Neutrosophic Code

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Abstract: The idea of neutrosophic code came into my mind at that time when i was reading the literature about linear codes and i saw that, if there is data transfremation between a sender and a reciever. They want to send 11 and 00 as codewords. They suppose 11 for true and 00 for false. When the sender sends the these two codewords and the error occures. As a result the reciever recieved 01 or 10 instead of 11 and 00. This story give a way to the neutrosophic codes and thus we introduced neutrosophic codes over finite field in this paper.

Introduction

Florentin Smarandache for the first time intorduced the concept of neutrosophy in 1995 which is basically a new branch of philosophy which actually studies the origion, nature, and scope of neutralities. The neutrosophic logic came into being by neutrosophy. In neutrosophic logic each proposition is approximated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F. Neutrosophic logic is an extension of fuzzy logic. In fact the neutrosophic set is the generalization of classical set, fuzzy conventional set, intuitionistic fuzzy set, and interal valued fuzzy set. Neutrosophic logic is used to overcome the problems of imperciseness, indeterminate, and inconsistentness of data etc. The theory of neutrosophy is so applicable to every field of agebra.W.B Vasantha Kandasamy and Florentin Smarandache introduced neutrosophic fields, neutrosophic rings, neutrosophic vectorspaces, neutrosophic groups, neutrosophic bigroups and neutrosophic N-groups, neutrosophic semigroups, neutrosophic bisemigroups, and neutrsosophic N-semigroups, neutrosophic loops, nuetrosophic biloops, and neutrosophic N-loops, and so on. Mumtaz ali $et\ al$. introduced nuetrosophic LA-semigoups.

Algebriac codes are used for data compression, cryptography, error correction and and for network coding. The theory of codes was first focused by Claude Shanon in 1948 and then gradually developed by time to time. There are many types of codes which is important to its algebriac structures such as Linear block codes, Hamming codes, BCH codes etc. The most common type of code is a linear code over the field F_q. There are also linear codes which are define over the finite rings. The linear codes over finite ring are initiated by Blake in a series of papers $[2]$, $[3]$, Spiegel $[4]$, $[5]$ and Forney et al. $[6]$. Klaus Huber defined codes over Gaussian integers.

In the further section, we intorduced the concept of neutrosophic code and establish the basic results of codes. We also developed the decoding procedures for neutrosophic codes and illustrate it with examples.

Basic concepts

Definition 1 Let A be a finite set of q symbols where $(q>1)$ and let $V=A^n$ be the set of n-tuples of elements of A where n is some positive integer greater than 1. In fact V is a vecotr space over A. Now let C be a non empty subset of V. Then C is called a q-ary code of length n over A.

Definition 2 Let F^n be a vector space over the field F, and $x, y \in F^n$ where $x = x_1x_2...x_n$, $y = y_1y_2...y_n$. The Hamming distance between the vectors x and y is denoted by $d(x, y)$, and is defined as $d(x, y) = |i : x_i \neq y_i|$.

Definition 3 The minimum distance of a code C is the smallest distance between any two distinct codewords in C which is denoted by $d(C)$, that is $d(C) = \min\{d(x, y) : x, y \in C, x \neq y\}$.

Definition 4 Let F be a finite field and n be a positive integer. Let C be a subspace of the vector space $V = F^n$. Then C is called a linear code over F.

Definition 5 The linear code C is called linear $[n,k]$-code if $\dim(C) = k$.

Definition 6 Let C be a linear $[n,k]$-code. Let G be a $k \times n$ matrix whose rows form basis of C. Then G is called generator matrix of the code C.

Definition 7 Let C be an $[n,k]$-code over F. Then the dual code of C is defined to be

$$C^\perp = \{y \in F^n : x \cdot y = 0, \forall x \in C\}$$

Definition 8 Let C be an $[n,k]$-code and let H be the generator matrix of the dual code $C^\perp$. Then H is called a parity-check matrix of the code C.

Definition 9 A code C is called self-orthogonal code if $C \subset C^\perp$.

Definition 10 Let C be a code over the field F and for every $x \in F^n$, the coset of C is defined to be

$$C = \{x + c : c \in C\}$$

Definition 11 Let C be a linear code over F. The coset leader of a given coset is defined to be the vector with least weight in that coset.

Definition 12 If a codeword x is transmitted and the vector y is received, then $e = y - x$ is called error vector. Therefore a coset leader is the error vector for each vector y lying in that coset.

Nuetrosophic code

Definition 13 Let C be a q-ary code of length n over the field F. Then the neutrosophic code is denoted by $N(C)$, and is defined to be

$$N(C) = \langle C \cup nI \rangle$$

where I is indeterminacy and called neutrosophic element with the property $I + I = 2I, I^2 = I$. For an integer n, $n + I$, nI are neutrosophic elements and $0.I = 0$. I^{-1}, the inverse of I is not defined and hence does not exist.

Example 1 Let $C = \{000,111\}$ be a binary code of length 3 over the field $F = Z_2$. Then the corresponding neutrosophic binary code is $N(C)$, where

$$N(C) = \langle C \cup III \rangle = \{000,111, III, I'I'I'\}$$

where $I' = (1 + I)$ which is called dual bit or partially determinate bit. In $1 + I$, 1 is determinate bit and I is indeterminate bit or multibit. This multibit is sometimes 0 and sometimes 1.

Theorem 1 A neutrosophic code $N(C)$ is a neutrosophic vector space over the field F.

Definition 14 Let $F^n(I)$ be a neutrosophic vector space over the field F and $x, y \in F^n(I)$, where

$x = x_1x_2...x_n$, $y = y_1y_2...y_n$. The Haming neutrosophic distance between the neutrosophic vectors x and y is denoted by $d_N(x, y)$, and is defined as $d_N(x, y) = \left| j : x_j \neq y_j \right|$.

Example 2 Let the neutrosophic code $N(C)$ be as in above example. Then the Hamming neutrosophic distance $d_N(x, y) = 3$, for all $x, y \in N(C)$.

Remark 1 The Hamming neutrosophic distance satisfies the three conditions of a distance function:

1) $d_N(x, y) = 0$ if and only if $x = y$.
2) $d_N(x, y) = d_N(y, x)$ for all $x, y \in F^n(I)$.
3) $d_N(x, z) \leq d_N(x, y) + d_N(y, z)$ for all $x, y, z \in F^n(I)$.

Definition 15 The minimum neutrosophic distance of a neutrosophic code $N(C)$ is the smallest distance between any two distinct neutrosophic codewords in $N(C)$. We denote the minimum neutrosophic distance by $d_N(N(C))$. Equivalently $d_N(N(C)) = \min\{d_N(x, y) : x, y \in N(C), x \neq y\}$.

Example 3 Let $N(C)$ be a neutrosophic code over the field $F = Z_2 = \{0,1\}$, where

$$N(C) = \langle C \cup III \rangle = \{000, 111, III, I'I'I'\}$$

with $I' = (1 + I)$. Then the minimum neutrosophic distance $d_N(N(C)) = 3$.

Definition 16 Let C be a linear code of lenght n over the field F. Then $N(C) = \langle C \cup nI \rangle$ is called neutrosophic linear code over F.

Example 4 In example (3), $C = \{000, 111\}$ is a linear binary code over the field $F = Z_2 = \{0,1\}$, and the required neutrosophic linear code is

$$N(C) = \langle C \cup III \rangle = \{000, 111, III, I'I'I'\}$$

where $I' = (1 + I)$.

Theorem 2 Let C be a linear code and $N(C)$ be a neutrosohic linear code. If $\dim(C) = k$, then $\dim(N(C)) = k + 1$.

Definition 17 The linear neutrosophic code $N(C)$ is called linear $[n, k+1]$-neutrosophic code if $\dim(N(C)) = k + 1$.

Theorem 3 The linear $[n, k+1]$-neutrosophic code $N(C)$ contains the linear $[n, k]$-code C.

Theorem 4 The linear code C is a subspace of the neutrosophic code $N(C)$ over the field F.

Theorem 5 The linear code C is a sub-neutrosophic code of the neutrosophic code $N(C)$ over the field F.

Theorem 6 Let B be a basis of a linear code C of lenght n over the field F. Then $B \cup nI$ is the neutrosophic basis of the neutrosophic code $N(C)$, where I is the neutrosophic element.

Theorem 7 If the linear code C has a code rate $\frac{k}{n}$, then the neutrosophic linear code $N(C)$ has code rate

$\frac{k+1}{n}$.

Theorem 8 If the linear code C has redundancy $n-k$, then the neureosophic code has redundancy $n-(k+1)$.

Definition 18 Let $N(C)$ be a linear $[n,k+1]$-neutrosophic code. Let $N(G)$ be a $(k+1)\times n$ matrix whose rows form basis of $N(C)$. Then $N(G)$ is called neutrosophic generator matrix of the neutrosophic code $N(C)$.

Example 5 Let $N(C)$ be the linear neutrosophic code of length 3 over the field F, where

$$N(C)=\langle C\cup III\rangle=\{000,111,III,I'I'I'\}$$

with $I'=1+I$.

Let $N(G)$ be a $(k+1)\times n$ neutrosophic matrixe where

$$N(G)=\begin{bmatrix}1 & 1 & 1\\ I & I & I\end{bmatrix}_{2\times 3}$$

Then clearly $N(G)$ is a neutrosophic generator matrix of the neutrosophic code $N(C)$ because the rows of $N(G)$ generates the linear neutrosophic code $N(C)$. In fact the rows of $N(G)$ form a basis of $N(G)$.

Remark 2 The neutrosophic generator matrix of a neutrosophic code $N(C)$ is not unique.
We take the folowing example to prove the remark.

Example 6 Let $N(C)$ be a linear neutrosophic code of length 3 over the field F, where

$$N(C)=\langle C\cup III\rangle=\{000,111,III,I'I'I'\}$$

with $I'=1+I$. Then clearly $N(C)$ has three neutrosophic generator matrices which are follows.

$$N(G_1)=\begin{bmatrix}1 & 1 & 1\\ I & I & I\end{bmatrix}_{2\times 3},\ N(G_2)=\begin{bmatrix}1 & 1 & 1\\ I' & I' & I'\end{bmatrix}_{2\times 3},$$
$$N(G_3)=\begin{bmatrix}I & I & I\\ I' & I' & I'\end{bmatrix}$$

Theorem 9 Let G be a generator matrix of a linear code C and $N(G)$ be the neutrosophic generator matrix of the neutrosophic linear code $N(C)$, then G is always contained in $N(G)$.

Definition 19 Let $N(C)$ be an $[n,k+1]$-neutrosophic code over F. Then the neutrosophic dual code of the neutrosophic code $N(C)$ is defined to be

$$N(C)^{\perp}=\{y\in F^n(I): xgy=0\,\forall x\in N(C)\}$$

Example 7 Let $N(C)$ be a linear neutrosophic code of length 2 over the neutrosophic field $F=Z_2$, where

$$N(C)=\{00,11,II,I'I'\}$$

with $I'=1+I$.
Since

$$F_2^{\,2}(I)=\begin{Bmatrix}00,01,0I,0I',10,11,1I,1I', \\ I0,I1,II,II',I'0,I'1,I'I,I'I'\end{Bmatrix}$$

where $I'=1+I$. Then the neutrosophic dual code $N(C)^{\perp}$ of the neutrosophic code $N(C)$ is given as follows,

$$N(C)^{\perp}=\{00,11,II,I'I'\}.$$

Theorem 10 If the neutrosophic code $N(C)$ has dimension $k+1$, then the neutrosophic dual code $N(C)^{\perp}$ has dimension $2n-(k+1)$.

Theorem 11 If $C^{\perp}$ is a dual code of the code C over F, then $N(C)^{\perp}$ is the neutrosophic dual code of the neutrosophic code $N(C)$ over the field F, where $N(C)^{\perp}=\langle C^{\perp}\cup nI\rangle$.

Definition 20 A neutrosophic code $N(C)$ is called self neutrosophic dual code if $N(C)=N(C)^{\perp}$.

Example 8 In example (7), the neutrosophic code $N(C)$ is self neutrosophic dual code because $N(C)=N(C)^{\perp}$.

Definition 21 Let $N(C)$ be an $[n,k+1]$-neutrosophic code and let $N(H)$ be the neutrosophic generator matrix of the neutrosophic dual code $N(C)^{\perp}$. Then $N(H)$ is called a neutrosophic parity-check matrix of the neutrosophi code $N(C)$.

Example 9 Let $N(C)$ be the linear neutrosophic code of length 3 over the field F, where

$$N(C)=\{000,111,III,I'I'I'\}$$

with $I'=1+I$. The neutrosophic generator matrix is

$$N(G)=\begin{bmatrix}1 & 1 & 1\\ I & I & I\end{bmatrix}_{2\times 3}$$

The neutrosophic dual code $N(C)^{\perp}$ of the above neutrosophic code $N(C)$ is as following,

$$N(C)^{\perp}=\begin{Bmatrix}000,011,101,110,1II',1I'I, \\ 0II,I0I,I1I',II0,I'0I',I'1I, \\ I'I1,I'I'0,II'1,0I'I'\end{Bmatrix}$$

The corresponding neutrosophic parity check matrix is given as follows,

$$N(H)=\begin{bmatrix}1 & 1 & 0\\ I & I' & I\\ 0 & I & I\\ I' & 0 & I\end{bmatrix}$$

Theorem 12 Let $N(C)$ be an $[n,k+1]$-neutrosophic code. Let $N(G)$ and $N(H)$ be neutrosophic generator matrix and neutrosophic parity check matrix of $N(C)$ respectively. Then

$$N(G)N(H)^T=0=N(H)N(G)^T$$

Remark 3 The neutrosophic parity check matrix $N(H)$ of a neutrosophic code $N(C)$ is not unique. To see the proof of this remark, we consider the following example.

Example 10 Let the neutrosophic code $N(C)$ be as in above example. The neutrosophic parity check matrices of $N(C)$ are given as follows.

$$N(H_1)=\begin{bmatrix}1 & 1 & 0\\ I & I' & 1\\ 0 & I & I\\ I' & 0 & I\end{bmatrix},N(H_2)=\begin{bmatrix}1 & 0 & 1\\ I & I' & 1\\ I & 0 & I\\ I' & 0 & I\end{bmatrix}$$

and so on.

Definition 22 A neutrosophic code $N(C)$ is called self-orthogonal neutrosophic code if $N(C)\subset N(C)^{\perp}$.

Example 11 Let $N(C)$ be the linear $[4,2]$-neutrosophic code of length 4 over the neutrosophic field $F=Z_2$, where

$$N(C)=\{0000,1111,IIII,I'I'I'I'\}$$

with $I'=1+I$. The neutrosophic dual code $N(C)^{\perp}$ of $N(C)$ is following;

$$N(C)^{\perp}=\begin{Bmatrix}0000,1100,1010,1001,0110,0101,0011,1111,IIII,\\ I'I'I'I',I'I'00,I'0I'0,I'00I',0I'I'0,0I'0I',00I'I',\ldots\end{Bmatrix}$$

Then clearly $N(C)\subset N(C)^{\perp}$. Hence $N(C)$ is self-orthogonal neutrosophic code.

Theorem 13 If C is self-orthognal code then $N(C)$ is self-orthognal neutrosophic code.

Pseudo Neutrosophic Code

Definition 23 A linear $[n,k+1]$-neutrosophic code $N(C)$ is called pseudo linear $[n,k+1]$-neutrosophic code if it does not contain a proper subset of S which is a linear $[n,k]$-code.

Example 12 Let $N(C)$ be the linear neutrosophic code of length 3 over the field $F = Z_2 = \{0,1\}$, where

$$N(C) = \{000,111,III,I'I'I'\}$$

with $I' = 1 + I$. Then clearly $N(C)$ is a pseudo linear $[3,2]$-neutrosophic code because it does not contain a proper subset of C which is a linear $[3,1]$-code.

Theorem 14 Every pseudo linear $[n,k+1]$-neutrosophic code $N(C)$ is a trivially a linear $[n,k+1]$-neutrosophic code but the converse is not true.

We prove the converse by taking the following example.

Example 13 Let $C = \{00,01,10,11\}$ be a linear $[2,2]$-code and $N(C)$ be the corresponding linear $[2,3]$-neutrosophic code of length 2 over the field $F = Z_2 = \{0,1\}$, where

$$N(C) = \{00,01,10,11,II,II',I'I,I'I'\}$$

with $I' = 1 + I$.

Then clearly $N(C)$ is not a pseudo linear $[2,3]$-neutrosophic code because $\{00,11\}$ is a proper subspace of C which is a code.

Strong or Pure Neutrosophic Code

Definition 24 A neutrosophic code $N(C)$ is called strong or pure neutrosophic code if $0 \neq y$ is neutrosophic codeword for all $y \in N(C)$.

Example 14 Let $N(C)$ be the linear neutrosophic code of length 3 over the field $F = Z_2 = \{0,1\}$, where

$$N(C) = \{000,III\}$$

Then clearly $N(C)$ is a strong or pure neutrosophic code over the field F.

Theorem 15 Every strong or pure neutrosophic code is trivially a neutrosophic code but the converse is not true.

For converse, let us see the following example.

Example 15 Let $N(C)$ be a neutrosophic code of length 3 over the field $F = Z_2 = \{0,1\}$, where

$$N(C) = \{000,111,III,I'I'I'\}$$

with $I' = 1 + I$. Then clearly $N(C)$ is not a strong or pure neutrosophic code.

Theorem 16 There is one to one correspondence between the codes and strong or pure neutrosophic codes.

Theorem 17 A neutrosophic vector space have codes, neutrosophic codes, and strong or pure neutrosophic codes.

Decoding Algorithem

Definition 25 Let $N(C)$ be a neutrosophic code over the field F and for every $x \in F^n(I)$, the neutrosophic coset of $N(C)$ is defined to be

$$N(C)_C = \{x + c : c \in N(C)\}$$

Theorem 18 Let $N(C)$ be a linear neutrosophic code over the field F and let $y \in F^n(I)$. Then the

neutrosophic codeword x nearest to y is given by $x = y - e$, where e is the neutosophic vector of the least weight in the neutrosophic coset containing y.
if the neutrosophic coset containing y has more than one neutrosophic vector of least weight, then there are more than one neutosophic codewords nearest to y.

Definition 26 Let $N(C)$ be a linear neutrosophic code over the field F. The neutrosophic coset leader of a given neutrosophic coset $N(C)_C$ is defined to be the neutrosophic vector with least weight in that neutrosophic coset.

Theorem 19 Let F be a field and $F(I)$ be the corresponding neutrosophic vector space. If $|F| = q$, then $|F(I)| = q^2$.

Proof It is obvious.

Algorithem

Let $N(C)$ be an $[n, k+1]$-neutrosophic code over the field F_q with $|F_q(I)| = q^2$. As $F_q^{\,n}(I)$ has q^{2n} elements and so there are q^{k+1} elements in the coset of $N(C)$. Therefore the number of distinct cosets of $N(C)$ are $q^{2n-(k+1)}$. Let the coset leaders be denoted by $e_1, e_2, \ldots, e_N$, where $N = q^{2n-(k+1)}$. We also consider that the neutrosophic coset leaders are arranged in ascending order of weight; i.e $w(e_i) \le w(e_{i+1})$ for all i and consequently $e_1 = 0$ is the coset leader of $N(C) = 0 + N(C)$. Let $N(C) = \{c_1, c_2, \ldots, c_M\}$, where $M = q^{k+1}$ and $c_1 = 0$. The q^{2n} vectors can be arranged in an $N \times M$ table, which is given below. In this table the (i, j)-entry is the neutrosophic vector $e_i + c_j$. The elements of the coset $e_i + N(C)$ are in ith row with the coset leader e_i as the first entry. The neutrosophic code $N(C)$ will be placed on the top row. The corresponding table is termed as the standard neutrosophic array for the neutrosophic code $N(C)$.

$e_1 = 0 = c_1$	c_2	$\cdots$	c_j	$\cdots$	c_M
e_2	$e_2 + c_2$	$\cdots$	$e_2 + c_j$	$\cdots$	$e_2 + c_M$
$\vdots$	$\vdots$		$\vdots$		$\vdots$
e_i	$e_i + c_2$	$\cdots$	$e_i + c_j$	$\cdots$	$e_i + c_M$
$\vdots$	$\vdots$		$\vdots$		$\vdots$
e_N	$e_N + c_2$	$\cdots$	$e_N + c_J$	$\cdots$	$e_N + c_M$

Table 1.

For decoding, the standard neutrosophic array can be used as following:
Let us suppose that a neutrosophic vector $y \in F_q^{\,n}(I)$ is recieved and then look at the position of y in the table. In the table if y is the (i, j)-entry, then $y = e_i + c_j$ and also e_i is the neutrosophic vector of least weight in the neutrosophic coset and by theoem it follows that $x = y - e_i = c_j$. Hence the recieved neutrosophic

vector y is decoded as the neutrosophic codeword at the top of the column in which y appears.

Definition 27 If a neutrosophic codeword x is transmitted and the neutosophic vector y is received, then $e = y - x$ is called neutrosophic error vector. Therefore a neutrosophic coset leader is the neutrosophic error vector for each neutrosophic vector y lying in that neutrosophic coset.

Example 16 Let $F_2^{\,2}(I)$ be a neutrosophic vector space over the field $F = Z_2 = \{0,1\}$, where

$$F^2{}_2(I) = \begin{Bmatrix} 00, 01, 0I, 0I', 10, 1I, 1I', \\ I0, I1, II, II', I'0, I'1, I'I, I'I' \end{Bmatrix}$$

with $I' = 1 + I$. Let $N(C)$ be a neutrosophic code over the field $F = Z_2 = \{0,1\}$, where

$$N(C) = \{00, 11, II, I'I'\}$$

The following are the neutrosophic cosets of $N(C)$:

$$00 + N(C) = N(C),$$
$$01 + N(C) = \{01, 10, II', I'I\} = 10 + N(C),$$
$$0I + N(C) = \{0I, 1I', I0, I'1\} = I0 + N(C),$$
$$0I' + N(C) = \{0I', 1I, I1, I'0\} = I'0 + N(C)$$

The standard neutrosophic array table is given as under;

00	11	II	$I'\ I'$
01	10	I I'	I' I
0I	1 I'	I0	I' 1
0 I'	1I	I1	I' 0

Table 2.

We want to decode the neutrosophic vector $1I'$. Since $1I'$ occures in the second coloumn and the top entry in that column is 11. Hence $1I'$ is decoded as the neutrosophic codeword 11.

Sydrome Decoding

Definition 28 Let $N(C)$ be an $[n, k+1]$-neutrosophic code over the field F with neutrosophic parity-check matrix $N(H)$. For any neutrosophic vector $y \in F_q^{\,n}(I)$, the syndrome of y is denoted by $S(y)$ and is defined to be

$$S(y) = yN(H)^T$$

Definition 29 A table with two columns showing the coset leaders e_i and the corresponding syndromes $S(e_i)$ is called syndrome table.
To decode a recieved neutrosophic vector y, compute the syndrome $S(y)$ and then find the neutrosophic coset leader e in the table for which $S(e)=S(y)$. Then y is decoded as $x=y-e$. This algorithem is known as syndrome decoding.

Example 17 Let $F_2^2(I)$ be a neutrosophic vector space over the field $F=Z_2=\{0,1\}$, where

$$F^2{}_2(I)=\begin{Bmatrix}00,01,0I,0I',10,1I,1I', \\ I0,I1,II,II',I'0,I'1,I'I,I'I'\end{Bmatrix}$$

with $I'=1+I$.
Let $N(C)$ be a neutrosophic code over the field $F=Z_2=\{0,1\}$, where

$$N(C)=\{00,11,II,I'I'\}$$

The neutosophic parity-check matrix of $N(C)$ is

$$N(H)=\begin{bmatrix}1 & 1\\ I & I\end{bmatrix}$$

First we find the neutrosophic cosets of $N(C)$:

$$00+N(C)=N(C),$$
$$01+N(C)=\{01,10,II',I'I\}=10+N(C),$$
$$0I+N(C)=\{0I,1I',I0,I'1\}=I0+N(C),$$
$$0I'+N(C)=\{0I',1I,I1,I'0\}=I'0+N(C)$$

These are the neutrosophic cosets of $N(C)$. After computing the syndrome $eN(H)^T$ for every coset leader, we get the following syndrome table.

Coset leaders	Syndrome
00	00
01	$1I$
$0I$	II
$0I'$	$I'0$

Table 3.

Let $y=10$, and we want do decode it. So

$$S(y) = yN(H)^T = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}1 & I \\ 1 & I\end{bmatrix}$$
$$= \begin{bmatrix}1 & I\end{bmatrix}$$

Hence $S(10) = S(01)$ and thus y is decoded as the neutrosophic codeword

$$x = y - e = 10 - 01$$
$$x = 11$$

Thus we can find all the decoding neutrosophic codewords by this way.

Advantages and Betterness of Neutrosophic code

1) The code rate of a neutrosophic code is better than the ordinary code. Since the code rate of a neutrosophic code is $\frac{k+1}{n}$, while the code rate of ordinary code is $\frac{k}{n}$.
2) The redundancy is decrease in neutrosophic code as compared to ordinary codes. The redundancy of neutrosophic code is $n-(k+1)$, while the redundancy of ordinary code is $n-k$.
3) The number of neutrosophic codewords in neutrosophic code is more than the number of codewords in ordinary code.
4) The minimum distance remains same for both of neutrosophic codes as well as ordinary codes.

Conclusion

In this paper we initiated the concept of neutrosophic codes which are better codes than other type of codes. We first construct linear neutrosophic codes and gave illustrative examples. This neutrosophic algebriac structure is more rich for codes and also we found the containement of corresponding code in neutrosophic code. We also found new types of codes and these are pseudo neutrosophic codes and strong or pure neutrosophic codes. By the help of examples, we illustrated in a simple way. We established the basic results for neutosophic codes. At the end, we developed the decoding proceedures for neutrosophic codes.

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