A Combined Use of Soft and Neutrosophic Sets for Student Assessment with Qualitative Grades

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Abstract: A hybrid assessment method of a group's overall performance with respect to a certain activity is developed in this paper using soft and neutrosophic sets as tools and it is applied for student assessment. The present method is compared with another method developed in an earlier authors' work, which uses soft sets and grey numbers as tools for the assessment.

Keywords: Fuzzy Set (FS), Neutrosophic Set (NS), Soft Set (SS), Assessment under fuzzy conditions

1. Introduction

The assessment of human or machine activities is a very important process, because it helps to correct mistakes and improve performance. Assessment takes place in two ways, either with the help of numerical or with the help of qualitative grades. The second way is usually preferred when more elasticity is desirable (as it frequently happens, for example, in case of student assessment), or when no exact numerical data are available.

When numerical grades are used, standard methods are applied for the overall assessment of the skills of a group of objects participating in a certain activity, like the calculation of the mean value of all the individual grades. In earlier works we have developed a series of methods for assessment with qualitative grades (therefore under fuzzy conditions), most of which are reviewed in [1]. Recently we have also develop a hybrid method for assessing analogical reasoning skills that uses soft sets and grey numbers (closed real intervals) as tools [2].

In the present paper we develop a new hybrid assessment method using soft and neutrosophic sets as tools and we apply it to student assessment. Our new method is compared with the previous one [2], to emphasize the different information given to the user in each case.. The rest of the paper is organized as follows: Section 2 contains the background information about grey numbers, neutrosophic sets and soft sets needed for the better understanding of the paper. The new hybrid assessment method is developed in Section 3 and is compared to the method developed in [2]. The paper closes with the final conclusions and some hints for future research, which are presented in its last Section 4.

2. Background Information

2.1 Grey Numbers

Deng [3] introduced in 1982 the theory of *grey systems* as a new tool for dealing with the uncertainty created by the use of approximate data. A system is characterized as grey if it lacks information about its structure, operation and/or its behavior. The use of *grey numbers (GNs)* is the tool for performing the necessary calculations in grey systems.

A GN A, is an interval estimate [x, y] of a real number, whose exact value within [x, y] is not known. We write then $A \in [x, y]$. A GN is frequently accompanied by a *whitenization function* g: $[x, y] \to [0, 1]$, such that, if $a \in [x, y]$, then the closer g(a) to 1, the better a approximates the unknown exact value of the GN. If no whitenization function is defined (then the GN coincides with the *closed real interval* [x, y]), it is logical to consider as a crisp approximation of unknown number A the real number

$$V(A) = \frac{x+y}{2} \quad (1)$$

The arithmetic operations on GNs are defined with the help of the known arithmetic of the real intervals [4]. In this work we'll only make use of the addition of GNs and of the scalar product of a GN with a positive number, which are defined as follows:

Let $A \in [x_1, y_1]$, $B \in [x_2, y_2]$ be two GNs and let k be a positive number. Then:

- The sum: A+B is the GN $A+B \in [x_1+y_1, x_2+y_2]$ (2)
- The scalar product kA is the GN $kA \in [kx_1, ky_1]$ (3)

2.2 Neutrosophic Sets

Zadeh defined the concept of *fuzzy set (FS)* in 1965 as follows [5]:

Definition 1: Let U be the universal set of the discourse, then a FS A in U is defined with the help of its *membership function* m: $U \to [0,1]$ as the set of the ordered pairs

$$A = \{(x, m(x)): x \in U\} \quad (4)$$

The real number m(x) is called the *membership degree* of x in A**.** The greater m(x), the more x satisfies the characteristic property of A. A crisp subset A of U is a FS on U with membership function taking the values m(x)=1 if x belongs to A and 0 otherwise. Whereas probability theory is suitable for tackling the uncertainty due to *randomness* (e.g. games of chance), FSs tackle successfully the uncertainty due to *vagueness*. Vagueness is created when one is unable to clearly differentiate between two classes, such as "a good player" and "a mediocre player". For general facts on FSs we refer to [6].

Atanassov in 1986 added to Zadeh's membership degree the *degree of non-membership* and introduced the concept of *intuitionistic fuzzy set (IFS)* as follows [7]:

Definition 2: An IFS A in the universe U is defined with the help of its membership function m: $U \to [0,1]$ and of its non-membership function n: $U \to [0,1]$ as the set of the ordered triples

$$A = \{(x, m(x), n(x)): x \in U, 0 \le m(x) + n(x) \le 1\} \quad (5)$$

One can write m(x) + n(x) + h(x) = 1, where h(x) is called the *hesitation* or *uncertainty degree* of x. When h(x) = 0, then the corresponding IFS is reduced to an ordinary FS. An IFS promotes the intuitionistic idea, as it incorporates the degree of hesitation.

For example, if A is the IFS of the diligent students of a class and (x, 0.7, 0.2) ∈ A, then there is a 70% probability for the student x to be diligent, a 20% probability to be not diligent, and a 10% hesitation to be characterized as either diligent or not.

IFSs, simulate successfully the existing *imprecision* in human thinking. For general facts on IFSs we refer to [8].

Smarandache, motivated by the various neutral situations appearing in real life - like <friend, neutral, enemy>, <positive, zero, negative>, <small, medium, high>, <male, transgender, female>, <win, draw, defeat>, etc. - introduced in 1995 the degree of *indeterminacy/neutrality* of the elements of the universal set U in a subset of U and defined the concept of NS as follows [9]:

Definition 3: A *single valued NS (SVNS)* A in U is of the form

$$A = \{(x,T(x),I(x),F(x)): x \in U,\ T(x),I(x),F(x) \in [0,1],\ 0 \le T(x)+I(x)+F(x) \le 3\} \quad (6)$$

In (6) T(x), I(x), F(x) are the degrees of *truth* (or membership), indeterminacy and *falsity* (or non-membership) of x in A respectively, called the *neutrosophic components* of x. For simplicity, we write A<T, I, F>.

The term "neutrosophy" comes from the adjective "neutral΄ and the Greek word "sophia" (wisdom) and means "the knowledge of neutral thought".

For example, let U be the set of the players of a basketball team and let A be the SVNS of the good players of U. Then each player x of U is characterized by a *neutrosophic triplet* (t, i, f) with respect to A, with t, i, f in [0, 1]. For example, x(0.7, 0.1, 0.4) ∈ A means that there is a 70% probability for x to be a good player, a 10% doubt if x could be characterized as a good player and a 40% probability for x to be a not a good player. In particular, x(0,1,0)∈ A means that we do not know absolutely nothing about x's affiliation with A.

Indeterminacy is understood to be in general everything which is between the opposites of truth and falsity [10]. In an IFS the indeterminacy coincides by default to hesitancy, i.e. we have I(x)=1- T(x) – F(x). Also, in a FS is I(x)=0 and F(x) = 1 – T(x), whereas in a crisp set is T(x)=1 (or 0) and F(x)= 0 (or 1). In other words, crisp sets, FSs and IFSs are special cases of SVNSs.

For general facts on SVNSs new refer to [11]

When the sum T(x) + I(x) + F(x) of the neutrosophic components of x ∈ U in a SVNS A on U is <1, then it leaves room for incomplete information about x, when is equal to 1 for complete information and when is greater than 1 for *paraconsistent* (i.e. contradiction tolerant) information about x. A SVNS may contain simultaneously elements leaving room to all the previous types of information.

When T(x) + I(x) + F(x)<1, ∀ x ∈ U, then the corresponding SVNS is usually referred as *picture FS (PiFS)* [12]. In this case 1- T(x)-I(x)-F(x) is called the degree of *refusal membership* of x in A. The PiFSs based models are adequate in situations where we face human opinions involving answers of types yes, abstain, no and refusal. Voting is a representative example of such a situation.

The difference between the *general definition of a NS* and the previously given definition of a SVNS is that in the general definition T(x), I(x) and F(x) may take values in the *non-standard unit interval*]−0, 1+[(including values <0 or >1). This could happen in real situations. For example, in a company with full-time work for its employees 35 hours per week, an employee, with respect to his/her work, belongs by $\frac{35}{35}=1$ to the company (full-time job) or by $\frac{20}{35}<1$ (part-time job) or by $\frac{40}{35}>1$ (over-time job). Assume further that a full-time employee caused a damage to his/her job's equipment, the cost of which must be taken from his salary. Then, if the cost is equal to $\frac{40}{35}$ of his/her weekly salary, the employee belongs this week to the company by $-\frac{5}{35}<0$.

NSs, apart from vagueness, manage as well the cases of uncertainty due to *ambiguity* and *inconsistency*. In the former case the existing information leads to several interpretations by different observers. For example, the statement "Boy no girl" written as "Boy, no girl" means boy, but written as "Boy no, girl" means girl. Inconsistency, on the other hand, appears when two or more pieces of information cannot be true at the same time. As a result the obtainable in this case information is conflicted or undetermined. For example, "the probability for being windy tomorrow is 90%, but this does not mean that the probability for not having strong winds is 10%, because they might be hidden meteorological conditions".

2.3 Soft Sets

A disadvantage connected to FSs is that there is not any exact rule for defining properly the membership function. The methods used are usually empirical or statistical and the definition of the membership function is not unique depending on the "signals" that each one receives from the environment, which are different from person to person. For example, defining the FS of "young people" one could consider as young all those being less than 30 years old and another one all those being less than 40 years old. As a result the two observers will assign different membership degrees to people with ages below those two upper bounds. The only restriction for the definition of the membership function is to be compatible to common logic; otherwise the resulting FS does not give a reliable description of the corresponding real situation. This could happen for instance, if in the FS of "young people", people aged over 70 years possess membership degrees ≥ 0.5.

The same difficulty appears to all generalizations of FSs in which membership functions are involved (e.g. IFSs, NSs, etc.). For this reason, the concept of *interval-valued FS (IVFS)* was introduced in 1975 [13], in which the membership degrees are replaced by sub-intervals of the unit interval [0, 1]. Alternative to FSs theories were also proposed, in which the definition of a membership function is either not necessary (grey systems/GNs [3]), or it is overpassed by considering a pair of sets which give the lower and the upper approximation of the original crisp set (*rough sets* [14]).

Molodstov, in order to deal with the uncertainty in a parametric manner, initiated in 1999 the concept of *soft set (SS)* as follows [15]:

Definition 4: Let E be a set of parameters, let A be a subset of E, and let f be a map from A into the power set P(U) of all subsets of the universe U. Then the SS (f, A) in U is defined to be the set of the ordered pairs

$$(f, A) = \{(e, f(e)): e \in A\} \quad (7)$$

The name "soft" was given because the form of (f, A) depends on the parameters of A. For each e ∈ A, its image f(e) is called the *value set* of e in (f, A), while f is called the *approximation function* of (f, A).

Example 1: Let U= $\{C_1, C_2, C_3\}$ be a set of cars and let E = $\{e_1, e_2, e_3\}$ be the set of the parameters e_1=cheap, e_2=hybrid (petrol and electric power) and e_3= expensive. Let us further assume that C_1, C_2 are cheap, C_3 is expensive and C_2, C_3 are the hybrid cars. Then, a map f: E $\rightarrow$ P(U) is defined by $f(e_1)=\{C_1, C_2\}$, $f(e_2)=\{C_2, C_3\}$ and $f(e_3)=\{C_3\}$. Therefore, the SS (f, E) in U is the set of the ordered pairs (f, E) = $\{(e_1, \{C_1, C_2\}), (e_2, \{C_2, C_3\}, (e_3, \{C_3\}\}$.

A FS in U with membership function y = m(x) is a SS in U of the form (f, [0, 1]), where

$f(\alpha)=\{x \in U: m(x) \geq \alpha\}$ is the corresponding $\alpha - cut$ of the FS, for each α in [0, 1].

As it has been already mentioned, an important advantage of SSs is that, by using the parameters, they pass through the existing difficulty of defining properly membership functions.

For general facts on soft sets we refer to [16].

3. The Hybrid Assessment Method

3. Assessment Using Soft Sets and Grey Numbers

We illustrate this method, developed in [2], with the following example:

Example 2: The teacher of mathematics of a high-school class consisting of 20 students evaluated their mathematical skills as follows: The first three of them are excellent students, the next five very good, the next six good, the following four mediocre students and the last two demonstrated a non-satisfactory performance. It is asked:

1. To represent the mathematical performance of the class in a parametric manner.
2. To estimate the mean mathematical level of the class.

Solution: 1. Let U = $\{s_1, s_2, \ldots, s_{20}\}$ be the set of the students of the class and let E = {A, B, C, D, E} be the set of the qualitative grades (parameters) A=excellent, B=very good, C=good, D=mediocre and F=not satisfactory. Then a function f: E → P(U) can be defined by f(A) = $\{s_1, s_2, s_3\}$, f(B) = $\{s_4, s_5, s_6, s_7, s_8\}$, f(C) = $\{s_9, s_{10}, s_{11}, s_{12}, s_{13}, s_{14}\}$, f(D) = $\{s_{15}, s_{16}, s_{17}, s_{18}\}$ and f(B) = $\{s_{19}, s_{20}\}$. A parametric representation of the performance of the class in mathematics is given, therefore, by the SS in U

(f, E) = {(A, f(A)), (B, f(B)), (C, f(C)), (D, f(D)), (F, f(F))}.

2. Translating the qualitative grades of E in the numerical scale 0-100 we assign to each qualitative grade a closed real interval (GN), denoted for simplicity by the same letter, as follows: A=[85, 100], B=[75, 84], C=[60, 74], D=[50, 59] and F=[49, 0]. Obviously, although it was performed according to generally accepted standards, this assignment is not unique, depending on the user's personal goals (more strict or more elastic assessment).

It is logical now to consider as a representative of the student mean performance the real interval M = $\frac{1}{20}$(3A+ 5B+6C+4D+2F). Using equations (2) and (3) it is straightforward to check that M = $\frac{1}{20}$[1190, 1498] = [59.5, 74.9]. Therefore, by equation (1) one finds that V(M)=67.2, which shows that the mean mathematical level of the class is good (C).

Remark 1: Case 2 of Example 2 could be also solved by using *triangular fuzzy numbers (TFNs)* instead of GNs. It can be shown that these two methods are equivalent to each other ([1], Sections 5, 6 and Remark 3-(1)).

3.2 Arithmetic Operations in Neutrosophic Sets

Writing the elements of a SVNS A in the form of neutrosophic triplets we define addition and scalar product in A as follows:
Let (t_1, i_1, f_1), (t_2, i_2, f_2) be in A and let k be appositive number. Then;

- The sum (t_1, i_1, f_1) + (t_2, i_2, f_2) = (t_1+ t_2, i_1+ i_2, f_1+ f_2) (8)
- The scalar product k(t_1, i_1, f_1) = (kt_1,k i_1, kf_1) (9)

3.3 Assessment Using Soft Sets and Neutrosophic Sets

We illustrate our new hybrid assessment method with the following example:
Example 3: Reconsider Example 2 and assume that the teacher is not sure about the individual assessment of the student mathematical skills. He/she decides, therefore, to characterize the set of excellent students using neutrosophic triplets as follows: s_1(1, 0, 0), s_2(0.9, 0.1, 0.1), s_3(0.8, 0.2, 0.1), s_4(0.4, 0.5, 0.8), s_5(0.4, 0.5, 0.8), s_6(0.3, 0.7, 0.8), s_7(0.3, 0.7, 0.8), s_8(0.2, 0.8, 0.9), s_9(0.1, 0.9, 0.9), s_{10}(0.1, 0.9, 0.9} and all the other students by (0, 0, 1). This means that the teacher is absolutely sure that s_1 is an excellent student, 90% sure that s_2 is an excellent student too, but he/she has a 10% doubt about it and there is also a 10% probability to be not excellent, etc. For the last 10 students the teacher is absolutely sure that they cannot be characterized as excellent.

1. It is asked to represent the mathematical performance of the class in a parametric manner.
2. What should be the teacher's conclusion about the class's mean mathematical level in this case?

Solution: 1. Work as in case 1 of Example 2.
2. It is logical to accept in this case that the class's mean mathematical level can be estimated by the neutrosophic triplet $\frac{1}{20}$[(1, 0, 0)+(0.9, 0.1, 0.1)+(0.8, 0.2, 0.1)+2(0.4, 0.5, 0.8)+2(0.3, 0.7, 0.8)+(0.2, 0.8, 0.9)+2(0.1, 0.9, 0.9)+10(0, 0, 1)], which by equations (8) and (9) is equal to $\frac{1}{20}$(4.5, 5.3, 16.3) = (0.225, 0.265, 0.815). This means that a random student of the class has a 22.5 % probability to be an excellent student, however, there exist also a 26.5% doubt about it and an 81.5% probability to be not an excellent student. Obviously this conclusion is characterized by inconsistency.

Remark 2: The teacher could work in the same way by considering the NSs of the very good, good, mediocre and weak students and get analogous results.

3.4 Comparison of the Assessment Methods

The use of SS enables in both cases a parametric/qualitative assessment of the class's performance. The use of GNs is appropriate when the teacher is absolutely sure for the assessment of the individual performance of each student and gives a creditable approximation of the class's mean performance. The use of NSs, on the contrary, is appropriate when the teacher has doubts about the student individual assessment. In this case, the information obtained depends on the choice of the corresponding NS (e.g. excellent students, good students, etc.) and it is possible to be characterized by inconsistency (e.g. in Example 3 a random student of the class has a 22.5 % probability to be an excellent student, but at the same timer a 81.5% probability to be not an excellent student).

4. Conclusions and Hints for Future Research

In the present paper two hybrid assessment methods under fuzzy conditions (with qualitative grades) were studied. The discussion performed leads to the following three important conclusions:

- The use of SSs enables a parametric/qualitative assessment of the class's overall performance.
- The use of closed real intervals (GNs) is suitable when the teacher is absolutely sure for the assessment of the individual performance of each student and gives a creditable approximation of the class's mean performance. Obviously, this approach is very useful when the performance of two or more classes must be compared.
- The use of NSs is suitable when the teacher has doubts about the student individual assessment. In this case, the information obtained depends on the choice of the corresponding NS (e.g. excellent students, good students, etc.) and it is possible to be characterized by inconsistency.

The results obtained in this and in older authors' papers (e.g. [2, 17], etc.) show that frequently a combination of two or more of the theories developed for dealing with the existing in real world fuzziness (e.g. see [18]) gives better results, not only in assessment cases, but also in decision making, for tackling the uncertainty, and possibly in many other human or machine activities. This is, therefore, a promising area for future research.

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