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Theory of Relativity on the Finsler Spacetime

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Abstract: Einstein's theory of special relativity and the principle of causality imply that the speed of any moving object cannot exceed that of light in a vacuum (c). Nevertheless, there exist various proposals for observing faster-than-c propagation of light pulses, using anomalous dispersion near an absorption line, nonlinear and linear gain lines, or tunnelling barriers. However, in all previous experimental demonstrations, the light pulses experienced either very large absorption or severe reshaping, resulting in controversies over the interpretation. Recently, L.J.Wang, A.Kuzmich and A.Dogariu use gain-assisted linear anomalous dispersion to demonstrate superluminal light propagation in atomic caesium gas. The group velocity of a laser pulse in this region exceeds c and can even become negative, while the shape of the pulse is preserved. The textbooks say nothing can travel faster than light, not even light itself. New experiments show that this is no longer true, raising questions about the maximum speed at which we can send information. On the other hand, the light speed reduction to 17 meters per second in an ultracold atomic gas. This shows that the light speed could taken on voluntariness numerical value, This paper shows that if ones think of the possibility of the existence of the superluminal-speeds (the speeds faster than that of light) and redescribe the special theory of relativity following Einstein's way, it could be supposed that the physical spacetime is a Finsler spacetime, characterized by the metric

$$ds^4 = g_{ijkl}dx^i dx^j dx^k dx^l.$$

If so, a new spacetime transformation could be found by invariant ds^4 and the theory of relativity is discussed on this transformation. It is possible that the Finsler spacetime $F(x, y)$ may be endowed with a catastrophic nature. Based on the different properties between the $ds^2$and ds^4, it is discussed that the flat spacetime will also have the catastrophe nature on the Finsler metric ds^4. The spacetime transformations and the physical quantities will suddenly change at the catastrophe set of the spacetime, the light cone. It will be supposed that only the dual velocities of the superluminal-speeds could be observed. If so, a particle with the superluminal-speeds $v > c$ could be regarded as its anti-particle with the dual velocity $v_1 = c^2/v < c$. On the other hand, it could be assumed that the horizon of the field of the general relativity is also a catastrophic set. If so, a particle with the superluminal-speeds could be projected near the horizon of these fields, and the particle will move on the spacelike curves. It is very interesting that, in the Schwarzschild fields, the theoretical calculation for

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the spacelike curves should be in agreement with the data of the superluminal expansion of extragalactic radio sources observed year after year.(see Cao,1992b)
The catastrophe of spacetime has some deep cosmological means. According to the some interested subjects in the process of evolution of the universe the catastrophe nature of the Finsler spacetime and its cosmological implications are discussed. It is shown that the nature of the universal evolution could be attributed to the geometric features of the Finsler spacetime (see Cao,1993).

Key words: Spacetime, catastrophe, Finsler metric, Finsler spacetime, speed faster than light.

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It is known that in his first paper on the special theory of relativity: "On the electrodynamics of moving bodies", Einstein clearly states (cf. Einstein, 1923) that 'Velocities greater than that of light have, no possibility of existence.' But he neglected to point out the applicable range of Lorentz transformation. In fact, his whole description must be based on velocities smaller than that of light which we call subluminal-speed. So, the special theory of relativity cannot negate that real motion at a speed greater than the speed of light in vacuum which we call superluminal-speed could exist. In this paper, it is shown that if we think of the possibility of existence of the superluminal-speed and redescribe the special theory of relativity following Einstein's way, a new theory would be founded on the Finsler spacetime. The new theory would retain all meaning of the special theory of relativity when matters move with subluminal-speed and would give new content when matters move with superluminal-speed. If we assume that the superluminal-speed will accord with the spacelike curves in the general theory of relativity, calculations indicate that the superluminal expansion of extragalactic radio sources exactly corresponds with the spacelike curves of the Schwarzschild geometry.

Our discussion is still based on the principle of relativity and on the principle of constancy of the velocity of light which have been defined by Einstein as follows:

(1)The laws by which the states of physical systems undergo change are not affected, whether these changes of state be referred to the one or the other of two systems of coordinates in uniform translatory motion (see Einstein, 1923;p.41).

(2)Any ray of light moves in the 'stationary' system of coordinates with the determined velocity c, whether the ray be emitted by stationary or by a moving body.

Note that these two postulates do not impose any constraint on the relative speed v of the two inertial observers.

§1 The General Theory of the Transformation of Spacetime

1.1 Definition of simultaneity and temporal order

In his description about definition of simultaneity, Einstein stated: "Let us take a system of coordinates in which the equations of Newtonian mechanics hold good", $\cdots$, "Let a ray of light start at the 'A time' t_A from A towards B, let it at the B time' t_B be reflected at B in the direction of A, and arrive again at A at the 'A time' t'_A." In accordance with definition, the two clocks synchronize if (see Einstein, 1923; p.40)

$$t_B - t_A = t'_A - t_B. \qquad (1.1)$$

"In agreement with experience we further assume the quantity

$$\frac{2AB}{t_B - t_A} = c, \qquad (1.2)$$

to be a universal constant - the velocity of light in empty space."

"It is essential to have time defined by means of stationary clocks in the stationary system, and the time now defined being appropriate to the stationary system we call it 'the time of the stationary system'." In this way, Einstein finished his definition of simultaneity. But he did not consider the applicable condition of this definition, still less the temporal order and as it appears to me these discussions are essential too. Let us continue these discussions following Einstein's way.

First and foremost, let us assume if the point B is moving with velocity v relative to the point A, in agreement with experience we must use the following equations instead of Equation:

$$\frac{2AB}{t_A - t_B} = \begin{cases} c - v, \quad when \quad B \quad is \quad leaving \quad A \quad (a) \\ c + v, \quad when \quad B \quad is \quad approaching \quad A \quad (b) \end{cases} \qquad (1.3)$$

Obviously, Equation (1.3a) is not always applicable, it must require v<c, but Equation (1.3b) is always applicable-i.e., for $v < c$ and $v > c$ Einstein's whole discussion is based on the following formulae:

$$t_B - t_A = \frac{r_{AB}}{c - v} \text{ and } t'_A - t_B = \frac{r_{AB}}{c + v}. \qquad (1.4)$$

It must require $v < c$, because t_B t_A must be larger than zero. Particularly, in order to get the Lorentz transformation, Einstein was based on the following formula (see Einstein, 1923; p.44)

$$\frac{1}{2}[\tau(0,0,0,t) + \tau(0,0,0,t + \tfrac{x'}{c-v} + \tfrac{x'}{c+v})] = \tau(x',0,0,t + \tfrac{x'}{c-v}), \qquad (1.5)$$

where $\frac{x'}{c-v}$ is just t_B t_A, so must require $v < c$, i.e., B must be the motion with the subluminal-speed. Then the Lorentz transformation only could be applied to the motion with subluminal-speed. It could not presage anything about the motion with the superluminal-speed, i.e., the special theory of relativity could not negate that the superluminal-speed would exist.

In order for our discussion to be applied to the motion with the superluminal-speed, we will only use Equation (1.3b), i.e., let the point B approach A. Now, let another ray of light (it must be distinguished from the first) start at the 'A time' t_{A1} from A towards B (when B will be at a new place B_1) let it at the 'B time' t_{B1} be reflected at B in the direction of A, and arrive again at A at the 'A time' t_{A1}.

According to the principle of relativity and the principle of the constancy of the velocity of light, we obtain the following formulas:

$$\frac{1}{2}(t'_A - t_A) = t_B - t_A = \frac{AB}{c+v}, \qquad (1.6)$$

$$\frac{1}{2}(t'_{A1} - t_{A1}) = t_{A1} - t_{B1} = \frac{AB_1}{c+v}, \qquad (1.7)$$

$$AB - AB_1 = v(t_{A1} - t_A). \qquad (1.8)$$

Let

$$\Delta t_A = t_{A1} - t_A, \Delta t_B = t_{B1} - t_B \quad \text{and} \quad \Delta t'_A = t'_{A1} - t'_A, \qquad (1.9)$$

where $\Delta t_A, \Delta t_B$, and $\Delta t'_A$ represent the temporal intervals of the emission from A, the reflection from B, and arrival at A for two rays of light, respectively. The symbols of the temporal intervals describe the temporal orders. When $\Delta t > 0$ it will be called the forward order and when $\Delta t < 0$, the backward order.

From Equations (1.6)-(1.9) we can get

$$\Delta t_B = \frac{c}{c+v}\Delta t_A, \qquad (1.10)$$

and

$$\Delta t'_A = \frac{c-v}{c+v}\Delta t_A. \qquad (1.11)$$

Then we assume that, if $\Delta t_A > 0$, i.e., two rays of light were emitted from A, successively we must have $\Delta t_B > 0$ i.e., for the observer at system A these two rays of light were reflected by the forward order from B. But

$$\Delta t'_A \geq 0, \text{ if and only if } v \leq c$$

and

$$\Delta t'_A < 0, \text{ if and only if } v > c.$$

It means that for the observer at system A these two rays of light arrived at A by the forward order only when the point B moves with subluminal-speed, and by the backward order only when with superluminal-speed. In other words, the temporal order is not always constant. It is constant only when v<c, and it is not constant when v>c.

Usually, one thinks that this is a backward flow of time. In fact, it is only a procedure of time in the system B with the superluminal-speed which gives the observer in the ‘stationary system’ A an inverse appearance of the procedure of the time. It is an inevitable outcome when the velocity of the moving body is faster than the transmission velocity of the signal. This outcome will be called the relativity of the temporal order. It is a new nature of the time when the moving body attains the supeluminal-speed. It is known that it is not spacetime that impresses its form on things, but the things and their physical laws that determine spacetime. So, the superluminal-speed need not be negated by the character of the spacetime of the special theory of relativity, but will represent the new nature of the spacetime, the relativity of the temporal order.

1.2 The temporal order and the chain of causation

In order to explain the disparity between the backward flow of time and the relativity of the temporal order, we will use spacetime figure (as Fig.1-1)

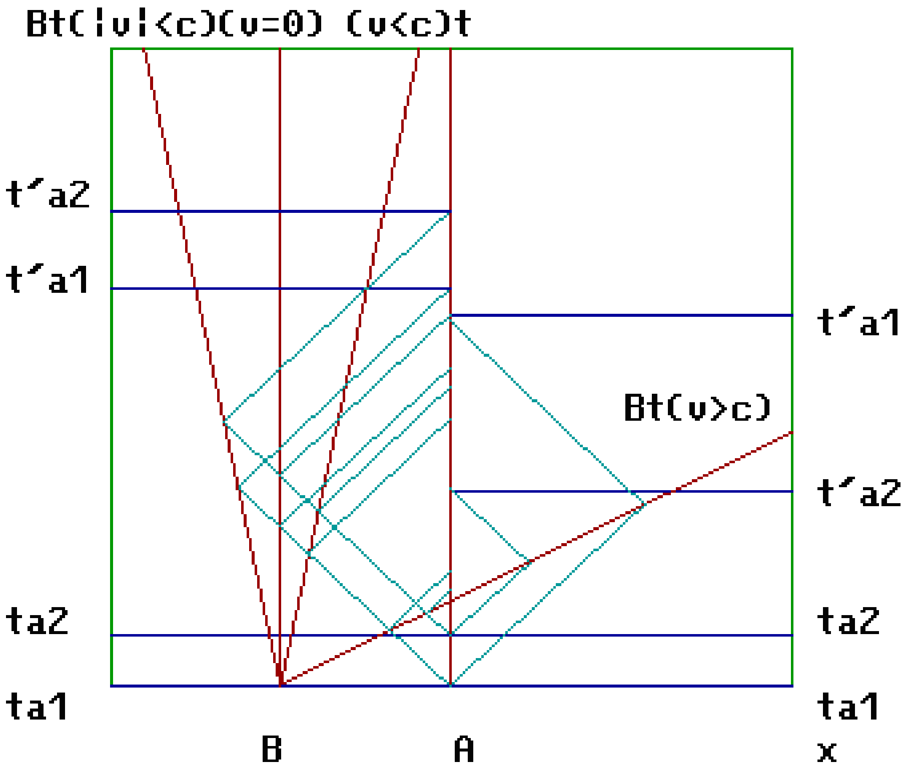

Fig.1-1. The spacetime figure

and take following definitions.

(1)The chain of the event, t_{A0},t_{A1},...,t_{Ai},.... The ith ray of light will be started at t_{Ai} and $\Delta t_{Ai} = t_{A(i+1)}$ t_{Ai} >0 It may or may not be chain of causality.

(2)The chains of the transference of the light t_{A0}, t_{B0}, t'_{A0}; t_{A1}, t_{B1}, t'_{A1};.... Every chain t_{Ai}, t_{Bi}, t'_{Ai} must be a chain of causality -i.e.

$$\frac{1}{2}(t'_{Ai} - t_{Ai}) = t_{Bi} - t_{Ai} = t'_{Ai} - t_{Bi} > 0. \qquad (1.12)$$

If they take a negative sign it will be the backward flow of time and will violate the principle of causality.

(3)The chains of the motion are the rays of the light, which will be reflected at B, but it will have different features when B moves with different velocity. Let us assume that:

(a) v >0 when B is approaching A;
(b) v <0 when B is leaving A;
(c) c >0 when the ray of light from A backwards B;
(d) c <0 when the ray of light from A towards B.

So, if v=0, we must have c <0. Then

$$t_{A(i+1)} - t_{Ai} = t_{B(i+1)} - t_{Bi} = t'_{A(i+1)} - t'_{Ai}. \qquad (1.13)$$

If $v < c$, we must have c <0 and when v >0,

$$t_{A(i+1)} - t_{Ai} > t_{B(i+1)} - t_{Bi} > t'_{A(i+1)} - t'_{Ai} > 0. \qquad (1.14)$$

But when $v < 0$,

$$0 < t_{A(i+1)} - t_{Ai} < t_{B(i+1)} - t_{Bi} < t'_{A(i+1)} - t'_{Ai}. \qquad (1.15)$$

Last of all, if $v > c$, must have v >0; and when c <0,

$$t_{A(i+1)} - t_{Ai} > t_{B(i+1)} - t_{Bi} > |t'_{A(i+1)} t'_{Ai}| > 0. \qquad (1.16)$$

But

$$t'_{A(i+1)} - t'_{Ai} < 0. \qquad (1.17)$$

When c >0,

$$0 < t_{A(i+1)} - t_{Ai} < |t_{B(i+1)} - t_{Bi}| < |t'_{A(i+1)} - t'_{Ai}| \qquad (1.18)$$

and

$$t_{B(i+1)} - t_{Bi} < 0 \quad and \quad t'_{A(i+1)} - t_{Ai} < 0. \qquad (1.19)$$

These are rigid relations of causality.

4.The chains of the observation t'_{A0},t'_{A1},...,t'_{Ai},... and t_{B0},t_{B1},...,t_{Bi},... are not chains of causality. The relativity of temporal order is just that they could be a positive when $v < c$ or a negative when $v > c$ and the vector v and c have the same direction.

In (1.4) when $v > c$, $t_B \;\; t_A$ <0 it does not mean that velocities greater than that of light have no possibility of existence but only that the ray of light cannot catch up with the body with superluminal-speed.

1.3 Theory of the transformation of coordinates

From equations (1.10) and (1.11) we can get

$$\Delta t_B = \frac{c}{c+v} \Delta t_A \qquad (1.20)$$

and

$$\Delta t_B = \frac{c}{c-v}\Delta t'_A. \quad quad \qquad (1.21)$$

It has been pointed out that Δt_A and $\Delta t'_A$ are measurable by observer of the system A, but Δt_B is unmeasurable. Accordingly, the observer must conjecture Δt_B from Δt_A or $\Delta t'_A$. In form, Δt_B in Equation (1.20) and Δt_B in (1.21) are different. If we can find a transformation of coordinates it will satisfy following equation:

$$\Delta\tau^2 = \Delta t_A \cdot \Delta t'_A \qquad (1.22)$$

and, according to Equations (1.10) and (1.11), could get

$$\Delta\tau^2 = \begin{cases} >0, & iff \quad v < c, \\ =0, & iff \quad v = c, \\ <0, & iff \quad v > c. \end{cases} \qquad (1.23)$$

Then, we get

$$\Delta t_B^2 = \frac{c^2}{c^2 - v^2}\Delta\tau^2$$

or

$$dt^2 = \frac{c^2}{c^2 - v^2}d\tau^2. \qquad (1.24)$$

Let $ds^2 = c^2 d\tau^2$. We get

$$ds^2 = c^2 d\tau^2 = (c^2 \ v^2)dt^2. \qquad (1.25)$$

So

$$ds^2 = \begin{cases} >0, & v < c \quad timelike, \\ =0, & v = c \quad lightlike, \\ <0, & v > c \quad spacelike. \end{cases} \qquad (1.26)$$

What merits special attention is that $ds^2 = (c^2 - v^2)dt^2$ and $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$ are not identical. Usually, the special theory of relativity does not recognize their difference because motion with subluminal-speed does not involve the relative change of temporal orders, so the symbol of ds^2 remains unchanged when the inertial system changes.

Now let

$$ds^2 = ds_v^2 + ds_0^2, \qquad (1.27)$$

where

$$ds_v^2 = (c^2 - v^2)dt^2, \qquad (1.28)$$

$$ds_0^2 = dx^2 + dy^2 + dz^2, \qquad (1.29)$$

then

$$ds^2 = \begin{cases} +ds_v^2 + ds_0^2, & v < c, \\ -ds_v^2 + ds_0^2, & v > c. \end{cases} \tag{1.30}$$

Between any two inertial systems

$$ds_v^2 + ds_0^2 = \begin{cases} +ds_v^2 + ds_0^2, & v < c, \\ -ds_v^2 + ds_0^2, & v > c. \end{cases} \tag{1.31}$$

According to classical mechanics, we can determine the state of a system with n degrees of freedom at time t by measuring the 2n position and momentum coordinates $q^i(t)$, $p_i(t)$, i=1,2,...,n. These quantities are commutative each other, i.e., $q^i(t) \quad p_j(t) = p_j \ (t)q^i(t)$. But, in quantum mechanics the situation is entirely different. The operators Q_{op} and P_{op} corresponding to the classical observable position vector q and momentum vector p. These operators are non-commutative each other, i.e.,

$$QP \neq PQ.$$

So, ones doubt whether the quantum mechanics is not a good theory at first. But, ones discover that the non-commutability of operators is closely related to the uncertainty principle, it is just an essential distinction between the classical and quantum mechanics.

So, I doubt that whether the non-positive definite metrics ds^2 is just the best essential nature in the relativity theory? But, it was cast aside in Einstein's theory. Now, we could assume that

$$ds^4 = ds_v^4 + ds_0^4. \tag{1.32}$$

In general, we could let

$$ds^4 = g_{ijkl} dx^i dx^j dx^k dx^l, \qquad i, j, k, l = 0, 1, 2, 3. \tag{1.33}$$

Equations (1.32) and (1.33) which are defined as a Finsler metric are the base of the spacetime transformations. From the physical point of view this means that a new symmetry between the timelike and the spacelike could exist.

In his memoir of 1854, Riemann discusses various possibilities by means of which an n-dimensional manifold may be endowed with a metric, and pays particular attention to a metric defined by the positive square root of positive definite quadratic differential form. Thus the foundations of Riemannian geometry are laid; nevertheless, it is also suggested that the positive fourth root of a fourth-order differential form might serve as metric function (see Rund, 1959; Introduction X).

In his book of 1977, Wolfgang Rindler stated: "Whenever the squared differential distance $d\sigma^2$ is given by a homogeneous quadratic differential form in the surface coordinates, as in (7.10), we say that $d\sigma^2$ is a Riemannian metric, and that the corresponding surface is Riemannian. It is, of course, not a foregone conclusion that all metrics must be of this form: one could define, for example, a non-Riemannian metric $d\sigma^2 = \sqrt{dx^4 + dy^4}$ for some two-dimensional space, and investigate the resulting geometry.(Such more general metrics give rise to 'Finsler' geometry.)" (see W. Rindler,1997).

§2 The Special Theory of Relativity on the Finsler Spacetime ds^4

2.1 Spacetime transformation group on the Finsler metric ds^4

If $v = v_x$, then, between any two inertial systems we have

$$\begin{aligned} &c^4dt^4 + dx^4 - 2c^2dt^2dx^2 + dy^4 + dz^4 + 2dy^2dz^2 \\ &\quad = c^4dt'^4 + dx'^4 - 2c^2dt'^2 + dy'^4 + dz'^4 + 2dy'^2dz'^2 \end{aligned} \tag{2.1}$$

From (2.1) we could get transformations

$$t = \frac{t' + \frac{v}{c^2}x'}{\sqrt[4]{1 - 2\beta^2 + \beta^4}}, \qquad x = \frac{x' + vt'}{\sqrt[4]{1-2\beta^2+\beta^4}}, \qquad y = y', \qquad z = z'. \tag{2.2}$$

These transformations are called spacetime transformations. All spacetime transformations form into a group, called the spacetime transformation group (The Lorentz transformations group is only subgroup of the spacetime transformation group). The inverse transformations are of the form

$$\pm t' = \frac{t - \beta\frac{x}{c}}{\sqrt[4]{1 - 2\beta^2 + \beta^4}}, \qquad \pm x' = \frac{x - vt}{\sqrt[4]{1-2\beta^2+\beta^4}}, \qquad y' = y, \qquad z' = z, \tag{2.3}$$

where $\beta = \frac{v}{c}$. We could also use dual velocity $v_1 = \frac{c^2}{v}$ to represent the spacetime transformations. In fact, the transformations (2.2) can be rewritten as

$$t = \frac{\beta_1 t' + \frac{x'}{c}}{\sqrt[4]{1 - 2\beta_1^2 + \beta_1^4}}, \qquad x = \frac{\beta_1 x' + ct'}{\sqrt[4]{1-2\beta_1^2+\beta_1^4}}, \qquad y = y', \qquad z = z'. \tag{2.4}$$

Their inverse transformations are of the form

$$\pm t' = \frac{\beta_1 t - \frac{x}{c}}{\sqrt[4]{1 - 2\beta_1^2 + \beta_1^4}}, \qquad \pm x' = \frac{\beta_1 x - ct}{\sqrt[4]{1-2\beta_1^2+\beta_1^4}}, \qquad y' = y, \qquad z' = z. \tag{2.5}$$

where $\beta_1 = \frac{v_1}{c} = \frac{c}{v} = \frac{1}{\beta}$.

It is very interesting that all spacetime transformations are applicable to both the subluminal-speed (i.e.,β<1 or β_1 >1) and the superluminal-speed (i.e.,β>1 or β_1 <1). Whether the velocity is superluminal- or subluminal-speed, it is characterized by minus or plus sign of their inverse transformations, respectively.

Lastly, all spacetime transformations have the same singularity as the Lorentz transformation when the $\beta = \beta_1 = 1$.

2.2 Kinematics on the ds^4 invariant

We shall now consider the question of the measurement of length and time increment. In order to find out the length of a moving body, we must simultaneously plot the coordinates of its ends in a fixed system. From Equation (2.2) and (2.4), an expression for the length of a moving scale $\Delta x'$ measured by a fixed observer follows as

$$\pm\Delta x' = \Delta x \sqrt[4]{1 - 2\beta^2 + \beta^4}, \qquad (2.6)$$

and

$$\pm\Delta x' = c\Delta t \sqrt[4]{1 - 2\beta_1^2 + \beta_1^4}, \qquad (2.7)$$

Einstein stated: "For $v = c$ all moving objects - viewed from the 'stationary' system - shrivel up into plain figures. For velocities greater than that of light our deliberations become meaningless." However, formula (2.6) can applied to the case for velocities greater than that of light. Fig.2.1 give the relation between the length of a moving scale L and the velocity.

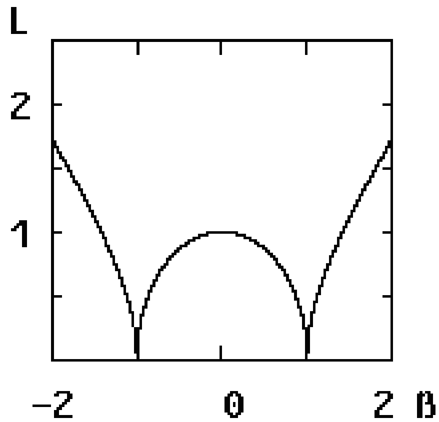

Fig.2.1. L-β curve

Let Δt be the time increment when the clock is at rest with respect to the stationary system, and $\Delta\tau$ be the time increment when the clock is at rest with respect to the moving system. Then

$$\pm\Delta\tau = \Delta t \sqrt[4]{1 - 2\beta^2 + \beta^4} \qquad (2.8)$$

and

$$\pm\Delta\tau = \frac{\Delta x}{c} \sqrt[4]{1 - 2\beta_1^2 + \beta_1^4}, \qquad (2.9)$$

Differentiating (2.3) or (2.5) and dividing dx' by dt' we obtain

$$\frac{dx'}{dt'} = v'_x = \frac{dx/dt - v}{1 - v/c^2 \, dx/dt} = \frac{v_x - v}{1 - vv_x/c^2}, \qquad (2.10)$$

Noting that $dy' = dy$, $dz' = dz$, we have a transformation of the velocity components perpendicular to v:

$$\frac{dy'}{dt'} = v'_y = \frac{v_y \sqrt[4]{1-2\beta^2+\beta^4}}{1-vv_x/c^2}, \frac{dz'}{dt'} = v'_z = \frac{v_z \sqrt[4]{1-2\beta^2+\beta^4}}{1-vv_x/c^2}, \quad (2.11)$$

where

$$v^2 = v_x^2 + v_y^2 + v_z^2. \quad (2.12)$$

From Equation (2.8), we could see that the composition of velocities have four physical implications: i.e.,

(1)A subluminal-speed and another subluminal-speed will be a subluminal-speed.

(2)A superluminal-speed and a subluminal-speed will be a superluminal-speed.

(3)The composition of two superluminal-speeds is a subluminal-speed.

(4)The composition of light-speed with any other speed (subluminal-,light-, or superluminal-speed) still is the light-speed.

There are the essential nature of the spacetime transformation group. The usual Lorentz transformation is a only subgroup of the spacetime transformation group.

It is necessary to point out that if $1 - vv_x/c^2 = 0$, i.e.,

$$v_x = v/c^2, \quad (2.13)$$

then $v_x \to \infty$. It implies that if two velocities are dual to each other and in opposite directions, then their composition velocity is an infinitely great velocity. We guess that it may well become an effective way to make an appraisal of a particle with the superluminal-speed.

2.3 Dynamics on the ds^4 invariant

The Lagrangian for a free particle with mass m is

$$L = -mc^2 \sqrt[4]{1-2\beta^2+\beta^4}, \quad (2.14)$$

The momentum energy, and mass of motion of the particle are of the forms:

$$p = \frac{mv}{\sqrt[4]{1-2\beta^2+\beta^4}}, E = \frac{mc^2}{\sqrt[4]{1-2\beta^2+\beta^4}}, M = \frac{m}{\sqrt[4]{1-2\beta^2+\beta^4}}. \quad (2.15)$$

Those could also be represented by dual velocity v_1:

$$p(v) = \frac{mv}{\sqrt[4]{1-2\beta^2+\beta^4}} = \frac{mc}{\sqrt[4]{1-2\beta_1^2+\beta_1^4}} = \frac{1}{c}E(v_1), \quad (2.16)$$

$$E(v) = \frac{mc^2}{\sqrt[4]{1-2\beta^2+\beta^4}} = \frac{mv_1 c}{\sqrt[4]{1-2\beta_1^2+\beta_1^4}} = cp(v_1), \quad (2.17)$$

$$M(v) = \frac{m}{\sqrt[4]{1-2\beta^2+\beta^4}} = \frac{\beta_1 m}{\sqrt[4]{1-2\beta_1^2+\beta_1^4}} = \beta_1 M(v_1). \quad (2.18)$$

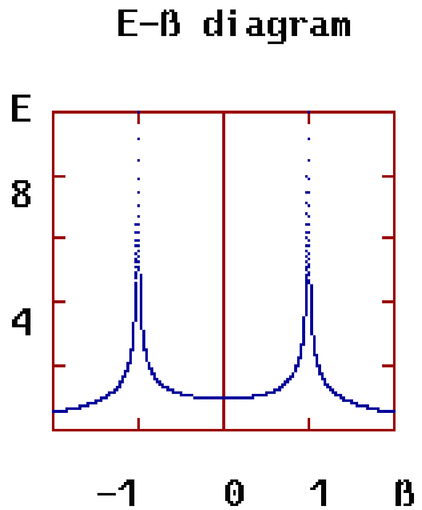

Fig.2.2. E-β diagram

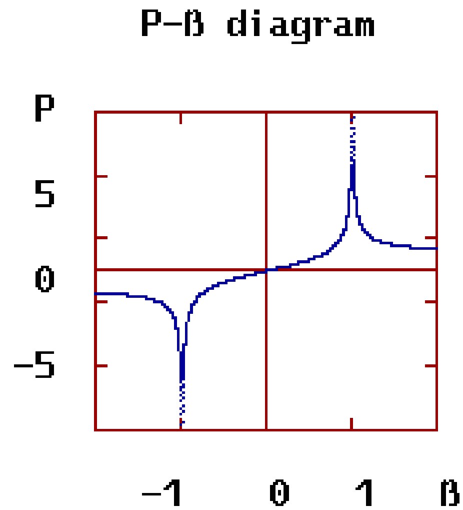

Fig. 2.3. p-β diagram

Einstein stated: “Thus, when $v = c$, E becomes infinite, velocities greater than that of light have - as in our previous results - no possibility of existence.” But, formula (2.7) can also applied to the case for velocities greater than that of light. Fig.2.2 give the relation between the energy of a moving particle and its velocity, and Fig.2.3 give the relation between the momentum of a moving particle and its velocity.

It is very interesting that the momentum (or energy) in the v's representation will change into the energy (or momentum) in the v_1's representation. From (2.15) (or (2.16) and (2.17)), we could get the following relation between the momentum and energy of a free material particle:

$$p(v) = \frac{v}{c^2}E(v) \quad or \quad p(v_1) = \frac{v_1}{c^2}E(v_1), \qquad (2.19)$$

where the relation (2.19) keeps up the same form as the special theory of relativity. But a new invariant will be obtained as

$$E^4 + c^4p^4 - 2c^2p^2E^2 = m^4c^8. \qquad (2.20)$$

The relation (2.20) is correct for both of the v's and the v_1's representations. It is a new relation on the ds^4 invariant.

2.4 A charged particle in an electromagnetic field on the Finsler spacetine ds^4

Let us now turn to the equations of motion for a charged particle in an electromagnetic field, A,Φ, E_e andH_e. Their Lagrangian is

$$L = -mc^2\sqrt[4]{1 - 2\beta^2 + \beta^4} + \frac{e}{c}Av - e\Phi. \qquad (2.21)$$

The derivative $\partial L/\partial v$ is the generalized momentum of the particle. We denote it by p_e

$$p_e = mv\sqrt[4]{1 - 2\beta^2 + \beta^4} + \frac{e}{c}A = p + \frac{e}{c}A. \qquad (2.22)$$

where p denotes momentum in the absence of a field.

From the Lagrangian we could find the Hamiltonian function for a particle in a field from the general formula

$$H = mc^2 \sqrt[4]{1 - 2\beta^2 + \beta^4} + e\Phi. \qquad (2.23)$$

However, the Hamiltonian must be expressed not in terms of the velocity, but rather in terms of the generalized momentum of the particle. From equations (2.2) and (2.3), we can get the relation

$$[(\frac{H - e\Phi}{c})^2 - (p - \frac{e}{c}A)^2]^2 = m^4c^4. \qquad (2.24)$$

Now we write the Hamilton-Jacobi equation for a particle in an electromagnetic field in the Finsler spacetime. It is obtained by replacing, in the equation for the Hamiltonian, P by $\partial S/\partial r$, and H by $-\partial S/\partial t$. Thus we get from (2.24)

$$[(\nabla S - \frac{e}{c}A)^2 - \frac{1}{c^2}(\frac{\partial S}{\partial t} + e\Phi)^2]^2 - m^4c^4 = 0. \qquad (2.25)$$

Now we consider the equation of motion of a charge in an electromagnetic field. It could be written by Lagrangian (2.21) as

$$\frac{d}{dt}\frac{mv}{\sqrt[4]{1 - 2\beta^2 + \beta^4}} = eE_e + \frac{e}{c}v \times H_e. \qquad (2.26)$$

where

$$E_e = -\frac{1}{c}\frac{\partial A}{\partial t} - grad\Phi, \qquad H_e = curlA. \qquad (2.27)$$

It is easy to check the $dE_e = vdP$, i.e.,

$$v\frac{d}{dt}\frac{mv}{\sqrt[4]{1 - 2\beta^2 + \beta^4}} = mc^2\frac{d}{dt}\frac{1}{\sqrt[4]{1 - 2\beta^2 + \beta^4}}. \qquad (2.28)$$

Then from (2.26) we have

$$\frac{dE}{dt} = eE_e v. \qquad (2.29)$$

Integrate (2.29) and get

$$\frac{mc^2}{\sqrt[4]{1 - 2\beta^2 + \beta^4}} - \frac{mc^2}{\sqrt[4]{1 - 2\beta_0^2 + \beta_0^4}} = eU. \qquad (2.30)$$

where

$$\beta_0 = \frac{v_0}{c}, \qquad U = \int_{r_0}^{r} E_e dr. \qquad (2.31)$$

From (2.26) and (2.29), if we write it in terms of components, it is easy to obtain the spacetime transformation equations for the field components, and we could obtain the field transformation equation

$$\begin{cases} H'_x = H_x, & E'_x = E_x, \\ H'_y = \frac{H_y + \beta E_z}{\sqrt[4]{1-2\beta^2+\beta^4}}, & E'_y = \frac{E_y - \beta H_z}{\sqrt[4]{1-2\beta^2+\beta^4}}, \\ H'_z = \frac{H_z - \beta E_y}{\sqrt[4]{1-2\beta^2+\beta^4}}, & E'_z = \frac{E_z + \beta H_y}{\sqrt[4]{1-2\beta^2+\beta^4}}. \end{cases} \tag{2.32}$$

We could also use dual velocity v_1 to represent the field transformation equation

$$\begin{cases} H'_x = H_x, & E'_x = E_x, \\ H'_y = \frac{\beta_1 H_y + E_z}{\sqrt[4]{1-2\beta_1^2+\beta_1^4}}, & E'_y = \frac{\beta_1 E_y - H_z}{\sqrt[4]{1-2\beta_1^2+\beta_1^4}}, \\ H'_z = \frac{\beta_1 H_z - E_y}{\sqrt[4]{1-2\beta_1^2+\beta_1^4}}, & E'_z = \frac{\beta_1 E_z + H_y}{\sqrt[4]{1-2\beta_1^2+\beta_1^4}}. \end{cases} \tag{2.33}$$

An invariant will be obtained as

$H_e^4 + E_e^4 - 2H_e^2E_e^2$ =constant,

of new nature for the electromagnetic field in Finsler spacetime.

§3 The Catastrophe of the Spacetime and Its Physical Meaning

3.1 Catastrophe of the spacetime on the Finsler metric ds^4

The functions $y = x^2$ and $y = x^4$ are topologically equivalent in the theory of the singularities of differentiable maps (see Arnold et al.,1985). But the germ $y = x^2$ is topologically (and even differentially) stable at zero. the germ $y = x^4$ is differentially (and even topologically) unstable at zero. So, there is a great difference between the theories of relativity on the ds^2 and the ds^4.

On the other hand, a great many of the most interesting macroscopic phenomena in nature involve discontinuities. The Newtonian theory and Einstein's relativity theory only consider smooth, continuous processes. The catastrophe theory, however, provides a universal method for the study of all jump transitions, discontinuities and sudden qualitative changes. The catastrophe theory is a program. The object of this program is to determine the change in the solutions to families of equations when the parameters that appear in these equations change.

In general, a small change in parameter values only has a small quantitative effect on the solutions of these equations. However, under certain conditions a small change in the value of some parameters has a very large quantitative effect on the solutions of these equations. Large quantitative changes in solutions describe qualitative changes in the behaviour of the system modeled.

Catastrophe theory is, therefore, concerned with determining the parameter values at which there occur qualitative changes in solutions of families of equations described by parameters.

The double-cusp is the simplest non-simple in the sense of Arnold (see Arnold et al.,1985), but the double-cusp is unimodal.

The double-cusp is compact, in the sense that the sets f≤constant are compact. In Arnold's notation, the double-cusp belongs to the family X9 and in that family there are three real types of germ, according as to whether the germ has 0,2, or 4 real roots. For example representatives of the three types are: type $1x^4 + y^4$, type $2x^4 \;\; y^4$, type $3x^4 + y^4 \;\; 2\delta x^2y^2$,respectively, and only the type 1 is compact.

Compact germs play an important role in application (see Zeeman, 1977), because any perturbation of a compact germ has a minimum; therefore if minima represent the stable equilibria of some system, then for each point of the unfolding space there exists a stable state of the system.

3.2 Catastrophe of the spacetime on the Finsler metric ds^4

In accordance with the Finsler metric ds^4 of the spacetime, we could

$$f(T,X,Y,Z) = T^4 + X^4 + Y^4 + Z^4 - 2T^2X^2 + 2Y^2Z^2, \tag{3.1}$$

here T=ct. Equation (3.1) that describes the behaviour of the spacetime is a smooth function.

As the catastrophe theory, first we must find the critical points of this function. Let $f = 0$, and $f' = 0$, here $f' = \partial f/\partial s, s = T, X, Y, Z$. i.e.,

$$\begin{aligned}
&f = T^4 + X^4 + Y^4 + Z^4 - 2T^2X^2 + 2Y^2Z^2 = 0,\\
&f'_T = \partial f/\partial T = 4T(T^2 - X^2) = 0,\\
&f'_X = \partial f/\partial X = 4X(X^2 - T^2) = 0,\\
&f'_Y = \partial f/\partial Y = 4Y(Y^2 + Z^2) = 0,\\
&f'_Z = \partial f/\partial Z = 4Z(Z^2 + Y^2) = 0.
\end{aligned}$$

So, the critical point are

$$X = \pm T, \qquad T = X = Y = Z = 0.$$

Then, we form the stability matrix $(\partial^2 f/\partial x^i \partial x^j)$. It is of the form

$$H(T,X,Y,Z) = \begin{bmatrix} 12T^2 - 4x^2 & -8Tx & 0 & 0 \\ -8Tx & 12x^2 - 4T^2 & 0 & 0 \\ 0 & 0 & 12y^2 + 4z^2 & 8yz \\ 0 & 0 & 8yz & 12z^2 + 4y^2 \end{bmatrix}.$$

Obviously, for the submatrix

$$H(Y,Z) = \begin{pmatrix} 12y^2 + 4z^2 & 8yz \\ 8yz & 12z^2 + 4y^2 \end{pmatrix},$$

its determinant does not vanish, unless Y=Z=0.

With the Thom theorem (splitting lemma), we could get

$$f_M(Y,Z) = Y^4 + Z^4 + 2Y^2Z^2, \tag{3.2}$$

$$f_{NM}(T,X) = T^4 + X^4 - 2T^2X^2, \tag{3.3}$$

where f_M Morse function, can be reduced to the Morse canonical form

$$M_0^2 = Y^2 + Z^2,$$

and f_{NM}, non-Morse function, is a degenerate form of the double-cusp catastrophe (see Zeeman, 1977). For another submatrix of $H(T, X, Y, Z)$

$$H(T,X) = \begin{vmatrix} 12T^2 - 4X^2 & -8TX \\ -8XT & 12X^2 - 4T^2 \end{vmatrix} = -48(T^4 + X^4 - 2T^2X^2).$$

So, the spacetime submanifold $M(T, X)$ will be divided into four parts by the different values of the $H(T, X)$:

$$\begin{array}{lll} H(T,X) \neq 0 & T^2 - X^2 < 0 & \textit{spacelike state} \\ (\textit{material states}) & T^2 - X^2 > 0 & \textit{timelike state} \\ H(T,X) = 0 & T = \pm X & \textit{lightlike state} \\ (\textit{singularities}) & T = X = 0 & \textit{the origin (indeterminate)}. \end{array} \tag{3.4}$$

It means that the light cone is just a catastrophe set on the spacetime manifold, and both the timelike state and spacelike state are possible states of moving particles.

So, from the point of view of the catastrophe theory, the light cone is just a set of degenerate critical points on the spacetime manifold. The spacetime is structurally unstable at the light cone. It means that a lightlike state could change suddenly into a timelike state and a spacelike state. Also, a timelike state and a spacelike state could change suddenly into a lightlike state. It very much resembles the fact that two photons with sufficient energy could change suddenly into a pair of a particle and an anti-particle and contrarily, a pair of a particle and an antiparticle could annihilate and change into two photons.

According to the nature of catastrophe of the spacetime, the spacetime transformations (2.2) could be resolved into two parts at the light cone:

$$t = \frac{t' + \frac{\beta}{c}x'}{\sqrt{1-\beta^2}}, x = \frac{x' + vt'}{\sqrt{1-\beta^2}}, y = y', z = z'; \quad \beta = \frac{v}{c} < 1. \tag{3.5}$$

and

$$t = \frac{t' + \frac{\beta}{c}x'}{\sqrt{\beta^2 - 1}}, x = \frac{x' + vt'}{\sqrt{\beta^2 - 1}}, y = y', z = z'; \quad \beta = \frac{v}{c} > 1. \tag{3.6}$$

In the same way, the transformation (2.4) could also be resolved into two parts at the light cone:

$$t = \frac{\beta_1 t' + \frac{1}{c}x'}{\sqrt{\beta_1^2 - 1}}, x = \frac{\beta_1 x' + ct'}{\sqrt{\beta_1^2 - 1}}, y = y', z = z'; \quad \beta_1 = \frac{v_1}{c} > 1. \tag{3.7}$$

and

$$t = \frac{\beta_1 t' + \frac{1}{c} x'}{\sqrt{1-\beta_1^2}}, x = \frac{\beta_1 x' + ct'}{\sqrt{1-\beta_1^2}}, y = y', z = z'; \quad \beta_1 = \frac{v_1}{c} < 1. \qquad (3.8)$$

It is very interesting that transformations (3.5) and (3.7) have two major features: Firstly, they keep the same sign between the ds^2 and the ds'^2;i.e.,

$$ds_v^2 = ds'^2_v. \qquad (3.9)$$

Secondly, their inverse transformations are of the form

$$t' = \frac{t - \frac{\beta}{c} x}{\sqrt{1-\beta^2}}, x' = \frac{x - vt}{\sqrt{1-\beta^2}}, y' = y, z' = z; \quad \beta < 1. \qquad (3.10)$$

and

$$t' = \frac{\beta_1 t - \frac{1}{c} x}{\sqrt{\beta_1^2 - 1}}, x' = \frac{\beta_1 x - ct}{\sqrt{\beta_1^2 - 1}}, y' = y, z' = z; \quad \beta_1 > 1. \qquad (3.11)$$

These transformations keep the same sign between x, t and x', t'. So, they will be called the timelike transformations and (3.5) will be called the timelike representation of the timelike transformation (TRTT),and (3.7) the spacelike representation of timelike transformation (SRTT).

In the same manner, transformations (3.6) and (3.8) have two common major features, too. Firstly, they will change the sign between ds^2 and ds'^2,i.e.,

$$-ds_v^2 = ds'^2_v. \qquad (3.12)$$

Secondly, their inverse transformations are of the form

$$-t' = \frac{t - \frac{\beta}{c} x}{\sqrt{\beta^2 = 1}}, -x' = \frac{x - vt}{\sqrt{\beta^2 - 1}}, y' = y, z' = z; \quad \beta > 1. \qquad (3.13)$$

and

$$-t' = \frac{\beta_1 t - \frac{1}{c} x}{\sqrt{1-\beta_1^2}}, -x' = \frac{\beta_1 x - ct}{\sqrt{1-\beta_1^2}}, y' = y, z' = z; \quad \beta_1 < 1. \qquad (3.14)$$

These transformations will change the sign between x, t andx', t'. They will be called the spacelike transformations and (3.6) will be called the spacelike representation of spacelike transformation (SRST); and (3.8) the timelike representation of spacelike transformation (TRST).

Now, we have had four types of form of the spacetime transformation under ds^2:

Type I. TRTT, (3.5), it is just the Lorentz transformation;

Type II. SRTT, (3.7), it is the spacelike representation of the Lorentz transformation with the dual velocity $v_1 = c^2/v$, it is larger than the velocity of light;

Type III. SRST, (3.6), it is just the superluminal Lorentz transformation (see Recami, 1986 and Sen Gupta, 1973);

Type IV. TRST, (3.8), it is the timelike representation of the superluminal Lorentz transformation with the dual velocity $v_1 = c^2/v$, but it is less than the velocity of light.

3.3 The catastrophe of physical quantities on the Finsler metric ds^4

Firstly, we shall consider the question of the catastrophe of the measurement of length and time increment. According to the nature of catastrophe of spacetime, the expression for the length of a moving scale $\Delta x'$ measured by a fixed observer (2.6)-(2.9) could be resolved into two parts,

$$\Delta x' = \Delta x\sqrt{1-\beta^2}, \quad \beta < 1. \tag{3.15}$$

$$-\Delta x' = \Delta x\sqrt{\beta^2-1}, \quad \beta > 1. \tag{3.16}$$

$$-\Delta x' = c\Delta t\sqrt{1-\beta_1^2}, \quad \beta_1 < 1. \tag{3.17}$$

$$\Delta x' = c\Delta t\sqrt{\beta_1^2-1}, \quad \beta_1 > 1. \tag{3.18}$$

The expression for the time increment $\Delta\tau$ of the clock at rest with respect to the moving system could be resolved into two parts at the light cone:

$$\Delta\tau = \Delta t\sqrt{1-\beta^2}, \quad \beta < 1, \tag{3.19}$$

$$-\Delta\tau = \Delta t\sqrt{\beta^2-1}, \quad \beta > 1. \tag{3.20}$$

$$-\Delta\tau = \frac{\Delta x}{c}\sqrt{1-\beta_1^2}, \quad \beta_1 < 1, \tag{3.21}$$

$$\Delta\tau = \frac{\Delta x}{c}\sqrt{\beta_1^2-1}, \quad \beta_1 > 1; \tag{3.22}$$

It is very interesting that the $\Delta x'$, (or Δx) will exchange with Δt (or $\Delta\tau$) in the expressions (3.17)-(3.18) and (3.21)-(3.22).

If we let (see the formula (3.20))

$$f(E,P) = E^4 + c^4P^4 - 2c^2E^2P^2 \tag{3.23}$$

as the catastrophe theory, we could find a catastrophe set

$$E = \pm P \tag{3.24}$$

and we could have four types of the representation for the momentum, the energy, and the mass of a moving particle with the rest mass m:

Type I. TRTT

$$p^T(v) = \frac{mv}{\sqrt{1-\beta^2}}, E^T(v) = \frac{mc^2}{\sqrt{1-\beta^2}}, M^T(v) = \frac{m}{\sqrt{1-\beta^2}}; \quad \beta < 1. \qquad (3.25)$$

Type II. SRTT

$$p^S\{v_1\} = \frac{mv_1}{\sqrt{\beta_1^2 - 1}}, E^S(v_1) = \frac{mc^2}{\sqrt{\beta_1^2 - 1}}, M^S(v_1) = \frac{m}{\sqrt{\beta_1^2 - 1}}; \quad \beta_1 > 1. \qquad (3.26)$$

Type III. SRST

$$p^S\{v\} = \frac{-mv}{\sqrt{\beta^2 - 1}}, E^S(v) = \frac{-mc^2}{\sqrt{\beta^2 - 1}}, M^S(v) = \frac{-m}{\sqrt{\beta^2 - 1}}; \quad \beta > 1. \qquad (3.27)$$

Type IV. TRST

$$p^S\{v_1\} = \frac{-mv_1}{\sqrt{1-\beta_1^2}}, E^S(v_1) = \frac{-mc^2}{\sqrt{1-\beta_1^2}}, M^S(v_1) = \frac{-m}{\sqrt{1-\beta_1^2}}; \quad \beta_1 < 1. \qquad (3.28)$$

The transformations between type I (or type II) and type III (or type IV) have the forms

$$p^T(v) = \frac{mv}{\sqrt{1-\beta^2}} = \frac{mc}{\sqrt{\beta_1^2 - 1}} = \frac{1}{c}E^T(v_1), \qquad (3.29)$$

$$E^T(v) = \frac{mc^2}{\sqrt{1-\beta^2}} = \frac{mv_1 c}{\sqrt{\beta_1^2 - 1}} = cp^T(v_1), \qquad (3.30)$$

$$M^T(v) = \frac{m}{\sqrt{1-\beta^2}} = \frac{\beta_1 m}{\sqrt{\beta_1^2 - 1}} = \beta_1 M^T(v_1) \qquad (3.31)$$

and

$$p^S(v) = \frac{-mv}{\sqrt{\beta^2 - 1}} = \frac{-mc}{\sqrt{1-\beta_1^2}} = \frac{1}{c}E^S(v_1), \qquad (3.32)$$

$$E^S(v) = \frac{-mc^2}{\sqrt{\beta^2 - 1}} = \frac{-mv_1 c}{\sqrt{1-\beta_1^2}} = cp^S(v_1), \qquad (3.33)$$

$$M^S(v) = \frac{-m}{\sqrt{\beta^2 - 1}} = \frac{-\beta_1 m}{\sqrt{1 - \beta_1^2}} = \beta_1 M^S(v_1). \tag{3.34}$$

With these forms above, we could get that when β=β_1=1,

$$cP(c) = E(c) = mc^2 \quad \textit{and} \quad M(c) = m. \tag{3.35}$$

Note that although all through Einstein's relativistic physics there occur indications that mass and energy are equivalent according to the formula

$$E = mc^2.$$

But it is only an Einstein's hypothesis.

It is very interesting that from type I and type IV we could get

$$E^2 - c^2p^2 = m^2c^4, \quad v < c \quad \text{and} \quad v_1 < c \quad (i.e., v > c) \tag{3.36}$$

and from type II and type III

$$E^2 - c^2p^2 = -m^2c^4, \quad v > c \quad \text{and} \quad v_1 > c \quad (i.e., v < c) \tag{3.37}$$

Here, we have forgotten the indices for the types in Equations (3.35) to (3.37). If we let the $H^2(E,P) = E^2 \;\; c^2P^2$, then we could get

$$f(H, mc) = H^4 - (mc^2)^4. \tag{3.38}$$

It is a type II of the double-cusp catastrophe, we could also get (3.36) and (3.37) from it.

3.4 The catastrophe a charged particle in an electromagnetic field on the Finsler spacetime ds^4

The Hamilton-Jacobi equation for a particle in an electromagnetic field in the Finsler spacetime, formula (2.25) is a type II of the double-cusp catastrophe. We could get that

$$c^2(\nabla S - \frac{e}{c}A)^2 - (\frac{\partial S}{\partial t} + c\Phi)^2 + m^2c^4 = 0 \tag{3.39}$$

for type I and type IV of the spacetime transformation.

$$c^2(\nabla S - \frac{e}{c}A)^2 - (\frac{\partial S}{\partial t} + c\Phi)^2 - m^2c^4 = 0 \tag{3.40}$$

for type II and type III of the spacetime transformation.

Now, we consider the catastrophe change of the equation of a charge in an electromagnetic field. By equation (2.26), we could get

$$\frac{d}{dt}\frac{mv}{\sqrt{1-\beta^2}} = eE_e + \frac{e}{c}v \times H_e, \qquad v < c \tag{3.41}$$

and

$$-\frac{d}{dt}\frac{mv}{\sqrt{\beta^2-1}} = eE_e + \frac{e}{c}v \times H_e, \qquad v > c\ . \tag{3.42}$$

If we integrate (3.41) and (3.42), then

$$\frac{mc^2}{\sqrt{1-\beta^2}} - \frac{mc^2}{\sqrt{1-\beta_0^2}} = eU, \qquad v_0 < c \tag{3.43}$$

and

$$\frac{mc^2}{\sqrt{\beta_0^2-1}} - \frac{mc^2}{\sqrt{\beta^2-1}} = eU, \qquad v_0 > c\ . \tag{3.44}$$

So, the velocity v has

$$v = c\sqrt{1 - \left(\frac{eU}{mc} + 1/\sqrt{1-\beta_0^2}\right)^{-2}} < c, \qquad iff \quad v_0 < c, \tag{3.45}$$

and

$$v = c\sqrt{1 + \left(\frac{eU}{mc} - 1/\sqrt{\beta_0^2-1}\right)^{-2}} > c, \qquad iff \quad v_0 > c\ . \tag{3.46}$$

The expressions (3.45) and (3.46) mean that if $v_0 < c$, then for the charged particle always $v < c$; and if $v_0 > c$, then $v > c$. The velocity of light will be a bilateral limit: i.e., it is both of the maximum for the subluminal-speeds and the minimum for the superluminal-speeds.

If we let

$$f(H_e, E_e) = H_e^4 + E_e^4 - 2H_e^2E_e^2, \tag{3.47}$$

we will get that the catastrophe set is

$$H_e = \pm E_e \tag{3.48}$$

and could obtain the spacetime transformation equations for the electromagnetic field components(by (2.31) and (2.32)):

Type I. TRTT

$$\begin{cases} H'_x = H_x, & E'_x = E_x, \\ H'_y = \frac{H_y+\beta E_z}{\sqrt{1-\beta^2}}, & E'_y = \frac{E_y-\beta H_z}{\sqrt{1-\beta^2}}, \\ H'_z = \frac{H_z-\beta E_y}{\sqrt{1-\beta^2}}, & E'_z = \frac{E_z+\beta H_y}{\sqrt{1-\beta^2}}. \end{cases} \quad (3.49)$$

Type II. SRTT

$$\begin{cases} H'_x = H_x, & E'_x = E_x, \\ H'_y = \frac{\beta_1 H_y+E_z}{\sqrt{\beta_1^2-1}}, & E'_y = \frac{\beta_1 E_y-H_z}{\sqrt{\beta_1^2-1}}, \\ H'_z = \frac{\beta_1 H_z-E_y}{\sqrt{\beta_1^2-1}}, & E'_z = \frac{\beta_1 E_z+H_y}{\sqrt{\beta_1^2-1}}. \end{cases} \quad (3.50)$$

Type III. SRST

$$\begin{cases} H'_x = H_x, & E'_x = E_x, \\ -H'_y = \frac{H_y+\beta E_z}{\sqrt{\beta^2-1}}, & -E'_y = \frac{E_y-\beta H_z}{\sqrt{\beta^2-1}}, \\ -H'_z = \frac{H_z-\beta E_y}{\sqrt{\beta^2-1}}, & -E'_z = \frac{E_z+\beta H_y}{\sqrt{\beta^2-1}}. \end{cases} \quad (3.51)$$

Type IV. TRST

$$\begin{cases} H'_x = H_x, & E'_x = E_x, \\ -H'_y = \frac{\beta_1 H_y+E_z}{\sqrt{1-\beta_1^2}}, & -E'_y = \frac{\beta_1 E_y-H_z}{\sqrt{1-\beta_1^2}}, \\ -H'_z = \frac{\beta_1 H_z-E_y}{\sqrt{1-\beta_1^2}}, & -E'_z = \frac{\beta_1 E_z+H_y}{\sqrt{1-\beta_1^2}}. \end{cases} \quad (3.52)$$

3.5 The interchange of the forces between the attraction and the rejection

Usually, because of the equivalence of energy and mass in the relativity theory, ones believe that an object has due to its motion will add to its mass. In other words, it will make it harder to increase its speed. This effect is only really significant for objects moving at speeds close to the speed of light. So, only light, or other waves that have no intrinsic mass, can move at the speed of light.

The mass is the measure of the gravitational and inertial properties of matter. Once thought to be conceivably different, gravitational mass and inertial mass have recently been shown to be the same to one part in 10^{11}.

Inertial mass is defined through Newton's second law, F=ma, in which m is mass of body. F is the force action upon it, and a is the acceleration of the body induced by the force. If two bodies are acted upon by the same force (as in the idealized case of connection with a massless spring), their instantaneous accelerations will be in inverse ratio to their masses.

Now, we need discuss the problem of defining mass m in terms of the force and acceleration. This, however, implies that force has already been independently defined, which is by no means the case.

3.5.1 Electromagnetic mass and electromagnetic force

It is well known that the mass of the electron is about 2000 times smaller than that of the hydrogen atom. Hence the idea occurs that the electron has, perhaps, no "ordinary" mass at all, but is nothing other than an "atom of electricity", and that its mass is entirely electromagnetic in origin. Then, the theory found strong support in refined observations of cathode rays and of the β-rays of radioactive substances, which are also ejected electrons. If magnetic action on these rays allows us to determine the ratio of the charge to the mass, $\frac{e}{m_{el}}$, and also their velocity v, and that at first a definite value for $\frac{e}{m_{el}}$ was obtained, which was independent of v if $v << c$. But, on proceeding to higher velocities, a decrease of $\frac{e}{m_{el}}$ was found. This effect was particularly clear and could be measured quantitatively in the case of the β-rays of radium, which are only slightly slower than light. The assumption that an electric charge should depend on the velocity is incompatible with the ideas of the electron theory. But, that the mass should depend on the velocity was certainly to be expected if the mass was to be electromagnetic in origin. To arrive at a quantitative theory, it is true, definite assumptions had to be made about the form of the electron and the distribution of the charge on it. M. Abraham (1903) regarded the electron as a rigid sphere, with a charge distributed on the one hand, uniformly over the interior, or, on the other, over the surface, and he showed that both assumptions lead to the same dependence of the electromagnetic mass on the velocity, namely, to an increase of mass with increasing velocity. The faster the electron travels, the more the electromagnetic field resists a further increase of velocity. The increase of m_{el} explains the observed decrease of $\frac{e}{m_{el}}$, and Abraham's theory agrees quantitatively very well with the results of measurement of Kaufmann (1901) if it is assumed that there is no "ordinary" mass present. But, the electromagnetic force $F = e[E + \frac{1}{c}(v \times H)]$ was believed to be a constant and be independent of the velocity v.

Note that if we support that the mass m is independent of the velocity v, but the electromagnetic force $F = e[E + \frac{1}{c}(v \times H)]$ is dependent of the velocity v, it will be incompatible with neither the ideas of the electron theory nor the results of measurement of Kaufmann. One further matter needs attention: the E and$H$ occurring in the formula for the force F are supposed to refer to that system in which the electron is momentarily at rest.

3.5.2 The mass and the force in the Einstein's special relativity

In the Einstein's special relativity, Lorentz's formula for the dependency of mass on velocity has a much more general significance than is the electromagnetic mass apparent. It must hold for every kind of mass, no matter whether it is of electrodynamics origin or not.

Experiments by Kaufmann (1901) and others who have deflected cathode rays by electric and magnetic fields have shown very accurately that the mass of electrons grows with velocity according to Lorentz's formula (**??**). On the other hand, these measurements can no longer be regarded as a confirmation of the assumption that all mass is of electromagnetic origin. For Einstein's theory of relativity shows that mass as such, regardless of its origin, must depend on velocity in the way described by Lorentz's formula.

Up to now, if we support that all kinds of the mass, m, are independent of the velocity v, but all forces are dependent of the velocity v, it will be incompatible with neither the ideas of the physical theory nor the results of measurement of physics. Could make some mew measurements of physics (or some observations of astrophysics) to support this viewed from another standpoint.

3.5.3 The interchange of the forces between the attraction and the rejection

Let us return to the Newton's second law, F=ma, we can see that the product of mass and acceleration is a quantity antisymmetric with respect to the two interaction particles B and C. We shall now make the hypothesis that the value of this quantity in any given case depends on the relative position of the particles and sometimes on their relative velocities as well as the time. We express this functional dependence by introducing a vector function $F_{BC}(r, \dot{r}, t)$, where r is the position vector of B with respect to C and $\dot{r}$ is the relative velocity. We then write

$$m_B a_{BC} = F_{BC}. \quad (3.53)$$

and define the function F_{BC} as the force acting on the particle B due to the particle C. It is worth while to stress the significance of the definition of force presented here. It will be noted that no merely anthropomorphic notion of push of pull is involved. Eq.(3.53) states that the product of mass and acceleration, usually known as the *kinetic reaction*, is equal to the *force*.

Now, if we explain the experiments by Kaufmann (1901) with here point of view, then, we could say that the electromagnetic force $F = e[E + \frac{1}{c}(v \times H)]$ is a function dependent of the velocity v, $F = F(v)$.

From the above mentioned, the relativity theory provides for an increase of apparent inertial mass with increasing velocity according to the formula

$$m = \frac{m_0}{\sqrt{1-\beta^2}}$$

could be understood equivalently as a decrease of the effective force of the fields with increasing relativistic velocity between the source of the field and the moving body according to the formula

$$F_{eff} = F\sqrt{1-\beta^2}.$$

Further, the negative apparent inertial mass could be understood equivalently as the effective forces of the fields have occurred the interchange between the attraction and the rejection according to the formula.

$$F_{eff} = -F\sqrt{\beta^2-1}.$$

3.5.4. The character velocity and effective forces for a forces

Up to now, one common essential feature for forces is neglected that the character velocities for forces. Ones commonly believe that if the resistance on the wagon with precisely the same force with which the horse pulls forward on the wagon then the wagon will keep the right line moving with a constant velocity. However, we could ask that if the resistance on the wagon is zero force then will the wagon be continue accelerated by the horse? How high velocity could be got by the wagon? It is very easy understood that the maximum velocity of the wagon, v_{max}, will be the fastest running velocity of the horse, v_{fst}. The velocity v_{fst} is just the character velocity, v_c, for the pulling force of the horse. When the velocity of the wagon is zero velocity, the pulling force of the horse to the wagon has the largest effective value $F_{eff} = F$. We assume that a decrease of the effective force with increasing velocity of the wagon, and $F_{eff} = 0$ if and only if $\beta = \frac{v_w}{v_c} = 1$. If $\beta = \frac{v_w}{v_c} > 1$ then $F_{eff}= -kF$. It means that when the velocity of the wagonv_w is larger the character velocity v_c, not that the horse pulls the wagon, but that the wagon pushes the horse.

If the interactions of the fields traverse empty space with the velocity of light, c, then the velocity of light is just the character velocity for all kinds of the interactions of the fields. We guess that the principle of the constancy of the velocity of light is just a superficial phenomenon of the character of the interactions of the fields.

3.5.5. One possible experiment for distinguish between moving mass and effective force

The Newtonian law of universal gravitation assumes that, two bodies attract each other with a force that is proportional to the mass of each body and is inversely proportional to the square of their distance apart:

$$F = G\frac{m_1 m_2}{r^2}. \tag{3.54}$$

According as Einstein's special relativity, if the body$_1$ is moving with constant speed v with respect to the body$_2$, then the mass of the body$_1$ will become with respect to the body$_2$ that

$$M_1 = \frac{m_1}{\sqrt{1-\frac{v^2}{c^2}}}. \tag{3.55}$$

According to the principle of equivalence the body's gravitational mass equal to its inertia mass. So, the force of gravitational interaction between the two bodies will be

$$F_{M.M.} = G\frac{m_1 m_2}{r^2\sqrt{1-\frac{v^2}{c^3}}}. \tag{3.56}$$

But, according as the theory of the effective force, the force of gravitational interaction between the two bodies will be

$$F_{E.F.} = G\frac{m_1 m_2}{r^2}\sqrt{1-\frac{v^2}{c^2}}. \tag{3.57}$$

We hope that could design some new experiments to discover this deviation.

3.6 Decay of particles

On the Einstein's special relativity theory, consider the spontaneous decay of a body of mass M into two parts with masses m_1 and m_2. The law of conservation of energy in the decay, applied in the system of reference in which the body is at rest, gives

$$M = E_{10} + E_{20}, \tag{3.58}$$

where E_{10} and E_{20} are the energies of the emerging particles. Since $E_{10} > m_1$ and $E_{20} > m_2$, the equality (120) can be satisfied only if M¿m_1+m_2, i.e. a body can disintegrate spontaneously into parts the sum of whose masses is less than the mass of the body. On the other hand, if M¡$m_1 + m_2$, the body is stable (with respect to the particular decay) and does not decay spontaneously. To cause the decay in this case, we would have to supply to the body from outside an amount of energy at least equal to its "binding energy" $(m_1 + m_2 - M)$.

Usually, ones believe that momentum as well as energy must be conserved in the decay process. Since the initial momentum of the body was zero, the sum of the momenta of the emerging particles must be zero: $\mathbf{p}_{10}+\mathbf{p}_{20}=0$ in the special relativity theory. Consequently $p_{10}^2 = p_{20}^2$, or

$$E_{10}^2 - m_1^2 = E_{20}^2 - m_2^2. \tag{3.59}$$

The two equations (3.58) and (3.59) uniquely determine the energies of the emerging particles

$$E_{10} = \frac{M^2 + m_1^2 - m_2^2}{2M}, \quad E_{20} = \frac{M^2 - m_1^2 + m_2^2}{2M}. \tag{3.60}$$

In a certain sense the inverse of this problem is the calculation of the total energy M of two colliding particles in the system of reference in which their total momentum is zero. (This is abbreviated as the "system of the center of inertia" or the "C-system".) The computation of this quantity gives a criterion for the possible occurrence of various inelastic collision processes, accompanied by a change in state of the colliding particles, or the "creation" of new particles. A process of this type can occur only if the sum of the masses of the "reaction products" does not exceed M.

Suppose that in the initial reference system (the "laboratory" system) a particle with mass m_1 and energy E_1 collides with a particle of mass m_2 which is at rest. The total energy of the two particles is

$$E = E_1 + E_2 = E_1 + m_2,$$

and their total momentum is $\boldsymbol{p}=\boldsymbol{p}_1+\boldsymbol{p}_2=\boldsymbol{p}_1$. Considering the two particles together as a single composite system, we find the velocity of its motion as a whole from (2.19):

$$V = \frac{p}{E} = \frac{p_1}{E_1 + m_2}. \tag{3.61}$$

This quantity is the velocity of the C-system with respect to the laboratory system (the L-system).

However, in determining the mass M, there is no need to transform from one reference frame to the other. Instead we can make direct use of formula (3.36), which is applicable to the composite system just as it is to each particle individually. We thus have

$$M^2 = E^2 - p^2 = (E_1 + m_2)^2 - (E_1^2 - m_1^2),$$

from which

$$M^2 = m_1^2 + m_2^2 + 2m_2E_1. \tag{3.62}$$

§4 Conclusions

From the discussion in this paper, we could get the following conclusions:

(1) The special theory of relativity cannot negate the possibility of the existence of superluminal-speed.

(2) The essential nature of the superluminal-speed is the relativity of the temporal order. If one does not know how to distinguish the temporal orders, a particle moving with superluminal-speed could be taken for one moving with a subluminal-speed of some unusual nature.

(3) The specal theory of relativity could be discussed in the Finsler spacetime. The spacetime transformation on the Finsler metric ds^4 contains a new symmetry between the timelike and spacelike.

(4) Some new invariants describe the catastrophe nature of the Finsler spacetime ds^4. They obey the double-cusp catastrophe. The timelike state cannot change smoothly into the spacelike state for a motion particle. But a lightlike state could change suddenly into a timelike state and spacelike state. Also, a timelike state and a spacelike state could change suddenly into a lightlike state.

(5) The length x will exchange the position with the time increment t between v's representation and v_1's representation. The momentum (or energy) in the timelike (or spacelike) representation will be transformed into the energy (or momentum) in the spacelike (or timelike) representation.

(6) The difference between the subluminal- and superluminal-speed would be described as follows: a particle with the subluminal-speed has positive momentum, energy, and moving mass, and a particle with the superluminal-speed has negative ones.

(7) Usually, it is believed that Tachyons have a spacelike energy-momentum four-vector so that

$$E^2 < c^2P^2.$$

Hence, the square of the rest mass m defined by

$$m^2c^4 = E^2 - c^2P^2 < 0$$

requires the 'rest mass' to be imaginary' (see Hawking and Ellis, 1973).

As has been said in this paper, from the expressions (3.25)-(3.28) it is clear that, no matter whether a particle is moving with a subluminal- or superluminal-speed, in the timelike representation it will obey Equation (3.36), but, in the spacelike representation it will obey

Equation (3.37). So, for a particle with superluminal-speed its mass $M(v)$ (energy $E(v)$, and momentum $P(v)$) is negative rather than imaginary. As expression (3.28)

$$E^S(v_1) = -mc^2$$

when $\beta \to 0$.

So the particle with superluminal-speed, in the timelike representation, will remain a negative 'rest-mass'. We shall write:

$$E = \begin{cases} +mc^2 & for \quad sublu\min al - speed, \qquad i.e., \quad v < c(\ or \quad v_1 > c), \\ -mc^2 & for \quad \sup erlu\min al - speed, \qquad i.e., v > c(\ or \quad v_1 < c). \end{cases}$$

It was just analyzed by Dirac for the anti-particle. So, we guess that a particle with the superluminal-speed$v > c$ could be regarded as its anti-particle with the dual velocity $v_1 = c^2/v < c$.

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