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Edge Detour Graphs with Edge Detour Number 2

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Abstract: For two vertices u and v in a graph $G = (V, E)$, the *detour distance* $D(u, v)$ is the length of a longest u–v path in G. A u–v path of length $D(u, v)$ is called a *u–v detour*. For any integer $k \geq 1$, a set $S \subseteq V$ is called a *Smarandache k-edge detour set* if every edge in G lies on at least k detours joining some pairs of vertices of S. The *Smarandache k-edge detour number* $dn_k(G)$ of G is the minimum order of its Smarandache k-edge detour sets and any Smarandache k-edge detour set of order $dn_k(G)$ is a *Smarandache k-edge detour basis* of G. A connected graph G is called a *Smarandache k-edge detour graph* if it has a Smarandache k-edge detour set for an integer k. Smarandache 1-edge detour graphs are refered to as edge detour graphs and in this paper, such graphs G with detour diameter $D \leq 4$ and $dn_1(G) = 2$ are characterized.

Key Words : Detour, Smarandache k-edge detour set, Smarandache k-edge detour number, edge detour set, edge detour graph, edge detour number.

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§1. Introduction

By a *graph* $G = (V, E)$, we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. For basic definitions and terminologies, we refer to [1], [6].

For vertices u and v in a connected graph G, the *detour distance* $D(u, v)$ is the length of a longest u–v path in G. A u–v path of length $D(u, v)$ is called a *u–v detour*. It is known that the detour distance is a metric on the vertex set V. Detour distance and detour center of a graph were studied by Chartrand et al. in [2], [5].

A vertex x is said to lie on a u–v detour P if x is a vertex of P including the vertices u and v. A set $S \subseteq V$ is called a *Smarandache k-detour set* if every vertex v in G lies on at least k detours joining some pairs of vertices of S. The *Smarandache k-detour number* $dnk(G)$ of G is the minimum order of a Smarandache k-detour set and any Smarandache k-detour set of order $dnk(G)$ is called a *Smarandache k-detour basis* of G. Smarandache 1-detour sets and Smarandache 1-detour number are nothing but the detour sets and the detour number $dn(G)$ of a graph as introduced and studied by Chartrand et al. in [3]. These concepts have interesting applications in Channel Assignment Problem in radio technologies [4], [7].

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An edge e of G is said to lie on a u–v detour P if e is an edge of P. In general, there are graphs G for which there exist edges which do not lie on a detour joining any pair of vertices of V. For the graph G given in Figure 1.1, the edge v_1v_2 does not lie on a detour joining any pair of vertices of V. This motivated us to introduce the concepts of *weak edge detour set* of a graph [8] and *edge detour graphs* [9].

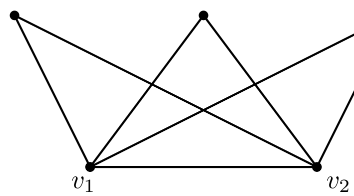
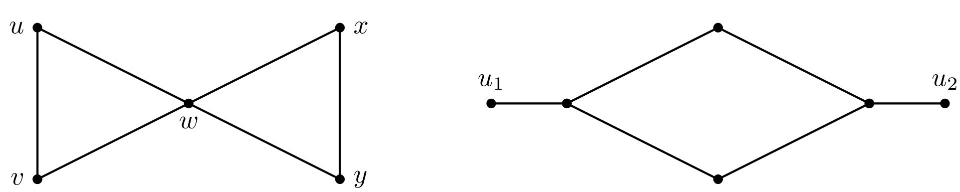


Figure 1.1: G

A set $S \subseteq V$ is called a *weak edge detour set* of G if every edge in G has both its ends in S or it lies on a detour joining a pair of vertices of S. The *weak edge detour number* $dn_w(G)$ of G is the minimum order of its weak edge detour sets and any weak edge detour set of order $dn_w(G)$ is called a *weak edge detour basis* of G. Weak edge detour sets and weak edge detour number of a graph were introduced and studied by Santhakumaran and Athisayanathan in [8].

A set $S \subseteq V$ is called an *edge detour set* of G if every edge in G lies on a detour joining a pair of vertices of S. The *edge detour number* $dn_1(G)$ of G is the minimum order of its edge detour sets and any edge detour set of order $dn_1(G)$ is an *edge detour basis* of G. A graph G is called an *edge detour graph* if it has an edge detour set. Edge detour graphs were introduced and studied in detail by Santhakumaran and Athisayanathan in [9], [10].

For the graph G given in Figure 1.2(a), the sets $S_1 = \{u, x\}$, $S_2 = \{u, w, x\}$ and $S_3 = \{u, v, x, y\}$ are detour basis, weak edge detour basis and edge detour basis of G respectively and hence $dn(G) = 2$, $dn_w(G) = 3$ and $dn_1(G) = 4$. For the graph G given in Figure 1.2(b), the set $S = \{u_1, u_2\}$ is a detour basis, weak edge detour basis and an edge detour basis so that $dn(G) = dn_w(G) = dn_1(G) = 2$. The graphs G given in Figure 1.2 are edge detour graphs. For the graph G given in Figure 1.1, the set $S = \{v_1, v_2\}$ is a detour basis and also a weak edge detour basis. However, it does not contain an edge detour set and so the graph G in Figure 1.1 is not an edge detour graph. Also, for the graph G given in Figure 1.3, it is clear that no



(a) (b)

Figure 1.2: G

two element subset of V is an edge detour set of G. It is easily seen that $S_1 = \{v_1, v_2, v_4\}$ is an edge detour set of G so that S_1 is an edge detour basis of G and hence $dn_1(G) = 3$. Thus G is an edge detour graph. Also $S_2 = \{v_1, v_2, v_5\}$ is another edge detour basis of G and thus there can be more than one edge detour basis for a graph G.

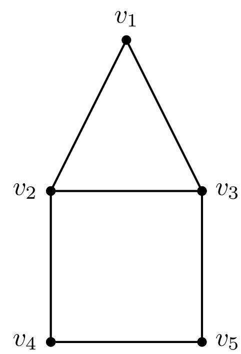


Figure 1.3: G

The following theorems are used in the sequel.

Theorem 1.1([9]) *Every end-vertex of an edge detour graph G belongs to every edge detour set of G. Also if the set S of all end-vertices of G is an edge detour set, then S is the unique edge detour basis for G.*

Theorem 1.2([9]) *If T is a tree with k end-vertices, then $dn_1(T) = k$.*

Theorem 1.3([9]) *If G is the complete graph K_2 or $K_p - e$ $(p \geqslant 3)$ or an even cycle C_n or a non-trivial path P_n or a complete bipartite graph $K_{m,n}$ $(m, n \geqslant 2)$, then G is an edge detour graph and $dn_1(G) = 2$.*

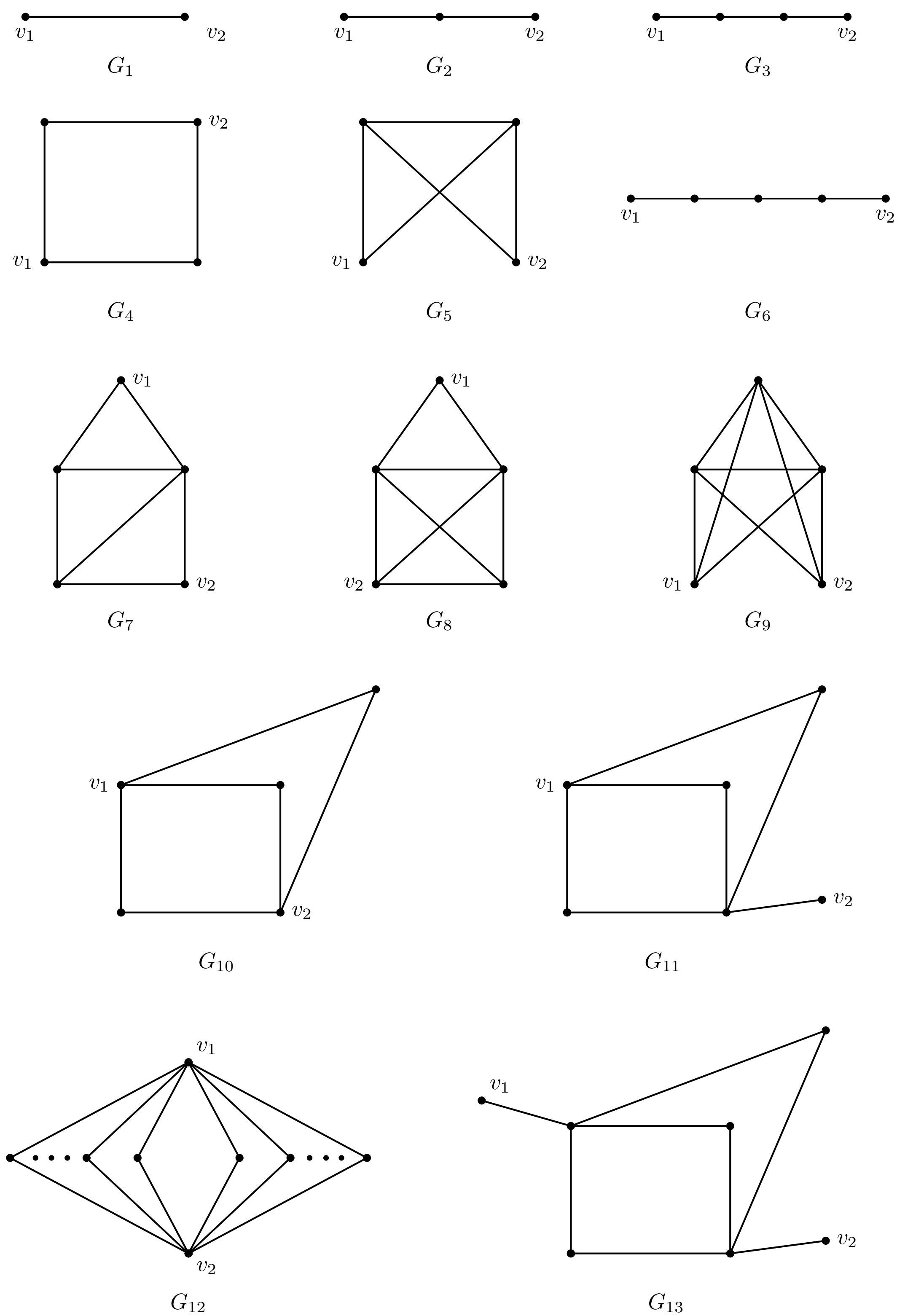
Theorem 1.4([9]) *If G is the complete graph K_p $(p \geqslant 3)$ or an odd cycle C_n, then G is an edge detour graph and $dn_1(G) = 3$.*

Theorem 1.1([9]) *Let $G = (K_{n_1} \cup K_{n_2} \cup \cdots \cup K_{n_r} \cup kK_1) + v$ be a block graph of order $p \geqslant 5$ such that $r \geqslant 2$, each $n_i \geqslant 2$ and $n_1 + n_2 + \cdots + n_r + k = p - 1$. Then G is an edge detour graph and $dn_1(G) = 2r + k$.*

Throughout this paper G denotes a connected graph with at least two vertices.

§2. Edge detour graphs G with $diam_D G \leq 4$ and $dn_1(G) = 2$

An edge detour set of an edge detour graph G needs at least two vertices so that $dn_1(G) \geq 2$ and the set of all vertices of G is an edge detour set of G so that $dn_1(G) \leq p$. Thus $2 \leq dn_1(G) \leq p$. The bounds in this inequality are sharp. For the complete graph $K_p (p = 2, 3), dn_1(K_p) = p$. The set of two end vertices of a path $P_n (n \geq 2)$ is its unique edge detour set so that $dn_1(P_n) = 2$. Thus the complete graphs $K_p (p = 2, 3)$ have the largest possible edge detour number p and the non-trivial paths have the smallest edge detour number 2. In the following, we characterize graphs G with detour diameter $D \leq 4$ for which $dn_1(G) = 2$. For this purpose we introduce the collection $\mathscr{H}$ of graphs given in Figure 2.1.



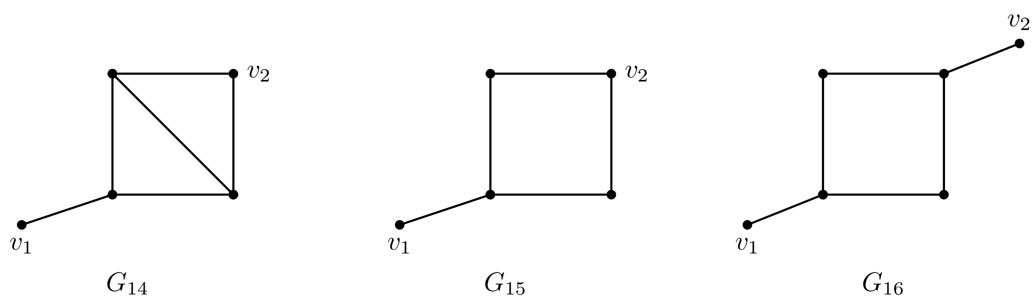


Figure 2.1: Graphs in family $\mathscr{H}$

Theorem 2.1 *Let G be an edge detour graph of order $p \geq 2$ with detour diameter $D \leq 4$. Then $dn_1(G) = 2$ if and only if $G \in \mathscr{H}$ given in Figure 2.1.*

Proof It is straightforward to verify that the set $\{v_1, v_2\}$ as marked in the graphs G_i $(1 \leq i \leq 16)$ given in $\mathscr{H}$ of Figure , is an edge detour set for each G_i. Hence $dn_1(G_i) = 2$ for all the graphs $G_i \in \mathscr{H}$ $(1 \leqslant i \leqslant 16)$ given in Figure 2.1.

For the converse, let G be an edge detour graph of order $p \geqslant 2$, $D \leqslant 4$ and $dn_1(G) = 2$.

If $D = 1$, then it is clear that $G_1 \in \mathscr{H}$ given in Figure 2.1 is the only graph for which $dn_1(G) = 2$.

Suppose $D = 2$. If G is a tree, then it follows from Theorem 1.2 that $G_2 \in \mathscr{H}$ given in Figure 2.1 is the only graph with $dn_1(G) = 2$. If G is not a tree, let $c(G)$ denote the length of a longest cycle in G. Since G is connected and $D = 2$, it is clear that $c(G) = 3$ and G has exactly three vertices so that $G = K_3$ and by Theorem 1.4, $dn_1(G) = 3$. Thus, when $D = 2$, $G_2 \in \mathscr{H}$ given in Figure 2.1 is the only graph that satisfies the requirements of the theorem.

Suppose $D = 3$. If G is a tree, then it follows from Theorem 1.2 that the path $G_3 \in \mathscr{H}$ given in Figure 2.1 is the only graph with $dn_1(G) = 2$. Assume that G is not a tree. Let $c(G)$ denote the length of a longest cycle in G. Since G is connected and $D = 3$, it follows that $p \geq 4$ and $c(G) \leq 4$. We consider two cases.

Case 1 Let $c(G) = 4$. Then, since G is connected and $D = 3$, it is clear that G has exactly four vertices. Hence G_4, $G_5 \in \mathscr{H}$ given in Figure 2.1 and K_4 are the only graphs with these properties. But by Theorem 1.3, $dn_1(G_4) = dn_1(G_5) = 2$ and by Theorem 1.4, $dn_1(K_4) = 3$. Thus in this case G_4, $G_5 \in \mathscr{H}$ given in Figure 2.1 are the only graphs that satisfy the requirements of the theorem.

Case 2: Let $c(G) = 3$. If G contains two or more triangles, then $c(G) = 4$ or $D \geqslant 4$, which is a contradiction. Hence G contains a unique triangle C_3: v_1, v_2, v_3, v_1. Now, if there are two or more vertices of C_3 having degree 3 or more, then $D \geqslant 4$, which is contradiction. Thus exactly one vertex in C_3 has degree 3 or more. Since $D = 3$, it follows that $G = K_{1,p-1} + e$ and so by Theorem 1.5 $dn_1(K_{1,p-1} + e) = p - 1 \geq 3$, which is a contradiction. Thus, in this case, there are no graphs that satisfying the requirements of the theorem.

Suppose $D = 4$. If G is a tree, then it follows from Theorem 1.2 that $G_6 \in \mathscr{H}$ given in Figure 2.1 is the only graph with $dn_1(G) = 2$. Assume that G is not a tree. Let $c(G)$ denote the length of a longest cycle in G. Since $D = 4$, it follows that $p \geq 5$ and $c(G) \leq 5$. We consider three cases.

Case 1 Let $c(G) = 5$. Then, since $D = 4$, it is clear that G has exactly five vertices. Now, it is easily verified that the graphs G_7, G_8, and $G_9 \in \mathscr{H}$ given in Figure 2.1 are the only graphs with $dn_1(G_i) = 2$ ($i = 7, 8, 9$) among all graphs on five vertices having a largest cycle of length 5.

Case 2 Let $c(G) = 4$. Suppose that G contains K_4 as an induced subgraph. Since $p \geqslant 5$, $D = 4$ and $c(G) = 4$, every vertex not on K_4 is pendant and adjacent to exactly one vertex of K_4. Thus the graph reduces to the graph G given in Figure 2.1.

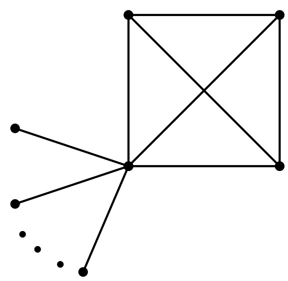
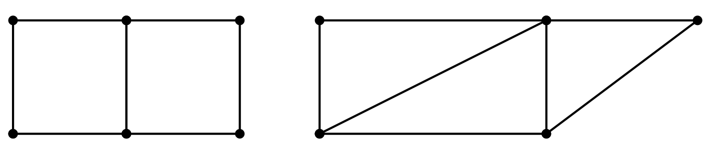


Figure 2.2: G

For this graph G, it follows from Theorem 1.5 that $dn_1(G) = p-1 \geq 4$, which is a contradiction.

Now, suppose that G does not contain K_4 as an induced subgraph. We claim that G contains exactly one 4-cycle C_4. Suppose that G contains two or more 4-cycles. If two 4-cycles in G have no edges in common, then it is clear that $D \geqslant 5$, which is a contradiction. If two 4-cycles in G have exactly one edge in common, then G must contain the graphs given in Figure 2.3 as subgraphs or induced subgraphs. In any case, $D \geqslant 5$ or $c(G) \geqslant 5$, which is a contradiction.



G_1 G_2

Figure 2.3: G

If two 4-cycles in G have exactly two edges in common, then G must contain only the graphs given in Figure 2.4 as subgraphs. It is easily verified that all other subgraphs having two edges in common will have cycles of length $\geqslant 5$ so that $D \geq 5$, which is a contradiction.

Now, if $G = H_1$, then $dn_1(G) = 2$ and it is nothing but the graph $G_{10} \in \mathscr{H}$ given in Figure 2.1. Assume first that G contains H_1 as a proper subgraph. Then there is a vertex x such that $x \notin V(H_1)$ and x is adjacent to at least one vertex of H_1. If x is adjacent to v_1, we get a path x, v_1, v_2, v_3, v_4, v_5 of length 5 so that $D \geqslant 5$, which is a contradiction. Hence x cannot be adjacent to v_1. Similarly x cannot be adjacent to v_3 and v_5. Thus x is adjacent to v_2 or v_4 or both. If x is adjacent only to v_2, then x must be a pendant vertex of G, for otherwise, we get a path of length 5 so that $D \geqslant 5$, which is a contradiction. Thus in this case, the graph G

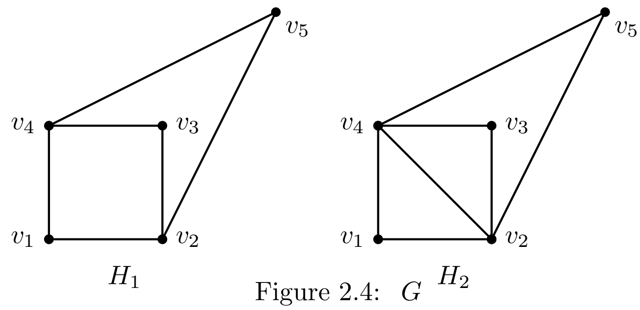


Figure 2.4: G

reduces to the one given in Figure 2.5.

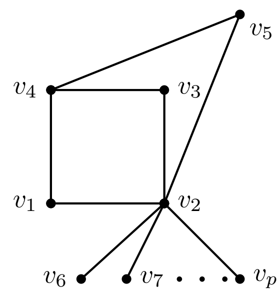


Figure 2.5: G

However, for this graph G, it follows from Theorem 1.1 that the set $\{v_4, v_6, v_7, \ldots, v_p\}$ is an edge detour basis so that $dn_1(G) = p - 4$. Hence $p = 6$ is the only possibility and the graph reduces to $G_{11} \in \mathscr{H}$ given in Figure 2.1 and satisfies the requirements of the theorem. If x is adjacent only to v_4, then we get a graph G isomorphic to the one given in Figure 2.5 and hence we get a graph G isomorphic to $G_{11} \in \mathscr{H}$ given in Figure 2.1 and satisfies the requirements of the theorem. If x is adjacent to both v_2 and v_4, then the graph reduces to the one given in Figure 2.6. This graph G is isomorphic to $G_{12} \in \mathscr{H}$ given in Figure and $\{v_1, v_2\}$ is an edge

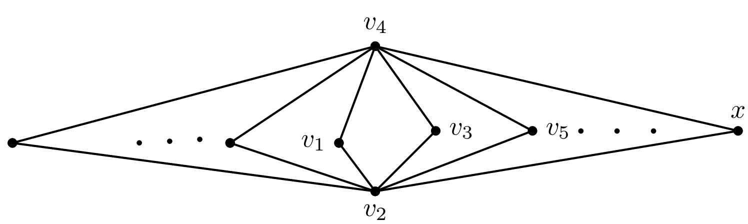


Figure 2.6: G

detour basis for G_{12} so that $dn_1(G) = 2$.

Next, if a vertex x not on H_1 is adjacent only to v_2 and a vertex y not on H_1 is adjacent only to v_4, then x and y must be pendant vertices of G, for otherwise, we get either a path or a cycle of length $\geqslant 5$ so that $D \geqslant 5$, which is a contradiction. Thus in this case, the graph reduces to the one given in Figure 2.7.

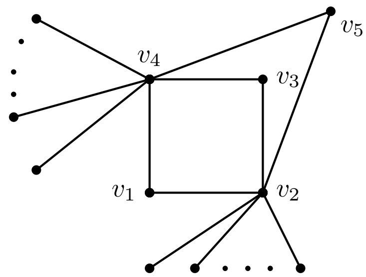


Figure 2.7: G

For this graph G, the set of all end-vertices is an edge detour basis so that by Theorem 1.1, $dn_1(G) = p-5$. Hence $p = 7$ is the only possibility and the graph reduces to $G_{13} \in \mathscr{H}$ given in Figure 2.1 and satisfies the requirements of the theorem. Thus, in this case, we have G_{10}, G_{11}, G_{12}, $G_{13} \in \mathscr{H}$ given in Figure 2.1 are the only graphs with H_1 as proper subgraph for which $dn_1(G) = 2$.

Next, if $G = H_2$ given in Figure 2.4, then the edge v_2v_4 does not lie on any detour joining a pair of vertices of G so that G is not an edge detour graph. If G contains H_2 as a proper subgraph, then as in the case of H_1, it is easily seen that the graph reduces to any one of the graphs given in Figure 2.8.

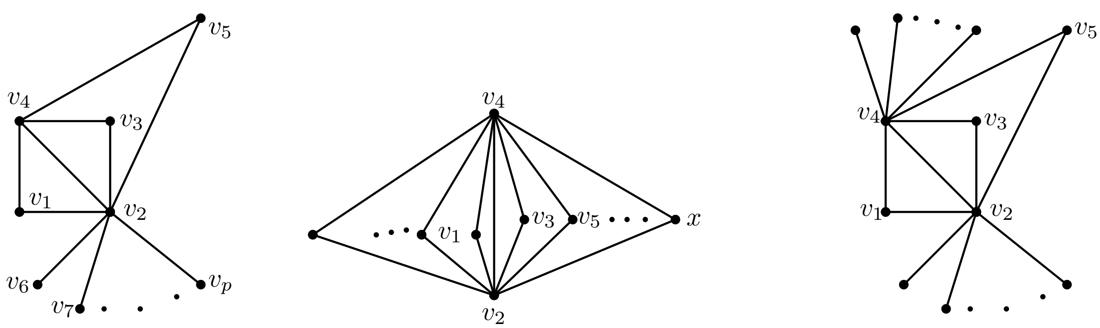


Figure 2.8: G

Since the edge v_2v_4 of G_i $(1 \leqslant i \leqslant 3)$ in Figure 2.8 does not lie on a detour joining any pair of vertices of G_i, these graphs are not edge detour graphs. Thus in this case there are no edge detour graphs G with H_2 as a proper subgraph satisfying the requirements of the theorem. Thus, if G does not contain K_4 as an induced subgraph, we have proved that G has a unique 4-cycle. Now we consider two subcases.

Subcase 1: The unique cycle C_4: v_1, v_2, v_3, v_4, v_1 contains exactly one chord v_2v_4. Since $p \geqslant 5$, $D = 4$ and G is connected, any vertex x not on C_4 is pendant and is adjacent to at least one vertex of C_4. The vertex x cannot be adjacent to both v_1 and v_3, for in this case, we get $c(G) = 5$, which is a contradiction. Suppose that x is adjacent to v_1 or v_3, say v_1. Also, if y is a vertex such that $y \neq x$, v_1, v_2, v_3, v_4, then y cannot be adjacent to v_2 or v_3 or v_4, for in each case $D \geqslant 5$, which is a contradiction. Hence y is a pendant vertex and cannot be adjacent to x or v_2 or v_3 or v_4 so that in this case the graph G reduces to the one given in Figure 2.9.

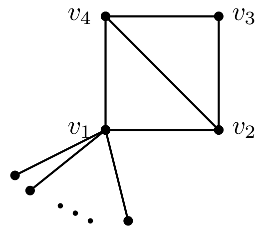


Figure 2.9: G

Since the set of all end vertices together with the vertex v_3 forms an edge detour basis for this graph G, it follows from Theorem 1.1 that $dn_1(G) = p - 3 \geq 2$. Hence $p = 5$ is the only possibility and the graph reduces to $G_{14} \in \mathscr{H}$ given in Figure 2.1 and satisfies the requirements of the theorem. Similarly, if x is adjacent to v_3, then also we get the graph $G_{14} \in \mathscr{H}$ given in Figure 2.1 and satisfies the requirements of the theorem.

Now, if x is adjacent to both v_2 and v_4, we get the graph H given in Figure 2.10 as a subgraph which is isomorphic to the graph H_2 given in Figure 2.4. Then, as in the first part of case 2, we see that there are no edge detour graphs which satisfy the requirements of the theorem.

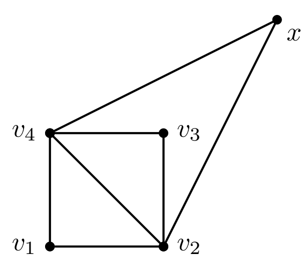


Figure 2.10: H

Thus x is adjacent to exactly one of v_2 or v_4, say v_2. Also, if y is a vertex such that $y \neq x$, v_1, v_2, v_3, v_4, then y cannot be adjacent to x or v_1 or v_3, for in each case $D \geqslant 5$, which is a contradiction. If y is adjacent to v_2 and v_4, then we get the graph H given in Figure 2.11 as a subgraph. Then exactly as in the first part of case 2 it can be seen that there are no graphs satisfying the requirements of the theorem.

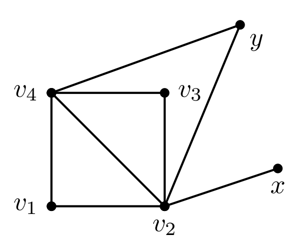


Figure 2.11: H

Thus y must be adjacent to v_2 or v_4 only. Hence we conclude that in either case the graph G

must reduce to the graph G_1 or G_2 as given in Figure 2.12. Similarly, if x is adjacent to v_4, then the graph G reduces to the graph G_1 or G_2 as given in Figure 2.12 and it is clear that $dn_1(G) = p - 2 \geq 3$, which is a contradiction.

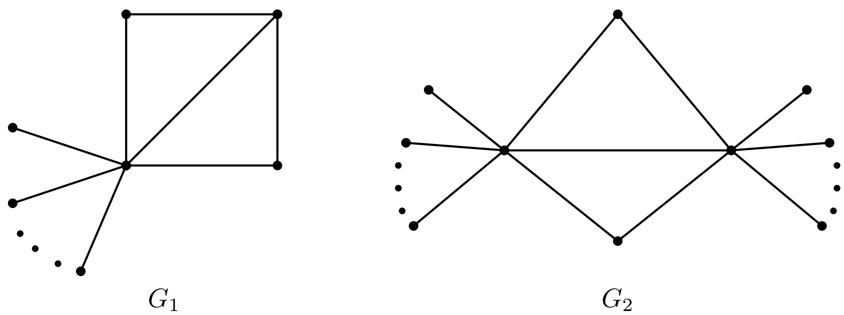


Figure 2.12: G

Thus, in this subcase 1, $G_{14} \in \mathscr{H}$ given in Figure 2.1 is the only graph satisfying the requirements of the theorem.

Subcase 2: The unique cycle C_4: v_1, v_2, v_3, v_4, v_1 has no chord. In this case we claim that G contains no triangle. Suppose that G contains a triangle C_3. If C_3 has no vertex in common with C_4 or exactly one vertex in common with C_4, we get a path of length at least 5 so that $D \geqslant 5$. If C_3 has exactly two vertices in common with C_4, we get a cycle of length 5. Thus, in all cases, we have a contradiction and hence it follows that G contains a unique chordless cycle C_4 with no triangles. Since $p \geqslant 5$, $D = 4$, $c(G) = 4$ and G is connected, any vertex x not on C_4 is pendant and is adjacent to exactly one vertex of C_4, say v_1. Also if y is a vertex such that $y \neq x$, v_1, v_2, v_3, v_4, then y cannot be adjacent to v_2 or v_4, for in this case, $D \geqslant 5$, which is a contradiction. Thus y must be adjacent to v_3 only. Hence we conclude that in either case G must reduce to the graphs H_1 or H_2 as given in Figure 2.13.

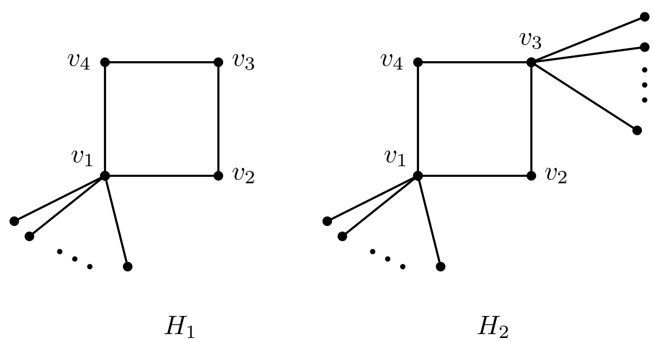


Figure 2.13: G

For these graphs H_1 and H_2 in Figure 2.13, it follows from Theorem 1.1 that $dn_1(H_1) = p - 3$ and $dn_1(H_2) = p - 4$. The only possible vaues are $p = 5$ for H_1 and $p = 6$ for H_2 so that H_1 reduces to $G_{15} \in \mathscr{H}$ and H_2 reduces to $G_{16} \in \mathscr{H}$ as given in Figure 2.1. Thus, in this subcase 2, G_{15}, $G_{16} \in \mathscr{H}$ as given in Figure 2.1 are the only graphs satisfying the requirements of the theorem. Thus, when $D = 4$ and $c(G) = 4$, the graphs satisfying the requirements of the theorem are G_{14}, G_{15}, $G_{16} \in \mathscr{H}$ as in Figure 2.1.

Case 3 Let $c(G) = 3$.

Case 3a G contains exactly one triangle C_3: v_1, v_2, v_3, v_1. Since $p \geqslant 5$, there are vertices not on C_3. If all the vertices of C_3 have degree three or more, then $p \geq 6$ and since $D = 4$, the graph G must reduce to the one given in Figure 2.14. It follows from Theorem 1.1 that $dn_1(G) = p - 3$. Since $p \geq 6$, this is a contradiction. Hence we conclude that at most two vertices of C_3 have degree $\geqslant 3$.

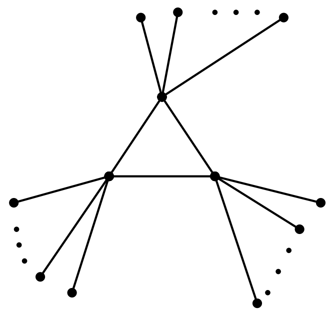


Figure 2.14: G

Subcase 1 Exactly two vertices of C_3 have degree 3 or more. Let $deg\ v_3 = 2$. Now, since $p \geqslant 5$, $D = 4$, $c(G) = 3$ and G is connected, we see that the graph reduces to the graph G given in Figure 2.15. For this graph G, it follows from Theorem 1.1 that $dn_1(G) = p - 2$. Since $p \geq 5$, this is a contradiction.

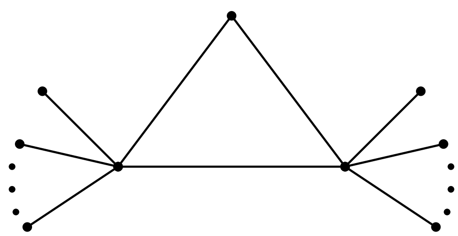


Figure 2.15: G

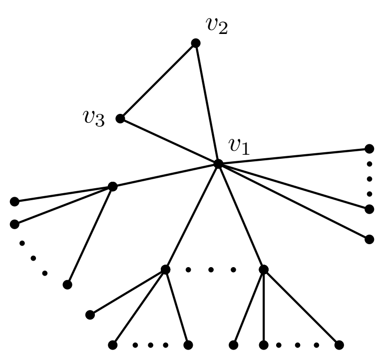


Figure 2.16: G

Subcase 2: Exactly one vertex v_1 of C_3 has degree 3 or more. Since G is connected, $p \geqslant 5$, $D = 4$ and $c(G) = 3$, the graph reduces to the one given in Figure 2.16. We claim that exactly one neighbor of v_1 other than v_2 and v_3 has degree ≥ 2. Otherwise, more than one neighbor of

v_1 other than v_2 and v_3 has degree $\geqslant 2$ so that $p \geq 7$ and set of all end-vertices together with v_2 and v_3 forms an edge detour set of G and so $dn_1(G) \geqslant 4$, which is a contradiction. Thus the graph reduces to the one given in Figure 2.17 and it is clear that $dn_1(G) = p - 2$. Since $p \geq 5$, this is a contradiction.

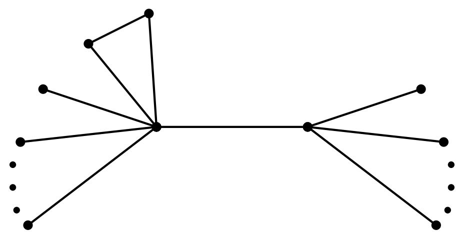


Figure 2.17: G

Case 3b: G contains more than one triangle. Since $D = 4$ and $c(G) = 3$, it is clear that all the triangles must have a vertex v in common. Now, if two triangles have two vertices in common then it is clear that $c(G) \geqslant 4$. Hence all triangles must have exactly one vertex in common. Since $p \geqslant 5$, $D = 4$, $c(G) = 3$ and G is connected, all the vertices of all the triangles are of degree 2 except v. Thus the graph reduces to the graphs given in Figure 2.18.

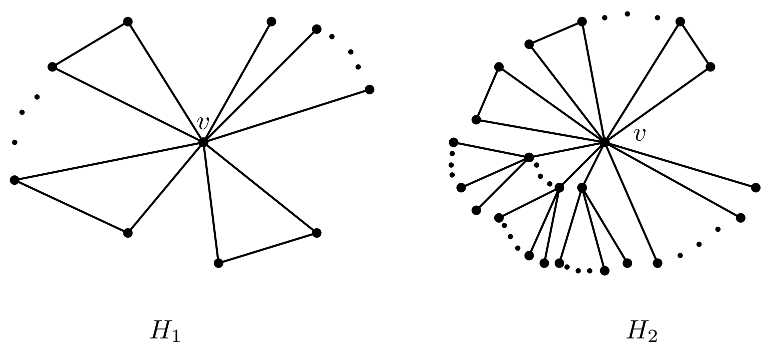


Figure 2.18: G

If $G = H_1$, then by Theorem 1.5, $dn_1(G) = p - 1$. Since $p \geq 5$, this is a contradiction. If $G = H_2$ and more than one neighbor of v not on the triangles has degree $\geqslant 2$, then $p \geq 9$ and the set of all end-vertices together with the all the vertices of all triangles except v forms an edge detour set of G. Hence $dn_1(G) \geqslant 6$, which is a contradiction.

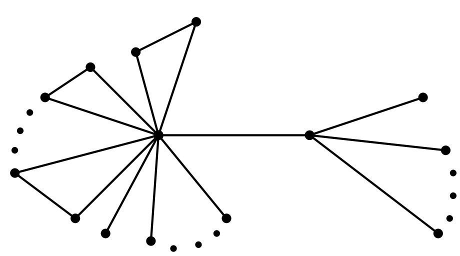


Figure 2.19: G

If $G = H_2$ and exactly one neighbor of v not on the triangles has degree $\geqslant 2$, then the graph reduces to the graph G given in Figure 2.19, and it is easy to verify that $dn_1(G) = p-2$. Since $p \geq 5$, this is a contradiction. Thus we see that when $D = 4$ and $c(G) = 3$, there are no graphs satisfying the requirements of the theorem. This completes the proof of the theorem.□

In view of Theorem 2.1, we leave the following problem as an open question.

Problem 2.2 *Characterize edge detour graphs G with detour diameter $D \geq 5$ for which $dn_1(G) = 2$.*

The following theorem is a characterization for trees.

Theorem 2.3 *For any tree T of order $p \geq 2$, $dn_1(G) = 2$ if and only if T is a path.*

Proof This follows from Theorem 2.1. □

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