# C-KÜRESEL GÖSTERGE EĞRİSİNİN SABBAN ÇATISINA GÖRE SMARANDACHE EĞRİLERİ C-Smarandache Curves According to the Sabban Frame of the Spheri- cal Indicatrix Curve



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# C-KÜRESEL GÖSTERGE EĞRİSİNİN SABBAN ÇATISINA GÖRE SMARANDACHE EĞRİLERİ

## C- Smarandache Curves According to the Sabban Frame of the Spherical Indicatrix Curve

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**ÖZET:** Bu çalışmada ilk olarak diferansiyellenebilir bir eğrinin $\{N, C, W\}$Alternatif çatısına ait C birim vektörünün birim küre üzerinde çizdiği küresel gösterge eğrisine ait Sabban çatısı ve bu çatıdan elde edilen Smarandache eğrileri tanımlandı. Son olarak tanımlanan eğrilerin Sabban çatıları ve geodezik eğrilikleri hesaplandı.

**Anahtar Kelimeler:** Sabban çatısı, Smarandache eğrisi, Alternatif çatı,geodezik eğrilik.

**ABSTRACT:** In this paper, we first, define Smarandache curves generated by the Sabban frame of the spherical indicatrix curve of theunit Darboux vector C belonging to the alternative frame of {N, C, W} of a differentiable curve. Then we calculate the geodesic curvatures and geodesic Sabban apparatus of these curves according to this new frame.

**Keywords:** Sabban frame, Smarandache curve, alternative frame, spherical indicatrix curve

### 1 Introduction

In differential geometry, special curves have an important role. One of these curves is a Smarandache curve. Smarandache curves are first defined by M. Turgut and S. Yılmaz in 2008 [7]. Special Smarandache curves also have been studied by some authors[1, 2, 3]. Let $\alpha = \alpha(s)$ be a regular unit speed curve in $E^3$. The Frenet frame and alternative frame of this curve are $\{T, N, B\}$ and $\{N, C, W\}$, respectively. Here, $N$ is normal vector, $W$ is unit Darboux vector and $C = W \wedge N$ [5]. In this paper, we created the Smarandache curves according to the alternative frame of the unit speed curve. We then introduced alternative frame and its properties. Finally we calculated geodesic curvature of these curves according to alternative frame.

## 2 Preliminaries

Let $\alpha = \alpha(s)$ be a regular curve with unit speed. Then the Frenet apparatus of the curve $(\alpha)$ [4]

$$T(s) = \alpha^{'}(s), \quad N(s) = \frac{T^{'}(s)}{\left\| T^{'}(s) \right\|}, \quad B(s) = T(s) \wedge N(s)$$

$(2.1)$ $$\kappa(s) = \left\| T^{'}(s) \right\|, \quad \tau(s) = \frac{\det(\alpha^{'}(s), \alpha^{''}(s), \alpha^{'''}(s))}{\left\| \alpha^{'}(t) \wedge \alpha^{''}(t) \right\|^{2}}$$

$(2.2)$

$$T' = \kappa N, \quad N' = -\kappa T + \tau B, \quad B' = -\tau N.$$

$(2.3)$

In Euclidean 3-space any regular curve $\alpha(s)$ depending on the Frenet vectors moves around the axis of Darboux vector and the Darboux vector and defining a unit vector field are given as[5]

$$W = \frac{\tau}{\sqrt{\kappa^2 + \tau^2}} T + \frac{\kappa}{\sqrt{\kappa^2 + \tau^2}} B, \quad C = W \wedge N = -\frac{\kappa}{\sqrt{\kappa^2 + \tau^2}} T + \frac{\tau}{\sqrt{\kappa^2 + \tau^2}} B$$

$(2.4)$

So build another orthonormal moving frame along the curve $\alpha(s)$. This frame defined as alternative frame and is represented by $\{N, C, W\}$.The derivative formulae of the alternative frame is given by[5]

$$N' = \beta C, \quad C' = -\beta N + \gamma W, \quad W' = -\gamma C, \quad \beta = \sqrt{\kappa^2 + \tau^2}, \quad \gamma = \frac{\kappa^2}{\kappa^2 + \tau^2} \left( \frac{\tau}{\kappa} \right)'.$$

$(2.5)$

The relationship between the Frenet frame and alternative frame is

$$C = \bar{\kappa} T + \bar{\tau} B, \quad W = \bar{\tau} T + \bar{\kappa} B, \quad T = -\bar{\kappa} C + \bar{\tau} W, \quad B = \bar{\tau} C + \bar{\kappa} W$$

$(2.6)$ where $\bar{\kappa} = \dfrac{\kappa}{\beta}, \quad \bar{\tau} = \dfrac{\tau}{\beta}$. Principal normal vektor $N$ is common both frames. Let $\gamma : I \to S^2$ be a unit speed spherical curve and $s$ arc-length parameter of $\gamma$. Let us denote $t(s) = \gamma'(s)$ and $d(s) = \gamma(s) \wedge t(s)$. This frame is called the Sabban frame of $\gamma$ on $S^2$. Then we have the follwing spherical Frenet formulae of $\gamma$

$$\gamma'(s)=t(s),\ t'(s)=-\gamma(s)+\kappa_g(s)d(s),\ d'(s)=-\kappa_g(s)t(s) \tag{2.7}$$

where $\kappa_g$ is the geodesic curvature of the curve of $\gamma$ on $S^2$, [6] .

$$\kappa_g(s)=\langle t'(s),d(s)\rangle \tag{2.8}$$

## 3 Smarandache Curves of Alternative Frame

In this section ,we investigated special Smarandache curves according to Sabban frame on $S^2$. Let $C=C(s)$ and $\alpha_C(s)=C(s)$ be a unit speed regular spherical curves on $S^2$ $\{C,T_C,C\wedge T_C\}$ and $\{C_{\alpha_C},T_{C\alpha_C},(C\wedge T_C)_{\alpha_C}\}$ be the Sabban frame of these curves, respectively. Let $\alpha_C(s)=C(s)$ and if we take the derivative of the equation, then $T_C$ vector is

$$\frac{d\alpha_C}{ds^*}\cdot\frac{ds^*}{ds}=C'(s)=-\beta N+\gamma W,$$

$$T_C=-\frac{\beta}{\sqrt{\beta^2+\gamma^2}}N+\frac{\gamma}{\sqrt{\beta^2+\gamma^2}}W,\quad \frac{ds^*}{ds}=\sqrt{\beta^2+\gamma^2}. \tag{3.1}$$

Considering the $C(s)$ and $T_C$ vectors we can whire,

$$C\wedge T_C=\frac{\gamma}{\sqrt{\beta^2+\gamma^2}}N+\frac{\beta}{\sqrt{\beta^2+\gamma^2}}W. \tag{3.2}$$

Accordingly, the $\{C,\ T_C,\ C\wedge T_C\}$ Sabban frame is obtained from the $C$ vector. If we take the derivative of the equation $(3.1)$, then $T_C'$ vector is

$$T_C'\frac{ds^*}{ds}=\left(-\frac{\beta}{\sqrt{\beta^2+\gamma^2}}\right)'N-\frac{\beta}{\sqrt{\beta^2+\gamma^2}}N'+\left(\frac{\gamma}{\sqrt{\beta^2+\gamma^2}}\right)'W+\frac{\gamma}{\sqrt{\beta^2+\gamma^2}}W',$$

$$T_C{}'\frac{ds^*}{ds}=\left(-\frac{\beta}{\sqrt{\beta^2+\gamma^2}}\right)'N-\sqrt{\beta^2+\gamma^2}C+\left(\frac{\gamma}{\sqrt{\beta^2+\gamma^2}}\right)'W\,,$$

$$T_C{}'=\frac{1}{\sqrt{\beta^2+\gamma^2}}\left(-\frac{\beta}{\sqrt{\beta^2+\gamma^2}}\right)'N-C+\frac{1}{\sqrt{\beta^2+\gamma^2}}\left(\frac{\gamma}{\sqrt{\beta^2+\gamma^2}}\right)'W. \quad (3.3)$$

From the equation $(2.8)$ ,$(3.2)$ and $(3.3)$, the geodesic curvature of $\alpha_C(s)=C(s)$ is

$$\kappa_g^C(s)=\left\langle T_C{}'(s),(C\wedge T_C)(s)\right\rangle,$$

$$\kappa_g^C(s)=\frac{\gamma}{\sqrt{\beta^2+\gamma^2}}\left(-\frac{\beta}{\sqrt{\beta^2+\gamma^2}}\right)'+\frac{\beta}{\sqrt{\beta^2+\gamma^2}}\left(\frac{\gamma}{\sqrt{\beta^2+\gamma^2}}\right),$$

$$\kappa_g^C(s)=\frac{\gamma^2}{\left(\beta^2+\gamma^2\right)^{\frac{5}{2}}}(\gamma'\beta-\gamma\beta')+\frac{\beta^2}{\left(\beta^2+\gamma^2\right)^{\frac{5}{2}}}(\gamma'\beta-\gamma\beta'),$$

$$\kappa_g^C(s)=\frac{\gamma'\beta-\gamma\beta'}{\left(\beta^2+\gamma^2\right)^{\frac{3}{2}}}. \quad (3.4)$$

Then from the equation $(2.7)$ we have the following spherical Frenet formulae of $\alpha_C(s)$,

$$C'=T_C,\quad T_C{}'=-C+\kappa_g^C T_C,\quad (C\wedge T_C)'=-\kappa_g^C T_C. \quad (3.5)$$

**Definition 3.1** Let $(C)$ be a spherical curve of $\alpha(s)$, $C$ and $T_C$ be Sabban vectors of $(C)$. Then $CT_C$ *– Smarandache curve can be identified as*

$$\alpha_{CT_C}=\frac{1}{\sqrt{2}}(C+T_C). \quad (3.6)$$

**Theorem 3.1** The geodesic curvature according to $CT_C$ *– Smarandache curve is*

$$\kappa_g^{CT_C}=\frac{1}{\left(2+\left(\kappa_g^C\right)^2\right)^{\frac{5}{2}}}\left(\lambda_1\kappa_g^C-\lambda_2\kappa_g^C+2\lambda_3\right) \quad (3.7)$$

where

$$\begin{cases}\lambda_1 = \kappa_g^C\left(\kappa_g^C\right)' - \left(\kappa_g^C\right)^2 - 2, \quad \lambda_2 = -\left(\kappa_g^C\right)^4 - 3\left(\kappa_g^C\right)^2 - \kappa_g^C\left(\kappa_g^C\right)' - 2, \\ \lambda_3 = \left(\kappa_g^C\right)^3 + 2\kappa_g^C + 2\left(\kappa_g^C\right)'. \end{cases} \qquad (3.8)$$

**Proof:** If we take the derivative of the equation $(3.6)$ then $T_{CT_C}$ vector is

$$T_{CT_C}\frac{ds^*}{ds} = \frac{1}{\sqrt{2}}\left(T_C - C + \kappa_g^C\left(C \wedge T_C\right)\right),$$

$$T_{CT_C} = \frac{1}{\sqrt{2+\left(\kappa_g^C\right)^2}}\left(-C + T_C + \kappa_g^C\left(C \wedge T_C\right)\right), \quad \frac{ds^*}{ds} = \frac{\sqrt{2+\left(\kappa_g^C\right)^2}}{\sqrt{2}}. \qquad (3.9)$$

Considering the equations $(3.6)$ and $(3.9)$, we have

$$\alpha_{CT_C} \wedge T_{CT_C} = \frac{1}{\sqrt{4+2\left(\kappa_g^C\right)^2}}\left(C + T_C\right) \wedge \left(-C + T_C + \kappa_g^C\left(C \wedge T_C\right)\right),$$

$$\alpha_{CT_C} \wedge T_{CT_C} = \frac{1}{\sqrt{4+2\left(\kappa_g^C\right)^2}}\left(\kappa_g^C C - \kappa_g^C T_C + 2\left(C \wedge T_C\right)\right). \qquad (3.10)$$

If we take the derivative of the equation $(3.9)$, then $T'_{CT_C}$ vector is

$$T'_{CT_C}\frac{ds^*}{ds} = \left(\frac{1}{\sqrt{2+\left(\kappa_g^C\right)^2}}\right)'\left(-C + T_C + \kappa_g^C\left(C \wedge T_C\right)\right) + \frac{1}{\sqrt{2+\left(\kappa_g^C\right)^2}}\left(-C + T_C + \kappa_g^C\left(C \wedge T_C\right)\right)'$$

$$T'_{CT_C}\frac{ds^*}{ds} = -\frac{\kappa_g^C\left(\kappa_g^C\right)'}{\left(2+\left(\kappa_g^C\right)^2\right)^{\frac{3}{2}}}\left(-C + T_C + \kappa_g^C\left(C \wedge T_C\right)\right)$$

$$+\frac{1}{\sqrt{2+\left(\kappa_g^C\right)^2}}\left(-C + \left(1+\left(\kappa_g^C\right)^2\right)T_C + \left(\kappa_g^C + \left(\kappa_g^C\right)'\right)\left(C \wedge T_C\right)\right),$$

$$T'_{CT_C}=\sqrt{2}\cdot\frac{\kappa_g^C\left(\kappa_g^C\right)'-\left(\kappa_g^C\right)^2-2}{\left(2+\left(\kappa_g^C\right)^2\right)^2}C-\sqrt{2}.\frac{\left(\kappa_g^C\right)^4+3\left(\kappa_g^C\right)^2+\kappa_g^C\left(\kappa_g^C\right)'+2}{\left(2+\left(\kappa_g^C\right)^2\right)^2}T_C$$

$$+\sqrt{2}\cdot\frac{\left(\kappa_g^C\right)^3+2\kappa_g^C+2\left(\kappa_g^C\right)'}{\left(2+\left(\kappa_g^C\right)^2\right)^2}\left(C\wedge T_C\right).$$

$(3.11)$

Using the equation $(2.8),(3.8),(3.10)$ and $(3.11)$, we can white $\kappa_g^{CT_C}$ geodesic curvature is

$$\kappa_g^{CT_C}=\left\langle T'_{CT_C},\alpha_{CT_C}\wedge T_{CT_C}\right\rangle=\frac{1}{\left(2+\left(\kappa_g^C\right)^2\right)^{\frac{5}{2}}}\left(\lambda_1\kappa_g^C-\lambda_2\kappa_g^C+2\lambda_3\right).$$

**Definition 3.2** Let $(C)$ be a spherical curve of $\alpha(s)$, $C$ and $C\wedge T_C$ be Sabban vectors of $(C)$. Then $C(C\wedge T_C)-$ *Smarandache curve can be identified as*

$$\alpha_{C(C\wedge T_C)}=\frac{1}{\sqrt{2}}(C+C\wedge T_C). \qquad (3.12)$$

**Theorem 3.2** The geodesic curvature according to $C(C\wedge T_C)-$ *Smarandache curve is* $\kappa_g^{C(C\wedge T_C)-}=\dfrac{1+\kappa_g^C}{1-\kappa_g^C}$ $(3.13)$

**Proof:** If we take the derivative of the equation $(3.13)$ then $T_{C(C\wedge T_C)}$ vector is

$$T_{C(C\wedge T_C)}\frac{ds^*}{ds}=\frac{1}{\sqrt{2}}\left(T_C-\kappa_g^C T_C\right),\ T_{C(C\wedge T_C)}=T_C,\quad \frac{ds^*}{ds}=\frac{1-\kappa_g^C}{\sqrt{2}} \qquad (3.14)$$

Considering the equations $(3.12)$ and $(3.14)$, we have

$$\alpha_{C(C\wedge T_C)}\wedge T_{C(C\wedge T_C)}=\frac{1}{\sqrt{2}}\left(-C+\left(C\wedge T_C\right)\right) \qquad (3.15)$$

If we take the derivative of the equation $(3.14)$, then $T'_{C(C\wedge T_C)}$ vector is

$$T'_{C(C\wedge T_C)}=\frac{\sqrt{2}}{1-\kappa_g^C}\left(-C+\kappa_g^C\left(C\wedge T_C\right)\right)$$

$(3.16)$

Using the equation $(2.8)$, $(3.15)$ and $(3.16)$, we can white $\kappa_g^{C(C\wedge T_C)}$ geodesic curvature is $\kappa_g^{C(C\wedge T_C)}=\dfrac{1+\kappa_g^C}{1-\kappa_g^C}$.

**Definition 3.3** Let $(C)$ be a spherical curve of $\alpha(s)$, $T_C$ and $C\wedge T_C$ be Sabban vectors of $(C)$. Then $T_C\left(C\wedge T_C\right)-$ *Smarandache curve can be identified as*

$$\alpha_{T_C(C\wedge T_C)}=\frac{1}{\sqrt{2}}(T_C+C\wedge T_C).$$

$(3.17)$

***Theorem 3.3*** *The geodesic curvature according to* $T_C\left(C\wedge T_C\right)-$ *Smarandache curve is*

$$\kappa_g^{T_C(C\wedge T_C)}=\frac{1}{\left(1+2\left(\kappa_g^C\right)^2\right)^{\frac{5}{2}}}\left(2\lambda_1\kappa_g^C-\lambda_2+\lambda_3\right)$$

$(3.18)$

where

$$\begin{cases}\lambda_1=2\kappa_g^C\left(\kappa_g^C\right)'+\kappa_g^C+2\left(\kappa_g^C\right)^3,\quad \lambda_2=-1-\left(\kappa_g^C\right)'-3\left(\kappa_g^C\right)^2-2\left(\kappa_g^C\right)^4,\\ \lambda_3=\left(\kappa_g^C\right)'-\left(\kappa_g^C\right)^2+2\left(\kappa_g^C\right)^4.\end{cases}$$

$(3.19)$

**Proof:** If we take the derivative of the equation $(3.8)$ then $T_{T_C(C\wedge T_C)}$ vector is

$$T_{T_C(C\wedge T_C)}\frac{ds^*}{ds}=\frac{1}{\sqrt{2}}\left(-C-\kappa_g^C T_C+\kappa_g^C\left(C\wedge T_C\right)\right),$$

$$T_{T_C(C\wedge T_C)}=\frac{1}{\sqrt{1+2\left(\kappa_g^C\right)^2}}\left(-C-\kappa_g^C T_C+\kappa_g^C\left(C\wedge T_C\right)\right),\quad \frac{ds^*}{ds}=\frac{\sqrt{1+2\left(\kappa_g^C\right)^2}}{\sqrt{2}}$$

$(3.20)$ Considering the equations $(3.17)$ and $(3.20)$, we have

$$\alpha_{T_C(C\wedge T_C)}\wedge T_{T_C(C\wedge T_C)}=\frac{1}{\sqrt{2+4\left(\kappa_g^C\right)^2}}\left(T_C+C\wedge T_C\right)\wedge\left(-C-\kappa_g^C T_C+\kappa_g^C\left(C\wedge T_C\right)\right),$$

$$\alpha_{T_C(C\wedge T_C)}\wedge T_{T_C(C\wedge T_C)}=\frac{1}{\sqrt{2+4\left(\kappa_g^C\right)^2}}\left(2\kappa_g^C C-T_C+\left(C\wedge T_C\right)\right).$$

$(3.21)$

If we take the derivative of the equation $(3.20)$, then $T'_{T_C(C\wedge T_C)}$ vector is

$$T'_{T_C(C\wedge T_C)}\frac{ds^*}{ds}=\left(\frac{1}{\sqrt{1+2\left(\kappa_g^C\right)^2}}\right)'\left(-C-\kappa_g^C T_C+\kappa_g^C\left(C\wedge T_C\right)\right)$$

$$+\frac{1}{\sqrt{1+2\left(\kappa_g^C\right)^2}}\left(-C-\kappa_g^C T_C+\kappa_g^C\left(C\wedge T_C\right)\right)',$$

$(3.22)$

$$T'_{T_C(C\wedge T_C)}=\frac{\sqrt{2}\left(2\kappa_g^C\left(\kappa_g^C\right)'+\kappa_g^C+2\left(\kappa_g^C\right)^3\right)}{\left(1+2\left(\kappa_g^C\right)^2\right)^2}C+\frac{\sqrt{2}\left(-1-\left(\kappa_g^C\right)'-3\left(\kappa_g^C\right)^2-2\left(\kappa_g^C\right)^4\right)}{\left(1+2\left(\kappa_g^C\right)^2\right)^2}T_C$$

$$+\frac{\sqrt{2}\left(\left(\kappa_g^C\right)'-\left(\kappa_g^C\right)^2+2\left(\kappa_g^C\right)^4\right)}{\left(1+2\left(\kappa_g^C\right)^2\right)^2}\left(C\wedge T_C\right).$$

Using the equation $(2.8)$, $(3.19)$, $(3.20)$ and $(3.21)$, we can white $\kappa_g^{CT_C}$ geodesic curvature is

$$\kappa_g^{T_C(C\wedge T_C)}=\frac{1}{\left(1+2\left(\kappa_g^C\right)^2\right)^{\frac{5}{2}}}\left(2\lambda_1\kappa_g^C-\lambda_2+\lambda_3\right).$$

**Definition 3.4** Let $(C)$ be a spherical curve of $\alpha(s)$ and $C, T_C$ and $C\wedge T_C$ be Sabban vectors of $(C)$. Then $CT_C(C\wedge T_C)-$*Smarandache curve can be identified as* $\alpha_{CT_C(C\wedge T_C)}=\dfrac{1}{\sqrt{2}}\left(C+T_C+C\wedge T_C\right).$ $\quad(3.23)$

***Theorem 3.4*** *The geodesic curvature according to* $CT_C(C\wedge T_C)-$*Smarandache curve is*

$$\kappa_g^{CT_C(C\wedge T_C)}=\frac{\sqrt{3}}{2\sqrt{2}\left(1-\kappa_g^C+\left(\kappa_g^C\right)^2\right)^{\frac{5}{2}}}\left(\left(-1+2\kappa_g^C\right)\lambda_1-\lambda_2+\left(1-\kappa_g^C\right)\lambda_3\right) \quad (3.24)$$

where

$$\begin{cases}\lambda_1=6\left(\kappa_g^C\right)'\left(-1+2\kappa_g^C\right)+3\left(\kappa_g^C-1\right)\sqrt{1-\kappa_g^C+\left(\kappa_g^C\right)^2}, \\ \lambda_2=-6\left(\kappa_g^C\right)'\left(-1+3\kappa_g^C-2\left(\kappa_g^C\right)^2\right)-3\left(1+\left(\kappa_g^C\right)'+\left(\kappa_g^C\right)^2\right)\sqrt{1-\kappa_g^C+\left(\kappa_g^C\right)^2}, \\ \lambda_3=6\left(\kappa_g^C\right)'\left(\kappa_g^C-2\left(\kappa_g^C\right)^2\right)-3\left(\kappa_g^C+\left(\kappa_g^C\right)^2-\left(\kappa_g^C\right)'\right)\sqrt{1-\kappa_g^C+\left(\kappa_g^C\right)^2}\end{cases} \quad (3.25)$$

**Proof:** If we take the derivative of the equation $(3.23)$ then $T_{CT_C(C\wedge T_C)}$ vector is

$$T_{CT_C(C\wedge T_C)}\frac{ds^*}{ds}=\frac{1}{\sqrt{3}}\left(-C+\left(1-\kappa_g^C\right)T_C+\kappa_g^C\left(C\wedge T_C\right)\right),$$

$$T_{CT_C(C\wedge T_C)}=\frac{\sqrt{3}\left(-C+\left(1-\kappa_g^C\right)T_C+\kappa_g^C\left(C\wedge T_C\right)\right)}{\sqrt{2}\sqrt{1-\kappa_g^C+\left(\kappa_g^C\right)^2}},$$

$$\frac{ds^*}{ds}=\frac{\sqrt{2}}{\sqrt{3}}\sqrt{1-\kappa_g^C+\left(\kappa_g^C\right)^2} \quad (3.26)$$

Considering the equations $(3.23)$ and $(3.26)$, we have

$$\alpha_{CT_C(C\wedge T_C)}\wedge T_{CT_C(C\wedge T_C)}=\sqrt{3}\frac{\left(C+T_C+C\wedge T_C\right)\wedge\left(-C+\left(1-\kappa_g^C\right)T_C+\kappa_g^C\left(C\wedge T_C\right)\right)}{\sqrt{2}\sqrt{1-\kappa_g^C+\left(\kappa_g^C\right)^2}}$$

$$\alpha_{CT_C(C\wedge T_C)}\wedge T_{CT_C(C\wedge T_C)}=\sqrt{3}\frac{\left(\left(-1+2\kappa_g^C\right)C-T_C+\left(1-\kappa_g^C\right)\left(C\wedge T_C\right)\right)}{\sqrt{2}\sqrt{1-\kappa_g^C+\left(\kappa_g^C\right)^2}}. \qquad (3.27)$$

If we take the derivative of the equation $(3.26)$, then $T'_{CT_C(C\wedge T_C)}$ vector is

$$T'_{CT_C(C\wedge T_C)}\frac{ds^*}{ds}=\left(\frac{\sqrt{3}}{\sqrt{2}\sqrt{1-\kappa_g^C+\left(\kappa_g^C\right)^2}}\right)'\left(-C+\left(1-\kappa_g^C\right)T_C+\kappa_g^C\left(C\wedge T_C\right)\right)$$
$$+\frac{\sqrt{3}}{\sqrt{2}\sqrt{1-\kappa_g^C+\left(\kappa_g^C\right)^2}}\left(-C+\left(1-\kappa_g^C\right)T_C+\kappa_g^C\left(C\wedge T_C\right)\right)', \qquad (3.28)$$

$$T'_{CT_C(C\wedge T_C)}=\frac{6\left(\kappa_g^C\right)'\left(-1+2\kappa_g^C\right)+3\left(\kappa_g^C-1\right)\sqrt{1-\kappa_g^C+\left(\kappa_g^C\right)^2}}{2\left(1-\kappa_g^C+\left(\kappa_g^C\right)^2\right)^{\frac{3}{2}}}C$$
$$-\frac{6\left(\kappa_g^C\right)'\left(-1+3\kappa_g^C-2\left(\kappa_g^C\right)^2\right)+3\left(1+\left(\kappa_g^C\right)'+\left(\kappa_g^C\right)^2\right)\sqrt{1-\kappa_g^C+\left(\kappa_g^C\right)^2}}{2\left(1-\kappa_g^C+\left(\kappa_g^C\right)^2\right)^{\frac{3}{2}}}T_C$$
$$+\frac{6\left(\kappa_g^C\right)'\left(\kappa_g^C-2\left(\kappa_g^C\right)^2\right)-3\left(\kappa_g^C+\left(\kappa_g^C\right)^2-\left(\kappa_g^C\right)'\right)\sqrt{1-\kappa_g^C+\left(\kappa_g^C\right)^2}}{2\left(1-\kappa_g^C+\left(\kappa_g^C\right)^2\right)^{\frac{3}{2}}}\left(C\wedge T_C\right)$$

Using the equation $(2.8),(3.25),(3.27)$ and $(3.28)$, we can white $\kappa_g^{CT_C(C\wedge T_C}$ geodesic curvature is

$$\kappa_g^{CT_C(C\wedge T_C)}=\frac{\sqrt{3}}{2\sqrt{2}\left(1-\kappa_g^C+\left(\kappa_g^C\right)^2\right)^{\frac{5}{2}}}\left(\left(-1+2\kappa_g^C\right)\lambda_1-\lambda_2+\left(1-\kappa_g^C\right)\lambda_3\right)$$

**Example 3.5:** Let $\gamma(s)=(\frac{9}{208}\sin 16s-\frac{1}{117}\sin 36s,-\frac{9}{208}\cos 16s+\frac{1}{117}\cos 36s,\frac{6}{65}\sin 10s)$ be a curve with the alternative frame of $\{N,C,W\}$ given as

$$N(s)=\left(\frac{12}{13}\cos 26s,-\frac{12}{13}\sin 26s,\frac{5}{13}\right),\qquad C(s)=(-\sin 26s,\cos 26s,0),$$

$$W(s)=\left(\frac{5}{13}\cos 26s,-\frac{5}{13}\sin 26s,\frac{12}{13}\right).$$

Then we have the following spherical indicatrix curve $(C)$ and smarandache curve according to Sabban frame on $S^2$.$\beta_1,\ \beta_2,\ \beta_3\ and\ \beta_4$(see Figure 2,3,4,5) as

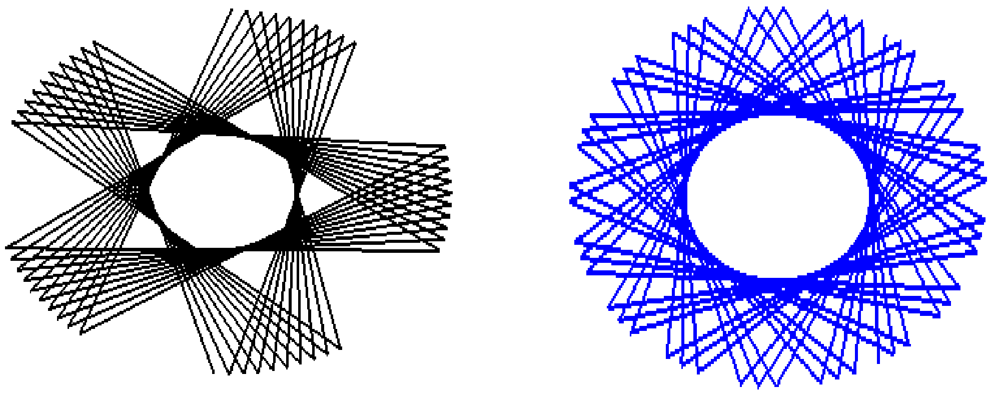
$$\beta_1=\frac{1}{\sqrt{2}}(C+T_C)=\frac{1}{\sqrt{2}}(-26\cos 26s-\sin 26s,\cos 26s-26\sin 26s,0),$$

$$\beta_2=\frac{1}{\sqrt{2}}(C+C\wedge T_C)=\frac{1}{\sqrt{2}}(-\sin 26s,\cos 26s,26),$$

$$\beta_3=\frac{1}{\sqrt{2}}(T_C+C\wedge T_C)=\frac{1}{\sqrt{2}}(-26\cos 26s,-26\sin 26s,26),$$

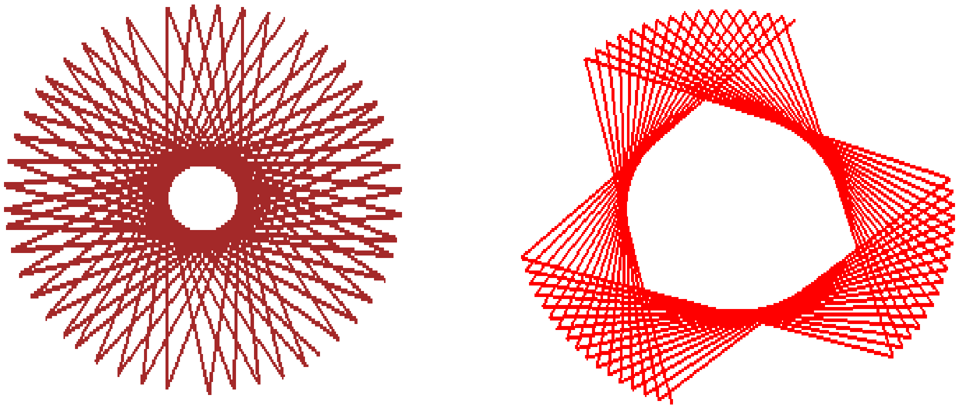
$$\beta_4=\frac{1}{\sqrt{3}}(C+T_C+C\wedge T_C)$$

$$=\frac{1}{\sqrt{3}}(-26\cos 26s-\sin 26s,\cos 26s-26\sin 26s,26).$$



**(a)** **(b)**

**Figure 1: a)** $\boldsymbol{\beta_1-Smarandache\ curve, s\in(0,2\pi)},\ \boldsymbol{b)\beta_2-Smarandache\ curve, s\in(0,\frac{4\pi}{3})}$



**(a)** **(b)**

**Figure 2: a)$\boldsymbol{\beta_3 - Smarandache\ eğrisi, s \in \left(-\pi, \frac{2\pi}{3}\right)}$, $\boldsymbol{b)\beta_4 - Smarandache\ eğrisi, s \in (-\frac{3\pi}{2}, \pi)}$**

**References:**


Ali A. T., Special Smarandache Curves in the Euclidean Space, International Journal of Mathematical Combinatorics, 2(2010),30-36.

Bektaş O., Yüce S., Special Smarandache Curves According to Darboux Frame in Euclidean 3- Space, Romanian Journal of Mathematics and Computer Science, 3(1)(2013),48-59.

Çetin M., Tuncer Y., Karacan M. K., SmarandacheCurvesAccordingtoBishopFrame in Euclidean 3- Space, Gen. Math. Notes, 20(2)(2014), 50-66.

Do Carmo, M. P.,DifferentialGeometry of CurvesandSurfaces, PrenticeHall, EnglewoodCliffs, NJ, 1976.

Kaya O., Önder M., New Partner Curves in theEuclidean 3-Space E^3, International Journal of Geometry, 6(2)(2017),41- 50.

Taşköprü K., Tosun M., Smarandache curves on S ^2 , Boletim da Sociedade Paranaense de Matematica 3 Srie., 32 (1)(2014), 51-59.

Turgut M., Yılmaz S., Smarandache Curves in Minkowski Space-time, International J. Math. Combin., 3(2008),51-55.