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Dynamical Knot and Their Fundamental Group

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Abstract: In this article, we introduce the fundamental group of the dynamical trefoil knot. Also the fundamental group of the limit dynamical trefoil knot will be achieved. Some types of conditional dynamical manifold restricted on the elements of a free group and their fundamental groups are presented. The dynamical trefoil knot of variation curvature and torsion of manifolds on their fundamental group are deduced .Theorems governing these relations are obtained.

Keywords: Dynamical trefoil knot, fundamental group, knot group, Smarandache multi-space.

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§1. Introduction

A means of describing how one state develops into another state over the course of time. Technically, a dynamical system is a smooth action of the reals or the integers on another object (usually a manifold). When the reals are acting, the system is called a continuous dynamical system, and when the integers are acting, the system is called a discrete dynamical system. If f is any continuous function, then the evolution of a variable x can be given by the formula $x_{n+1} = f(x_n)$. This equation can also be viewed as a difference equation $x_{n+1} - x_n = f(x_n) - x_n$, so defining $g(x) \equiv f(x) - x$ gives $x_{n+1} - x_n = g(x_n) * 1$,which can be read "as n changes by 1 unit, x changes by $g(x)$. This is the discrete analog of the differential equation $x'(n) = g(x(n))$.

In other words; a dynamic system is a set of equations specifying how certain variables change over time. The equations specify how to determine (compute) the new values as a function of their current values and control parameters. The functions, when explicit, are either difference equations or differential equations. Dynamic systems may be stochastic or deterministic. In a stochastic system, new values come from a probability distribution. In a deterministic system, a single new value is associated with any current value $[1, 11]$.

The dynamical systems were discussed in $[1, 9, 11]$. The fundamental groups of some types of a manifold were studied in $[2, 6-8, 10]$.

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§1. Definitions

1. The set of homotopy classes of loops based at the point x_0 with the product operation $[f][g] = [f.g]$ is called the fundamental group and denoted by $\pi_1(X, x_0)$ [3].

2. Given spaces X and Y with chosen points $x_0 \in X$ and $y_0 \in Y$, then the wedge sum $X \vee Y$ is the quotient of the disjoint union $X \cup Y$ obtained by identifying x_0 and y_0 to a single point [5].

3. A knot is a subset of 3-space that is homeomorphic to the unit circle and a trefoil knot is the simplest nontrivial knot, it can be obtained by joining the loose ends of an overhand knot [5].

4. A Smarandache multi-space is a union of n spaces equipped with some different structures for an integer $n \geq 2$, which can be used for discrete or connected space [4].

5. Given a knot k, the fundamental group $\pi_1(R^3 - k)$ is called the knot group of k [5].

6. A dynamical system in the space X is a function $q = f(p, t)$ which assigns to each point p of the space X and to each real number$t$, $\infty < t < \infty$ a definite point $q \in X$ and possesses the following three properties :

a- Initial condition : $f(p, 0) = p$ for any point$p \in X$.

b- Property of continuity in both arguments simultaneously:

$$\lim_{\substack{p \to p_0 \\ t \to t_0}} f(p, t) = f(p_0, t_0)$$

c- Group property $f(f(p, t_1), t_2) = f(p, t_1 + t_2)$[11].

§2. The Main Results

Aiming to our study, we will introduce the following:

Theorem 3.1 *Let K be a trefoil knot then there are two types of dynamical trefoil knot $D_i : K \to \overline{K}$, $i = 1, 2$, $D_i(K) \neq K$,which induces dynamical trefoil knot $\bar{D}_i : \pi_1(K) \to \pi_1(\overline{K})$ such that $\bar{D}_i(\pi_1(K))$ is a free group of rank ≤ 4 or identity group.*

Proof Let $D_1 : K \to \overline{K}$ be a dynamical trefoil knot such that $D_1(K)$ is dynamical crossing i.e. the point of upper arc crossing touch the point of lower crossing, where $D_1(c) = p_1$ as in FIGURE 1(a) then we have the induced dynamical trefoil knot $\bar{D}_1 : \pi_1(K) \to \pi_1(\overline{K})$ such that $\bar{D}_1(\pi_1(K)) = \pi_1(D_1(K)) \approx \pi_1(S_1^1) * \pi_1(S_2^1)$, thus $\bar{D}_1(\pi_1(K)) \approx Z * Z$, so $\bar{D}_1(\pi_1(K))$is a free group of rank $= 2$. Also, if $D_1 : K \to \overline{K}$ such that $D_1(c) = p_1$, $D_1(b) = p_2$ then $D_1(K)$ is space as in FIGURE 1(b) and so $\bar{D}_1(\pi_1(K)) = \pi_1(D_1(K)) \approx \pi_1(S_1^1) * \pi_1(S_2^1) * \pi_1(S_3^1)$, thus $\bar{D}_1(\pi_1(K))$ is a free group of rank $= 3$. Moreover, if $D_1 : K \to \overline{K}$ such that $D_1(c) = p_1$, $D_1(b) = p_2, D_1(a) = p_3$, then $D_1(K)$ is space as in FIGURE 1(c) and so $\bar{D}_1(\pi_1(K)) =$

$\pi_1(D_1(K)) \approx \pi_1(S_1^1) * \pi_1(S_2^1) * \pi_1(S_3^1) * \pi_1(S_4^1)$, hence $\bar{D}_1(\pi_1(K))$is a free group of rank $= 4$. There is another type $D_2 : K \to \overline{K}$ such that $D_2(K)$ is dynamical trefoil knot with singularity as in FIGURE 1(d) then we obtain the induced dynamical trefoil knot $\bar{D}_2 : \pi_1(K) \to \pi_1(\overline{K})$ such that $\bar{D}_2(\pi_1(K)) = \pi_1(D_2(K)) = 0$. Therefore, $\bar{D}_i(\pi_1(K))$ is a free group of rank≤ 4 or identity group. □

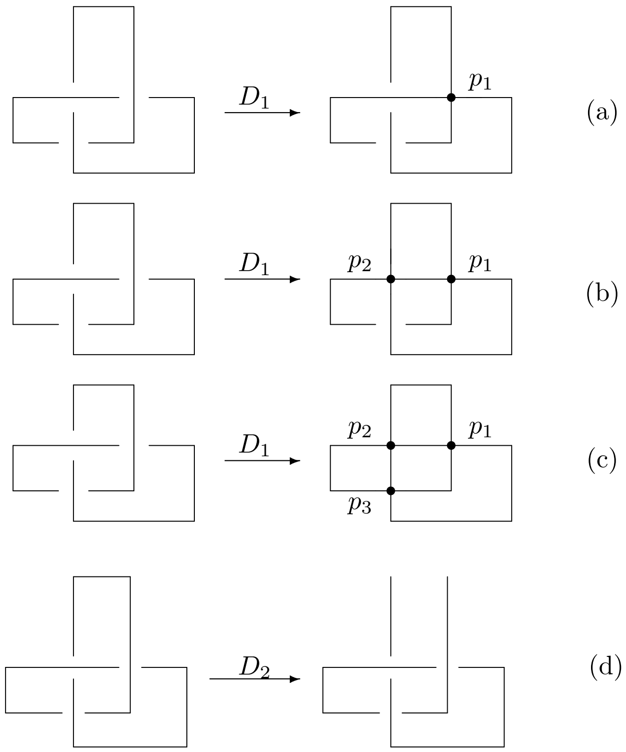

FIGURE 1

Theorem 3.2 *The fundamental group of the limit dynamical trefoil knot is the identity group.*

Proof Let $D_1 : K \to K_1$, $D_2 : D_1(K) \to D_1(K_2)$,..., $D_n : D_{n-1}(D_{n-2})\ldots(D_1(K) \to D_{n-1}(D_{n-2})\ldots(D_1(K_n)$ such that $\lim_{n\to\infty}(D_n(D_{n-1})\ldots(D_1(K)\ldots)$ is a point as in FIGURE 2 (a,b) ,then $\pi_1(\lim_{n\to\infty}(D_n(D_{n-1})\ldots(D_1(K)\ldots)) = 0$. □

Theorem 3.3 *There are different types of dynamical link graph L which represent a trefoil knot ,where $D(L) \neq L$ such that $\pi_1(D(L))$ is a free group of rank ≤ 3.*

Proof Let L be a link graph which represent a trefoil knot and consider the following dynamical edges $D(e) = a, D(f) = c, D(g) = b$ as in FIGURE 3(a) then $\pi_1(D(L)) \approx \pi_1(S^1)$ and so $\pi_1(D(L))$ is a free group of rank 1. Now, if $D(e) \neq e, D(f) \neq f, D(g) \neq g$ as in FIGURE 3(b)we get the same result. Also, if $D(e) = e, D(f) = f, D(g) \neq g$ as in FIGURE 3(c) then, $\pi_1(D(L)) \approx \pi_1(S_1^1) * \pi_1(S_2^1) * \pi_1(S_3^1)$, thus $\pi_1(D(L))$ is a free group of rank 3.Moreover, if

$D(e) = e, D(f) \neq f, D(g) \neq g$ as in FIGURE 3(d)then $\pi_1(D(L)) \approx \pi_1(S_1^1) * \pi_1(S_2^1)$ .Hence $\pi_1(D(L))$ is a free group of rank 2. Therefore $\pi_1(D(L))$ is a free group of rank ≤ 3. □

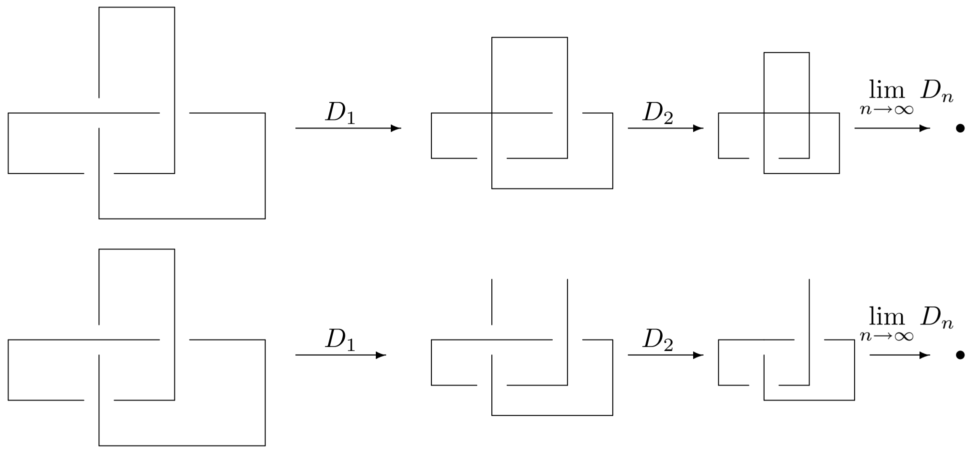

FIGURE 2

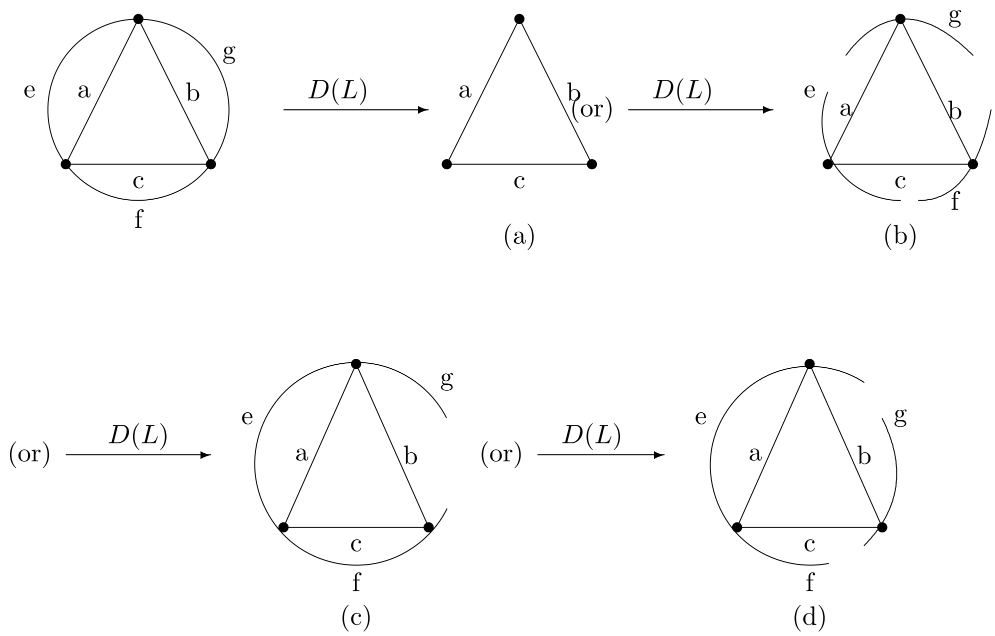

FIGURE 3

Theorem 3.4 *The fundamental group of limit dynamical link graph of n vertices is a free group of rank n.*

Proof Let K be link graph of n vertices , then $\lim_{n\to\infty}(D(K))$ is a graph with only one vertex and n-loops as in FIGURE 4 ,for n=3 and so $\pi_1(\lim\limits_{n\to\infty}(D(K))) = \pi_1(\overset{n}{\underset{i=1}{\vee}} S_i^1) \approx \underbrace{Z * Z * ... * Z}_{n \text{ terms}}$. Hence, $\pi_1(\lim\limits_{n\to\infty}(D(K)))$ is a free group of rank n. □

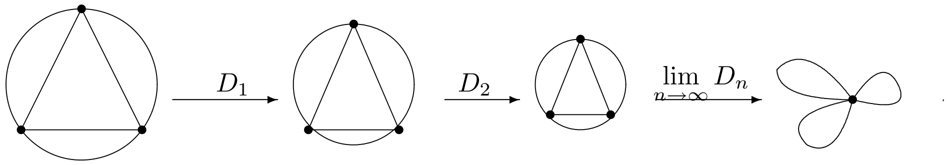

FIGURE 4

Theorem 3.5 *Let I be the closed interval $[0,1]$. Then there is a sequence of dynamical manifolds $D_i : I \to I_i,\ i = 1, 2, ..., n$ with variation curvature and torsion such that $\lim\limits_{n\to\infty} D_n(I)$ is trefoil knot and $\pi_1(R^3 - \lim\limits_{n\to\infty} D_n(I)) \approx Z$.*

Proof Consider the sequence of dynamical manifolds with variation curvature and torsion : $D_1 : I \to I_1, D_2 : I_1 \to I_2, ..., D_n : I_{n-1} \to I_n$ such that $\lim\limits_{n\to\infty} D_n(I)$ is a trefoil knot as in FIGURE 5, Therefore, $\pi_1(R^3 - \lim\limits_{n\to\infty} D_n(I)) \approx Z.$ □

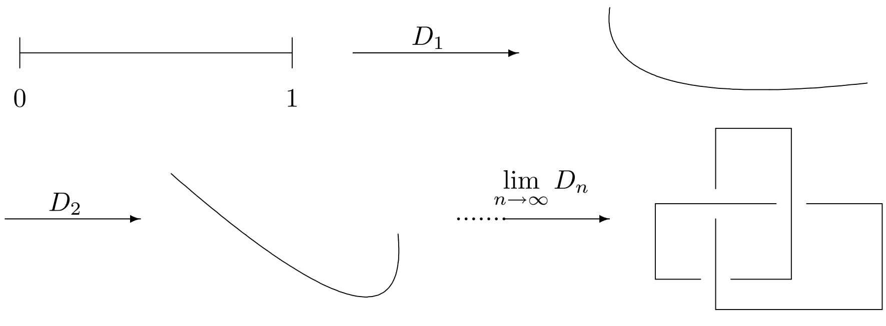

FIGURE 5

Theorem 3.6 *The knot group of the limit dynamical sheeted trefoil knot is either isomorphic to Z or identity group.*

Proof Let $\overline{K}$ be a sheet trefoil knot with boundary $\{A, B\}$as in FIGURE 6 and $D : \overline{K} \to \overline{K}$ is dynamical sheeted trefoil knot of $\overline{K}$ into itself,then we get the following sequence: $D_1 : \overline{K} \to \overline{K}, D_2 : D_1(\overline{K}) \to D_1(\overline{K})$, . . . ,
$D_n : (D_{n-1}) \dots (D_1(\overline{K}) \dots) \to (D_{n-1}) \dots (D_1(\overline{K}) \dots)$
such that $\lim\limits_{n\to\infty}(D_n(D_{n-1}) \dots (D_1(\overline{K}) \dots) = k$ where, k is a trefoil knot as in FIGURE 6(a) then $\pi_1(R^3 - k) \approx Z$..Also, if $\lim\limits_{n\to\infty}(D_n(D_{n-1}) \dots (D_1(\overline{K}) \dots)$ =point as in FIGURE 6(b,c) then$\pi_1(R^3 - \lim\limits_{n\to\infty}(D_n(D_{n-1}) \dots (D_1(\overline{K}) \dots)) = \pi_1(R^3 - \textit{one point})$. Hence,

$$\pi_1(R^3 - \lim_{n\to\infty}(D_n(D_{n-1})\dots(D_1(\overline{K})\dots)) = 0.$$

Therefore, the knot group of the limit dynamical sheeted trefoil knot is either isomorphic to Z or identity group.

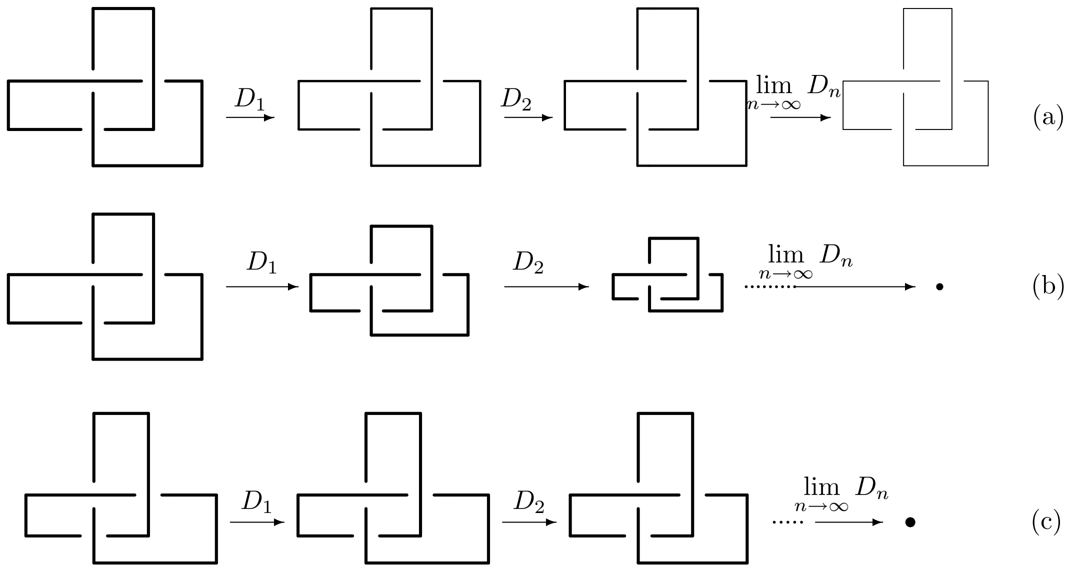

FIGURE 6

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