International J.Math. Combin. Vol.1(2013), 85-98

Geometric Mean Labeling
Of Graphs Obtained from Some Graph Operations

A.Durai Baskar, S.Arockiaraj and B.Rajendran

Department of Mathematics, Mepco Schlenk Engineering College

Mepco Engineering College (PO)-626005, Sivakasi, Tamil Nadu, India

E-mail: a.duraibaskar@gmail.com, sarockiaraj_77@yahoo.com, drbr58msec@hotmail.com

Abstract: A function f is called a geometric mean labeling of a graph $G(V,E)$ if $f : V(G) \to \{1,2,3,\ldots,q+1\}$ is injective and the induced function $f^* : E(G) \to \{1,2,3,\ldots,q\}$ defined as

$$f^*(uv) = \left\lfloor \sqrt{f(u)f(v)} \right\rfloor, \ \forall uv \in E(G),$$

is bijective. A graph that admits a geometric mean labeling is called a geometric mean graph. In this paper, we have discussed the geometric meanness of graphs obtained from some graph operations.

Key Words: Labeling, geometric mean labeling, geometric mean graph.

AMS(2010): 05C78

§1. Introduction

Throughout this paper, by a graph we mean a finite, undirected and simple graph. Let $G(V,E)$ be a graph with p vertices and q edges. For notations and terminology, we follow [3]. For a detailed survey on graph labeling, we refer [2].

Cycle on n vertices is denoted by C_n and a path on n vertices is denoted by P_n. A tree T is a connected acyclic graph. Square of a graph G, denoted by G^2, has the vertex set as in G and two vertices are adjacent in G^2 if they are at a distance either 1 or 2 apart in G. A graph obtained from a path of length m by replacing each edge by C_n is called as mC_n-snake, for $m \geq 1$ ad $n \geq 3$.

The total graph $T(G)$ of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent if and only if either they are adjacent vertices of G or adjacent edges of G or one is a vertex of G and the other one is an edge incident on it. The graph Tadpoles $T(n,k)$ is obtained by identifying a vertex of the cycle C_n to an end vertex of the path P_k. The H-graph is obtained from two paths $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \cdots, v_n$ of equal length by joining an edge $u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}$ when n is odd and $u_{\frac{n+2}{2}} v_{\frac{n}{2}}$ when n is even. An arbitrary supersubdivision $P(m_1, m_2, \cdots, m_{n-1})$ of a path P_n is a graph obtained by replacing each i^{th} edge of P_n by identifying its end vertices of the edge with a partition of K_{2,m_i} having 2 elements, where m_i is

[1]Received November 23, 2012. Accepted March 12, 2013.

any positive integer. $G \odot K_1$ is the graph obtained from G by attaching a new pendant vertex to each vertex of G.

The study of graceful graphs and graceful labeling methods was first introduced by Rosa [5]. The concept of mean labeling was first introduced by S.Somasundaram and R.Ponraj [6] and it was developed in [4,7]. S.K.Vaidya et al. [11] have discussed the mean labeling in the context of path union of cycle and the arbitrary supersubdivision of the path P_n. S.K.Vaidya et al. [8-10] have discussed the mean labeling in the context of some graph operations. In [1], A.Durai Baskar et al. introduced geometric mean labeling of graph.

A function f is called a geometric mean labeling of a graph $G(V, E)$ if $f : V(G) \to \{1, 2, 3, \cdots, q+1\}$ is injective and the induced function $f^* : E(G) \to \{1, 2, 3, \cdots, q\}$ defined as

$$f^*(uv) = \left\lfloor \sqrt{f(u)f(v)} \right\rfloor, \quad \forall uv \in E(G),$$

is bijective. A graph that admits a geometric mean labeling is called a geometric mean graph.

In this paper we have obtained the geometric meanness of the graphs, union of two cycles C_m and C_n, union of the cycle C_m and a path P_n, P_n^2, mC_n-snake for $m \geq 1$ and $n \geq 3$, the total graph $T(P_n)$ of P_n, the Tadpoles $T(n, k)$, the graph obtained by identifying a vertex of any two cycles C_m and C_n, the graph obtained by identifying an edge of any two cycles C_m and C_n, the graph obtained by joining any two cycles C_m and C_n by a path P_k, the H-graph and the arbitrary supersubdivision of a path $P(1, 2, \cdots, n-1)$.

§2. Main Results

Theorem 2.1 *Union of any two cycles C_m and C_n is a geometric mean graph.*

Proof Let $u_1, u_2, \cdots, u_m$ and $v_1, v_2, \cdots, v_n$ be the vertices of the cycles C_m and C_n respectively. We define $f : V(C_m \cup C_n) \to \{1, 2, 3, \cdots, m+n+1\}$ as follows:

$$f(u_i) = \begin{cases} i & \text{if } 1 \leq i \leq \left\lfloor \sqrt{m+2} \right\rfloor - 1 \\ i+1 & \text{if } \left\lfloor \sqrt{m+2} \right\rfloor \leq i \leq m-1, \end{cases}$$

$$f(u_m) = m+2 \text{ and}$$

$$f(v_i) = \begin{cases} m+n+3-2i & \text{if } 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\ m+1 & \text{if } i = \left\lfloor \frac{n}{2} \right\rfloor + 1 \\ m-n+2i & \text{if } \left\lfloor \frac{n}{2} \right\rfloor + 2 \leq i \leq n. \end{cases}$$

The induced edge labeling is as follows:

$$f^*(u_iu_{i+1}) = \begin{cases} i & \text{if } 1 \leq i \leq \left\lfloor \sqrt{m+2} \right\rfloor - 1 \\ i+1 & \text{if } \left\lfloor \sqrt{m+2} \right\rfloor \leq i \leq m-1, \end{cases}$$

$$f^*(u,u_m) = \left\lfloor\sqrt{m+2}\right\rfloor,$$

$$f^*(v_iv_{i+1}) = \begin{cases} m+n+1-2i & \text{if } 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor \\ m+1 & \text{if } i = \left\lfloor \frac{n}{2} \right\rfloor + 1 \text{ and } n \text{ is odd} \\ m+2 & \text{if } i = \left\lfloor \frac{n}{2} \right\rfloor + 1 \text{ and } n \text{ is even} \\ m-n+2i & \text{if } \left\lfloor \frac{n}{2} \right\rfloor + 2 \le i \le n-1 \end{cases}$$

and $$f^*(v_1v_n) = m+n.$$

Hence, f is a geometric mean labeling of the graph $C_m \cup C_n$. Thus the graph $C_m \cup C_n$ is a geometric mean graph, for any $m, n \ge 3$. □

A geometric mean labeling of $C_7 \cup C_{10}$ is shown in Fig.1.

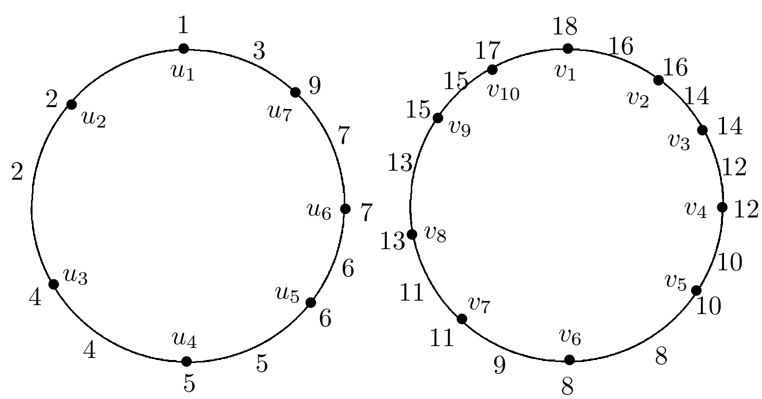

Fig.1

The graph $C_m \cup nT, n \ge 2$ cannot be a geometric mean graph. But the graph $C_m \cup T$ may be a geometric mean graph.

Theorem 2.2 *The graph $C_m \cup P_n$ is a geometric mean graph.*

Proof Let $u_1, u_2, \cdots, u_m$ and $v_1, v_2, \cdots, v_n$ be the vertices of the cycle C_m and the path P_n respectively. We define $f : V(C_m \cup P_n) \to \{1, 2, 3, \cdots, m+n\}$ as follows:

$$f(u_i) = \begin{cases} m+n+2-2i & \text{if } 1 \le i \le \left\lfloor \frac{m}{2} \right\rfloor \\ n & \text{if } i = \left\lfloor \frac{m}{2} \right\rfloor + 1 \\ n-m-1+2i & \text{if } \left\lfloor \frac{m}{2} \right\rfloor + 2 \le i \le m, \end{cases}$$

$f(v_i) = i,$ for $1 \le i \le n-1$ and

$f(v_n) = n+1.$

The induced edge labeling is as follows:

$$f^*(u_iu_{i+1}) = \begin{cases} m+n-2i & \text{if } 1 \leq i \leq \left\lfloor \frac{m}{2} \right\rfloor \\ n & \text{if } i = \left\lfloor \frac{m}{2} \right\rfloor + 1 \text{ and } m \text{ is odd} \\ n+1 & \text{if } i = \left\lfloor \frac{m}{2} \right\rfloor + 1 \text{ and } m \text{ is even} \\ n-m-1+2i & \text{if } \left\lfloor \frac{m}{2} \right\rfloor + 2 \leq i \leq m-1, \end{cases}$$

$$f^*(u_1u_m) = m+n-1 \text{ and}$$

$$f^*(v_iv_{i+1}) = i, \text{ for } 1 \leq i \leq n-1.$$

Hence, f is a geometric mean labeling of the graph $C_m \cup P_n$. Thus the graph $C_m \cup P_n$ is a geometric mean graph, for any $m \geq 3$ and $n \geq 2$. □

A geometric mean labeling of $C_{12} \cup P_7$ is shown in Fig.2.

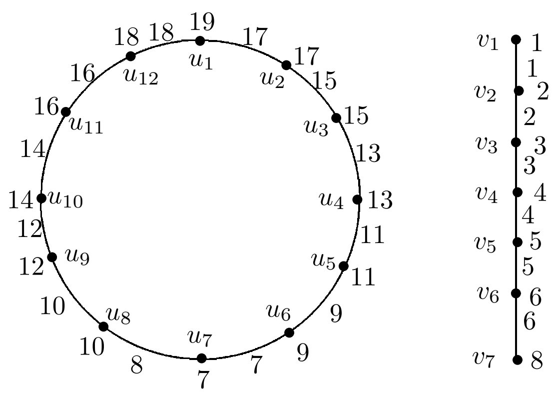

Fig.2

The T-graph T_n is obtained by attaching a pendant vertex to a neighbor of the pendant vertex of a path on $(n-1)$ vertices.

Theorem 2.3 *For a T-graph T_n, $T_n \cup C_m$ is a geometric mean graph, for $n \geq 2$ and $m \geq 3$.*

Proof Let $u_1, u_2, \cdots, u_{n-1}$ be the vertices of the path P_{n-1} and u_n be the pendant vertex identified with u_2. Let $v_1, v_2, \cdots, v_m$ be the vertices of the cycle C_m.

$$V(T_n \cup C_m) = V(C_m) \cup V(P_n) \cup \{u_n\} \text{ and}$$

$$E(T_n \cup C_m) = E(C_m) \cup E(P_n) \cup \{u_2u_n\}.$$

We define $f : V(T_n \cup C_m) \to \{1, 2, 3, \cdots, m+n\}$ as follows:

$$f(u_i) = i+1, \text{ for } 1 \leq i \leq n-2,$$

$$f(u_{n-1}) = n-1,$$

$$f(u_n) = 1,$$

$$f(v_i)=\begin{cases} m+n+2-2i & \text{if } 1\le i\le \left\lfloor \dfrac{m}{2}\right\rfloor \\ n & \text{if } i=\left\lfloor \dfrac{m}{2}\right\rfloor+1 \\ n-m-1+2i & \text{if } \left\lfloor \dfrac{m}{2}\right\rfloor+2\le i\le m. \end{cases}$$

The induced edge labeling is as follows:

$$f^*(u_iu_{i+1})=i+1,\ \text{for } 1\le i\le n-2,$$
$$f^*(u_2u_n)=1,$$
$$f^*(v_iv_{i+1})=\begin{cases} m+n-2i & \text{if } 1\le i\le \left\lfloor \dfrac{m}{2}\right\rfloor \\ n & \text{if } i=\left\lfloor \dfrac{m}{2}\right\rfloor+1 \text{ and } m \text{ is odd} \\ n+1 & \text{if } i=\left\lfloor \dfrac{m}{2}\right\rfloor+1 \text{ and } m \text{ is even} \\ n-m-1+2i & \text{if } \left\lfloor \dfrac{m}{2}\right\rfloor+2\le i\le m-1 \end{cases} \quad \text{and}$$
$$f^*(v_1v_m)=m+n-1.$$

Hence f is a geometric mean labeling of $T_n\cup C_m$. Thus the graph $T_n\cup C_m$ is a geometric mean graph, for $n\ge 2$ and $m\ge 3$. □

A geometric mean labeling of $T_7\cup C_6$ is as shown in Fig.3.

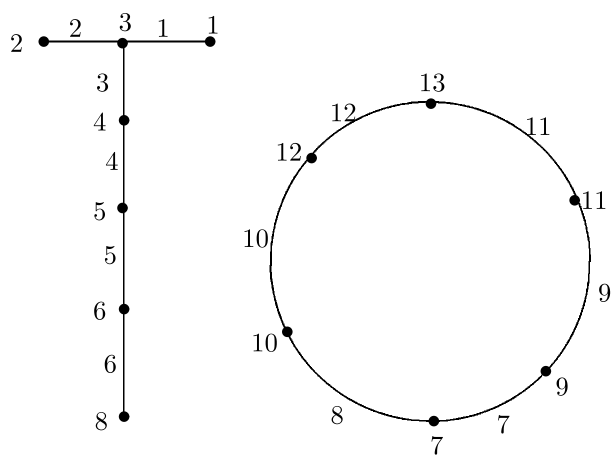

Fig.3

Theorem 2.4 *P_n^2 is a geometric mean graph, for $n\ge 3$.*

Proof Let $v_1,v_2,\cdots,v_n$ be the vertices of the path P_n. We define $f:V(P_n^2)\to\{1,2,3,\cdots,2(n-1)\}$ as follows:

$$f(v_i)=2i-1,\ \text{for } 1\le i\le n-1 \text{ and}$$
$$f(v_n)=2(n-1).$$

The induced edge labeling is as follows:

$$f^*(v_iv_{i+1}) = 2i-1, \text{ for } 1 \le i \le n-1 \text{ and}$$
$$f^*(v_iv_{i+2}) = 2i, \text{ for } 1 \le i \le n-2.$$

Hence, f is a geometric mean labeling of the graph P_n^2. Thus the graph P_n^2 is a geometric mean graph, for $n \ge 3$. □

A geometric mean labeling of P_9^2 is shown in Fig.4.

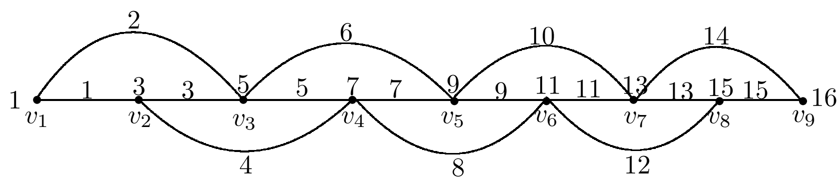

Fig.4

Theorem 2.5 *mC_n-snake is a geometric mean graph, for any $m \ge 1$ and $n = 3, 4$.*

Proof The proof is divided into two cases.

Case 1 $n = 3$.

Let $v_1^{(i)}, v_2^{(i)}$ and $v_3^{(i)}$ be the vertices of the i^{th} copy of the cycle C_3, for $1 \le i \le m$. The mC_3-snake G is obtained by identifying $v_3^{(i)}$ and $v_1^{(i+1)}$, for $1 \le i \le m-1$. We define $f : V(G) \to \{1, 2, 3 \cdots, 3m+1\}$ as follows:

$$f(v_1^{(i)}) = 3i-2, \text{ for } 1 \le i \le m$$
$$f(v_2^{(i)}) = 3i, \text{ for } 1 \le i \le m \text{ and}$$
$$f(v_3^{(i)}) = 3i+1, \text{for } 1 \le i \le m.$$

The induced edge labeling is as follows:

$$f^*(v_1^{(i)}v_2^{(i)}) = 3i-2, \text{ for } 1 \le i \le m,$$
$$f^*(v_2^{(i)}v_3^{(i)}) = 3i, \text{ for } 1 \le i \le m \text{ and}$$
$$f^*(v_1^{(i)}v_3^{(i)}) = 3i-1, \text{ for } 1 \le i \le m.$$

Hence, f is a geometric mean labeling of the graph mC_3-snake. For example, a geometric mean labeling of $6C_3$-snake is shown in Fig.5.

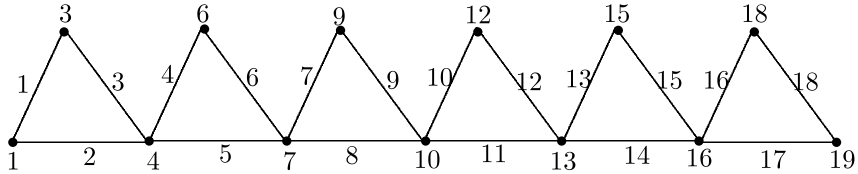

Fig.5

Case 2 $n = 4$.

Let $v_1^{(i)}, v_2^{(i)}, v_3^{(i)}$ and $v_4^{(i)}$ be the vertices of the i^{th} copy of the cycle C_4, for $1 \leq i \leq m$. The mC_4-snake G is obtained by identifying $v_4^{(i)}$ and $v_1^{(i+1)}$, for $1 \leq i \leq m-1$. We define $f : V(G) \to \{1, 2, 3, \cdots, 4m+1\}$ as follows:

$$f(v_1^{(i)}) = 4i - 3, \text{ for } 1 \leq i \leq m,$$
$$f(v_2^{(i)}) = 4i - 1, \text{ for } 1 \leq i \leq m,$$
$$f(v_3^{(i)}) = 4i, \text{ for } 1 \leq i \leq m \text{ and}$$
$$f(v_4^{(i)}) = 4i + 1, \text{for } 1 \leq i \leq m.$$

The induced edge labeling is as follows:

$$f^*(v_1^{(i)} v_2^{(i)}) = 4i - 3, \text{ for } 1 \leq i \leq m,$$
$$f^*(v_2^{(i)} v_3^{(i)}) = 4i - 1, \text{ for } 1 \leq i \leq m$$
$$f^*(v_3^{(i)} v_4^{(i)}) = 4i, \text{ for } 1 \leq i \leq m \text{ and}$$
$$f^*(v_1^{(i)} v_4^{(i)}) = 4i - 2, \text{ for } 1 \leq i \leq m.$$

Hence, f is a geometric mean labeling of the graph mC_4-snake. □

A geometric mean labeling of $5C_4$-snake is shown in Fig.6.

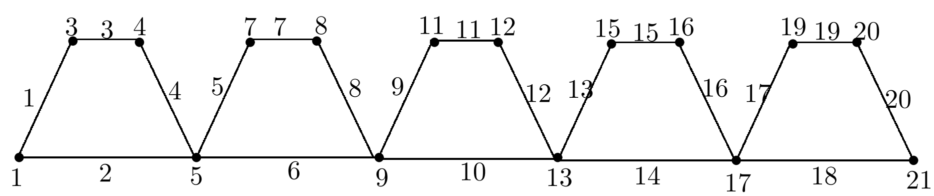

Fig.6

Theorem 2.6 *$T(P_n)$ is a geometric mean graph, for $n \geq 2$.*

Proof Let $V(P_n) = \{v_1, v_2, \cdots, v_n\}$ and $E(P_n) = \{e_i = v_i v_{i+1}; 1 \leq i \leq n-1\}$ be the vertex set and edge set of the path P_n. Then

$$V(T(P_n)) = \{v_1, v_2, \ldots, v_n, e_1, e_2, \cdots, e_{n-1}\} \text{ and}$$
$$E(T(P_n)) = \{v_i v_{i+1}, e_i v_i, e_i v_{i+1}; 1 \leq i \leq n-1\} \cup \{e_i e_{i+1}; 1 \leq i \leq n-2\}.$$

We define $f : V(T(P_n)) \to \{1, 2, 3, \cdots, 4(n-1)\}$ as follows:

$$f(v_i) = 4i - 3, \text{ for } 1 \leq i \leq n-1,$$
$$f(v_n) = 4n - 4 \text{ and}$$
$$f(e_i) = 4i - 1, \text{ for } 1 \leq i \leq n-1.$$

The induced edge labeling is as follows:

$$\begin{aligned} f^*(v_iv_{i+1}) &= 4i-2, \text{ for } 1 \le i \le n-1, \\ f^*(e_ie_{i+1}) &= 4i, \text{ for } 1 \le i \le n-2, \\ f^*(e_iv_i) &= 4i-3, \text{ for } 1 \le i \le n-1 \text{ and} \\ f^*(e_iv_{i+1}) &= 4i-1, \text{ for } 1 \le i \le n-1. \end{aligned}$$

Hence, f is a geometric mean labeling of the graph $T(P_n)$. Thus the graph $T(P_n)$ is a geometric mean graph, for $n \ge 2$. □

A geometric mean labeling of $T(P_5)$ is shown in Fig.7.

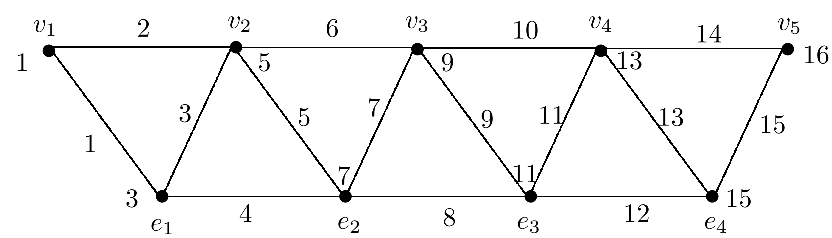

Fig.7

Theorem 2.7 *Tadpoles $T(n,k)$ is a geometric mean graph.*

Proof Let $u_1, u_2, \cdots, u_n$ and $v_1, v_2, \cdots, v_k$ be the vertices of the cycle C_n and the path P_k respectively. Let $T(n,k)$ be the graph obtained by identifying the vertex u_n of the cycle C_n to the end vertex v_1 of the path P_k. We define $f : V(T(n,k)) \to \{1, 2, 3, \cdots, n+k\}$ as follows:

$$f(u_i) = \begin{cases} i & \text{if } 1 \le i \le \lfloor \sqrt{n+1} \rfloor - 1 \\ i+1 & \text{if } \lfloor \sqrt{n+1} \rfloor \le i \le n \end{cases} \quad \text{and}$$

$$f(v_i) = n+i, \text{ for } 2 \le i \le k.$$

The induced edge labeling is as follows:

$$f^*(u_iu_{i+1}) = \begin{cases} i & \text{if } 1 \le i \le \lfloor \sqrt{n+1} \rfloor - 1 \\ i+1 & \text{if } \lfloor \sqrt{n+1} \rfloor \le i \le n-1, \end{cases}$$

$$f^*(u_1u_n) = \lfloor \sqrt{n+1} \rfloor \text{ and}$$

$$f^*(v_iv_{i+1}) = n+i, \text{ for } 1 \le i \le k-1.$$

Hence, f is a geometric mean labeling of the graph $T(n,k)$. Thus the graph $T(n,k)$ is a geometric mean graph. □

A geometric mean labeling of the Tadpoles $T(7,5)$ is shown in Fig.8.

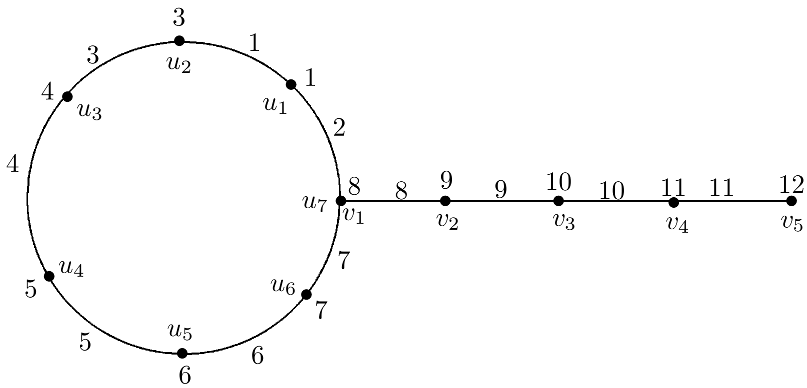

Fig.8

Theorem 2.8 *The graph obtained by identifying a vertex of any two cycles C_m and C_n is a geometric mean graph.*

Proof Let $u_1, u_2, \cdots, u_m$ and $v_1, v_2, \cdots, v_n$ be the vertices of the cycles C_m and C_n respectively. Let G be the resultant graph obtained by identifying the vertex u_m of the cycle C_m to the vertex v_n of the cycle C_n. We define $f : V(G) \to \{1, 2, 3, \cdots, m+n+1\}$ as follows:

$$f(u_i) = \begin{cases} i & \text{if } 1 \le i \le \lfloor\sqrt{m+1}\rfloor - 1 \\ i+1 & \text{if } \lfloor\sqrt{m+1}\rfloor \le i \le m \end{cases} \quad \text{and}$$

$$f(v_i) = \begin{cases} m+1+i & \text{if } 1 \le i \le \left\lfloor\sqrt{(m+1)(m+n+1)}\right\rfloor - m - 2 \\ m+2+i & \text{if } \left\lfloor\sqrt{(m+1)(m+n+1)}\right\rfloor - m - 1 \le i \le n-1. \end{cases}$$

The induced edge labeling is as follows:

$$f^*(u_iu_{i+1}) = \begin{cases} i & \text{if } 1 \le i \le \lfloor\sqrt{m+1}\rfloor - 1, \\ i+1 & \text{if } \lfloor\sqrt{m+1}\rfloor \le i \le m-1, \end{cases}$$

$$f^*(v_iv_{i+1}) = \begin{cases} m+1+i & \text{if } 1 \le i \le \left\lfloor\sqrt{(m+1)(m+n+1)}\right\rfloor - m - 2, \\ m+2+i & \text{if } \left\lfloor\sqrt{(m+1)(m+n+1)}\right\rfloor - m - 1 \le i \le n-2, \end{cases}$$

$$f^*(u_1u_m) = \lfloor\sqrt{m+1}\rfloor,$$

$$f^*(v_{n-1}v_n) = \left\lfloor\sqrt{(m+1)(m+n+1)}\right\rfloor \text{ and}$$

$$f^*(v_1v_n) = m+1.$$

Hence, f is a geometric mean labeling of the graph G. Thus the resultant graph G is a geometric mean graph. □

A geometric mean labeling of the graph G obtained by identifying a vertex of the cycles C_8 and C_{12}, is shown in Fig.9.

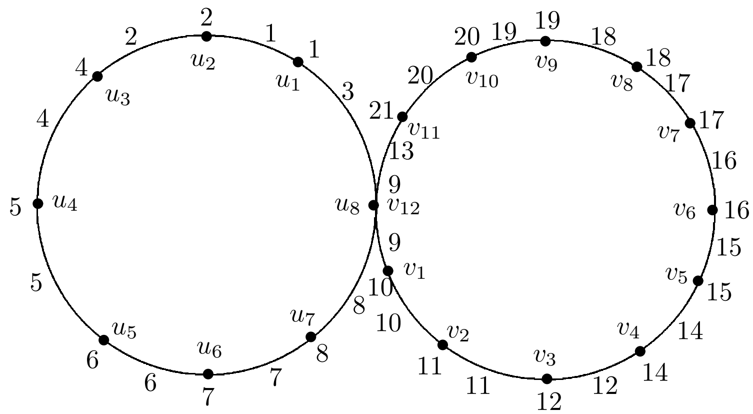

Fig.9

Theorem 2.9 *The graph obtained by identifying an edge of any two cycles C_m and C_n is a geometric mean graph.*

Proof Let $u_1, u_2, \cdots, u_m$ and $v_1, v_2, \cdots, v_n$ be the vertices of the cycles C_m and C_n respectively. Let G be the resultant graph obtained by identifying an edge $u_{m-1}u_m$ of cycle C_m with an edge $v_{n-1}v_n$ of the cycle C_n. We define $f : V(G) \to \{1, 2, 3, \cdots, m+n\}$ as follows:

$$f(u_i) = \begin{cases} i & \text{if } 1 \le i \le \left\lfloor \sqrt{m+1} \right\rfloor - 1 \\ i+1 & \text{if } \left\lfloor \sqrt{m+1} \right\rfloor \le i \le m \end{cases} \quad \text{and}$$

$$f(v_i) = \begin{cases} m+1+i & \text{if } 1 \le i \le \left\lfloor \sqrt{m(m+n)} \right\rfloor - m - 2 \\ m+2+i & \text{if } \left\lfloor \sqrt{m(m+n)} \right\rfloor - m - 1 \le i \le n-2. \end{cases}$$

The induced edge labeling is as follows:

$$f^*(u_iu_{i+1}) = \begin{cases} i & \text{if } 1 \le i \le \left\lfloor \sqrt{m+1} \right\rfloor - 1 \\ i+1 & \text{if } \left\lfloor \sqrt{m+1} \right\rfloor \le i \le m-1, \end{cases}$$

$$f^*(v_iv_{i+1}) = \begin{cases} m+1+i & \text{if } 1 \le i \le \left\lfloor \sqrt{m(m+n} \right\rfloor - m - 2 \\ m+2+i & \text{if } \left\lfloor \sqrt{m(m+n)} \right\rfloor - m - 1 \le i \le n-3, \end{cases}$$

$$f^*(u_1u_m) = \left\lfloor \sqrt{m+1} \right\rfloor,$$

$$f^*(v_1v_n) = m+1 \text{ and}$$

$$f^*(v_{n-2}v_{n-1}) = \left\lfloor \sqrt{m(m+n)} \right\rfloor.$$

Hence, f is a geometric mean labeling of the graph G. Thus the resultant graph G is a geometric mean graph. □

A geometric mean labeling of the graph G obtained by identifying an edge of the cycles C_{10} and C_{13}, is shown in Fig.10.

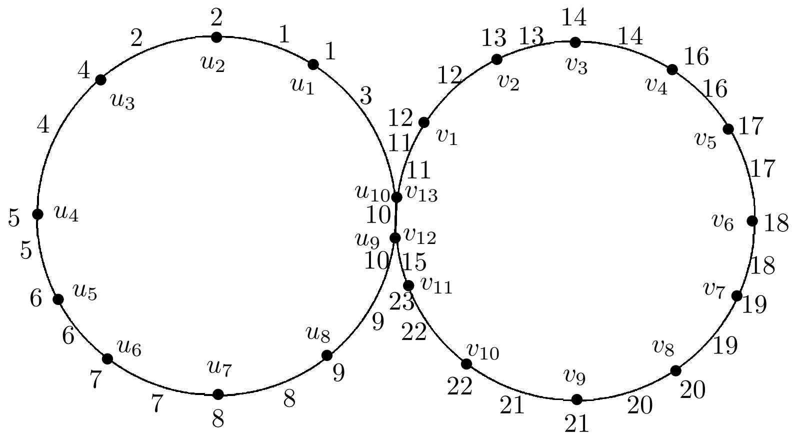

Fig.10

Theorem 2.10 *The graph obtained by joining any two cycles C_m and C_n by a path P_k is a geometric mean graph.*

Proof Let G be a graph obtained by joining any two cycles C_m and C_n by a path P_k. Let $u_1, u_2, \cdots, u_m$ and $v_1, v_2, \cdots, v_n$ be the vertices of the cycles C_m and C_n respectively. Let $w_1, w_2, \ldots, w_k$ be the vertices of the path P_k with $u_m = w_1$ and $w_k = v_n$. We define $f : V(G) \to \{1, 2, 3, \cdots, m+k+n\}$ as follows:

$$f(u_i) = \begin{cases} i & \text{if } 1 \leq i \leq \left\lfloor \sqrt{m+1} \right\rfloor - 1 \\ i+1 & \text{if } \left\lfloor \sqrt{m+1} \right\rfloor \leq i \leq m, \end{cases}$$

$$f(w_i) = m + i, \text{ for } 2 \leq i \leq k \text{ and}$$

$$f(v_i) = \begin{cases} m+k+i & \text{if } 1 \leq i \leq \left\lfloor \sqrt{(m+k)(m+k+n)} \right\rfloor - m - k - 1 \\ m+k+1+i & \text{if } \left\lfloor \sqrt{(m+k)(m+k+n)} \right\rfloor - m - k \leq i \leq n-1. \end{cases}$$

The induced edge labeling is as follows:

$$f^*(u_i u_{i+1}) = \begin{cases} i & \text{if } 1 \leq i \leq \left\lfloor \sqrt{m+1} \right\rfloor - 1 \\ i+1 & \text{if } \left\lfloor \sqrt{m+1} \right\rfloor \leq i \leq m-1, \end{cases}$$

$$f^*(w_i w_{i+1}) = m + i, \text{ for } 1 \leq i \leq k-1,$$

$$f^*(v_i v_{i+1}) = \begin{cases} m+k+i & \text{if } 1 \leq i \leq \left\lfloor \sqrt{(m+k)(m+k+n)} \right\rfloor - m - k - 1 \\ m+k+1+i & \text{if } \left\lfloor \sqrt{(m+k)(m+k+n)} \right\rfloor - m - k \leq i \leq n-2, \end{cases}$$

$$f^*(u_1 u_m) = \left\lfloor \sqrt{m+1} \right\rfloor,$$

$$f^*(v_n v_{n-1}) = \left\lfloor \sqrt{(m+k)(m+k+n)} \right\rfloor \text{ and}$$

$$f^*(v_1 v_n) = m + k.$$

Hence, f is a geometric mean labeling of the graph G. Thus the resultant graph G is a geometric mean graph. □

A geometric mean labeling of the graph G obtained by joining two cycles C_7 and C_{10} by a path P_4, is shown in Fig.11.

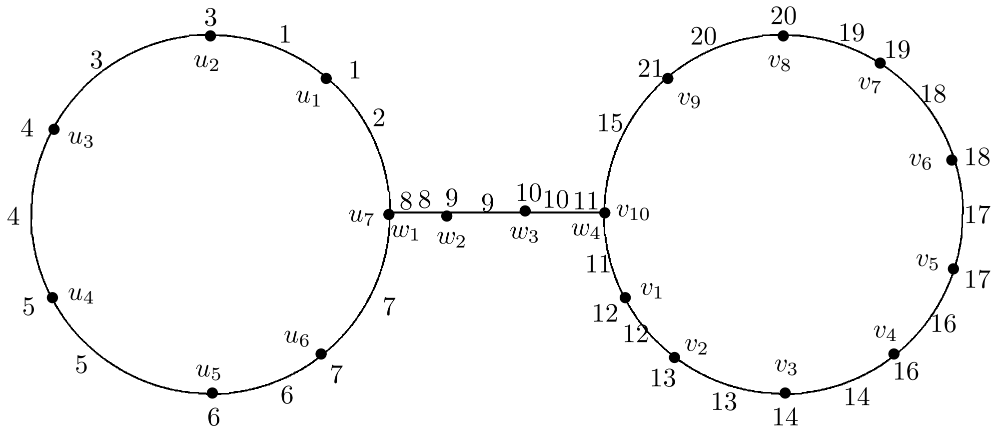

Fig.11

Theorem 2.11 *Any H-graph G is a geometric mean graph.*

Proof Let $u_1, u_2, \cdots, u_n$ and $v_1, v_2, \cdots, v_n$ be the vertices on the paths of length n in G.

Case 1 n is odd.

We define $f : V(G) \to \{1, 2, 3, \cdots, 2n\}$ as follows:

$$f(u_i) = i, \text{ for } 1 \le i \le n \text{ and}$$

$$f(v_i) = \begin{cases} n + 2i & \text{if } 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor \\ n + 2i - 1 & \text{if } i = \left\lfloor \frac{n}{2} \right\rfloor + 1 \\ 3n + 1 - 2i & \text{if } \left\lfloor \frac{n}{2} \right\rfloor + 2 \le i \le n. \end{cases}$$

The induced edge labeling is as follows:

$$f^*(u_i u_{i+1}) = i, \text{ for } 1 \le i \le n - 1,$$

$$f^*(u_i v_i) = n, \text{ for } i = \left\lfloor \frac{n}{2} \right\rfloor + 1 \text{ and}$$

$$f^*(v_i v_{i+1}) = \begin{cases} n + 2i & \text{if } 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor \\ 3n - 1 - 2i & \text{if } \left\lfloor \frac{n}{2} \right\rfloor + 1 \le i \le n - 1. \end{cases}$$

Case 2 n is even.

We define $f : V(G) \to \{1, 2, 3, \cdots, 2n\}$ as follows:

$$f(u_i) = i, \text{ for } 1 \le i \le n \text{ and}$$

$$f(v_i) = \begin{cases} n + 2i & \text{if } 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor \\ 3n + 1 - 2i & \text{if } \left\lfloor \frac{n}{2} \right\rfloor + 1 \le i \le n. \end{cases}$$

The induced edge labeling is as follows:

$$f^*(u_iu_{i+1}) = i, \text{ for } 1 \le i \le n-1,$$
$$f^*(u_{i+1}v_i) = n, \text{ for } i = \left\lfloor \frac{n}{2} \right\rfloor \text{ and}$$
$$f^*(v_iv_{i+1}) = \begin{cases} n+2i & \text{if } 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor - 1 \\ 3n-1-2i & \text{if } \left\lfloor \frac{n}{2} \right\rfloor \le i \le n-1. \end{cases}$$

Hence, H-graph admits a geometric mean labeling. □

A geometric mean labeling of H-graphs G_1 and G_2 are shown in Fig.12.

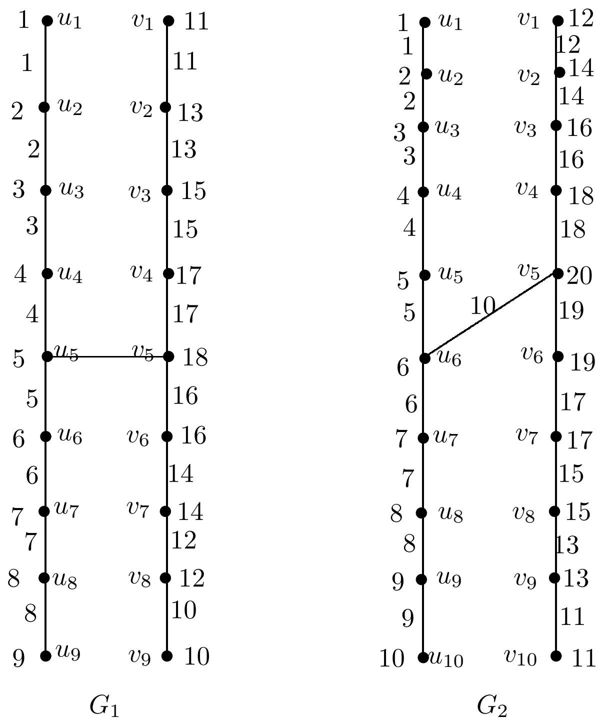

Fig.12

Theorem 2.12 *For any $n \ge 2, P(1,2,3,\cdots,n-1)$ is a geometric mean graph.*

Proof Let $v_1, v_2, \cdots, v_n$ be the vertices of the path P_n and let u_{ij} be the vertices of the partition of K_{2,m_i} with cardinality $m_i, 1 \le i \le n-1$ and $1 \le j \le m_i$. We define $f : V(P(1,2,\cdots,n-1)) \to \{1,2,3,\ldots,n(n-1)+1\}$ as follows:

$$f(v_i) = i(i-1)+1, \text{ for } 1 \le i \le n \text{ and}$$
$$f(u_{ij}) = i(i-1)+2j, \text{ for } 1 \le j \le i \text{ and } 1 \le i \le n-1.$$

The induced edge labeling is as follows:

$$f^*(v_iu_{ij}) = i(i-1)+j, \text{ for } 1 \le j \le i \text{ and } 1 \le i \le n-1$$
$$f^*(u_{ij}v_{i+1}) = i^2+j, \text{ for } 1 \le j \le i \text{ and } 1 \le i \le n-1.$$

Hence, f is a geometric mean labeling of the graph $P(1, 2, \cdots, n-1)$. Thus the graph $P(1, 2, \cdots, n-1)$ is a geometric mean graph. □

A geometric mean labeling of $P(1, 2, 3, 4, 5)$ is shown in Fig.13.

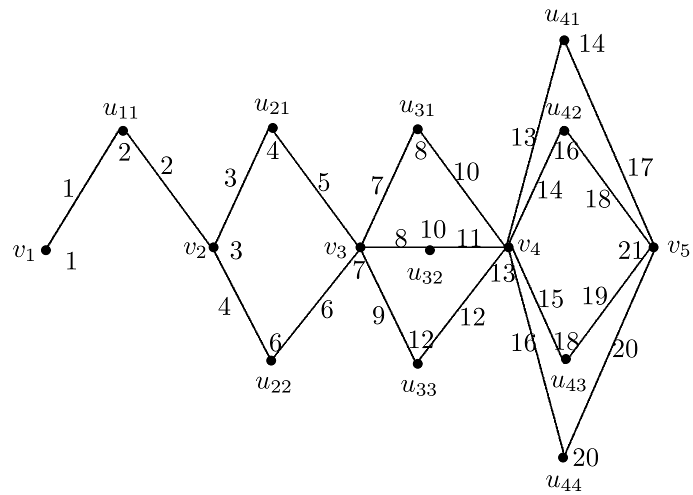

Fig.13

References

[1] A.Durai Baskar, S.Arockiaraj and B.Rajendran, *Geometric Mean Graphs* (Communicated).

[2] J.A.Gallian, A dynamic survey of graph labeling, *The Electronic Journal of Combinatorics*, **17**(2011).

[3] F.Harary, *Graph Theory*, Addison Wesely, Reading Mass., 1972.

[4] R.Ponraj and S.Somasundaram, Further results on mean graphs, *Proceedings of Sacoeference*, (2005), 443-448.

[5] A.Rosa, On certain valuation of the vertices of graph, International Symposium, Rome, July 1966, Gordon and Breach, N.Y. and Dunod Paris (1967), 349-355.

[6] S.Somasundaram and R.Ponraj, Mean labeling of graphs, *National Academy Science Letter*, **26**(2003), 210-213.

[7] S.Somasundaram and R.Ponraj, Some results on mean graphs, *Pure and Applied Mathematika Sciences*, **58**(2003), 29-35.

[8] S.K.Vaidya and Lekha Bijukumar, Mean labeling in the context of some graph operations, *International Journal of Algorithms, Computing and Mathematics*, **3**(2010).

[9] S.K.Vaidya and Lekha Bijukumar, Some netw families of mean graphs, *Journal of Mathematics Research*, **2**(3) (2010).

[10] S.K.Vaidya and Lekha Bijukumar, Mean labeling for some new families of graphs, *Journal of Pure and Applied Sciences*, **8** (2010), 115-116.

[11] S.K.Vaidya and K.K.Kanani, Some new mean graphs, *International Journal of Information Science and Computer Mathematics*, **1**(1) (2010), 73-80.