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On Variation of Edge Bimagic Total Labeling

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Abstract: An edge magic total labeling of a graph $G(V,E)$ with p vertices and q edges is a bijection $f : V(G) \cup E(G) \to \{1, 2, \cdots, p+q\}$ such that $f(u)+f(v)+f(uv)$ is a constant k for any edge $uv \in E(G)$. If there exist two constants k_1 and k_2 such that the above sum is either k_1 or k_2, it is said to be an edge bimagic total labeling. A total edge magic (edge bimagic) graph is called a super edge magic (super edge bimagic) if $f(V(G)) = \{1, 2, \cdots, p\}$ and it is called superior edge magic(bimagic) if $f(E(G)) = \{1, 2, \cdots, q\}$. In this paper, we investigate and exhibit super and superior edge bimagic labeling for some classes of graphs.

Key Words: Graph, labeling, bimagic labeling, bijective function.

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§1. Introduction

All graphs considered in this article are finite, simple and undirected. A labeling of a graph G is an assignment of labels to either the vertices or the edges, or both subject to certain conditions. Labeled graphs are becoming an increasingly useful family of mathematical models from a broad range of applications such as coding theory, X-ray, Crystallography, radar, astronomy, circuit design, communication networks and data base management. Graph labeling was first introduced in the late 1960s. A useful survey on graph labeling by Gallian (2012) can be found in [4]. We follow the notation and terminology of [5].

A graph $G = (V,E)$ with p vertices and q edges is called total edge magic if there is a bijection $f : V(G) \cup E(G) \to \{1, 2, \cdots, p+q\}$ such that $f(u) + f(v) + f(uv) = k$ for any edge $uv \in E(G)$. The original concept of total edge-magic graph is due to Kotzig and Rosa [6] who called it magic graph. A total edge-magic graph is called a super edge-magic if $f(V(G)) = \{1, 2, \cdots, p\}$.

Wallis [7] called super edge-magic as strongly edge-magic. The notion of edge bimagic labeling was introduced by Baskar Babujee [1]. A graph G with p vertices and q edges is called total edge bimagic if there exists a bijection $f : V(G) \cup E(G) \to \{1, 2, \cdots, p+q\}$ and two

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constants k_1 and k_2 such that $f(u)+f(v)+f(uv)$ is either k_1 or k_2 for any edge $uv \in E(G)$. A total edge-bimagic graph is called super edge-bimagic if $f(V(G)) = \{1, 2, \cdots, p\}$and it is called superior edge bimagic if $f(E(G)) = \{1, 2, \cdots, q\}$. In this article, $C_n\ \hat{o}\ C_n^+$, $C_n\ \hat{e}\ C_n^+$, $C_n\ \hat{o}\ C_n$, $C_n\ \hat{e}\ C_n$, $(K_{1,m} + K_1)\ \hat{o}\ C_n^+$ and $G\ \hat{o}\ (P_2 + mK_1)$ are shown to admit super and superior edge bimagic labeling.

Definition 1.1([2]) *A bijection $f : V(G) \cup E(G) \to \{1, 2, 3, \cdots, p + q\}$ is said to be super edge bimagic total labeling of G if there exist two constants k_1 and k_2 such that $f(u) + f(v) + f(uv)$ is either k_1 or k_2 for any edge $uv \in E(G)$ and $f(V(G)) = \{1, 2, \cdots, p\}$.*

Definition 1.2([8]) *A graph G with p vertices and q edges is called superior edge magic if there is a bijection $f : V(G) \cup E(G) \to \{1, 2, 3, \cdots, p+q\}$ such that $f(u)+f(v)+f(uv)$ is a constant for any edge $uv \in E(G)$, where $f(E(G) = \{1, 2, \cdots, q\}$. If $f(u) + f(v) + f(uv)$ are all distinct for all $uv \in E(G)$, then the graph is called superior edge antimagic total labeling.*

Definition 1.3([3]) *If $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$ are two connected graphs then the graph obtained by superimposing any selected vertex of G_2 on any selected vertex of G_1 is denoted by $G_1\ \hat{o}\ G_2$. The resultant graph $G = G_1\ \hat{o}\ G_2$ contains p_1 +p_2 - 1 vertices and $q_1 + q_2$ edges. In general, there are p_1p_2 possibilities of getting from G_1 and G_2.*

Definition 1.4 *$G_1\ \hat{e}\ G_2$ is obtained from G_1 and G_2 by introducing an edge between an arbitrary vertex of G_1 and an arbitrary vertex of G_2. If $G_1(p_1, q_1)$ has p_1 vertices and q_1 edges and $G_2(p_2, q_2)$ has p_2 vertices and q_2 edges then $G_1\ \hat{e}\ G_2$ will have $(p_1 + p_2)$ vertices and $(q_1 + q_2 + 1)$ edges. If $G_1 = K_{1,m}$, $G_2 = P_n$.*

Interesting graph structures $K_{1,m}\hat{e}P_n$ is obtained respectively using our operation defined above and we prove the following results.

§2. Main Results

In this section, we obtain super and superior edge bimagic labeling from connected magic graphs.

Theorem 2.1 *There exists at least one graph G from the class $C_n\hat{o}C_n^+$, $(n \geq 3)$ when n is odd that admits super edge bimagic labeling.*

Proof Let the graph G is obtained by superimposing a vertex of C_n on a pendant vertex of C_n^+ is denoted by $C_n\ \hat{o}\ C_n^+$. We define that the vertex set $V(G) = \{v_1^j, v_2^j; 1 \leq j \leq n\} \cup \{u_1^k; 1 \leq k \leq n-1\}$ and edge set $E(G) = E_1 \cup E_2 \cup E_3 \cup E_4$ where $E_1 = \{v_1^j v_2^j; 1 \leq j \leq n\}$, $E_2 = \{v_2^j v_2^{j+1}; 1 \leq j \leq n-1\}$, $E_3 = \{u_1^k u_1^{k+1}; 1 \leq k \leq n-2\}$, $E_4 = \{v_2^1 v_2^n, v_1^1 u_1^1, v_1^1 u_1^{n-1}\}$. Define a bijective function $f : V(G) \bigcup E(G) \to \{1, 2, 3, \cdots, 6n-1\}$ is as follows:

For $j = 1$ to n, let $f(v_1^j) = n-1+j$; For $j = 1$ to n, when $j \equiv 1(mod\ 2)$, let $f(v_2^j) = \dfrac{5n-j}{2}$, $f(v_1^j v_2^j) = \dfrac{10n-j-1}{2}$ and when $j \equiv 0(mod\ 2)$, let $f(v_2^j) = \dfrac{6n-j}{2}$, $f(v_2^j v_2^{j+1}) = 3n + j - 1$.

For $k=1$ to $n-2$, let $f(u_1^k u_1^{k+1})=6n-k$; For k = 1 to n-1 ; when $k\equiv 1(mod\ 2)$, let $f(u_1^k)=\dfrac{n+k}{2}$ and when $k\equiv 0(mod\ 2)$, let $f(u_1^k)=\dfrac{k}{2}$.

Let $f(u_1^1 v_1^1)=5n$, $f(v_2^1 v_2^n)=4n-1$, $f(v_1^1 u_1^{n-1})=5n+1$.

In the following cases, it is justified that the above assignment results in the required labeling.

Case 1 For edges in E_1, when $j\equiv 1(mod\ 2)$, we have

$$\begin{aligned} f(v_1^j)+f(v_2^j)+f(v_1^j v_2^j) &= n-1+j+\frac{5n-j}{2}+\frac{10n-j-1}{2}\\ &= \frac{17n-3}{2}=k_1 \end{aligned}$$

and when $j\equiv 0(mod\ 2)$, we have

$$\begin{aligned} f(v_1^j)+f(v_2^j)+f(v_1^j v_2^j) &= n-1+j+\frac{6n-j}{2}+\frac{9n-j-1}{2}\\ &= \frac{17n-3}{2}=k_1. \end{aligned}$$

Case 2 For edges in E_2, when $j\equiv 1(mod\ 2)$, we have

$$\begin{aligned} f(v_2^j)+f(v_2^{j+1})+f(v_2^j v_2^{j+1}) &= \frac{5n-j}{2}+\frac{6n-j-1}{2}+3n-1+j\\ &= \frac{17n-3}{2}=k_1 \end{aligned}$$

and when $j\equiv 0(mod\ 2)$, we have

$$\begin{aligned} f(v_2^j)+f(v_2^{j+1})+f(v_2^j v_2^{j+1}) &= \frac{6n-j}{2}+\frac{5n-j-1}{2}+3n-1+j\\ &= \frac{17n-3}{2}=k_1. \end{aligned}$$

Case 3 For edges in E_3, when $k\equiv 1(mod\ 2)$, we have

$$\begin{aligned} f(u_1^k)+f(u_1^{k+1})+f(u_1^k u_1^{k+1}) &= \frac{n+k}{2}+\frac{k+1}{2}+6n-k\\ &= \frac{13n+1}{2}=k_2 \end{aligned}$$

and when $k\equiv 0(mod\ 2)$, we have

$$\begin{aligned} f(u_1^k)+f(u_1^{k+1})+f(u_1^k u_1^{k+1}) &= \frac{k}{2}+\frac{n+k+1}{2}+6n-k\\ &= \frac{13n+1}{2}=k_2. \end{aligned}$$

Case 4 For the edges in E_4, we have

$$f(v_2^n)+f(v_2^1)+f(v_2^1v_2^n)=2n+\frac{5n-1}{2}+4n-1=\frac{17n-3}{2}=k_1,$$
$$f(u_1^1)+f(v_1^1)+f(u_1^1v_1^1)=\frac{n+1}{2}+n+5n=\frac{13n+1}{2}=k_2,$$
$$f(v_1^1)+f(v_1^{n-1})+f(v_1^1u_1^{n-1})=n+\frac{n-1}{2}+5n+1=\frac{13n+1}{2}=k_2.$$

We observe that there are two constants k_1 and k_2 such that for each edge $uv\in E(G)$, $f(u)+f(v)+f(uv)$ is either k_1 or k_2. From the above cases we have two constants $k_1=\frac{17n-3}{2}$ and $k_2=\frac{13n+1}{2}$. Hence the resultant graph admits super edge bimagic labeling. □

Illustration 1 The graph $C_9\hat{o}C_9^+$ is given in Figure 1. It is super edge bimagic labelling is also indicated in the same figure.

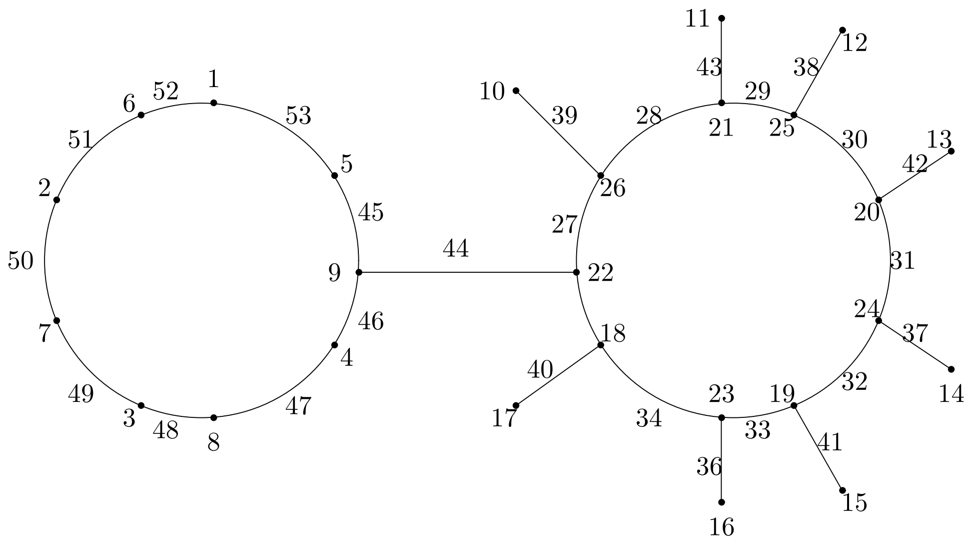

Figure 1 $k_1=75\quad k_2=59$

Theorem 2.2 *There exists at least one graph G from the class $C_n\hat{e}C_n^+$, $(n\geq 3)$, when n is odd that admits super edge bimagic total labeling.*

Proof Let the graph G is obtained by introducing an edge between a vertex of C_n and a pendant vertex of C_n^+ is denoted by $C_n\ \hat{e}\ C_n^+$. We define that the vertex set $V(G)=\{u_i,v_j,w_j;1\leq i,j\leq n\}$ and edge set $E(G)=E_1\cup E_2\cup E_3\cup E_4$ where $E_1=\{u_iu_{i+1};1\leq i\leq n-1\}$, $E_2=\{v_jw_j;1\leq j\leq n\}$, $E_3=\{w_jw_{j+1};1\leq j\leq n-1\}$, $E_4=\{u_1u_n,v_1u_n,w_1w_n\}$. Define a bijective function $f:V(G)\bigcup E(G)\rightarrow\{1,2,3,\cdots,6n-1\}$ is as follows:

For $i=1$ to $n-1$, let $f(u_iu_{i+1})=6n-i+1$; For $i=1$ to n, when $i\equiv 1(mod\ 2)$, let $f(u_i)=\frac{i+1}{2}$ and when $i\equiv 0(mod\ 2)$, let $f(u_i)=\frac{n+i+1}{2}$. For $j=1$ to $n-1$, let

$f(w_jw_{j+1}) = 3n+j$; For $j = 1$ to n, when $j \equiv 1(mod\ 2)$, let $f(w_j) = \dfrac{5n-j+2}{2}$ and when $j \equiv 0(mod\ 2)$, let $f(w_j) = \dfrac{6n-j+2}{2}$. For $j=1$ to n, when $j \equiv 1(mod\ 2)$, let $f(v_j) = n+j$, $f(w_jv_j) = \dfrac{10n-j+1}{2}$ and when $j \equiv 0(mod\ 2)$, let $f(v_j) = n+j$, $f(w_jv_j) = \dfrac{19n-j+1}{2}$. Let $f(u_1u_n) = 6n+1$, $f(v_1u_n) = 5n+1$ and $f(w_1w_n) = 4n$.

In the following cases, it is justified that the above assignment results in the required labeling.

Case 1 For edges in E_1, when $i \equiv 1(mod\ 2)$, we obtain

$$\begin{aligned} f(u_i)+f(u_{i+1})+f(u_iu_{i+1}) &= \frac{i+1}{2}+\frac{n+i+2}{2}+6n-i+1 \\ &= \frac{13n+5}{2} = k_1 \end{aligned}$$

and when $i \equiv 0(mod\ 2)$, we have

$$\begin{aligned} f(u_i)+f(u_{i+1})+f(u_iu_{i+1}) &= \frac{n+i+1}{2}+\frac{i+2}{2}+6n-i+1 \\ &= \frac{13n+5}{2} = k_1. \end{aligned}$$

Case 2 For edges in E_2, when $j \equiv 1(mod\ 2)$, we obtain

$$\begin{aligned} f(v_j)+f(w_j)+f(v_jw_j) &= n+j+\frac{5n-j+2}{2}+\frac{10n-j+2}{2}+\frac{10n-j+1}{2} \\ &= \frac{17n+3}{2} = k_2 \end{aligned}$$

and when $j \equiv 0(mod\ 2)$, we obtain

$$\begin{aligned} f(v_j)+f(w_j)+f(v_jw_j) &= n+j+\frac{6n-j+2}{2}+\frac{9n-j+1}{2} \\ &= \frac{17n+3}{2} = k_2. \end{aligned}$$

Case 3 For edges in E_3, when $j \equiv 1(mod\ 2)$, we obtain

$$\begin{aligned} f(w_j)+f(w_{j+1})+f(w_jw_{j+1}) &= \frac{5n-j+2}{2}+\frac{6n-j+1}{2}+3n+j \\ &= \frac{17n+3}{2} = k_2 \end{aligned}$$

and when $j \equiv 0(mod\ 2)$, we have

$$\begin{aligned} f(w_j)+f(w_{j+1})+f(w_jw_{j+1}) &= \frac{6n-j+2}{2}+\frac{5n-j+1}{2}+3n+j \\ &= \frac{17n+3}{2} = k_2. \end{aligned}$$

Case 4 For the edges in E_4, we have,

$$f(u_1)+f(u_n)+f(u_1u_n)=1+\frac{n+i}{2}+6n+1=\frac{13n+5}{2}=k_1,$$
$$f(v_1)+f(u_n)+f(v_1u_n)=n+1+\frac{n+1}{2}+5n+1=\frac{13n+5}{2}=k_1,$$
$$f(w_1)+f(w_n)+f(w_1w_n)=2n+1+\frac{5n+1}{2}+4n=\frac{17n+3}{2}=k_2.$$

We observe that there are two constants k_1 and k_2 such that for each edge $uv \in E(G)$, $f(u)+f(v)+f(uv)$ is either k_1 or k_2. From the above cases we have two constants $k_1=\frac{13n+5}{2}$ and $k_2=\frac{17n+3}{2}$. Hence the graph $C_n\hat{e}C_n^+$, $(n\geq 3)$ admits super edge bimagic labeling. □

Illustration 2 The graph $C_{11}\hat{e}C_{11}^+$ is given in Figure 2. It is super edge bimagic labelling is also indicated in the same figure.

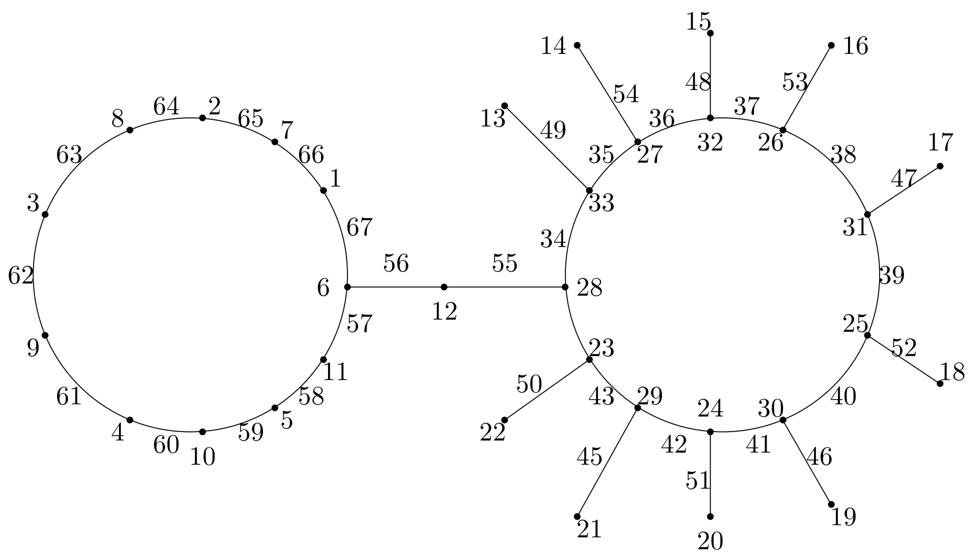

Figure 2 $k_1=74$ $k_2=95$

Theorem 2.3 *There exists at least one graph G from the class $C_n\hat{o}C_n$, $(n\geq 3)$ when n is odd that admits superior edge bimagic total labeling.*

Proof Let the graph G is obtained by superimposing a vertex of C_n on a vertex of the same copy denoted by $C_n\ \hat{o}\ C_n$. Now, we define that the vertex set $V(G)=\{u_i,v_j;1\leq i\leq n\},1\leq j\leq n-1\}$ and edge set $E(G)=E_1\cup E_2\cup E_3$ where $E_1=\{u_iu_{i+1};1\leq i\leq n-1\}$, $E_2=\{v_jv_{j+1};1\leq j\leq n-2\}$, $E_3=\{u_1u_n,v_1u_n,u_nv_{n-1}\}$. A bijective function $f:V(G)\bigcup E(G)\rightarrow\{1,2,3,\cdots,4n-1\}$ is given bellow:

For $i=1$ to $n-1$, let $f(u_iu_{i+1})=6n-i+1$; For $i=1$ to n, when $i\equiv 1(mod\ 2)$, let $f(u_i)=\frac{i+1}{2}$ and when $i\equiv 0(mod\ 2)$, let $f(u_i)=\frac{n+i+1}{2}$. For $i=1$ to $n-1$, let

$f(u_iu_{i+1}) = i$; For $i = 1$ to n, when $i \equiv 1(mod\ 2)$, let $f(u_i) = \dfrac{7n-i}{2}$ and when $i \equiv 0(mod\ 2)$, let $f(u_i) = \dfrac{8n-i}{2}$. For $j = 1$ to $n-2$, let $f(v_jv_{j+1}) = n+2+j$; For $j = 1$ to $n-1$, when $j \equiv 1(mod\ 2)$, let $f(v_j) = \dfrac{5n-j}{2}$, when $j \equiv 0(mod\ 2)$, let $f(v_j) = \dfrac{6n-j}{2}$. Let $f(v_{n-1}) = \dfrac{5n+1}{2}$, $f(u_nv_{n-1}) = n+1$, $f(v_1u_n) = n+2$, $f(u_1u_n) = n$.

The above assigned labels are justified in the following cases.

Case 1 For edges in E_1, when $i \equiv 1(mod\ 2)$, we obtain

$$\begin{aligned} f(u_i) + f(u_{i+1}) + f(u_iu_{i+1}) &= \frac{7n-i}{2} + \frac{8n-i-1}{2} + i \\ &= \frac{15n-1}{2} = k_1 \end{aligned}$$

and when $i \equiv 0(mod\ 2)$, we obtain

$$\begin{aligned} f(u_i) + f(u_{i+1}) + f(u_iu_{i+1}) &= \frac{8n-i}{2} + \frac{7n-i-1}{2} + i \\ &= \frac{15n-1}{2} = k_1. \end{aligned}$$

Case 2 For edges in E_2, when $j \equiv 1(mod\ 2)$, we obtain

$$\begin{aligned} f(v_j) + f(v_{j+1}) + f(v_jv_{j+1}) &= \frac{5n-j}{2} + \frac{6n-j-1}{2} + n + 2 + j \\ &= \frac{13n+3}{2} = k_2; \end{aligned}$$

when $j \equiv 0(mod\ 2)$, we obtain

$$\begin{aligned} f(v_j) + f(v_{j+1}) + f(v_jv_{j+1}) &= \frac{6n-j}{2} + \frac{5n-j-1}{2} + n + 2 + j \\ &= \frac{13n+3}{2} = k_2. \end{aligned}$$

Case 3 For the edges in E_3, we have

$$f(u_1) + f(u_n) + f(u_1u_n) = \frac{7n-1}{2} + 3n + n = \frac{15n-1}{2} = k_1,$$
$$f(v_1) + f(u_n) + f(v_1u_n) = \frac{5n-1}{2} + 3n + n + 2 = \frac{13n+3}{2} = k_2,$$
$$f(u_n) + f(v_{n-1}) + f(u_uu_{n-1}) = 3n + \frac{5n+1}{2} + n + 1 = \frac{13n+3}{2} = k_2.$$

Therefore, when we observe from the above cases, we have the constant $k_1 = \dfrac{15n-1}{2}$ and $k_2 = \dfrac{13n+3}{2}$. Hence the graph $G = C_n\ \hat{o}\ C_n$, $(n \geq 3)$ admits superior edge bimagic total labeling. □

Theorem 2.4 *There exists at least one graph G from the class $C_n\ \hat{e}\ C_n$, $(n \geq 3)$ when n is* *odd that admits super edge bimagic total labeling.*

Proof Let the graph G is obtained by introducing an edge between a vertex of C_n and a vertex of the same copy denoted by $C_n \hat{e} C_n$. Now, we define that the vertex set $V(G) = \{u_i, v_j; 1 \le i \le n\}, 1 \le j \le n\}$ and edge set $E(G) = E_1 \cup E_2 \cup E_3$ where $E_1 = \{u_i u_{i+1}; 1 \le i \le n-1\}$, $E_2 = \{v_j v_{j+1}; 1 \le j \le n-1\}$, $E_3 = \{u_1 v_n, u_1 u_n, v_1 v_n\}$. A bijective function $f : V(G) \bigcup E(G) \to \{1, 2, 3, \cdots, 4n+1\}$ is given bellow:

For $i = 1$ to $n-1$, let $f(u_i u_{i+1}) = 3n - i$; For $i = 1$ to n, when $i \equiv 1(mod\ 2)$, let $f(u_i) = \dfrac{2n+i+1}{2}$; when $i \equiv 0(mod\ 2)$, let $f(u_i) = \dfrac{3n+i+1}{2}$. For $j = 1$ to $n-1$, let $f(v_j v_{j+1}) = 4n+1+j$; For $j = 1$ to n, when $j \equiv 1(mod\ 2)$, let $f(v_j) = \dfrac{j+1}{2}$, when $j \equiv 0(mod\ 2)$, let $f(v_j) = \dfrac{n+j+1}{2}$. Let $f(u_1 u_n) = 3n$, $f(u_1 v_n = 3n+1)$, $f(v_1 v_n = 4n+1)$.

The above assigned labels are justified in the following cases.

Case 1 For edges in E_1, when $i \equiv 1(mod 2)$, we obtain

$$\begin{aligned} f(u_i) + f(u_{i+1}) + f(u_i u_{i+1}) &= \frac{2n+i+1}{2} + \frac{3n+i+2}{2} + 3n - i \\ &= \frac{11n+3}{2} = k_1 \end{aligned}$$

and when $i \equiv 0(mod\ 2)$, we obtain

$$\begin{aligned} f(u_i) + f(u_{i+1}) + f(u_i u_{i+1}) &= \frac{3n+i+1}{2} + \frac{2n+i+2}{2} + 3n - i \\ &= \frac{11n+3}{2} = k_1. \end{aligned}$$

Case 2 For edges in E_2, when $j \equiv 1(mod\ 2)$, we obtain

$$\begin{aligned} f(v_j) + f(v_{j+1}) + f(v_j v_{j+1}) &= \frac{j+1}{2} + \frac{n+j+2}{2} + 4n + 1 - j \\ &= \frac{9n+5}{2} = k_2. \end{aligned}$$

When $j \equiv 0(mod\ 2)$, we obtain

$$\begin{aligned} f(v_j) + f(v_{j+1}) + f(v_j v_{j+1}) &= \frac{n+j+1}{2} + \frac{j+2}{2} + 4n + 1 - j \\ &= \frac{9n+5}{2} = k_2. \end{aligned}$$

Case 3 For the dges in E_3, we have

$$f(u_1) + f(u_n) + f(u_1 u_n) = n + 1 + \frac{3n+1}{2} + 3n = \frac{11n+3}{2} = k_1,$$
$$f(v_1) + f(u_n) + f(v_1 u_n) = 1 + \frac{n+1}{2} + 4n + 1 = \frac{9n+5}{2} = k_2,$$
$$f(u_1) + f(v_n) + f(u_1 v_n) = n + 1 + \frac{n+1}{2} + 3n + 1 = \frac{9n+5}{2} = k_2.$$

Therefore, $k_1 = \dfrac{11n+3}{2}$ and $k_2 = \dfrac{9n+5}{2}$. Hence the graph $G = C_n\hat{e}C_n$, $(n \geq 3)$ admits super edge bimagic total labeling. □

Theorem 2.5 *There exists at least one graph G' from the class $G\hat{o}C_n^+$, $(n \geq 3)$, (when n is odd) that admits super edge bimagic total labeling, where G is any graph from $K_{1,m} + K_1, (m \geq 2)$.*

Proof Let the graph G' is obtained by merging of two graphs with a vertex of above degree 2 in G and a pendant vertex of C_n^+. We define the graph $G\hat{o}C_n^+$ with vertex set $V(G) = \{u_i, v_i; 1 \leq i \leq n\} \cup \{w_1\} \cup \{w_2^j; 1 \leq j \leq m\}$ and edge set $E(G) = E_1 \cup E_2 \cup E_3$ where $E_1 = \{u_iv_i; 1 \leq i \leq n\}$, $E_2 = \{v_iv_{i+1}; 1 \leq i \leq n-1\}$, $E_3 = \{u_1w_2^j; 1 \leq j \leq m\} \cup \{w_1w_2^j; 1 \leq j \leq m\} \cup \{u_1w_1, v_1v_n\}$. A bijective function $f : V(G) \bigcup E(G) \to \{1, 2, 3, \cdots, 4n + 3m + 2\}$ is given bellow:

For $i = 1$ to $n-1$, let $f(v_iv_{i+1}) = 2n + 1 + m + i$; For $i = 1$ to n, when $i \equiv 1(mod\ 2)$, let $f(v_i) = \dfrac{3n+4-i}{2} + m$. When $i \equiv 0(mod\ 2)$, let $f(v_i) = \dfrac{4n-i+4}{2} + m$. For $i = 1$ to n, let $f(u_i) = m + 1 + i$; For $i = 1$ to n, when $i \equiv 1(mod\ 2)$, let $f(u_iv_i) = \dfrac{8n-i+3}{2}$. When $i \equiv 0(mod\ 2)$, let $f(u_iv_i) = \dfrac{7n+3-i}{2}$. For $i = 1$ to n, let $f(v_1v_n) = 3n + m + 1$ and for $j = 1$ to m, let $f(w_2^j) = 1 + j$, $f(u_1w_2^j) = \dfrac{7n+1}{2} + 3m - j$, $f(w_1w_2^j) = \dfrac{7n+3}{2} + 4m - j$. Let $f(w_1) = 1$, $f(u_1w_1) = \dfrac{7n+1}{2} + 3m$.

In the following cases, it is justified that the above assignment results in the required labeling.

Case 1 For any edge $u_iv_i \in E_1$, when $i \equiv 1(mod\ 2)$, we obtain

$$\begin{aligned} f(u_i) + f(v_i) + f(u_iv_i) &= m + 1 + i + \frac{3n+4-i}{2} + m + \frac{8n-i+3}{2} + m \\ &= \frac{6m+11n+9}{2} = k_1. \end{aligned}$$

When $i \equiv 0(mod\ 2)$, we obtain

$$\begin{aligned} f(u_i) + f(v_i) + f(u_iv_i) &= m + 1 + i + \frac{n-i+3}{2} + m + 2n + m + 1 + i \\ &= \frac{6m+11n+9}{2} = k_1. \end{aligned}$$

Case 2 For any edge $v_iv_{i+1} \in E_2$, when $i \equiv 1(mod\ 2)$, we obtain

$$\begin{aligned} f(v_i) + f(v_{i+1}) + f(v_iv_{i+1}) &= \frac{3n+4-i}{2} + m + \frac{4n-i-3}{2} + m + 2n + m + 1 + i \\ &= \frac{6m+11n+9}{2} = k_1. \end{aligned}$$

When $i \equiv 0(mod\ 2)$, we obtain

$$\begin{aligned} f(v_i)+f(v_{i+1})+f(v_iv_{i+1}) &= \frac{4n+4-i}{2}+m+\frac{3n-i-3}{2}+m+2n+m+1+i \\ &= \frac{6m+11n+9}{2}=k_1. \end{aligned}$$

Case 3 For the edges in E_3, we have

$$f(u_1)+f(w_2^j)+f(u_1w_2^j)=m+2+1+j+\frac{7n+1}{2}+3m-j=\frac{8m+7n+7}{2}=k_2,$$
$$f(w_1)+f(w_2^j)+f(w_1w_2^j)=1+1+j+\frac{7n+3}{2}+4m-j=\frac{8m+7n+7}{2}=k_2,$$
$$f(v_1)+f(v_n)+f(v_1v_n)=\frac{3n+3}{2}+m+n+2+m+3n+m+1=\frac{6m+11n+9}{2}=k_1,$$
$$f(u_1)+f(w_1)+f(u_1w_1)=m+2+1+\frac{7n+1}{2}+3m=\frac{8m+7n+7}{2}=k_2.$$

We observe that there are two common counts k_1 and k_2 such that for each edge $uv \in E(G)$, $f(u)+f(v)+f(uv)$ is either k_1 or k_2. From the above cases we have two constants $k_1=\frac{6m+11n+9}{2}$ and $k_2=\frac{8m+7n+7}{2}$. Hence as per our construction G' admits super edge bimagic labeling.

Illustration 3 The graph $(K_{1,6}+K_1)\hat{o}C_9^+$ is given in figure 3. It is super edge bimagic labelling is also indicated in the same figure.

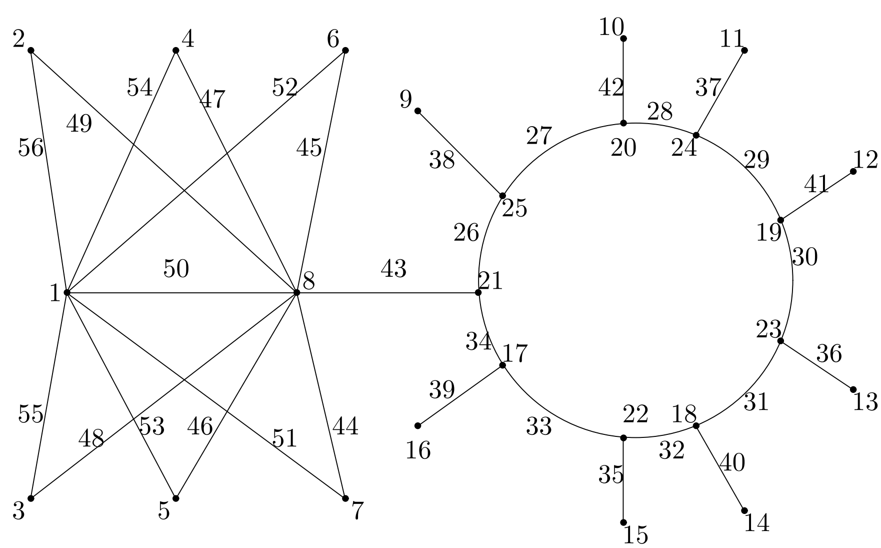

Figure 3 $k_1=72 \quad k_2=59$

Theorem 2.6 *If G is an arbitrary graph that admits total edge magic labeling then there exists at least one graph from the class $G\hat{o}(P_2+mK_1)$ admits edge bimagic total labeling.*

Proof Let G(p, q) be total edge magic graph with the bijective function $f: V(G) \cup E(G) \rightarrow \{1, 2, 3, \cdots, p+q\}$ such that $f(u)+f(v)+f(uv)=k_1$. Let $w \in V(G)$ must be vertex whose label $f(w)=p+q$ is the maximum value. Consider the graph (P_2+mK_1) with vertex set $\{u_0, v_0, u_i : 1 \leq i \leq m\}$ and edge set $E(G)=\{u_0u_i, v_0u_i : 1 \leq i \leq m\} \cup \{u_0v_0\}$. We superimpose the vertex v_0 is degree more than two of the (P_2+mK_1) graph on the vertex $w \in V(G)$ of G. Now we define the new graph $G^{'} = G\ \hat{o}\ (P_2+mK_1) : 1 \leq i \leq m$ and edge set $E^{'}(G^{'}) = E \cup E_1 \cup E_2 \cup E_3$ where $E_1=\{u_0u_i : 1 \leq i \leq m\}$, $E_2=\{wu_i : 1 \leq i \leq m\}$, $E_3=\{u_0w\}$. Consider the bijection $g: V^{'}(G^{'}) \cup E^{'}(G^{'}) = \{1, 2, 3, \cdots, p+q+3m+2\}$ defined by $g(v)=f(v)$ for all $v \in V(G)$ and $g(uv)=f(uv)$ for all $uv \in E(G)$.

From our construction of new graph $G^{'}$, the labels are defined as follows:

$f(w)=g(v_0)=g(w)=p+q$, $g(u_i)=p+q+i$, for $1 \leq i \leq m$;

$g(wu_i)=p+q+3m+3-i$, for $1 \leq i \leq m$;

$g(u_0u_i)=p+q+2m+2-i$, for $1 \leq i \leq m$;

$g(u_0)=p+q+m+1$ and $g(u_0w)=p+q+2m+2$.

Since the graph G is total edge magic with constant k_1 and implies that $g(u)+g(uv)+g(v) = k_2$ for all $uv \in E^{'}(G^{'})$.

Next, we have to prove that the remaining edges w and u_0 joining with $\{u_i : 1 \leq i \leq m\}$ have the constant k_2.

For the edges in $E_1 \cup E_2 \cup E_3$,

$$\begin{aligned}
g(u_0)+g(u_0u_i)+g(u_i) &= p+q+m+1+p+q+2m+2-i+p+q+i \\
&= 3(p+q+m+1) = k_2, \\
g(w)+g(u_i)+g(wu_i) &= p+q+p+q+i+p+q+3m+3-i \\
&= 3(p+q+m+1) = k_2 \text{ and} \\
g(u_0)+g(u_0w)+g(w) &= p+q+m+1+p+q+p+q+2m+2 \\
&= 3(p+q+m+1) = k_2.
\end{aligned}$$

Therefore, the resultant graph $G\hat{o}(P_2+mK_1)$ has two common counts k_1 and k_2. Hence the graph admits edge bimagic total labeling. □

Conclusion In our present study, we have investigated super and superior edge bimagic labeling for some special graphs. Investigating super and superior edge bimagic total labeling for the graph from the class $G_1\hat{o}G_2$ and $G_1\hat{e}G_2$ for some arbitrary graph G_1 and G_2 with this conditions. This is our future plan.

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