ and $u_{\frac{n}{2}}$ if n is even. The corona of a graph G on p vertices $v_1, v_2, \cdots, v_p$ is the graph obtained from G by adding p new vertices $u_1, u_2, \ldots, u_p$ and the new edges u_iv_i for $1 \leq i \leq p$. The corona of G is denoted by $G \odot K_1$. The 2-corona of a graph G, denoted by $G \odot S_2$ is a graph obtained from G by identifying the center vertex of the star S_2 at each vertex of G. The disjoint union of two graphs G_1 and G_2 is the graph $G_1 \cup G_2$ with $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$.

The concept of mean labeling was introduced and studied by S. Somasundaram and R. Ponraj [5]. Some new families of mean graphs are studied by S.K. Vaidya et al. [6]. Further some more results on mean graphs are discussed in [4,7,8]. A graph G is said to be a mean graph if there exists an injective function f from $V(G)$ to $\{0, 1, 2, \cdots, q\}$ such that the induced map f^* from $E(G)$ to $\{1, 2, 3, \cdots, q\}$ defined by $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is a bijection.

[1]Received June 4, 2014, Accepted March 2, 2015.

In [2], K. Manickam and M. Marudai introduced odd mean labeling of a graph. A graph G is said to be odd mean if there exists an injective function f from $V(G)$ to $\{0, 1, 2, 3, \cdots, 2q-1\}$ such that the induced map f^* from $E(G)$ to $\{1, 3, 5, \cdots, 2q-1\}$ defined by $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is a bijection. Some more results on odd mean graphs are discussed in [9,10].

The concept of skolem difference mean labeling was introduced and studied by K. Murugan and A. Subramanian [3]. A graph $G = (V, E)$ with p vertices and q edges is said to have skolem difference mean labeling if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1, 2, 3, \cdots, p+q$ in such a way that for each edge $e = uv$, let $f^*(e) = \left\lceil \frac{|f(u)-f(v)|}{2} \right\rceil$ and the resulting labels of the edges are distinct and are from $1, 2, 3, \cdots, q$. A graph that admits a skolem difference mean labeling is called a skolem difference mean graph.

The concept of skolem difference odd mean labeling was introduced in [11]. A graph with p vertices and q edges is said to have a skolem difference odd mean labeling if there exists an injective function $f : V(G) \to \{1, 2, 3, \cdots, 4q-1\}$ such that the induced map $f^* : E(G) \to \{1, 3, 5, \cdots, 2q-1\}$ defined by $f^*(uv) = \left\lceil \frac{|f(u)-f(v)|}{2} \right\rceil$ is a bijection. A graph that admits a skolem difference odd mean labeling is called a skolem difference odd mean graph.

A skolem difference odd mean labeling of $B_{4,7}$ is shown in Figure 1.

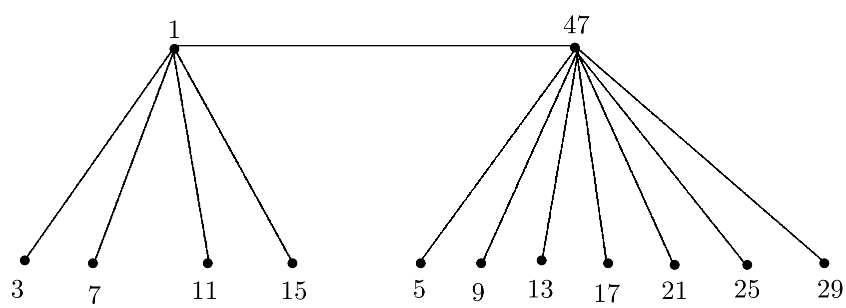

Figure 1

In this paper, we prove that the H-graph, corona of a H-graph, 2-corona of a H-graph are skolem difference odd mean graph. Also we prove that union of any two skolem difference odd mean H-graphs is also a skolem difference odd mean graph.

§2. Skolem Difference Odd Mean Graphs

Theorem 2.1 *The H-graph G is a skolem difference odd mean graph.*

Proof Let $v_1, v_2, \cdots, v_n$ and $u_1, u_2, \cdots, u_n$ be the vertices of the H-graph G. The graph G has $2n$ vertices and $2n-1$ edges.

Define $f : V(G) \to \{1, 2, 3, \cdots, 4q-1 = 8n-5\}$ as follows:

$$f(v_i) = \begin{cases} 2i-1, & 1 \le i \le n \text{ and } i \text{ is odd} \\ 8n-2i-1, & 1 \le i \le n \text{ and } i \text{ is even} \end{cases}$$

$$f(u_i) = \begin{cases} 6n-2i-1, & \text{if } n \text{ is odd, } 1 \le i \le n \text{ and } i \text{ is odd} \\ 2n+2i-1, & \text{if } n \text{ is odd, } 1 \le i \le n \text{ and } i \text{ is even} \\ 2n+2i-1, & \text{if } n \text{ is even, } 1 \le i \le n \text{ and } i \text{ is odd} \\ 6n-2i-1, & \text{if } n \text{ is even, } 1 \le i \le n \text{ and } i \text{ is even.} \end{cases}$$

For the vertex labeling f, the induced edge labeling f^* is given as follows:

$$f^*(v_i v_{i+1}) = 4n-2i-1, \quad 1 \le i \le n-1$$
$$f^*(u_i u_{i+1}) = 2n-2i-1, \quad 1 \le i \le n-1$$

$$f^*\left(v_{\frac{n+1}{2}} u_{\frac{n+1}{2}}\right) = 2n-1 \quad \text{if n is odd and}$$
$$f^*\left(v_{\frac{n}{2}+1} u_{\frac{n}{2}}\right) = 2n-1 \quad \text{if n is even.}$$

Thus, f is a skolem difference odd mean labeling and hence the H-graph G is a skolem difference odd mean graph. □

For example, a skolem difference odd mean labeling of H-graphs G_1 and G_2 are shown in Figure 2.

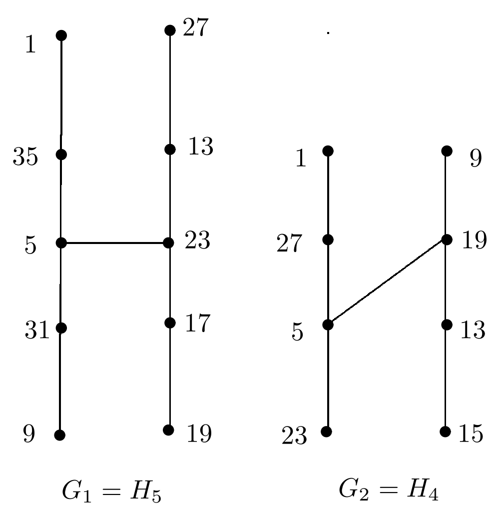

Figure 2

Theorem 2.2 *For a H-graph G, $G \odot K_1$ is a skolem difference odd mean graph.*

Proof By Theorem 2.1, there exists a skolem difference odd mean labeling f for G. Let

$v_1, v_2, \cdots, v_n$ and $u_1, u_2, \cdots, u_n$ be the vertices of G.

$$\text{Let } V(G \odot K_1) = V(G) \cup \{v_1', v_2', \ldots, v_n'\} \cup \{u_1', u_2', \ldots, u_n'\}$$
$$\text{and } E(G \odot K_1) = E(G) \cup \{v_i v_i', u_i u_i' : 1 \leq i \leq n\}.$$

Case 1. n is odd.

Define $g : V(G \odot K_1) \to \{1, 2, \cdots, 16n - 5\}$ as follows:

$$\begin{aligned}
g(v_{2i-1}) &= f(v_{2i-1}), \quad 1 \leq i \leq \frac{n+1}{2} \\
g(v_{2i}) &= f(v_{2i}) + 8n, \quad 1 \leq i \leq \frac{n-1}{2} \\
g(u_{2i-1}) &= f(u_{2i-1}) + 8n, \quad 1 \leq i \leq \frac{n+1}{2} \\
g(u_{2i}) &= f(u_{2i}) \\
g(v_{2i-1}') &= g(u_n) - 4n - 4(i-1), \quad 1 \leq i \leq \frac{n+1}{2} \\
g(v_{2i}') &= g(u_{n-1}) + 4n + 4i, \quad 1 \leq i \leq \frac{n-1}{2} \\
g(u_{2i-1}') &= g(u_n) - 2n + 4(i-1), \quad 1 \leq i \leq \frac{n+1}{2} \\
g(u_{2i}') &= g(u_{n-1}) + 2n - 4(i-1), \quad 1 \leq i \leq \frac{n-1}{2}.
\end{aligned}$$

For the vertex labeling g, the induced edge labeling g^* is given as follows:

$$\begin{aligned}
g^*(v_i v_{i+1}) &= f^*(v_i v_{i+1}) + 4n, \quad 1 \leq i \leq n-1 \\
g^*(u_i u_{i+1}) &= f^*(u_i u_{i+1}) + 4n, \quad 1 \leq i \leq n-1 \\
g^*(v_i v_i') &= 4n + 1 - 2i, \quad 1 \leq i \leq n \\
g^*(u_i u_i') &= 2n + 1 - 2i, \quad 1 \leq i \leq n \\
g^*\left(v_{\frac{n+1}{2}} u_{\frac{n+1}{2}}\right) &= 3f^*\left(v_{\frac{n+1}{2}} u_{\frac{n+1}{2}}\right) + 2.
\end{aligned}$$

Case 2. n is even.

Define $g : V(G \odot K_1) \to \{1, 2, 3, \cdots, 16n - 5\}$ as follows:

$$\begin{aligned}
g(v_{2i-1}) &= f(v_{2i-1}), \quad 1 \leq i \leq \frac{n}{2} \\
g(v_{2i}) &= f(v_{2i}) + 8n, \quad 1 \leq i \leq \frac{n}{2} \\
g(u_{2i-1}) &= f(u_{2i-1}), \quad 1 \leq i \leq \frac{n}{2} \\
g(u_{2i}) &= f(u_{2i}) + 8n, \quad 1 \leq i \leq \frac{n}{2} \\
g(v_{2i-1}') &= g(u_{n-1}) + 4n + 6 - 4i, \quad 1 \leq i \leq \frac{n}{2}
\end{aligned}$$

$$g(v'_{2i}) = g(u_n) - 4n - 2 + 4i, \quad 1 \leq i \leq \frac{n}{2}$$
$$g(u'_{2i-1}) = g(u_{n-1}) + 2(n+1) - 4(i-1), \quad 1 \leq i \leq \frac{n}{2}$$
$$g(u'_{2i}) = g(u_n) - 2(n+1) + 4i, \quad 1 \leq i \leq \frac{n}{2}.$$

For the vertex labeling g, the induced edge labeling g^* is obtained as follows:

$$g^*(v_iv_{i+1}) = f^*(v_iv_{i+1}) + 4n$$
$$g^*(u_iu_{i+1}) = f^*(u_iu_{i+1}) + 4n$$
$$g^*(v_iv'_i) = 4n + 1 - 2i$$
$$g^*(u_iu'_i) = 2n + 1 - 2i$$
$$g^*\left(v_{\frac{n}{2}+1}u_{\frac{n}{2}}\right) = 3f^*\left(v_{\frac{n}{2}+1}u_{\frac{n}{2}}\right) + 2.$$

Thus, g is a skolem difference odd mean labeling and hence $G \odot K_1$ is a skolem difference odd mean graph. □

For example, a skolem difference odd mean labeling of H-graphs $G_1, G_2, G_1 \odot K_1$ and $G_2 \odot K_1$ are shown in Figure 3.

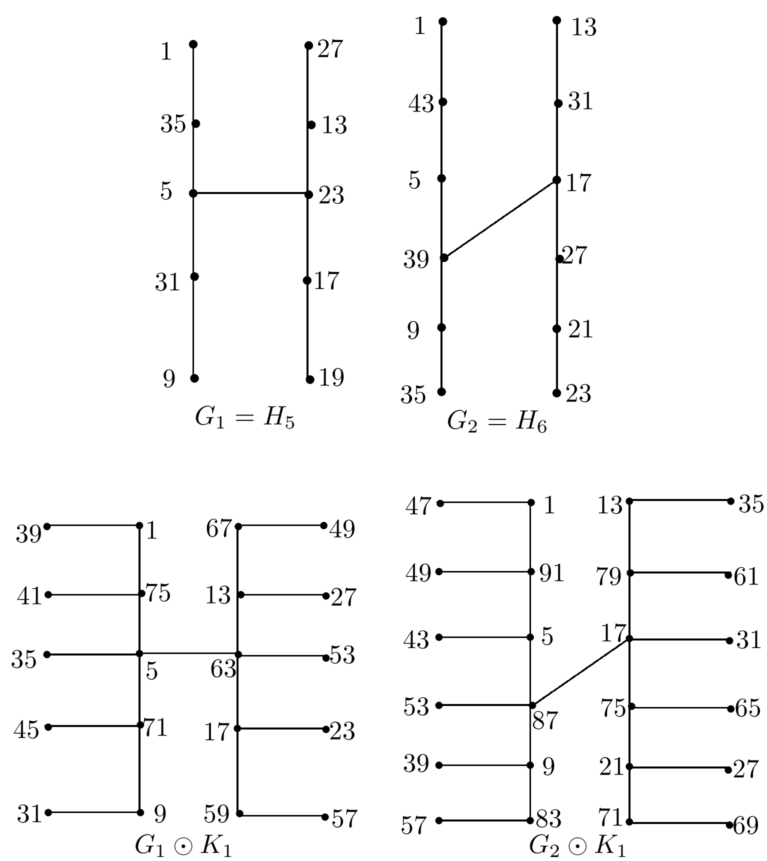

Figure 3

Theorem 2.3 *For a H-graph G, $G \odot S_2$ is a skolem difference odd mean graph.*

Proof By Theorem 2.1, there exists a skolem difference odd mean labeling f for G. Let $v_1, v_2, \cdots, v_n$ and $u_1, u_2, \cdots, u_n$ be the vertices of G. Let $V(G)$ together with $v'_1, v'_2, \cdots, v'_n, v''_1, v''_2, \cdots, v''_n, u'_1, u'_2, \cdots, u'_n$ and $u''_1, u''_2, \cdots, u''_n$ form the vertex set of $G \odot S_2$ and the edge set is $E(G)$ together with $\{v_i v'_i, v_i v''_i, u_i u'_i, u_i u''_i : 1 \le i \le n\}$.

Case 1. n is odd.

Define $g : V(G \odot S_2) \to \{1, 2, 3, \cdots, 24n - 5\}$ as follows:

$$\begin{aligned}
g(v_{2i-1}) &= f(v_{2i-1}), \quad 1 \le i \le \frac{n+1}{2} \\
g(v_{2i}) &= f(v_{2i}) + 16n, \quad 1 \le i \le \frac{n-1}{2} \\
g(u_{2i-1}) &= f(u_{2i-1}) + 16n, \quad 1 \le i \le \frac{n+1}{2} \\
g(u_{2i}) &= f(u_{2i}), \quad 1 \le i \le \frac{n-1}{2} \\
g(v'_{2i-1}) &= g(u_n) - 4n - 12(i-1), \quad 1 \le i \le \frac{n+1}{2} \\
g(v'_{2i}) &= g(u_{n-1}) + 4n - 4 + 12i, \quad 1 \le i \le \frac{n-1}{2} \\
g(v''_{2i-1}) &= g(v'_{2i-1}) - 4, \quad 1 \le i \le \frac{n+1}{2} \\
g(v''_{2i}) &= g(v'_{2i}) + 4, \quad 1 \le i \le \frac{n-1}{2} \\
g(u'_{2i-1}) &= g(u_n) - 6n + 12(i-1), \quad 1 \le i \le \frac{n+1}{2} \\
g(u'_{2i}) &= g(u_{n-1}) + 4n + 18 - 12i, \quad 1 \le i \le \frac{n-1}{2} \\
g(u''_{2i-1}) &= g(u'_{2i-1}) + 4, \quad 1 \le i \le \frac{n+1}{2} \\
g(u''_{2i}) &= g(u'_{2i}) - 4, \quad 1 \le i \le \frac{n-1}{2}.
\end{aligned}$$

For the vertex labeling g, the induced edge labeling g^* is given as follows:

$$\begin{aligned}
g^*(v_i v_{i+1}) &= f^*(v_i v_{i+1}) + 8n, \quad 1 \le i \le n-1 \\
g^*(u_i u_{i+1}) &= f^*(u_i u_{i+1}) + 8n, \quad 1 \le i \le n-1 \\
g^*(v_i v'_i) &= 8n + 3 - 4i, \quad 1 \le i \le n \\
g^*(v_i v''_i) &= 8n + 1 - 4i, \quad 1 \le i \le n \\
g^*(u_i u'_i) &= 4n + 3 - 4i, \quad 1 \le i \le n \\
g^*(u_i u''_i) &= 4n + 1 - 4i, \quad 1 \le i \le n \\
g^*\left(v_{\frac{n+1}{2}} u_{\frac{n+1}{2}}\right) &= 5f^*\left(v_{\frac{n+1}{2}} u_{\frac{n+1}{2}}\right) + 4.
\end{aligned}$$

Case 2. n is even.

Define $g : V(G \odot S_2) \to \{1, 2, 3, \cdots, 24n-5\}$ as follows:

$$g(v_{2i-1}) = f(v_{2i-1}), \quad 1 \le i \le \frac{n}{2}$$
$$g(v_{2i}) = f(v_{2i}) + 16n, \quad 1 \le i \le \frac{n}{2}$$
$$g(u_{2i-1}) = f(u_{2i-1}), \quad 1 \le i \le \frac{n}{2}$$
$$g(u_{2i}) = f(u_{2i}) + 16n, \quad 1 \le i \le \frac{n}{2}$$
$$g(v'_{2i-1}) = g(u_{n-1}) + 12n + 14 - 12i, \quad 1 \le i \le \frac{n}{2}$$
$$g(v'_{2i}) = g(u_n) - 12n - 6 + 12i, \quad 1 \le i \le \frac{n}{2}$$
$$g(v''_{2i-1}) = g(v'_{2i-1}) - 4, \quad 1 \le i \le \frac{n}{2}$$
$$g(v''_{2i}) = g(v'_{2i}) + 4, \quad 1 \le i \le \frac{n}{2}$$
$$g(u'_{2i-1}) = g(u_{n-1}) + 6n + 14 - 12i, \quad 1 \le i \le \frac{n}{2}$$
$$g(u'_{2i}) = g(u_n) - 6n - 6 + 12i, \quad 1 \le i \le \frac{n}{2}$$
$$g(u''_{2i-1}) = g(u'_{2i-1}) - 4, \quad 1 \le i \le \frac{n}{2}$$
$$g(u''_{2i}) = g(u'_{2i}) + 4, \quad 1 \le i \le \frac{n}{2}.$$

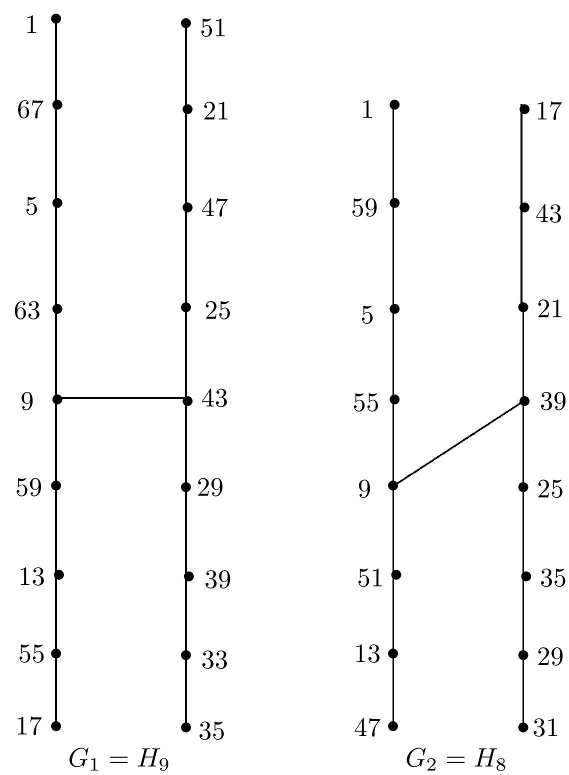

Figure 4

For the vertex labeling g, the induced edge labeling g^* is obtained as follows:

$$g^*(v_iv_{i+1}) = f^*(v_iv_{i+1}) + 8n, \quad 1 \le i \le n-1$$
$$g^*(u_iu_{i+1}) = f^*(u_iu_{i+1}) + 8n, \quad 1 \le i \le n-1$$
$$g^*(v_iv_i') = 8n + 3 - 4i, \quad 1 \le i \le n$$
$$g^*(v_iv_i'') = 8n + 1 - 4i, \quad 1 \le i \le n$$
$$g^*(u_iu_i') = 4n + 3 - 4i, \quad 1 \le i \le n$$
$$g^*(u_iu_i'') = 4n + 1 - 4i, \quad 1 \le i \le n$$
$$g^*\left(v_{\frac{n}{2}+1}u_{\frac{n}{2}}\right) = 5f^*\left(v_{\frac{n}{2}+1}u_{\frac{n}{2}}\right) + 4.$$

Thus, f is a skolem difference odd mean labeling and hence the graph $G \odot S_2$ is a skolem difference odd mean graph. □

For example, a skolem difference odd mean labeling of H-graphs $G_1, G_2, G_1 \odot S_2$ and $G_2 \odot S_2$ are shown in Figures 4 and 5.

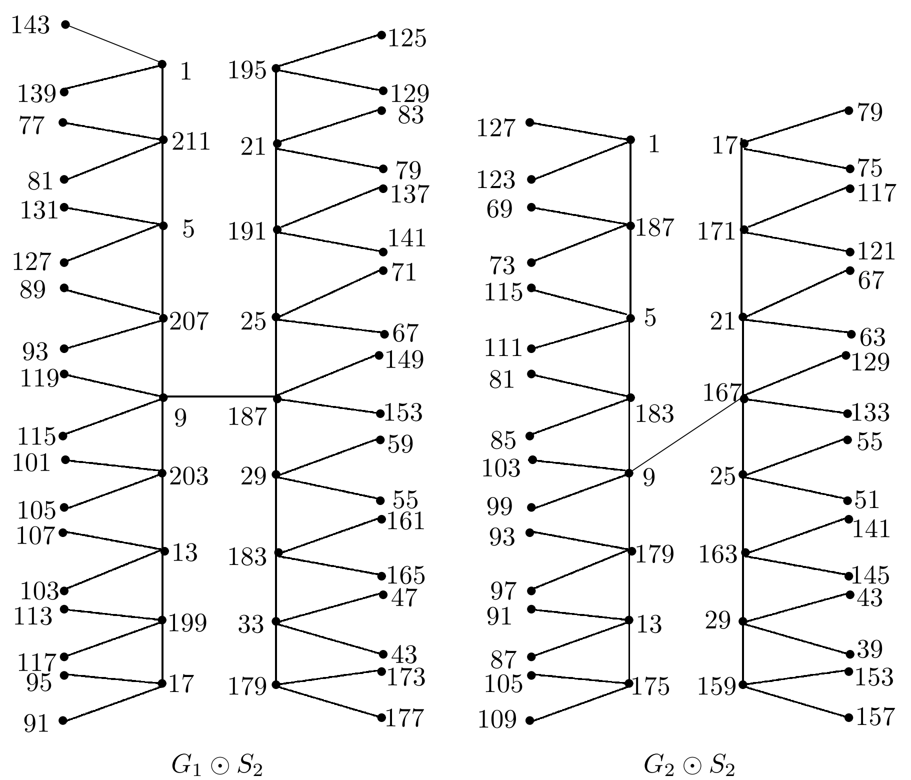

Figure 5

Theorem 2.4 *If G_1 and G_2 are skolem difference odd mean H-graphs, then $G_1 \cup G_2$ is also a skolem difference odd mean graph.*

Proof Let $V(G_1) = \{u_i, v_i : 1 \leq i \leq n\}$ and $V(G_2) = \{s_j, t_j : 1 \leq j \leq m\}$ be the vertices of the H-graphs G_1 and G_2 respectively. Then the graph $G_1 \cup G_2$ has $2(n+m)$ vertices and $2(n+m-1)$ edges. Let $f : V(G_1) \to \{1, 2, 3, \cdots, 8n-5\}$ and $g : V(G_2) \to \{1, 2, 3, \cdots, 8m-5\}$ be a skolem difference odd mean labeling of G_1 and G_2 respectively.

Define $h : V(G_1 \cup G_2) \to \{1, 2, 3, \cdots, 4q-1 = 8(n+m)-9\}$ as follows:

For $1 \leq i \leq n$ and $n \geq 1$,

$$h(u_i) = \begin{cases} f(u_i) & \text{if } i \text{ is odd} \\ f(u_i) + 8m - 4 & \text{if } i \text{ is even} \end{cases}$$

$$h(v_i) = \begin{cases} f(v_i) + 8m - 4 & \text{if } n \text{ is odd and } i \text{ is odd} \\ f(v_i) & \text{if } n \text{ is odd and } i \text{ is even} \\ f(v_i) & \text{if } n \text{ is even and } i \text{ is odd} \\ f(v_i) + 8m - 4 & \text{if } n \text{ is even and } i \text{ is even.} \end{cases}$$

For $1 \leq j \leq m$ and $m \geq 1$,

$$h(s_j) = g(s_j) + 2$$
$$h(t_j) = g(t_j) + 2.$$

For the vertex labeling h, the induced edge labeling h^* is given as follows:

For $1 \leq i \leq n-1$ and $n \geq 1$,

$$h^*(u_i u_{i+1}) = f^*(u_i u_{i+1}) + 4m - 2$$
$$h^*(v_i v_{i+1}) = f^*(v_i v_{i+1}) + 4m - 2$$
$$h^*\left(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right) = f^*\left(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right) + 4m - 2 \quad \text{if n is odd and}$$
$$h^*\left(u_{\frac{n}{2}+1} v_{\frac{n}{2}}\right) = f^*\left(u_{\frac{n}{2}+1} v_{\frac{n}{2}}\right) + 4m - 2 \quad \text{if n is even.}$$

For $1 \leq j \leq m-1$ and $m \geq 1$,

$$h^*(s_j s_{j+1}) = g^*(s_j s_{j+1})$$
$$h^*(t_j t_{j+1}) = g^*(t_j t_{j+1})$$

$$h^*\left(s_{\frac{m+1}{2}} t_{\frac{m+1}{2}}\right) = g^*\left(s_{\frac{m+1}{2}} t_{\frac{m+1}{2}}\right) \quad \text{if m is odd}$$
$$h^*\left(s_{\frac{m}{2}+1} t_{\frac{m}{2}}\right) = g^*\left(s_{\frac{m}{2}+1} t_{\frac{m}{2}}\right) \quad \text{if m is even.}$$

Thus, h is a skolem difference odd mean labeling of $G_1 \cup G_2$ and hence the graph $G_1 \cup G_2$ is a skolem difference odd mean graph. □

For example, a skolem difference odd mean labeling of $G_1 \cup G_2$ where $G_1 = H_3$; $G_2 =$

H_5, $G_1 = H_5$; $G_2 = H_6$, $G_1 = H_4$; $G_2 = H_6$ and $G_1 = H_4$; $G_2 = H_4$ are shown in Figures 6-9 following.

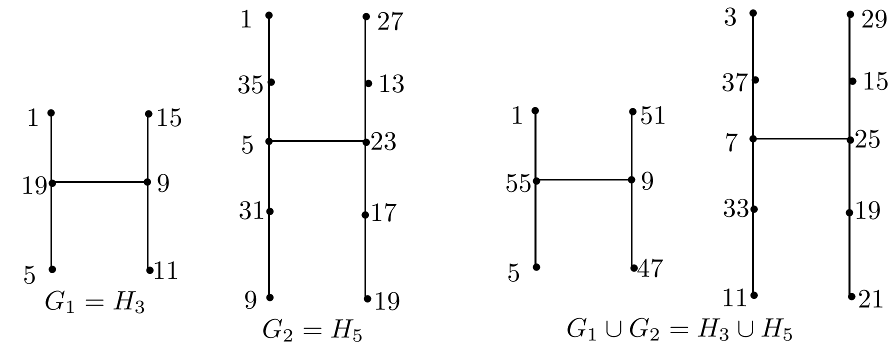

Figure 6

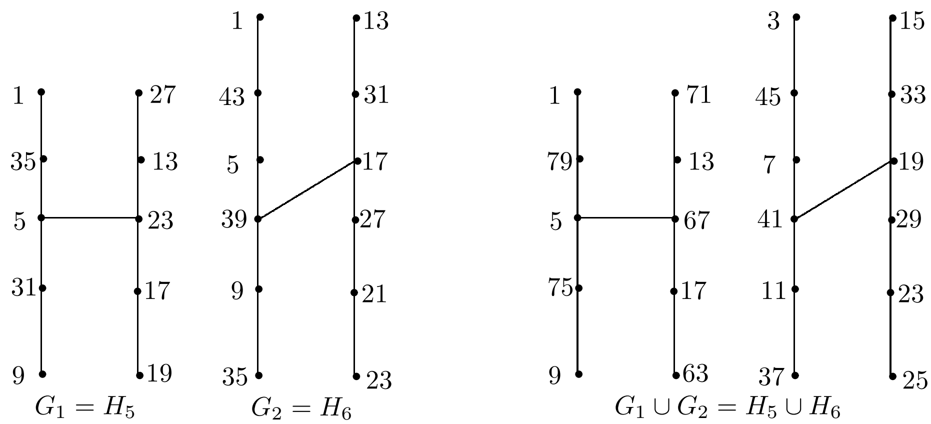

Figure 7

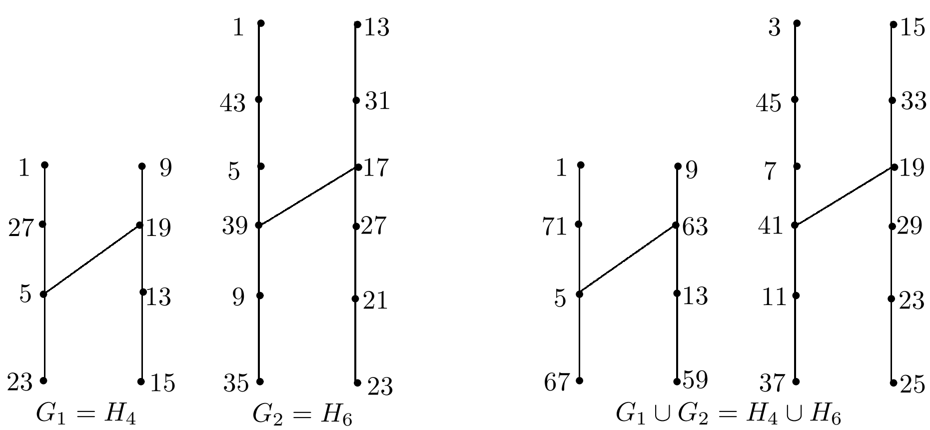

Figure 8

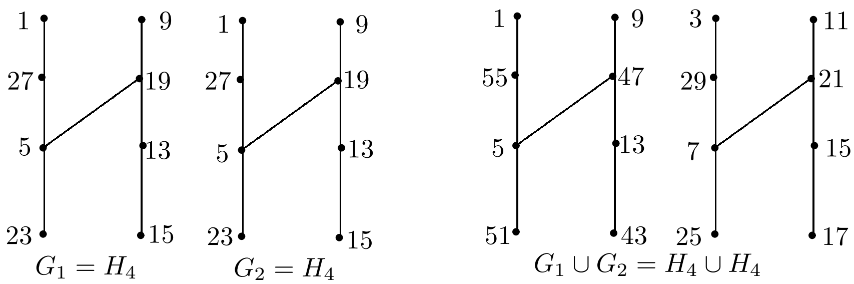

Figure 9

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