ON THE SMARANDACHE PIERCED CHAIN SEQUENCE

A.A.K. MAJUMDAR & Hary GUNARTO
Ritsumeikan Asia-Pacific University, 1-1 Jumonjibaru, Beppu-shi 874-8577, Japan

ABSTRACT

Let $\{SPC(n)\}_{n=1}^{\infty}$ be the Smarandache pierced chain sequence. In this paper, we show that, except for the first element, none of the remaining elements of the sequence $\{SPC(n)/101\}_{n=1}^{\infty}$ is prime.

AMS (1991) Subject Classification : 11A41, 11A51
Key Words : Smarandache pierced chain sequence, prime and composite numbers

INTRODUCTION

The Smarandache pierced chain sequence, denoted by $\{SPC(n)\}_{n=1}^{\infty}$, is defined by (Ashbacher, 1996)

101, 1010101, 10101010101, 101010101010101, …,

which is obtained by successively concatenating the string 0101 to the right of the preceding terms of the sequence, starting with SPC(1)=101.

As has been pointed out by Ashbacher (1996), without proof, all the terms of the sequence $\{SPC(n)\}_{n=1}^{\infty}$ are divisible by 101; in fact, the elements of the sequence $\{SPC(n)/101\}_{n=1}^{\infty}$ are

1, 10001, 100010001, 1000100010001, ….

Smarandache raised the question : How many terms of the sequence $\{SPC(n)/101\}_{n=1}^{\infty}$ are prime?

We find that the second term of the sequence $\{SPC(n)/101\}_{n=1}^{\infty}$ is not prime, since

$$SPC(2)/101=10001=73\times137.$$

Also, since SPC(3)/101=100010001 with the sum of digits equals to 3, it follows (Bernard and Child, 1947) that SPC(3)/101 is divisible by 3; in fact

$$SPC(3)/101=3\times33336667.$$

However, for $n\geq3$, the numbers become so large that it is impossible, even with the help of a computer, to test whether the successive terms of the sequence $\{SPC(n)/101\}_{n=1}^{\infty}$ are prime or not.

In this paper, we give answer to the question posed by Smarandache by showing that, starting from the second term, all the successive terms of the sequence

$\{SPC(n)/101\}_{n=1}^{\infty}$ are composite numbers.

MAIN RESULT

We first observe that the elements of the Smarandache pierced chain sequence, $\{SPC(n)\}_{n=1}^{\infty}$, satisfy the following recurrence relation :

$$SPC(n+1)=10^4\, SPC(n)+101,\ n\geq 2;\ SPC(1)=101. \qquad (1)$$

Lemma 1 : The elements of the sequence $\{SPC(n)\}_{n=1}^{\infty}$ are

$$SPC(n)=101[10^{4(n-1)}+10^{4(n-2)}+\ldots+10^4+1],\ n\geq 1. \qquad (2)$$

Proof : The result is clearly true for n=1. To proceed to prove the lemma by induction, we assume that the result is true for some n.

Now, from (1) together with the induction hypothesis, we see that

$$SPC(n+1)=10^4\, SPC(n)+101$$
$$=10^4[101(10^{4(n-1)}+10^{4(n-2)}+\ldots+10^4+1)]+101$$
$$=101(10^{4n}+10^{4(n-1)}+\ldots+10^4+1).$$

Thus, the result is true for n+1, which completes induction. □

Lemma 1 shows that SPC(n) is divisible by 101 for all n≥1. Another consequence of Lemma 1 is the following corollary.

Corollary 1 : The elements of the sequence $\{SPC(n)/101\}_{n=1}^{\infty}$ are

$$SPC(n)/101=x^{n-1}+x^{n-2}+\ldots+1,\ n\geq 1, \qquad (3)$$

where $x\equiv 10^4$.

In the following theorem, we prove the main result of this paper by finding out two factors of SPC(n)/101 for all n≥3.

Theorem 1 : For all n≥3, SPC(n)/101 is a composite number.

Proof : The result is true if n is even as is shown below : If n (≥4) is even, let n=2m for some integer m (≥2). Then, from (3),

$$SPC(2m)/101=x^{2m-1}+x^{2m-2}+\ldots+x+1$$
$$=x^{2m-2}(x+1)+\ldots+(x+1)$$
$$=(x+1)(x^{2m-2}+x^{2m-4}+\ldots+1), \qquad (4)$$

which shows that SPC(2m)/101 is a composite number for all m (≥2).

Thus, it is sufficient to consider the case when n is odd. First we consider the case when n is prime, say n=p, where p (≥3) is a prime. In this case, from (3),

$$SPC(p)/101=x^{p-1}+x^{p-2}+\ldots+1=(x^p-1)/(x-1). \qquad (5)$$

Let $y=10^2$ (so that $x=y^2$). Then,

$$x-1=y^2-1=(y+1)(y-1), \qquad (6)$$
$$x^p-1=(y^2)^p-1=(y^p+1)(y^p-1).$$

Using the Binomial expansions,

$$y^p+1=(y+1)(y^{p-1}-y^{p-2}+y^{p-3}-\ldots+1),$$

$$y^p-1=(y-1)(y^{p-1}+y^{p-2}+y^{p-3}+ \ldots+1),$$

x^p-1 can be expressed as

$$x^p-1=(y+1)(y-1)(y^{p-1}-y^{p-2}+y^{p-3}- \ldots+1)(y^{p-1}+y^{p-2}+y^{p-3}+ \ldots+1). \qquad (7)$$

Plugging in (5) the expressions for $x-1$ and x^p-1, given respectively by (6) and (7), we get, after canceling out the common factors $(y+1)(y-1)$ from the numerator and the denominator,

$$SPC(p)/101=(y^{p-1}-y^{p-2}+y^{p-3}- \ldots+1)(y^{p-1}+y^{p-2}+y^{p-3}+ \ldots+1), \qquad (8)$$

(where $y=10^2$). The expression (8) shows that SPC(p)/101 is a composite number for each prime p (≥3).

Finally, we consider the case when n is odd but composite. Then, by the Unique Factorization Theorem (Bernard and Child, 1947), n has at least one prime factor. Let n=pr, where p is the largest prime factor of n and r (≥2) is an integer. Then,

$$\begin{aligned} SPC(n)/101 &= SPC(pr)/101 \\ &= x^{pr-1}+x^{pr-2}+\ldots+1 \\ &= x^{p(r-1)}(x^{p-1}+x^{p-2}+\ldots+1)+x^{p(r-2)}(x^{p-1}+x^{p-2}+\ldots+1)+\ldots \\ &\quad +(x^{p-1}+x^{p-2}+\ldots+1) \\ &= (x^{p-1}+x^{p-2}+\ldots+1)(x^{p(r-1)}+x^{p(r-2)}+\ldots+1), \qquad (9) \end{aligned}$$

and hence, SPC(n)/101=SPC(pr)/101 is also a composite number.

By virtue of (4), (8) and (9), the proof of the theorem is complete. □

ACKNOWLEDGEMENT

The research leading to the present paper was done under a research grant from the Ritsumeikan Center for Asia-Pacific Studies of the Ritsumeikan Asia-Pacific University. The first two authors gratefully acknowledge the financial support.

REFERENCES

Ashbacher, C. 1996. Collection of Problems on Smarandache Notions. Erhus University Press, AZ, U.S.A.

Bernard, S. and Child, J.M. 1947. Higher Algebra, Macmillan Book Co. Ltd., U.K.