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Corrigendum: *On Set-Semigraceful Graphs*

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In this short communication we rectify certain errors which are in the paper, On Set-Semigraceful Graphs, *International J. Math. Combin.*, Vol.2(2012), 59-70. The following are the correct versions of the respective results.

Remark 3.2 (5) The Double Stars $ST(m, n)$ where $|V|$ is not a power of 2, are set-semigraceful by Theorem 2.13.

Remark 3.5 (3) The Double Stars $ST(m, n)$ where m is odd and $m + n + 2 = 2^l$, are not set-semigraceful by Theorem 2.12.

Delete the following sentence below Remark 3.9: "In fact the result given by Theorem 3.3 holds for any set-semigraceful graph as we see in the following".

Theorem 4.8([3]) *Every graph can be embedded as an induced subgraph of a connected set-graceful graph.*

Since every set-graceful graph is set-semigraceful, from the above theorem it follows that

Theorem 4.8*A* *Every graph can be embedded as an induced subgraph of a connected set-semigraceful graph.*

However, below we prove:

Theorem 4.8*B* *Every graph can be embedded as an induced subgraph of a connected set-semigraceful graph which is not set-graceful.*

Proof Any graph H with $o(H) \leq 5$ and $s(H) \leq 2$ and the graphs P_4, $P_4 \cup K_1$, $P_3 \cup K_2$ and P_5 are induced subgraphs of the set-semigraceful cycle C_{10} which is not set-graceful. Again any

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graph H' with $3 \le o(H') \le 5$ and $3 \le s(H') \le 9$ can be obtained as an induced subgraph of $H_1 \vee K_1$ for some graph H_1 with $o(H_1) = 5$ and $3 \le s(H_1) \le 9$. Then $3 < \log_2(|E(H_1 \vee K_1)| + 1) < 4$, since $8 \le s(H_1 \vee K_1) < 15$ and hence $H_1 \vee K_1$ is not set-graceful. By Theorem 2.4,

$$\begin{aligned} 4 &= \lceil \log_2(|E(H_1 \vee K_1)| + 1) \rceil \le \gamma(H_1 \vee K_1) \\ &\le \gamma(K_6) \quad \text{(by Theorem 2.5)} \\ &= 4 \quad \text{(by Theorem 2.19)} \end{aligned}$$

So that $H_1 \vee K_1$ is set-semigraceful. Further, note that K_5 is set-semigraceful but not set-graceful.

Now let $G = (V, E)$; $V = \{v_1, \ldots, v_n\}$ be a graph of order $n \ge 6$. Consider a set-indexer g of G with indexing set $X = \{x_1, \ldots, x_n\}$ defined by $g(v_i) = \{x_i\}$; $1 \le i \le n$. Let $S = \{g(e) : e \in E\} \cup \{g(v) : v \in V\}$. Note that $|S| = |E| + n$. Now take a new vertex u and join with all the vertices of G. Let m be any integer such that $2^{n-1} < m < 2^n - (|E| + n + 1)$. Since $|E| \le \dfrac{n(n-1)}{2}$ and $n \ge 6$, such an integer always exists. Take m new vertices $u_1, \ldots, u_m$ and join all of them with u. A set-indexer f of the resulting graph G' can be defined as follows:

$$f(u) = \emptyset, \quad f(v_i) = g(v_i); \quad 1 \le i \le n.$$

Besides, f assigns the vertices $u_1, \ldots, u_m$ with any m distinct elements of $2^X \setminus (S \cup \emptyset)$. Thus, $\gamma(G') \le n$. But we have $2^n > |E| + n + m + 1 > m > 2^{n-1}$ so that $\gamma(G') \ge n$, by Theorem 2.4. Hence,

$$\log_2(|E(G')| + 1) < \lceil \log_2(|E(G')| + 1) \rceil = n = \gamma(G').$$

This shows that G' is set-semigraceful, but not set-graceful. □

Corollary 4.16 *The double fan $P_k \vee K_2$ where $k = 2^n - m$ and $2^n \ge 3m$; $n \ge 3$ is set-semigraceful.*

Proof Let $G = P_k \vee K_2$; $K_2 = (u_1, u_2)$. By Theorem 2.4, $\gamma(G) \ge \lceil \log_2(|E| + 1) \rceil = \lceil \log_2(3(2^n - m) + 1) \rceil = n + 2$. But, $3m \le 2^n \Rightarrow m < 2^{n-1} - 1$. Therefore,

$$\begin{aligned} & 2^n - (2^{n-1} - 2)) \le 2^n - m < 2^n - 1 \\ & \Rightarrow 2^{n-1} + 1 \le 2^n - m - 1 < 2^n - 2 \\ & \Rightarrow 2^{n-1} + 1 \le k - 1 < 2^n - 2; \quad k = 2^n - m \\ & \Rightarrow 2^{n-1} + 1 \le |E(P_k)| < 2^n \\ & \Rightarrow \lceil \log_2(|E(P_k)| + 1) \rceil = n \\ & \Rightarrow \gamma(P_k) = n \end{aligned}$$

since P_k is set-semigraceful by Remark 3.2(3). □

Let f be a set-indexer of P_k with indexing set $X = \{x_1, \ldots, x_n\}$. Define a set-indexer g of G with indexing set $Y = X \cup \{x_{n+1}, x_{n+2}\}$ as follows:

$g(v) = f(v)$ for every $v \in V(P_k)$, $g(u_1) = \{x_{n+1}\}$ and $g(u_2) = \{x_{n+2}\}$.

Corollary 4.17 *The graph $K_{1,2^n-1} \vee K_2$ is set-semigraceful.*

Proof The proof follows from Theorems 4.15 and 2.33. □

Theorem 4.18 *Let C_k where $k = 2^n - m$ and $2^n + 1 > 3m$; $n \geq 2$ be set-semigraceful. Then the graph $C_k \vee K_2$ is set-semigraceful.*

Proof Let $G = C_k \vee K_2$; $K_2 = (u_1, u_2)$. By theorem 2.4, $\gamma(G) \geq \lceil \log_2(|E| + 1) \rceil = \lceil \log_2(3(2^n - m) + 2) \rceil = n + 2$. But, $3m \leq 2^n + 1 \Rightarrow m < 2^{n-1}$. Therefore,

$$\begin{aligned}
&2^n - (2^{n-1} - 1)) \leq 2^n - m < 2^n \\
&\Rightarrow 2^{n-1} + 1 \leq k < 2^n; \quad k = 2^n - m \\
&\Rightarrow 2^{n-1} + 1 \leq |E(C_k)| < 2^n \\
&\Rightarrow \lceil \log_2(|E(C_k)| + 1) \rceil = n \\
&\Rightarrow \gamma(C_k) = n
\end{aligned}$$

since C_k is set-semigraceful. □

Let f be a set-indexer of C_k with indexing set $X = \{x_1, \ldots, x_n\}$. Define a set-indexer g of G with indexing set $Y = X \cup \{x_{n+1}, x_{n+2}\}$ as follows:

$g(v) = f(v)$ for every $v \in V(C_k)$, $g(u_1) = \{x_{n+1}\}$ and $g(u_2) = \{x_{n+2}\}$.

Corollary 4.21 *W_n where $2^m - 1 \leq n \leq 2^m + 2^{m-1} - 2$; $m \geq 3$ is set-semigraceful.*

Proof The proof follows from Theorem 3.15 and Corollary 4.20. □

Theorem 4.22 *If W_{2k} where $\frac{2^{n-1}}{3} \leq k < 2^{n-2}$; $n \geq 4$ is set-semigraceful, then the gear graph of order $2k + 1$ is set-semigraceful.*

Proof Let G be the gear graph of order $2k + 1$. Then by theorem 2.4,

$$\begin{aligned}
&\lceil \log_2(3k + 1) \rceil \leq \gamma(G) \leq \gamma(W_{2k}) \quad \text{(by Theorem 2.5)} \\
&= \lceil \log_2(4k + 1) \rceil \quad (\text{since } \mathrm{W_{2k}} \text{ is set} - \text{semigraceful}) \\
&= \lceil \log_2(3k + 1) \rceil
\end{aligned}$$

since

$$\begin{aligned}
\frac{2^{n-1}}{3} \leq k < 2^{n-2} \qquad &\Rightarrow 2^{n-1} \leq 3k < 4k < 2^n \\
&\Rightarrow 2^{n-1} + 1 \leq 3k + 1 < 4k + 1 \leq 2^n.
\end{aligned}$$

Thus

$$\gamma(G) = \lceil \log_2(|E| + 1) \rceil.$$

So that G is set-semigraceful. □