

Total Domination in Lict Graph

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Abstract: For any graph $G = (V, E)$, lict graph $\eta(G)$ of a graph G is the graph whose vertex set is the union of the set of edges and the set of cut vertices of G in which two vertices are adjacent if and only if the corresponding edges are adjacent or the corresponding members of G are incident. A dominating set of a graph $\eta(G)$, is a total lict dominating set if the dominating set does not contains any isolates. The total lict dominating number $\gamma_t(\eta(G))$ of the graph G is a minimum cardinality of total lict dominating set of graph G . In this paper many bounds on $\gamma_t(\eta(G))$ are obtained and its exact values for some standard graphs are found in terms of parameters of G . Also its relationship with other domination parameters is investigated.

Key Words: Smarandachely k -dominating set, total lict domination number, lict graph, edge domination number, total edge domination number, split domination number, non-split domination number.

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§1. Introduction

The graphs considered here are finite, connected, undirected without loops or multiple edges and without isolated vertices. As usual ' p ' and ' q ' denote the number of vertices and edges of a graph G . For any undefined term or notation in this paper can be found in Harary [1].

A set $D \subseteq V$ of G is said to be a *Smarandachely k -dominating set* if each vertex of G is dominated by at least k vertices of S and the *Smarandachely k -domination number* $\gamma_k(G)$ of G is the minimum cardinality of a Smarandachely k -dominating set of G . Particularly, if $k = 1$, such a set is called a dominating set of G and the Smarandachely 1-domination number of G is called the *domination number* of G and denoted by $\gamma(G)$ in general.

The lict graph $\eta(G)$ of a graph G is the graph whose vertex set is the union of the set of edges and the set of cut vertices of G in which two vertices are adjacent if and only if the corresponding edges are adjacent or the corresponding members of G are incident. A dominating

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set of a graph $\eta(G)$, is a total list dominating set if the dominating set does not contain any isolates. The total list dominating number $\gamma_t(\eta(G))$ of G is a minimum cardinality of total list dominating set of G .

The vertex independence number $\beta_0(G)$ is the maximum cardinality among the independent set of vertices of G . $L(G)$ is the line graph of G , $\gamma'_e(G)$ is the complementary edge domination number, $\gamma_s(G)$ is the split dominating number, $\gamma'_t(G)$ is the total edge dominating number, $\gamma_{ns}(G)$ is the non-split dominating number, $\chi(G)$ is the chromatic number and $\omega(G)$ is the clique number of a graph G . The degree of an edge $e = uv$ of G is $\deg(e) = \deg(u) + \deg(v) - 2$. The minimum (maximum) degree of an edge in G is denoted by $\delta'(\Delta')$. A subdivision of an edge $e = uv$ of a graph G is the replacement of an edge e by a path (u, v, w) where $w \in E(G)$. The graph obtained from G by subdividing each edge of G exactly once is called the subdivision graph of G and is denoted by $S(G)$. For any real number X , $\lceil X \rceil$ denotes the smallest integer not less than X and $\lfloor X \rfloor$ denotes the greatest integer not greater than X .

In this paper we established the relationship of this concept with the other domination parameters. We use the following theorems for our later results.

Theorem A([2]) For any graph G , $\gamma_e(G) \geq \left\lceil \frac{q}{\Delta' + 1} \right\rceil$.

Theorem B([2]) For any graph G of order $p \geq 3$,

- (i) $\beta_1(G) + \beta_1(\bar{G}) \leq 2 \left\lceil \frac{p}{2} \right\rceil$.
- (ii) $\beta_1(G) * \beta_1(\bar{G}) \leq \left\lceil \frac{p}{2} \right\rceil^2$.

Theorem C([3]) For any graph G ,

- (i) $\gamma'_t(S(K_p)) = 2 \left\lceil \frac{p}{2} \right\rceil$.
- (ii) $\gamma'_t(S(K_{p,q})) = 2q(p \leq q)$.
- (iii) $\gamma'_t(S(G)) = 2(p - \beta_1)$.

Theorem D([4]) For every graph G of order p ,

- (i) $\chi(G) \geq \omega(G)$.
- (ii) $\chi(G) \geq \frac{q}{\beta_0}(G)$.

Theorem E([5]) For any connected graph G with $p \geq 3$ vertices, $\gamma'_t(G) \leq \left\lceil \frac{2p}{3} \right\rceil$.

Theorem F([5]) If G is a connected graph G with $p \geq 4$ vertices and q edges then $\frac{q}{\Delta'} \leq \gamma'_t(G)$, further equality holds for every cycle C_p where $p = 4n, n \geq 1$.

§2. Main Results

Theorem 1 First list out the exact values of $\gamma_t(\eta(G))$ for some standard graphs:

(i) For any cycle C_p with $p \geq 3$ vertices,

$$\gamma_t(\eta(C_p)) = \begin{cases} p/2 & \text{if } p \equiv 0(\text{mod}4). \\ \left\lfloor \frac{p}{2} \right\rfloor + 1 & \text{otherwise.} \end{cases}$$

(ii) For any path P_p with $p \geq 4$ vertices, $\gamma_t(\eta(P_p)) = \left\lfloor \frac{2p}{3} \right\rfloor$.

(iii) For any star graph $K_{1,p}$ with $p \geq 3$ vertices, $\gamma_t(\eta(K_{1,p})) = 2$.

(iv) For any wheel graph W_p with $p \geq 4$ vertices, $\gamma_t(\eta(W_p)) = \left\lfloor \frac{p}{2} \right\rfloor$.

(v) For any complete graph K_p with $p \geq 3$ vertices, $\gamma_t(\eta(K_p)) = \left\lfloor \frac{2p}{3} \right\rfloor$.

(vi) For any friendship graph F_p with k blocks, $\gamma_t(\eta(F_p)) = k$.

Initially we obtain a lower bound of total list domination number with edge and total edge domination number.

Theorem 2 For any graph G , $\gamma_t(\eta(G)) \geq \gamma_e(G)$.

Proof Let D be a γ_e set of graph G , if D is a total list dominating set of a graph G , then for every edge $e_1 \in D$ there exists an edge $e_2 \in D$, $e_1 \neq e_2$ such that e_1 is adjacent to e_2 . Hence $\gamma_t(\eta(G)) = \gamma_e(G)$. Otherwise for each isolated edge $e_i \in D$, choose an edge $e_j \in N(e_i)$. Let $E_1 = \{e_j/e_j \in N(e_i)\}$, then $D \cup E_1$ is a total list dominating set of G and $|D \cup E_1| \geq |D|$. Hence, $\gamma_t(\eta(G)) \geq \gamma_e(G)$. $\square$

Theorem 3 For any graph G $\gamma_t(\eta(G)) \geq \gamma'_t(G)$, equality holds if G is non-separable.

Proof Let D be a γ'_t set of G , if all the cut vertices of G are incident with at least one edge of D , then $\gamma_t(\eta(G)) = \gamma'_t(G)$. Otherwise there exists at least one cut vertex v_c of graph G which is not incident with any edge of D , then $\gamma_t(\eta(G)) \geq |D \cup e| \geq \gamma'_t(G) + 1$, where e is an edge incident with v_c and $e \in N(D)$. Thus, $\gamma_t(\eta(G)) \geq \gamma'_t(G)$.

For the equality, note that if the graph G is non-separable, then $\eta(G) = L(G)$. Thus $\gamma_t(\eta(G)) = \gamma_t(L(G)) = \gamma'_t(G)$. $\square$

Next we obtain an inequality of total list domination in terms of number of vertices, number of edges and maximum edge degree of graph G .

Theorem 4 For any connected graph G with $p \geq 3$ vertices, then $\gamma_t(\eta(G)) \leq 2 \left\lceil \frac{q}{3} \right\rceil$.

Proof Let $E(G) = \{e_1, e_2, e_3, \dots, e_l\}$ and let $D = \{e_i/1 \leq i \leq l \text{ and } i \neq 0(\text{mod}3)\} \cup \{e_{l-1}\}$. Then D is total list dominating set of G and $|D| = 2 \left\lceil \frac{q}{3} \right\rceil$. Hence, $\gamma_t(\eta(G)) \leq 2 \left\lceil \frac{q}{3} \right\rceil$. $\square$

Theorem 5 For any non-separable graph G ,

(i) $\gamma_t(\eta(G)) \leq \left\lceil \frac{2p}{3} \right\rceil$, $p \geq 3$.

(ii) $\frac{q}{\Delta} \leq \gamma_t(\eta(G))$, $p \geq 4$ vertices, equality holds for every cycle C_p , where $p = 4n$, $n \geq 1$.

Proof Let G be a non-separable graph, then $\gamma_t(\eta(G)) = \gamma'_t(G)$. Using Theorems E and F, the result follows. $\square$

Theorem 6 For any connected graph G , $\gamma_t(\eta(G)) \leq q - \Delta'(G) + 1$, where Δ' is a maximum degree of an edge.

Proof Let e be an edge with degree Δ' and let S be a set of edges adjacent to e in G . Then $E(G) - S$ is the list dominating set of graph G . We consider the following two cases.

Case 1 If $\langle E(G) - S \rangle$ contains at least one isolate in $\eta(G)$ other than the vertex corresponding to e in $\eta(G)$.

Let E_1 be the set of all such isolates, then for each isolate $e_i \in E_1$, let $E_2 = \{e_j / e_j \in (N(e_i) \cap N(e))\}$, then $F = [\{(E(G) - S) - E_1\} \cup E_2]$ is a total list dominating set of graph G . Thus, $\gamma_t(\eta(G)) \leq q - \Delta'(G)$.

Case 2 If $\langle E(G) - S \rangle$ contains only e as an isolate in $\eta(G)$.

Then for an edge $e_i \in N(e)$, $\{(E(G) - S) \cup e_i\}$ is a total list dominating set of a graph G . Thus, $\gamma_t(\eta(G)) \leq |(E(G) - S) \cup e_i| = q - \Delta'(G) + 1$.

From Cases 1 and 2, the result follows. $\square$

Theorem 7 For any connected graph G , $\gamma_t(\eta(G)) \geq \left\lceil \frac{q}{\Delta' + 1} \right\rceil$.

Proof Using Theorem 2 and Theorem A, the result follows. $\square$

Theorem 8 For any connected graph G , $\gamma_t(\eta(G)) \leq p - 1$.

Proof Let T be a spanning tree of a graph G . Let $A = \{e_1, e_2, e_3, \dots, e_k\}$ be the set of edges of spanning tree T , A covers all the vertices and cut vertices of a graph $\eta(G)$. Hence, $\gamma_t(\eta(G)) \leq A = p - 1$. $\square$

Now we obtain the relationship between total list domination and total domination of a line graph.

Theorem 9 For any graph G , with k number of cut vertices,

$$\gamma_t(\eta(G)) \leq \gamma_t(L(G)) + k.$$

Proof We consider the following two cases.

Case 1 $k = 0$.

Then the graph G is non-separable, and in that case $\eta(G) = L(G)$. Hence, $\gamma_t(\eta(G)) = \gamma_t(L(G))$.

Case 2 $k \neq 0$.

Let D be a total dominating set of $L(G)$ and let S be the set of cut vertices which is not incident with any edge of D , then for each cut vertex $v_c \in S$, choose exactly one edge in

E_1 , where $E_1 = \{e_j \in E(G)/e_j \text{ is incident with } v_c \text{ and } e_j \in N(D)\}$ with $|E_1| = |v_c|$. Hence, $\gamma_t(\eta(G)) \leq \gamma_t(L(G)) + |E_1| = \gamma_t(L(G)) + |v_c| = \gamma_t(L(G)) + k$.

From Cases 1 and 2, the result follows. $\square$

In the following theorems we obtain total lict domination of any tree in terms of different parameters of G .

Theorem 10 *For any tree T with k number of cut vertices, $\gamma_t(\eta(G)) \leq k + 1$, further equality holds if $T = K_{1,p}$, $p \geq 3$.*

Proof Let $A = \{v_1, v_2, v_3, \dots, v_k\} \subset V(G)$ be the set of all cut vertices of a tree T with $|A| = k$. Since every edge in T is incident with at least one element of A , A covers all the edges and cut vertices of $\eta(G)$, if for every cut vertex $v \in A$ there exists a vertex $u \in A, u \neq v$, such that v is adjacent to u . Otherwise let $e_1 \in E(G)$ such that e_1 is incident with A , so that $\gamma_t(\eta(G)) \leq |A \cup e_1| = |A| + 1 = k + 1$.

To prove the equality, let $K_{1,p}$ be a star and C be the cut vertex and e be any edge of $K_{1,p}$. Then $D = \{C \cup e\}$ is the γ_t set of $\eta(G)$ with cardinality $k + 1$. $\square$

Theorem 11 *For any tree T , $\gamma_t(\eta(T)) \geq \chi(T)$ and equality holds for all star graph $K_{1,p}$.*

Proof $\chi(T) = 2$ and $2 \leq \gamma_t(T) \leq p$. Hence, $\gamma_t(\eta(T)) \geq \chi(T)$. For $T = K_{1,p}$, clearly $\chi(T) = 2$. Using Theorem 1(iii), the equality follows. $\square$

Theorem 12 *For any tree T , $\gamma_t(\eta(T)) \geq \omega(T)$.*

proof The result follows from Theorem 11 and Theorem D. $\square$

Theorem 13 *For any tree T , $\gamma_t(\eta(T)) \geq \frac{q}{\beta_0(T)}$.*

Proof The result follows from Theorem 11 and Theorem D. $\square$

Theorem 14 *For any tree T , $\gamma_t(\eta(T)) \leq \gamma_t(T)$.*

Proof Let T be a tree and D be γ_t of T . Let E_1 denotes the edge set of the induced graph $\langle D \rangle$. Let F be the set of cut vertices which are not incident with any edge of E_1 . we consider the following two cases.

Case 1 If $F = \Phi$, and in $\eta(T)$ if E_1 does not contains any isolates then E_1 is a total lict dominating set of T . Otherwise for each isolated edge $e_i \in E_1$, choose exactly one edge in E_2 , where $E_2 = \{e_j \in E(T)/e_j \in N(e_i)\}$. Then $D^* = E_1 \cup E_2$ is a total lict dominating set of tree T . Hence, $\gamma_t(\eta(T)) \leq |D^*| \leq |D| = \gamma_t(T)$.

Case 2 If $F \neq \Phi$, then for each cut vertex $v_c \in F$. Let $E_2 = \{e_j \in E(T)/e_j \in N(v_c) \text{ and incident with } v_c\}$. Then $D^* = E_1 \cup E_2$ is a total lict dominating set of tree T . Hence, $\gamma_t(\eta(T)) \leq |D^*| \leq |D| = \gamma_t(T)$.

From Cases 1 and 2, the result follows. $\square$

Theorem 15 For any tree T with $p \geq 3$, in which every non-end vertex is incident with an end vertex, then $\gamma_t(\eta(T)) \leq \beta_0(T)$.

Proof We consider the following two cases.

Case 1 $T = K_{1,p}$.

Noticing that $\beta_0(T) = p - 1 \geq 2$ for $p \geq 3$, and using Theorem 1(iii), the result follows. Hence, $\gamma_t(\eta(T)) \leq \beta_0(T)$.

Case 2 $T \neq K_{1,p}$.

Let $B = \{v_1, v_2, v_3, \dots, v_m\} \subset V(G)$ such that $|B| = \beta_0(T)$. Let $S \subseteq B$ be the set of k end vertices of T and $N \subseteq B$ be the set of l non-end vertices of T such that $S \cup N = B$. In T , for each vertex $v_i \in S$ there exists cut vertex $C_i \in N(v_i)$. Then in $\eta(T)$ the cut vertex C_i covers the edges incident with cut vertex C_i of T where $i = 1, 2, 3, 4, 5, \dots, k$ and for each vertex $v_i \in N$ in T , a vertex $v_j \in \eta(T)$ which is a cut vertex of T covers all the edges incident with v_j where $j = 1, 2, 3, 4, 5, \dots, l$. Thus $\{C_i\}_{i=1}^k \cup \{v_j\}_{j=1}^l$ forms a total list dominating set of T . Hence $\gamma_t(\eta(T)) \leq |S \cup N| \leq |B| = \beta_0(T)$.

From case(1) and case(2) the result follows. $\square$

Theorem 16 Let T be any order $p \geq 3$ and n be the number of pendent edges of T , then $n \leq \gamma_t(\eta(S(T))) \leq 2(p - 1) - n$ and equality holds for all $K_{1,p}$.

Proof Let $u_1v_1, u_2v_2, u_3v_3, u_4v_4, \dots, u_nv_n$ be the pendent edges of T . Let w_i be the vertex set of $S(T)$ that subdivides the edges $u_iv_i, i = 1, 2, 3, 4, \dots, n$. Any total list dominating set of $S(T)$ contains the edges $u_iw_i, i = 1, 2, 3, 4, \dots, n$ and hence $\gamma_t(\eta(S(T))) \geq n$. Further $E(S(T)) - S$, where S is the set of all pendent edges of $S(T)$ forms a total list dominating set of $S(T)$. Hence, $\gamma_t(\eta(S(T))) \leq 2(p - 1) - n$.

Notice that the edges of $D = \{u_iw_i\}, i = 1, 2, 3, \dots, n$ will forms a γ_t of $\eta(S(T))$ for $K_{1,p}$. Thus, the equality $\gamma_t(\eta(S(T))) = n$. Similarly, the set $\{E(S(T)) - S\}$ will forms a γ_t of $\eta(S(T))$ for $K_{1,p}$. So $\gamma_t(\eta(S(T))) = 2(p - 1) - n$. $\square$

Now we obtain the relation between total list domination in terms of complimentary edge domination, total domination and split domination and non-split domination.

Theorem 17 For any graph G if $\gamma_e(G) = \gamma'_e(G)$, then $\gamma_t(\eta(G)) \geq \gamma'_e(G)$.

Proof Let us consider the graph G , with $\gamma_e(G) = \gamma'_e(G)$ and using Theorem 2.2, the result follows. $\square$

Corollary 1 Let D be the γ_e set of a non-separable graph G then, $\gamma_t(\eta(G)) \geq \gamma'_e(G)$.

Proof Since every complementary edge dominating set is an edge dominating set, the follows from Theorem 2. $\square$

Theorem 18 For any non-separable graph G with $p \geq 3$, then $\gamma_t(G) \leq \gamma_t(\eta(G))$, equality holds for all cycle C_p .

Proof Let $D = \{v_1, v_2, v_3, \dots, v_k\}$ be a γ_t set of a graph G . Let $E^* = \{e_i \in E(G) / e_i \text{ is incident with } v_i, i = 1, 2, 3, 4, \dots, k\}$. Then every edge in $\langle E(G) - E^* \rangle$ is adjacent to at least one edge in E^* . Clearly E^* covers all the vertices in $\eta(G)$, and $\langle E^* \rangle$ does not contain any isolates, E^* is a total list dominating set of graph G and $|D| \leq |E^*|$. Hence, $\gamma_t(G) \leq \gamma_t(\eta(G))$.

For any cycle C_p , $\eta(G) = L(G)$, $\gamma_t(L(G)) = \gamma_t(G)$. Hence, $\gamma_t(G) = \gamma_t(\eta(G))$. $\square$

Theorem 19 For any cycle C_p $p \geq 3$, $\gamma_s(C_p) \leq \gamma_t(\eta(C_p)) \leq \gamma_{ns}(C_p)$.

Proof We consider the following two cases.

Case 1 $\gamma_s(C_p) \leq \gamma_t(\eta(C_p))$.

Let $A = \{v_1, v_2, v_3, \dots, v_k\}$ be a γ_s dominating set of cycle C_p . For any cycle C_p , $\eta(G) = L(G)$, the corresponding edges $B = \{e_1, e_2, e_3, \dots, e_k\}$ will be a split dominating set of $\eta(G)$. Since $\langle B \rangle$ is disconnected, $\gamma_t(\eta(C_p)) \leq \gamma_s(C_p) + 1$. Hence, $\gamma_s(C_p) \leq \gamma_t(\eta(C_p))$.

Case 2 $\gamma_t(\eta(C_p)) \leq \gamma_{ns}(C_p)$.

Let $A = \{v_1, v_2, v_3, \dots, v_k\}$ be a γ_{ns} dominating set of cycle C_p . For any cycle C_p , $\eta(G) = L(G)$, the corresponding edges $B = \{e_1, e_2, e_3, \dots, e_k\}$ will be a split dominating set of $\eta(G)$. Since $\langle B \rangle$ is connected. Hence, $\gamma_t(\eta(C_p)) \leq \gamma_{ns}(C_p)$.

The result follows from Cases 1 and 2. $\square$

Now we obtain the total list dominating number in terms of independence number and edge covering number.

Theorem 20 For any graph G , $\gamma_t(\eta(G)) \leq 2\beta_1(G)$.

Proof Let S be a maximum independent edge set in a graph G . Then every edge in $E(G) - S$ is adjacent to at least one edge in S . Let D be the set of cut vertices that is not incident with any edge of S and let $E_1 = \{e_i \in E(G) - S / e_i \in N(S)\}$. We consider the following two cases.

Case 1 If $D = \phi$, then for each edge $e_j \in S$, pick exactly one edge $e_i \in E_1$, such that $e_i \in N(e_j)$. Let D_1 be the set of all such edges with $|D_1| \leq |S|$. Then $F = S \cup D_1$ is a total list dominating set of G . Hence, $\gamma_t(\eta(G)) \leq |S \cup D_1| = |S| + |D_1| \leq |S| + |S| = 2\beta_1(G)$.

Case 2 If $D \neq \phi$, then for each cut vertex $v_c \in D$. Let $E_2 = \{e_i \in E(G) - S / e_i \in N(S) \text{ and incident with } v_c\}$, $E_3 = \{e_k \in S / e_k \in N(E_2)\}$ and $D_2 = S - E_3$. Now for each edge $e_l \in D_2$, pick exactly one edge in $e_i \in E_1$, such that e_l is adjacent to e_i . Let D_3 be the set of all such edges. Then $F = D_2 \cup D_3 \cup E_2 \cup E_3$ is a total list dominating set of G . Hence,

$$\begin{aligned} \gamma_t(\eta(G)) &\leq |F| = |D_2 \cup E_3 \cup D_3 \cup E_2| \\ &\leq |D_2 \cup E_3| + |D_3 \cup E_2| \\ &= |S| + |S| = 2|S| = 2\beta_0(G) \end{aligned}$$

From Cases 1 and 2, the result follows. $\square$

Theorem 21 For any graph G , $\gamma_t(\eta(G)) \leq 2\alpha_0(G)$.

Proof Let $S = \{v_1, v_2, v_3, v_4, \dots, v_k\} \subset V(G)$ such that $|S| = \alpha_0(G)$. Then for each vertex v_i , choose exactly one edge in E_1 where $E_1 = \{e_i \in E(G)/e_i \text{ is incident with } v_i\}$ such that $|E_1| \leq |S|$. Let D be the set of cut vertices that is not incident with any edge of E_1 and let $E_2 = \{e_j \in E(G) - E_1/e_j \in N(E_1)\}$. We consider the following two cases.

Case 1 If $D = \phi$, then for each edge $e_i \in E_1$, pick exactly one edge $e_j \in E_2$ such that $e_j \in N(e_i)$. Let D_1 be the set of all such edges with $|D_1| \leq |E_1| = |S|$. Then $F = E_1 \cup D_1$ is a total list dominating set of G . Hence, $\gamma_t(\eta(G)) \leq |E_1 \cup D_1| = |E_1| + |D_1| \leq |S| + |S| = 2\alpha_0(G)$.

Case 2 If $D \neq \phi$, then for each cut vertex $v_c \in D$. Let $E_3 = \{e_l \in E(G) - E_1/e_l \in N(E_1) \text{ and incident with } v_c\}$,

$E_4 = \{e_k \in E_1/e_k \in N(E_3)\}$ and $D_3 = E_1 - E_4$. Now for each edge $e_r \in D_2$, pick exactly one edge in $e_j \in E_2$, such that e_r is adjacent to e_j . Let D_3 be the set of all such edges. Then $F = D_2 \cup D_3 \cup E_3 \cup E_4$ is a total list dominating set of G . Hence,

$$\begin{aligned} \gamma_t(\eta(G)) &\leq |F| = |D_2 \cup E_4 \cup D_3 \cup E_3| \\ &\leq |D_2 \cup E_4| + |D_3 \cup E_3| \\ &= |E_1| + |E_1| = |S| = 2\alpha_0(G) \end{aligned}$$

From Cases 1 and 2, the result follows. $\square$

Now we obtain the total list dominating number of a subdivision graph of a graph G in terms of edge independence number and number of vertices of a graph G .

Theorem 22 For any graph G , $\gamma_t(\eta(S(G))) \leq 2q - 2\beta_1 + p_0$, where p_0 is the number of vertices that subdivides β_1 .

Proof Let $A = \{u_i v_i / 1 \leq i \leq n\}$ be the edge set of a graph G . Let $X = \{u_i v_i / 1 \leq i \leq n\}$ be a maximum independent edge set of graph G . Then X is edge dominating set of a graph G . Let w_i be the vertex set of $S(G)$ and let $p_0 \in w_i$ be the set of vertices that subdivides X . Then for each vertex p_0 , choose exactly one edge in E_1 , where $E_1 = \{u_i w_i \text{ or } w_i v_i \in S(G)/u_i w_i \text{ or } w_i v_i \text{ is incident with } p_0 \text{ and adjacent to } A - X\}$. Let $F = \{\{A - \{X\}\} - \{E_1\}\}$ covers all the edges and cut vertices of $S(G)$. Hence, $\gamma_t(\eta(S(G))) \leq |F| = |A - X - E_1| = 2q - 2\beta_1 + p_0$. $\square$

Theorem 23 For any non-separable graph G ,

- (i) $\gamma_t(\eta(S(K_p))) = 2\lceil \frac{p}{2} \rceil$.
- (ii) $\gamma_t(\eta(S(K_{p,q}))) = 2q(p \leq q)$.
- (iii) $\gamma_t(\eta(S(G))) = 2(p - \beta_1)$.

Proof Using the definitions of total list dominating set and total edge dominating set of a graph, the result follows from Theorem C. $\square$

Next, we obtain the Nordhus-Gaddam results for a total domination number of a list graph.

Theorem 24 For any connected graph G of order $p \geq 3$ vertices,

- (i) $\gamma_t(\eta(G)) + \gamma_t(\eta(\bar{G})) \leq 4\lceil \frac{p}{2} \rceil$.
- (ii) $\gamma_t(\eta(G)) * \gamma_t(\eta(\bar{G})) \leq 4\lceil \frac{p}{2} \rceil^2$.

Proof The result follows from Theorem B and Theorem 20. □

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