On the Symbolic 2-Plithogenic Rings

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Abstract

The objective of this paper is to study for the first time the algebraic properties of symbolic 2-plithogenic rings generated from the fusion of symbolic plithogenic sets with algebraic rings, where we study some of the elementary properties and substructures of symbolic 2-plithogenic rings such as AH-ideals, AH-homomorphisms, and AHS-isomorphisms. Also, the idempotency and nilpotency of symbolic 2-plithogenic elements in terms of theorems have been discussed. Besides, many examples to clarify the validity of our work have been covered.

Keywords: Symbolic plithogenic set; 2-plithogenic ring; AH-ideal; AH-homomorphism

1. Introduction

Algebraic structures of all kinds are useful tools in many scientific disciplines, especially those related to theoretical mathematics, topology, data analysis, and cryptographic theory.
The most famous examples of the use of symbolization in the generalization of algebraic structures are neutrosophic rings, refined neutrosophic rings, neutrosophic spaces, matrices, and Turiyam structures [1-12,19-20, 22-29].
The concept of symbolic plithogenic structures was presented by Smarandache [13-18], in a similar way of n-refined neutrosophic structures with some differences in the definition of multiplication operation [30].
The n-plithogenic number is defined as follows:
$PN = a_0 + a_1P_1 + \cdots . +a_nP_n$, with a_i are taken from any algebraic structure.
For each value of the positive integer n, we get a new approach for a different algebraic structure. In this work, we study the 2-plithogenic rings by many algebraic aspects, where we present the concept of 2-plithogenic AH-ideal, 2-plithogenic AH-homomorphism, and 2-plithogenic powers and nilpotency. Besides, we support this work with several examples illustrating the ideas and concepts that were put forward and discussed.

Main Concepts and Discussion

Definition.

Let R be a ring, the symbolic 2-plithogenic ring is defined as follows:
$2 - SP_R = \{a_0P_0 + a_1P_1 + a_2P_2;\ a_i \in R, {P_j}^2 = P_j, P_1 \times P_2 = P_{max(1,2)} = P_2\}$.
Smarandache has defined algebraic operations on $2 - SP_R$ as follows:
Addition:
$[a_0 + a_1P_1 + a_2P_2] + [b_0 + b_1P_1 + b_2P_2] = (a_0 + b_0) + (a_1 + b_1)P_1 + (a_2 + b_2)P_2$.
Multiplication:
$[a_0 + a_1P_1 + a_2P_2].[b_0 + b_1P_1 + b_2P_2] = a_0b_0 + a_0b_1P_1 + a_0b_2P_2 + a_1b_0{P_1}^2 + a_1b_2P_1P_2 + a_2b_0P_2 + a_2b_1P_1P_2 + a_2b_2{P_2}^2 + a_1b_1P_1P_1 = a_0b_0 + (a_0b_1 + a_1b_0 + a_1b_1)P_1 + (a_0b_2 + a_1b_2 + a_2b_0 + a_2b_1 + a_2b_2)P_2$.
It is clear that $(2 - SP_R)$ is a ring.
Also, if R is commutative, then $2 - SP_R$ is commutative, and if R has a unity (1), than $2 - SP_R$ has the same unity (1).

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Example.
Consider the ring $R = Z_4 = \{0,1,2,3\}$, the corresponding $2 - SP_R$ is:
$2 - SP_R = \{a + bP_1 + cP_2; a, b, c \in Z_4\}$.
If $X = 1 + 2P_1 + 3P_2, Y = P_1 + 2P_2$, then:
$X + Y = 1 + 3P_1 + P_2, X - Y = 1 + P_1 + P_2, X.Y = P_1 + 2P_2 + 2P_1 + 4P_2 + 3P_2 + 6P_2 = 3P_1 + 3P_2$.
Invertibility.
Theorem.
Let $2 - SP_R$ be a 2-plithogenic symbolic ring, with unity (1).
Let $X = x_0 + x_1P_1 + x_2P_2$ be an arbitrary element, then:

1. X is invertible if and only if $x_0, x_0 + x_1, x_0 + x_1 + x_2$ are invertible.
2. $X^{-1} = x_0{}^{-1} + [(x_0 + x_1)^{-1} - x_0{}^{-1}]P_1 + [(x_0 + x_1 + x_2)^{-1} - (x_0 + x_1)^{-1}]P_2$.

Proof.

1. Assume that X is invertible, than there exists $Y = y_0 + y_1P_1 + y_2P_2$ such that $X.Y = 1$, hence:

$$\begin{cases} x_0y_0 = 1 \ldots (1) \\ x_0y_1 + x_1y_0 + x_1y_1 = 0 \ldots (2) \\ x_0y_2 + x_2y_0 + x_2y_2 + x_1y_2 + x_2y_1 = 0 \ldots (3) \end{cases}$$

Equation (1), means that x_0 is invertible.
By adding (1) to (2), we get $(x_0 + x_1)(y_0 + y_1) = 1$, thus $x_0 + x_1$ is invertible.
By adding (1) to (2) to (3), we get $(x_0 + x_1 + x_2)(y_0 + y_1 + y_2) = 1$, hence $x_0 + x_1 + x_2$ is invertible. The converse holds by the same.

2. From the previous approach, we can see that:

$y_0 = x_0{}^{-1}, y_0 + y_1 = (x_0 + x_1)^{-1}, y_0 + y_1 + y_2 = (x_0 + x_1 + x_2)^{-1}$, then:
$Y = x_0{}^{-1} + [(x_0 + x_1)^{-1} - x_0{}^{-1}]P_1 + [(x_0 + x_1 + x_2)^{-1} - (x_0 + x_1)^{-1}]P_2 = X^{-1}$.
Example.
Take $R = Z_5 = \{0,1,2,3,4\}$, $2 - SP_{Z_5}$ is the corresponding symbolic 2-plithogenic ring, consider $X = 2 + 4P_1 + 2P_2 \in 2 - SP_{Z_5}$, then:
$x_0 = 2$ is invertible with $x_0{}^{-1} = 3, x_0 + x_1 = 1$ is invertible with $(x_0 + x_1)^{-1} = 1$, $x_0 + x_1 + x_2 = 3$ is invertible with $(x_0 + x_1 + x_2)^{-1} = 2$, hence:
$X^{-1} = 3 + (1 - 3)P_1 + (2 - 1)P_2 = 3 + 3P_1 + P_2$.
Idempotency.
Definition.
Let $X = a + bP_1 + cP_2 \in 2 - SP_R$, then X is idempotent if and only if $X^2 = X$.
Theorem.
Let $X = a + bP_1 + cP_2 \in 2 - SP_R$, then X is idempotent if and only if $a, a + b, a + b + c$ are idempotent.
Proof.
$X^2 = X.X = (a + bP_1 + cP_2)(a + bP_1 + cP_2) = a^2 + (ab + ba + b^2)P_1 + (ac + bc + ca + cb + c^2)P_2$

$$X^2 = X.X \text{ equivalents } \begin{cases} a^2 = a \ldots (1) \\ ab + ba + b^2 = b \ldots (2) \\ ac + bc + ca + cb + c^2 = c \ldots (3) \end{cases}$$

Equation (1) means that a is idempotent.
By adding (1) to (2), we get $(a + b)^2 = a + b$, hence $a + b$ is idempotent.
By adding (1) to (2) to (3), we get $(a + b + c)^2 = a + b + c$, hence $a + b + c$ is idempotent, thus the proof is complete.
Example.
Take $R = Z_6 = \{0,1,2,3,4,5\}$, $2 - SP_{Z_6}$ is the corresponding symbolic 2-plithogenic ring, consider $X = 3 + P_1 + 5P_2 \in 2 - SP_{Z_5}$, we have:
$X^2 = 9 + 6P_1 + P_1 + 30P_2 + 25P_2 + 10P_2 = 3 + P_1 + 5P_2 = X$.
Theorem.
Let $2 - SP_R$ be a commutative symbolic 2-plithogenic ring, hence if $X = a + bP_1 + cP_2$, then $X^n = a^n + [(a + b)^n - a^n]P_1 + [(a + b + c)^n - (a + b)^n]P_2$ for every $n \in Z^+$.
Proof.
For $n = 1$, it holds easily. Assume that it is true for $n = k$, we prove it for $n = k + 1$.

$$\begin{aligned} X^{k+1} = X.X^k &= (a + bP_1 + cP_2)(a^k + [(a + b)^k - a^k]P_1 + [(a + b + c)^k - (a + b)^k]P_2) \\ &= a^{k+1} + [a(a + b)^k - a^{k+1} + ba^k + b(a + b)^k - ba^k]P_1 \\ &+ [a(a + b + c)^k - a(a + b)^k + ca^k + c(a + b)^k - ca^k + c(a + b + c)^k - c(a + b)^k]P_2 \\ &= a^{k+1} + [(a + b)^{k+1} - a^{k+1}]P_1 + [(a + b + c)^{k+1} - (a + b)^{k+1}]P_2 \end{aligned}$$

So, that proof is complete by induction.

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Example.

Take $R = Z$, the ring of integers. Let $2 - SP_Z$ be the corresponding symbolic 2-plithogenic ring, hence $X = 1 + 2P_1 + 3P_2, X^3 = 1^3 + P_1[(3)^3 - 1^3] + P_2[(6)^3 - 3^3] = 1 + 26P_1 + 189P_2$

Definition.

X is called nilpotent if there exists $n \in Z^+$ such that $X^n = 0$.

Theorem.

Let $X \in 2 - SP_R$, where R is a commutative ring, then X is nilpotent id and only if $a, a + b, a + b + c$ are nilpotent.

Proof.

$X = a + bP_1 + cP_2$ is nilpotent if and only if there exists $n \in Z^+$ such that $X^n = 0$, hence:

$$\begin{cases} a^n = 0 \\ (a+b)^n - a^n = 0 \\ (a+b+c)^n - (a+b)^n = 0 \end{cases} \Leftrightarrow \begin{cases} a^n = 0 \\ (a+b)^n = 0 \\ (a+b+c)^n = 0 \end{cases}$$, thus the proof is complete.

Example.

Take $R = Z_4 = \{0,1,2,3\}$, find all nilpotent elements in $2 - SP_{Z_4}$.

Let $X = a + bP_1 + cP_2 \in 2 - SP_{Z_4}$, then X is nilpotent if and only if $a, a + b, a + b + c \in \{0,2\}$, we discuss the possible cases:

Case 1.

$a = a + b = a + b + c = 0$, this implies $X = 0$.

Case 2.

$a = 0, a + b = 0, a + b + c = 2$, this implies $X = 2P_2$.

Case 3.

$a = 0, a + b = 2, a + b + c = 0$, this implies $X = 2P_1 + 2P_2$.

Case 4.

$a = 2, a + b = 0, a + b + c = 0$, this implies $X = 2 + 2P_1$.

Case 5.

$a = 2, a + b = 2, a + b + c = 2$, this implies $X = 2$.

Case 6.

$a = 2, a + b = 2, a + b + c = 0$, this implies $X = 2 + 2P_2$.

Case 7.

$a = 2, a + b = 2, a + b + c = 2$, this implies $X = 2 + 2P_1 + 2P_2$.

Case 8.

$a = 0, a + b = 2, a + b + c = 2$, this implies $X = 2P_1$.

The ring $2 - SP_{Z_4}$ has exactly 8 nilpotent elements**.**

Definition.

Let Q_0, Q_1, Q_2 be ideals of the ring R, we define the symbolic 2-plithogenic AH-ideal as follows:

$Q = Q_0 + Q_1P_1 + Q_2P_2 = \{x_0 + x_1P_1 + x_2P_2;\ x_i \in Q_i\}$.

If $Q_0 = Q_1 = Q_2$, then Q is called an AHS-ideal.

Example.

Let $R = Z$ be the ring of integers, then $Q_0 = 2Z, Q_1 = 3Z, Q_2 = 5Z$ are ideals of R.

$Q = \{2m + 3nP_1 + 5tP_2;\ m.n.t \in Z\}$ is an AHS-ideal of $2 - SP_Z$.

$M = \{2m + 2nP_1 + 2tP_2;\ m.n.t \in Z\}$ is an AHS-ideal of $2 - SP_Z$.

Theorem.

Let Q be an AHS- ideal of $2 - SP_R$, then Q is an ideal by the classical meaning.

Proof.

Q can be written as $Q = Q_0 + Q_0P_1 + Q_0P_2$, where Q_0 is an ideal of R. It is clear that $(Q, +)$ is a subgroup of $(2 - SP_R, +)$.

Let $S = s_0 + s_1P_1 + s_2P_2 \in 2 - SP_R$, then if $X = a + bP_1 + cP_2 \in Q$, we have:

$S.X = s_0a + (s_0b + s_1a + s_1b)P_1 + (s_0c + s_1c + s_2a + s_2b + s_2c)P_2 \in Q$, that is because:

$s_0a \in Q_0, s_0b + s_1a + s_1b \in Q_0, s_0c + s_1c + s_2a + s_2b + s_2c \in Q_0$.

Definition.

Let R, T be two rings, $2 - SP_R, 2 - SP_T$ are the corresponding symbolic 2-plithogenic rings, let $f_0, f_1, f_2 : R \to T$ be three homomorphisms, we define the AH-homomorphism as follows:

$f : 2 - SP_R \to 2 - SP_T$ such that:

$f(a + bP_1 + cP_2) = f_0(a) + f_1(b)P_1 + f_2(c)P_2$

If $f_0 = f_1 = f_2$, then f is called AHS-homomorphism.

Remark.

If f_0, f_1, f_2 is isomorphisms, then f is called AH-isomorphism.

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Example.
Take $R = Z, T = Z_6, f_0, f_1: R \to T$ such that:
$f_0(x) = x(mod\ 6), f_1(2) = 3x(mod\ 6)$. It is clear that f_0, f_1 are homomorphism.
We define $f: 2 - SP_R \to 2 - SP_T$, where:
$f(x + yP_1 + zP_2) = f_0(x) + f_1(y)P_1 + f_2(z)P_2 = x(mod\ 6) + y(mod\ 6)P_1 + (3z\ mod\ 6)P_2$
Which is an AH-homomorphism.
For example. If $X = 15 + 3P_1 + 4P_2$, we get:
$f(X) = 15(mod\ 6) + (3\ mod\ 6)P_1 + (12\ mod\ 6)P_2 = 3 + 3P_1$.
Theorem.
Let $f = f_0 + f_1P_1 + f_2P_2: 2 - SP_R \to 2 - SP_T$ be a mapping, then:

1. If f is an AHS-homomorphism, then f is a ring homomorphism by the classical meaning.
2. If f is an AHS-homomorphism, then it is an isomorphism by the classical meaning.

Proof.

1. Assume that f is an AHS-homomorphism, then $f_0 = f_1 = f_2$ are homomorphisms.

Let $X = x_0 + x_1P_1 + x_2P_2, Y = y_0 + y_1P_1 + y_2P_2 \in 2 - SP_R$, we have:
$f(X + Y) = f_0(x_0 + y_0) + f_0(x_1 + y_1)P_1 + f_0(x_2 + y_2)P_2 = f(X) + f(Y)$
$f(X.Y) = f_0(x_0y_0) + f_0(x_0y_1 + x_1y_0 + x_1y_1)P_1 + f_0(x_0y_2 + x_2y_0 + x_2y_2 + x_2y_1 + x_1y_2)P_2 = f_0(x_0)f_0(y_0) + \big(f_0(x_0)f_0(y_1) + f_0(x_1)f_0(y_0) + f_0(x_1)f_0(y_1)\big)P_1 + \big(f_0(x_0)f_0(y_2) + f_0(x_2)f_0(y_0) + f_0(x_2)f_0(y_2) + f_0(x_2)f_0(y_1) + f_0(x_1)f_0(y_2)\big)P_2 = [f_0(x_0) + f_0(x_1)P_1 + f_0(x_2)P_2][f_0(y_0) + f_0(y_1)P_1 + f_0(y_2)P_2] = f(X) + f(Y)$.
So that, the [roof is complete.

2. By a similar discussion of statement 1, we get the proof.

Definition.
Let $f = f_0 + f_1P_1 + f_2P_2: 2 - SP_R \to 2 - SP_T$ be an AH-homomorphism, we define:

1. AH-$ker(f) = ker(f_0) + ker(f_1)P_1 + ker(f_2)P_2 = \{m_0 + m_1P_1 + m_2P_2;\ m_i \in ker(f_i)\}$.
2. AH-factor $2 - SP_R/\text{AH} - ker(f) = R/ker(f_0) + R/ker(f_1)\,P_1 + R/ker(f_2)\,P_2$

If $f_0 = f_1 = f_2$, then we get an AHS- $ker(f)$ and AHS-factor.
Example.
Take $R = Z_{10}, f_0: R \to T, f_0(x) = (x\ mod\ 10), ker(f_0) = 10Z$.
The corresponding AHS-homomorphism is $f = f_0 + f_1P_1 + f_2P_2: 2 - SP_R \to 2 - SP_T$, such that:
$f(x_0 + x_1P_1 + x_2P_2) = f_0(x_0) + f_0(x_1)P_1 + f_0(x_2)P_2 = (x_0\ mod\ 10) + (x_1\ mod\ 10)P_1 + (x_2 mod\ 10)P_2$
AHS-$ker(f) = 10Z + 10ZP_1 + 10ZP_2 = \{10x + 10yP_1 + 10zP_2;\ x, y, z \in Z\}$
AHS-factor= $Z/10Z + Z/10Z\,P_1 + Z/10Z\,P_2$
Remark.
AH-$ker(f)$ is an AH-ideal of $2 - SP_R$, that is because $ker(f_0), ker(f_1), ker(f_2)$ are ideals of R.
Definition.
Let $(F, +, .)$ be a field, then $(2 - SP_F, +, .)$ Is called a symbolic 2-plithogenic field.
$(2 - SP_F, +, .)$ Is not a field in the algebraic meaning, that is because P_1, P_2 are not invertible, but it is a ring.
Example.
Let $F = Q$ the field of rational numbers, then the corresponding symbolic 2-plithogenic field
$2 - SP_Q = \{a + bP_1 + cP_2; a, b, c \in Q\}$.
Remark.
The $2 - SP_F$ has only the following AH-ideals:
$\{0\}, 2 - SP_F, FP_1 + FP_2, F + FP_1, F + FP_2, FP_1, FP_2, F$.
That is because the field F has only two ideals $\{0\}$ and F.
Example.
Find all AH-ideals in $2 - SP_C$, where C is the complex field.
Solution.
$L_1 = \{0\}, L_2 = C, L_3 = C + CP_1 = \{x + yP_1; x, y \in C\}, L_4 = C + CP_2 = \{x + yP_2; x, y \in C\}, L_5 = 2 - SP_C$
$L_6 = CP_1 + CP_2 = \{xP_1 + yP_2; x, y \in C\}, L_7 = CP_1 = \{xP_1; x \in C\}, L_8 = CP_2 = \{yP_2; y \in C\}$.
The group of units problem.
It is known that any ring with unity (1) has a special related subgroup under
$U(R) = \{x \in R, x\ is\ invertible\}$
$U(R)$ is a group under multiplication and it is called the group of units.
We have shown before that if $X = a + bP_1 + cP_2 \in 2 - SP_R$ then it is invertible (unit) if and only if $a, a + b, a + b + c$ are invertible (units).
Now, we classify the group of units $U(2 - SP_R)$.
Theorem.
Let $2 - SP_R$ be a symbolic 2-plithogenic ring with unity 1, then $U(2 - SP_R) \cong U(R) \times U(R) \times U(R)$.

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Proof.
Since R is a ring, we can build a refined neutrosophic ring:
$R(I_1, I_2) = \{a + bI_1 + cI_2; a, b, c \in R, I_i^2 = I_i, I_1.I_2 = I_2.I_1 = I_1\}$.
We define $f: R(I_1, I_2) \to 2 - SP_R$, such that $f(a + bI_1 + cI_2) = a + bP_1 + c$.
$\forall\, X = x_0 + x_1I_1 + x_2I_2, Y = y_0 + y_1I_1 + y_2I_2 \in R(I_1, I_2)$, we have:

1. If $X = Y$, then$x_0 = y_0, x_1 = y_1, x_2 = y_2$, thus:
$x_0 + x_1P_1 + x_2P_2 = y_0 + y_1P_1 + y_2P_2$, so that $f(X) = f(Y)$.
2. $f(X + Y) = [f(x_0 + y_0) + f(x_1 + y_1)I_1 + f(x_2 + y_2)I_2] = x_0 + y_0 + x_1 + y_1P_1 + (x_2 + y_2)P_2 = f(X) + f(Y)$.
3. $f(X.Y) = f[(x_0y_0) + (x_0y_1 + x_1y_0 + x_1y_1 + x_1y_2 + x_2y_1)I_1 + (x_0y_2 + x_2y_0 + x_2y_2)I_2] = (x_0y_0) + (x_0y_2 + x_2y_0 + x_2y_2)P_1 + (x_0y_1 + x_1y_0 + x_1y_1 + x_1y_2 + x_2y_1)P_2 = (x_0 + x_2P_1 + x_1P_2)(y_0 + y_2P_1 + y_1P_2) = f(X).f(Y)$

Also, f is a one-to-one mapping, this implies that $R(I_1, I_2) \cong 2 - SP_R$.
According to the literature, $U\big(R(I_1, I_2)\big) \cong U(R) \times U(R) \times U(R)$, thus $U(2 - SP_R) \cong U(R) \times U(R) \times U(R)$.
Remark.
If we have a $2 - SP_F$ (symbolic 2-plithogenic field), then $U(2 - SP_R) \cong F^* \times F^* \times F^*$.
Example.
Find all units in $2 - SP_{Z_4}$.
Solution.
Let $X = a + bP_1 + cP_2$ be a unit in $2 - SP_{Z_4}$, then $a \in \{1,3\}, a + b \in \{1,3\}, a + b + c \in \{1,3\}$, we have the following:
Case 1.
$a = 1, a + b = 1, a + b + c = 1$, then $X = 1$.
Case 2.
$a = 1, a + b = 3, a + b + c = 1$, then $X = 1 + 2P_1 + 2P_2$.
Case 3.
$a = 1, a + b = 1, a + b + c = 3$, then $X = 1 + 2P_2$.
Case 4.
$a = 1, a + b = 3, a + b + c = 3$, then $X = 1 + 2P_1$.
Case 5.
$a = 3, a + b = 1, a + b + c = 1$, then $X = 3 + 2P_1$.
Case 6.
$a = 3, a + b = 3, a + b + c = 3$, then $X = 3$.
Case 7.
$a = 3, a + b = 1, a + b + c = 3$, then $X = 3 + 2P_1 + 2P_2$.
Case 8.
$a = 3, a + b = 3, a + b + c = 1$, then $X = 3 + 2P_2$.
$U\big(2 - SP_{Z_4}\big) = \{1,3,1 + 2P_1, 1 + 2P_2, 1 + 2P_1 + 2P_2, 3 + 2P_1 + 2P_2, 3 + 2P_1, 3 + 2P_2\} \cong Z_2 \times Z_2 \times Z_2$
Open problems.

1. Classify all maximal/minimal ideals in $2 - SP_R$.
2. Find Triplets/Duplets in $2 - SP_R$.
3. Find on algorithm to solve linear/quadratic equations in $2 - SP_F$.
4. Find on algorithm to solve Diophantine equations in $2 - SP_Z$.

An Explanation of the suggested problems:
Problem (1):
Let X be a non-empty subset of the 2-plithogenic ring, when can we say that X is an ideal of $2 - SP_R$, when can we say that X is maximal or minimal ideal, i.e. what are the conditions that make X be maximal or minimal ideals.
Problem (2):
Triplets and duplets are neutrosophic elements with interesting properties. What are the conditions that govern triplets and duplets in the 2-plithogenic rings.
Problem (3):
If F is a field then $2 - SP_F$ is called a 2-plithogenic symbolic field, a very important concept is the linear/quadratic equations. Now, can we find a strong algorithm that explains how can we solve the previous equations by turning it into the classical equations.
Problem (4):

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If Z is the ring of integers ring then $2-SP_Z$ is called a 2-plithogenic symbolic ring of integers, a very important concept is the Diophantine equations. Now, can we find a strong algorithm that explains how can we solve the previous equations by turning it into the classical Diophantine equations.

The following tables clarifies the relationships between 2-plithogenic rings and some finite rings by regarding many points such as the order, the order of the group of units, and the classification of the group of units.

Table 1: A Comparison between some finite 2-plithogenic rings and classical rings

Classical ring	**Its order**	**The Group Of Units Order**	**The Group Of Units classification**	**The Corresponding 2-plithogenic ring**	**Its order**	**The group Of Units Order**	**The Group Of Units classification**
Z_2	2	1	Trivial	$2-SP_{Z_2}$	8	1	Trivial
Z_3	3	2	Z_2	$2-SP_{Z_2}$	27	8	$Z_2{*}Z_2*Z_2$
Z_4	4	2	Z_2	$2-SP_{Z_4}$	64	8	$Z_2{*}Z_2*Z_2$
Z_5	5	4	Z_4	$2-SP_{Z_5}$	125	64	$Z_4{*}Z_4*Z_4$
Z_6	6	2	Z_2	$2-SP_{Z_6}$	216	8	$Z_2{*}Z_2*Z_2$
Z_7	7	6	Z_6	$2-SP_{Z_7}$	343	216	$Z_6{*}Z_6*Z_6$
Z_8	8	4	$Z_2{*}Z_2$	$2-SP_{Z_8}$	512	64	$Z_2{*}Z_2{*}Z_2{*}Z_2*Z_2{*}Z_2$
Z_9	9	6	Z_6	$2-SP_{Z_9}$	729	216	$Z_6{*}Z_6*Z_6$
Z_{10}	10	4	Z_4	$2-SP_{Z_{10}}$	1000	64	$Z_4{*}Z_4*Z_4$

Table 2: A Comparison between some finite 2-plithogenic rings and neutrosophic rings

Neutrosophic ring	**Its order**	**The Group Of Units Order**	**The Group Of Units classification**	**The 2-plithogenic ring**	**Its order**	**The group Of Units Order**	**The Group Of Units classification**
$Z_2(I)$	4	1	Trivial	$2-SP_{Z_2}$	8	1	Trivial
$Z_3(I)$	9	4	Z_2*Z_2	$2-SP_{Z_2}$	27	8	$Z_2{*}Z_2*Z_2$
Z_4(I)	16	4	Z_2*Z_2	$2-SP_{Z_4}$	64	8	$Z_2{*}Z_2*Z_2$
$Z_5(I)$	25	16	Z_4*Z_4	$2-SP_{Z_5}$	125	64	$Z_4{*}Z_4*Z_4$
Z_6(I)	36	4	Z_2*Z_2	$2-SP_{Z_6}$	216	8	$Z_2{*}Z_2*Z_2$
Z_7(I)	49	36	Z_6*Z_6	$2-SP_{Z_7}$	343	216	$Z_6{*}Z_6*Z_6$
Z_8(I)	64	16	$Z_2{*}Z_2*Z_2{*}Z_2$	$2-SP_{Z_8}$	512	64	$Z_2{*}Z_2{*}Z_2{*}Z_2*Z_2{*}Z_2$
Z_9(I)	81	36	Z_6*Z_6	$2-SP_{Z_9}$	729	216	$Z_6{*}Z_6*Z_6$
Z_{10}(I)	100	16	Z_4*Z_4	$2-SP_{Z_{10}}$	1000	64	$Z_4{*}Z_4*Z_4$

5. Conclusion

In this paper, we have studied the concept of 2-plithogenic rings, where we have discussed many algebraic properties such as invertibility, idempotency, and nilpotency. Also, we have presented 2-plithogenic AH-ideals and AH-homomorphisms with many theorems that describe their elementary properties. As a future research direction, we aim to study other 2-plithogenic algebraic structures such as symbolic 2-plithogenic spaces and modules.

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