# Some types of Smarandache filters of a Smarandache BH-algebra

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# Some types of Smarandache filters of a Smarandache BH-algebra

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**Abstract**

In this paper, the notions of a Smarandache p-filter, a Smarandache $n$-fold $p$-filter, Smarandache $q$-filter, a Smarandache-$n$-fold $q$-filter of a Smarandache BH-Algebra are introduced. Some properties of them with some theorems, proportions and examples are given.

*Keywords:* BCK-algebra, BH-algebra, Smarandache filter.
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## 1. Introduction

The idea of BCK-algebras was formulated first in [4, 5]. In the same year another algebraic structure called BCI-algebra which was a popularization of a BCK-algebra was given by K. Iséki [6]. In 1983, Hu and Li introduced the notion of a BCH-algebra which was a popularization of BCK/BCI-algebras [8, 11]. Hoo show that the notions of an ideal and a filter in a BCI-algebra [7]. A BH-algebra is an algebraic structure introduced by Jun et al in [10] which was a popularization of BCH/BCI/BCK-algebras. The notions of a Smarandache BCI-algebra, Smarandache ideal of a Smarandache BCI-algebra are given by Jun in [9]. Abbass and Dahham introduced the concept of completely closed filter of a BH-algebra in [1]. Abbass and Luhaib introduced the idea of Smarandache filter of a Smarandache BH-Algebra in [3]. In this paper, the notions of a Smarandache-$p$-filter, a Smarandache $n$-fold $p$-filter, Smarandache $q$-filter, a Smarandache-$n$-fold $q$-filter and of a Smarandache BH-Algebra are given.

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## 2. Preliminaries

In this section, several basic connotations about a BCI-algebra, a BCK-algebra, a Smarandache BH-algebra, and a Smarandache filter of a Smarandache are reviewed.

**Definition 2.1.** *[9] A BCI-algebra is an algebra* $(X, \square, 0)$ *, where* $X$ *is a nonempty set,* $\square$ *is a binary operation and 0 is a constant, for all* $x, y, z \in X$*, satisfying the following axioms:*

**i.** $((x \square y) \square (x \square z)) \square (z \square y) = 0$,

**ii.** $(x \square (x \square y)) \square y = 0$,

**iii.** $x \square x = 0$,

**iv.** $x \square y = 0$ *and* $y \square x = 0$ *imply* $x = y$.

**Definition 2.2.** *[8] BCK-algebra is a BCI-algebra satisfying the axiom:* $0 \square x = 0$*, for all* $x \in X$.

**Definition 2.3.** *[10] A BH-algebra is a nonempty set* $X$ *with a constant 0 and a binary operation* $\square$ *satisfying the following conditions:*

**i.** $x \square x = 0$*, for all* $x \in X$.

**ii.** $x \square y = 0$ *and* $y \square x = 0$ *imply* $x = y$*, for all* $x, y \in X$.

**iii.** $x \square 0 = x$*, for all* $x \in X$.

**Definition 2.4.** *[10] A nonempty subset* $S$ *of a BH-algebra* $X$ *is called a subalgebra of* $X$ *if* $x \square y \in S$*, for all* $x, y \in S$.

**Definition 2.5.** *[1] A filter of a BH-algebra* $X$ *is a non-empty subset* $F$ *of* $X$ *such that:*

$(F_1)$ *if* $x \in F$ *and* $y \in F$*, then* $y \square (y \square x) \in F$ *and* $x \square (x \square y) \in F$.

$(F_2)$ *If* $x \in F$ *and* $x \square y = 0$ *then* $y \in F$ *for all* $y \in X$.

*Further* $F$ *is a closed filter if* $0 \square x \in F$*, for all* $x \in F$.

**Definition 2.6.** *[2] Let* $X$ *be a BH-algebra and* $F$ *be a filter of* $X$*. Then* $F$ *is called a p-**filter** denoted by* $p - f$ *if it satisfies:*

$$\text{if } x, y \in F \text{ imply } (x \square z) \square (y \square z) \in F \text{ for all } y, z \in X.$$

**Definition 2.7.** *[2] Let* $F$ *be a filter of a BH-algebra* $X$*. If* $x, y \in F$ *and there exists a fixed* $n \in N$ *such that* $z^n \in X$ *imply* $(x \square z^n) \square (y \square z^n) \in F$*, for all* $z \in X$*. Then* $F$ *is said to be a **n-fold p-filter** of* $X$.

**Definition 2.8.** *[2] Let* $X$ *be a BH-algebra and* $F$ *be a filter of* $X$*. Then* $F$ *is called a **q-filter** denoted by **q-f** if it satisfies:*

$$\text{If } x \square z \in F, y \in F \text{ imply } x \square (y \square z) \in F, \text{ for all } x, z \in X.$$

**Definition 2.9.** *[2] Let* $X$ *be a BH-algebra,* $F$ *be a filter of* $X$*, and there exists a fixed* $n \in N$ *such that* $x \square z^n \in F, y \in F$*, for all* $x, z \in \in$ *imply* $x \square (y \square z^n) \in F$*. Then* $F$ *is called a n-fold q-filter of* $X$.

**Definition 2.10.** *[3] A Smarandache BH-algebra is defined to be a BH-algebra* $X$ *in which there exists a proper subset* $Q$ *of* $X$ *denoted by **S. BH-algebra** such that*

**i.** $0 \in Q$ *and* $|Q| \geq 2$.

**ii.** $Q$ *is a BCK-algebra under the operation of* $X$.

**Definition 2.11.** *[3] A non-empty subset $F$ of a S. BH-algebra $\not{X}$ is called a **Smarandache filter** of $\not{X}$ denoted by **S.f**, if it satisfies $(F_1)$ and*

$$(F_3) \text{ If } x \in F \text{ and } x \square y = 0 \text{ then } y \in F\ ,\ \forall y \in Q.$$

**Proposition 2.12.** *[3] Let $\not{X}$ be a S. BH-algebra and let $\{F_\beta, \beta \in \Omega\}$ be a family of S.f of $\not{X}$. Then $\bigcap\limits_{\beta \in \Omega} F_\beta$ is an S.f of $\not{X}$.*

**Proposition 2.13.** *[3] Let $\not{X}$ be a S.f and let $\{F_i, i \in \lambda\}$ be a chain of S.f of $\not{X}$. Then $\bigcup\limits_{\beta \in \Omega} F_\beta$ is a S.f of $\not{X}$.*

**Theorem 2.14.** *[3] Let $\not{X}$ be a S. BH-algebra, and $F$ be a S.f of $\not{X}$ such that $x \square y \neq 0$, for all $y \notin F$ and $x \in F$. Then $F$ is a filter of $\not{X}$.*

## 3. Main Results

In this section, the notions of a Smarandache-$p$-filter, a Smarandache $n$-fold $p$-filter, Smarandache $q$-filter, a Smarandache-$n$-fold $q$-filter and of a Smarandache BH-Algebra of a Smarandache BH-Algebra are introduced. Also, some properties of these notions are studied.

**Definition 3.1.** *Let $\not{X}$ be a S. BH-algebra and $F$ be a Smarandache filter of $\not{X}$. Then $F$ is called a Smarandache p-filter of $\not{X}$ and denoted by **S.p-f** of $\not{X}$ if it satisfies:*

$$\text{If } x, y \in F \text{ imply } (x \square z) \square (y \square z) \in F \text{ for all } z \in Q.$$

*Further $F$ is a Smarandache closed p-filter if $0 \square x \in F$, for all $x \in F$ .*

**Example 3.2.** *Let $\not{X} = \{0,1, 2, 3\}$. Define $\square$ as follows:*

| $\square$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 0 | 2 | 3 |
| 1 | 1 | 0 | 1 | 2 |
| 2 | 2 | 2 | 0 | 1 |
| 3 | 3 | 3 | 2 | 0 |

*where $Q = \{0, 1\}$, the subset $F = \{0, 1, 2\}$ is a S.P.f of $X$. But is not p.f of $X$, since $z = 3, x = 3, y = 0$, $(3 \square 3) \square (0 \square 3) = 3 \notin F$.*

**Proposition 3.3.** *Let $\not{X}$ be a S. BH-algebra and $F$ be a p-f of $X$. Then $F$ is a S.p-f of $\not{X}$.*

**Proof .** Directly since $Q \subseteq \not{X}$. $\square$

**Theorem 3.4.** *Let $X$ be a S. BH-algebra, and $F$ be a S.p-f of $X$ such that $x \square y \neq 0$, $y \notin F$ if $(x \square z) \square (y \square z) \notin F$ and $x \in F, z \in X$. Then $F$ is a p.f of $\not{X}$.*

**Proof .** Let $F$ be a S.p-f of $X$ it follows that By Definition3.1 is a S.f of $\text{X}$. Since $x \,\square\, y \neq 0, y \notin F, x \in F$, By Theorem 2.14, $F$ is a filter of $\text{X}$.

Now, let $x, y \in F, z \in X$, then we have two cases:

**Case (I):** If $z \in Q$, imply $(x \,\square\, z) \,\square\, (y \,\square\, z) \in F$ because by definition 3.1 $F$ is S.p-f of $\text{X}$,

**Cases(II):** If $z \notin Q$, then either $(x \,\square\, z) \,\square\, (y \,\square\, z) \in \notin F$ or $(x \,\square\, z) \,\square\, (y \,\square\, z) \in F$.

Suppose $(x \,\square\, z) \,\square\, (y \,\square\, z) \notin F$, then $y \notin F$, this is a contradiction. Thus $(x \,\square\, z) \,\square\, (y \,\square\, z) \in F$. Therefore, is a p.f of $\text{X}$. □

**Proposition 3.5.** *Let $\text{X}$ be a Smarandache BH-algebra, and let $\{F_\beta,\ \beta \in \Omega\}$ be a family of S.p-fs of $\text{X}$. Then $\bigcap_{\beta \in \Omega} F_\beta$ is a S.p-f of $\text{X}$.*

**Proof .** Let $\{F_\beta,\ \beta \in \Omega\}$ be a family of S.p-fs of $\text{X}$, imply $\{F_\beta, \beta \in \Omega\}$ be a family of Smarandache filters of $X$. Hence, By Proposition 2.12, $\bigcap_{\beta \in \Omega} F_\beta$ is a S.f of $\text{X}$. Now, let $x, y \in \bigcap_{\beta \in \Omega} F_\beta$ and $z \in Q$. Then $x, y \in F_\beta$ and $z \in Q, \forall \beta \in \Omega$ implies that $(x \,\square\, z) \,\square\, (y \,\square\, z) \in F_\beta$, $\forall \beta \in \Omega$, because $F_\beta$ is a S.p-f of $\text{X}$, for all $\beta \in \Omega$, this mean that $(x \,\square\, z) \,\square\, (y \,\square\, z) \in \bigcap_{\beta \in \Omega} F_\beta$. Therefore $\bigcap_{\beta \in \Omega} F_\beta$ is a S.p-f of $\text{X}$. □

**Example 3.6.** *Let $\text{X} = \{0, 1, 2, 3, 4, 5\}$. Define □ as follows:-*

| □ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| 2 | 2 | 2 | 0 | 0 | 1 | 0 |
| 3 | 3 | 2 | 2 | 0 | 1 | 1 |
| 4 | 4 | 4 | 4 | 4 | 0 | 1 |
| 5 | 5 | 5 | 5 | 5 | 5 | 0 |

*where $Q = \{0, 2\}$. The subset $F_1 = \{0, 2, 3\}$ and $F_2 = \{0, 2, 5\}$ are two S.p-f of $X$, but $F_1 \cup F_2 = \{0, 2, 3, 5\}$ is not a S.p-f of $\text{X}$, since $x = 3, y = 5, z = 0 \in Q$ but $(3 \,\square\, 0) \,\square\, (5 \,\square\, 0) = 1 \notin F_1 \cup F_2$,*

**Proposition 3.7.** *Let $\text{X}$ be a S. BH-algebra, and let $\{F_\beta,\ \beta \in \Omega\}$ be a chain of S.P.f of $X$. Then $\bigcup_{\beta \in \Omega} F_\beta$ is a S.P.f of $\text{X}$.*

**Proof .** Let $\{F_\beta,\ \beta \in \Omega\}$ be a chain of S.P.f of $X$. it follows that $\{F_\beta,\ \beta \in \Omega\}$ be a chain of Smarandache filters of $X$ [By definition 3.1]. This together with Proposition (2.13) implies that $\bigcup_{\beta \in \Omega} F_\beta$ is a Smarandache filter of $X$.

Now, let $x, y \in \bigcup_{\beta \in \Omega} F_\beta, z \in Q$, then there exists $F_n, F_m \in \{F_\beta,\ \beta \in \Omega\}$, such that $x \in F_j$ and $y \in F_k$. Then either $F_n \subseteq F_m$ or $F_m \subseteq F_n$. If $F_n \subseteq F_m$, it follows that $x, y \in F_m$ and $z \in Q$. So, there exists $m \in \Omega$ such that $(x \,\square\, z) \,\square\, (y \,\square\, z) \in F_m$, because $F_i$ is a S.P.f of $X$,($\forall \beta \in \Omega$). Then $(x \,\square\, z) \,\square\, (y \,\square\, z) \in \bigcup_{\beta \in \Omega} F_\beta$. Similarly, $F_m \subseteq F_n$ implies that $\bigcup_{\beta \in \Omega} F_\beta$ is a S.P.f of $\text{X}$. □

**Definition 3.8.** *Let $F$ be a Smarandache filter of a S. BH-algebra $\cancel{X}$. If $x, y \in F$ and there exists a fixed $n \in N$ such that $z^n \in Q$ imply $(x \square z^n) \square (y \square z^n) \in F$, for all $z \in Q$. Then $F$ is said to be a Smarandache n-fold p-filter of $X$, denoted by a* ***S. n-fold. p-f*** *of $\cancel{X}$.*

**Example 3.9.** *Let $\cancel{X} = \{0, 1, 2, 3, 4\}$ be as in example 3.6. The filter $F = \{0, 2, 3\}$ is a S. 2-fold. p-f of $\cancel{X}$.*

**Theorem 3.10.** *Let $\cancel{X}$ be a S. BH-algebra, and $F$ be a S. n-fold. p-f of $X$ such that $x \square y \neq 0, y \notin F$ if $(x \square z^n) \square (y \square z^n) \notin F$ and $x \in F, z^n \in \cancel{X}$, for a fixed $n \in N$. Then $F$ is a n-fold p-filter of $\cancel{X}$.*

**Proof .** Let $F$ be a S. $n$-fold. P.f of $\cancel{X}$, then By Definition 3.8, $F$ is a S.f of $\cancel{X}$. Since $x \square y \neq 0, y \notin F, x \in F$, By Theorem 2.14, $F$ is a filter of $X$. Now, let $x, y \in F, z^n \in \cancel{X}$, then we have the following two cases:

**Case (I):** If $z^n \in Q$, then $(x \square z^n) \square (y \square z^n) \in F$, because by Definition 3.8, $F$ is S. $n$-fold. P.f of $\cancel{X}$,

**Cases(II):** If $z^n \notin Q$, then either $(x \square z^n) \square (y \square z^n) \notin F$ or $(x \square z^n) \square (y \square z^n) \in F$.

Suppose that $(x \square z^n) \square (y \square z^n) \notin F$, then $y \notin F$, this a contradiction. Thus $(x \square z^n) \square (y \square z^n) \in F$, consequently $F$ is a $n$-fold p-filter of $\cancel{X}$. □

**Proposition 3.11.** *Let $X$ be a S. BH-algebra, and let $\{F_\beta,\ \beta \in \Omega\}$ be a family of S. n-fold. p-f of $\cancel{X}$. Then $\bigcap_{\beta \in \Omega} F_\beta$ is a S. n-fold. p-f of $\cancel{X}$.*

**Proof .** Straightforward. □

**Proposition 3.12.** *Let $\cancel{X}$ be a Smarandache BH-algebra, and let $\{F_\beta,\ \beta \in \Omega\}$ be a chain of S. n-fold. p-f of $X$. Then $\bigcup_{\beta \in \Omega} F_\beta$ is a* ***S. n-fold. p-f*** *of $\cancel{X}$.*

**Proof .** Straightforward. □

**Definition 3.13.** *Let $X$ be a S. BH-algebra and $F$ be a Smarandache filter of $X$. Then $F$ is called* ***a Smarandache q-filter*** *and denoted by a* ***S.q-f*** *of $X$ if it satisfies:- If $x \square z \in F, y \in F$ imply $x \square (y \square z) \in F$ for all $x, z \in Q$.*

**Example 3.14.** *Let $\cancel{X} = \{0, 1, 2, 3, 4\}$. Define $\square$ as follows:*

| $\square$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 2 |
| 2 | 2 | 2 | 0 | 2 | 0 |
| 3 | 3 | 1 | 3 | 0 | 3 |
| 4 | 4 | 4 | 4 | 4 | 0 |

*Where $Q = \{0, 2\}$. The subset $F = \{0, 1, 2\}$ is a S.q-f of $\cancel{X}$ but it is not a q-filter of $X$. Since $x = 3, y = 0, z = 3$ and $3 \square (0 \square 3) = 3 \notin F$*

**Proposition 3.15.** *Let $X$ be a S. BH-algebra and $F$ is a q-filter of $X$. Then $F$ is a S.q.f of $\not X$ .*

**Proof .** Since $Q \subseteq \not X$, the proof is clear. □

**Remark 3.16.** *Consider the $Q_1-$ S. BH-algebra and $Q_2$-Smarandache BH-algebra $X$ such that $Q_1 \subseteq Q_2$. The $Q_1$-Smarandache q-filter of $\not X$ may be not a $Q_2-$Smarandache q-filter of $X$ as in the following example. Consider $X = \{0, 1, 2, 3\}$ in example 3.14, where $Q_1 = \{0, 1\}$, $Q_2 = \{0, 2, 3\}$ are BCK-algebras and $Q_1 \subseteq Q2$ : $F = \{0, 1, 2\}$ is a $Q_1$-Smarandache q-filter of $X$, but it is not $Q_2$-Smarandache q-filter of $X$. Since $x = 3, y = 2, z = 3$ implies that $3 \square (2 \square 3) = 3$, but $3 \notin F$.*

**Proposition 3.17.** *Let $\not X$ be a S. BH-algebra and $F$ be a S.q-f of $X$, such that $F \subseteq Q$. Then $F$ is a subalgebra of $\not X$.*

**Proof .** Let $x, y \in F$. Since $z \in Q$, choose $z = 0$, we have $x = x \square 0 \in F, y \in F, x, 0 \in Q$, because $F \subseteq Q$. This Implies that $x \square (y \square 0) \in F$, because by Definition 3.13, $F$ is a S.q.f of $X$. Then $x \square y \in F$. Hence, F is a subalgebra. □

**Theorem 3.18.** *Let $\not X$ be a S. BH-algebra, and be a S.q-f of $\not X$ such that $x \square y \neq 0, x \square z \notin F$, and $y \notin F if x \square (y \square z) \notin F$ and $x \in F, z \in \not X$. Then $F$ is a q-filter of $\not X$.*

**Proof .** Let $F$ be a S.q.f of $\not X$, then By Definition 3.13, it is a S.f of $X$. Since $x \square y \neq 0$, $y \notin F$, $x \in F$, By Theorem 2.14, $F$ is a filter of $X$.

Now, let $x \square z \in F, y \in F, x, z \in \not X$, then we have the following two cases:

**Case (I):** If $x, z \in Q$, then by Definition 3.13, $x \square (y \square z) \in F$,

**Cases(II):** If $x, z \notin Q$, then either $x \square (y \square z) \notin F$ or $x \square (y \square z) \in F$.

If $x \square (y \square z) \notin F$, then $y =\in F$, or x$\square z \notin F$, contradiction. Since $x \square z \in F, y \in F$, we have $x \square (y \square z) \in F$. Hence, it is a $q$-filter of $\not X$. □

**Proposition 3.19.** *Let $\not X$ be a S. BH-algebra, and let $\{F_\beta,\ \beta \in \Omega\}$ be a family of S.q-f of $\not X$. Then $\bigcap_{\beta \in \Omega} F_\beta$ is a S.q-f of $\not X$.*

**Proof .** Let $\{F_\beta,\ \beta \in \Omega\}$ be a family of S.q-fs of $\not X$, then By Definition 3.13, $\{F_\beta,\ \beta \in \Omega\}$ be a family of S.f of $X$. Thus, By Proposition 2.12, $\bigcap_{\beta \in \Omega} F_\beta$ is a S.f of $X$.

Now, let $x \square z \in \bigcap_{\beta \in \Omega} F_\beta$, $y \in \bigcap_{\beta \in \Omega} F_\beta$ such that $x, z \in Q$, it follows that $x \square z \in F_\beta, y \in F_\beta$, such that $x, z \in Q$, imply $x \square (y \square z) \in F_\beta$, $(\forall \beta \in \Omega)$, bevause $F_i$ is a S.q-f of $\not X$. Hence, $x \square (y \square z) \in \bigcap_{\beta \in \Omega} F_\beta$.

Therefore, $\bigcap_{\beta \in \Omega} F_\beta$ is a S.q-f of $\not X$. □

**Remark 3.20.** *Let $X$ be a S. BH-algebra and let $f_1, f_2$ be a S.q.f of $X$. Then $f_1 \cup f_2$ is not necessary a S.q.f of $\not X$.*

**Example 3.21.** *Consider $X = \{0, 1, 2, 3, 4, 5\}$ be as in example 3.6, where $Q = \{0, 1\}$. The subset $F_1 = \{0, 1, 3\}$ and $F_2 = \{0, 1, 4\}$ are two S.q-fs of $\not X$, but $F_1 \cup F_2 = \{0, 1, 3, 4\}$ is not a S.q-f of $X$, because $3, 4 \in F_1 \cup F_2$ , but $3 \square (3 \square 4) = 2 \notin F_1 \cup F_2$. Then $F_1 \cup F_2$ it is not a S.q-f.*

**Proposition 3.22.** *Let $\mathcal{X}$ be a S. BH-algebra and let $\{F_\beta,\ \beta \in \Omega\}$ be a chain of S.q.f of $X$. Then $\bigcup_{\beta\in\Omega} F_\beta$ is a S.q.f of $\mathcal{X}$.*

**Proof .** Let $\{F_\beta,\ \beta \in \Omega\}$ be a chain of S.q.f of $\mathcal{X}$. Then by Definition 3.13 $\{F_\beta,\ \beta \in \Omega\}$ is a chain of S.f of $X$. Thus, by Proposition 2.13, $\bigcup_{\beta\in\Omega} F_\beta$ is a S.f of $X$,

Now, let $x \square z \in \bigcup_{\beta\in\Omega} F_\beta,\ y \in \bigcup_{\beta\in\Omega} F_\beta$, such that $x, z \in Q$, then there exist $F_n, F_m \in \{F_\beta :\ \beta \in \Omega\}$, such that $x \square z \in F_n$ and $y \in F_m$. Thus either $F_n \subseteq F_m$ or $F_m \subseteq F_n$.

If $F_n \subseteq F_m$, then $x \square z \in F_m, y \in F_m$, such that $x, z \in Q$, thus there exists $m \in \Omega$ such that $x \square (y \square z) \in F_m$, because $F_\beta$ is a S.q.f of $X$, for all $\beta \in \Omega$. Consequently, $x \square (y \square z) \in \bigcup_{\beta\in\Omega} F_\beta$. Similarly, $F_m \subseteq F_n$. Hence, $\bigcup_{\beta\in\Omega} F_\beta$ is a S.$q$-f of $X$. $\square$

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