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Remark on how to solve Maxwell-Dirac Isomorphism problem in a Realism Interpretation of Wave Mechanics based on Dirac equation

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Abstract From time to time, some eminent physicists began to ask: What is the reality behind quantum mechanical predictions? Is there realism interpretation of Quantum Physics? This paper is intended to explore such a possibility of realism interpretation of QM, based on known problem called isomorphism between Maxwell-Dirac equation, which does not exist according to a number of theoretical physicists. In other article, we discuss a derivation of Maxwell equations in Quaternion Space (Progress in Physics, 2010), and it can be shown that this new theory is capable to describe electromagnetic in a harmonious way between classical and quantum theory in elegant and straightforward way. Second, we discuss how we can arrive at one-to-one correspondence between Maxwell and Dirac equations by starting again with a derivation of Maxwell equations using Dirac decomposition, introduced by Gersten (1999). Thirdly, we also discuss a second plausible approach, by describing Geometric Algebra in real Euclidean 3D space. Further observations are of course recommended in order to refute or verify some implications of this proposition.

1 Introduction

From time to time, some eminent physicists began to ask: What is the reality behind quantum mechanical predictions? Is there realism interpretation of Quantum Physics? For instance, in a recent paper Prof. G. 't Hooft asks: "The author has made his own analysis of the known facts, and came to the conclusion that the Copenhagen doctrine, that is, the consensus reached by many of the world experts at the beginning of the 20th century, partly during their numerous gatherings in the Danish capital, has it almost right: there is a wave function, or rather, something we call a quantum state, being a vector in Hilbert space, which obeys a Schrodinger equation. The absolute squares of the vector components may be used to describe probabilities whenever we wish to predict or explain something. Powerful techniques were developed, enabling one to guess the right Schrodinger equation if one knows how things evolve classically, that is, in the old theories where quantum mechanics had not yet been incorporated. It all works magnicently well. According to Copenhagen, however, there is one question one should not ask: "What does reality look like of whatever moves around in our experimental settings?", or: what is really going on? According to Copenhagen, such a question can never be addressed by means of any experiment, so it has no answer within the set of logical statements we can make of the world. Period, "schluss, ni." Those questions are senseless. It is this answer that we dispute. Even if this kind of questions cannot be answered by experiments, we can still in theory try to build credible models of reality."('t Hooft, [16]) This paper is intended to explore such a possibility of realism interpretation of QM, especially based on a derivation of Maxwell equations in Quaternion Space, an approach we developed quite sometime ago based on Gersten's method. In this regards, firstly we begin with discussion with Prof V. Simulik, on how he believes that it is impossible to argue for Maxwell-Dirac equations isomorphism. However, we can argue here that actually we can arrive to tenable arguments, as follows: (i) based on Quaternion space and its respective Quaternion Relativity (it also may be called as Rotational Relativity) as it has been defined in a number of papers including [1], and it can be shown that this new theory is capable to describe relativistic motion in elegant and straightforward way. Nonetheless there are subsequent theoretical developments which remains an open issue, for instance to derive Maxwell equations in Q-space [1]. Therefore the purpose of the present paper is to review our previous derivation of Maxwell equations in Q-space. (ii) Thirdly, we will discuss Geometric Algebra in Euclidean 3D real space. Further observations are of course recommended in order to refute or verify some implications of what we discuss here.

2 Problem of Maxwell-Dirac isomorphism. Basic aspects of Quaternion space

In this section, allow us to cite a discussion via email with Prof. V. Simulik a few years ago, he is a specialist in classical electromagnetic theories, including Maxwell theory etc. In response to one of us (VC)'s question: "Could it be that Max-Dir isomorphism exists only in matrix formulation or quaternionic expression? A quote from Bocker and Frieden paper at Heliyon, 2018: "As it turns out, the Dirac spacetime matrix equation is equivalent to four new vector equations, which are similar in form to the four Maxwell vector equations. These new equations will be referred to as the Dirac spacetime vector equations. This allows these new vector equations to be as readily solved as solving a set of Maxwell vector equations." Then may be the Dirac equation which has four vector is mathematically equivalent to Maxwell set of four vectors. Alternatively, in your book on electron (2005). There is a paper on (3,3) spacetime representation on electromagnetic theory." Then he wrote in June 2020: "The Maxwell-Dirac Isomorphism does not exist in mathematics. By fingers: eight real components of Dirac function - six real components of Maxwell functions (electromagnetic field strengths). Or four complex Dirac functions - three complex Maxwell fields. Isomorphism is eight to eight or four to four. And plus linearity... Therefore, the title Maxwell-Dirac Isomorphism of Sallhofer was not good. See Chapter 2, Section.2.4, especially 2.4.1 So it will be better if you can refer to it in future. This is the reason why I have sent two first chapters to you. Keller suggested mapping I used relation or relationship. 2) My big step forward (and today I am working with this as well) is the investigation of the situation, where Maxwell-Dirac Isomorphism really exists. It is four complex Maxwell fields or eight real. So called slightly generalized Maxwell equations for the system of electromagnetic and scalar fields. The first publication was in Advances in Applied Clifford Algebras in 1996 - today I do not have electronic copy." Then he wrote again in the same day, 15th June 2020: "The isomorphism does not exist. The main p[oint is one-to-one correspondence. Therefore, if we have 4 and 3, or 8 and 6 - no isomorphism. 4 cannot be isomorphic to 3." (cf. Richard P. Bocker, B. Roy Frieden. A new matrix formulation of the Maxwell and Dirac equations. Heliyon, Volume 4, Issue 12 (2018) e01033. doi: 10.1016/j.heliyon.2018.e01033; and also V. Simulik et al. 2019. Slightly generalized Maxwell system and longitudinal components of solution, J. Phys.: Conf. Ser. 1416 012033; also V. Simulik (ed.) What is the electron, Montreal: Apeiron C. Roy Keys Inc., 2005). And in the next section, we will argue in favor of one-to-one correspondence between Maxwell equations in quaternion space and quaternionic Dirac equations. And in the last section, we will argue that in response to Simulik's claim: "4 cannot be isomorphic to 3"; actually it is possible to find Dirac equation in real Euclidean 3D space representation, in terms of Geometric algebra (it is called: Cl(3,0) or simply Cl(3).) First of all, we will review some basic definitions of quaternion number [1]. Quaternion number belongs to the group of "very good" algebras: of real, complex, quaternion, and octonion [1], and normally defined as follows [1]:

$$Q \equiv a + bi + cj + dk. \tag{1}$$

Where a,b,c,d are real numbers, and i,j,k are imaginary quaternion units. These Q-units can be represented either via 2x2 matrices or 4x4 matrices. There is quaternionic multiplication rule which acquires compact form [1]:

$$1q_k = q_k 1 = q_k, \quad q_j q_k = -\delta_{jk} + \varepsilon_{jkn} q_n, \tag{2}$$

Where $\delta_{kn}$and ε_{jkn}represents 3-dimensional symbols of Kronecker and Levi-Civita, respectively.

In the context of Quaternion Space [1], it is also possible to write the dynamics equations of classical mechanics for an inertial observer in constant Q-basis. SO(3,R)-invariance of two vectors allow to represent these dynamics equations in Q-vector form [1]:

$$m\frac{d^2}{dt^2}(x_k q_k) = F_k q_k. \tag{3}$$

One advantage of this Quaternion Space representation is that it enables to describe rotational motion with great clarity.

After this short review of Q-space, next we will discuss a simplified method to derive Maxwell equations from Lorentz force, in a similar way with Feynman's derivation method using commutative relation [2][10].

3 A new derivation of Maxwell equations in Quaternion Space by virtue of Dirac decomposition.

In this section we present a derivation of Maxwell equations in Quaternion space based on Gersten's method to derive Maxwell equations from one photon equation by virtue of Dirac decomposition [4].

We know that Dirac deduces his equation from the relativistic condition linking the Energy E, the mass m and the momentum p [5]:

$$(E^2 - c^2\vec{p}^2 - m^2c^4)I^{(4)}\Psi = 0, \tag{4}$$

Where $I^{(4)}$is the 4x4 unit matrix and $\Psi$is a 4-component column (bispinor) wavefunction. Dirac then decomposes equation (7) by assuming them as a quadratic equation:

$$(A^2 - B^2)\Psi = 0 \tag{5}$$

Where

$$A = E, \tag{6}$$

$$B = c\vec{p} + mc^2. \tag{7}$$

The decomposition of equation (8) is well known, i.e. (A+B)(A-B)=0, which is the basic of Dirac's decomposition method into 2x2 unit matrix and Pauli matrix [4][12].

By virtue of the same method with Dirac, Gersten found in 1999 [4] a decomposition of one photon equation from relativistic energy condition (for massless photon [5]):

$$(\frac{E^2}{c^2} - \vec{p}^2)I^{(3)}\Psi = 0, \tag{8}$$

Where $I^{(3)}$ is the 3x3 unit matrix and $\Psi$is a 3-component column wavefunction. Gersten then found [4] equation (11) decomposes into the form:

$$[\frac{E}{c}I^{(3)} - \vec{p}\cdot\vec{S}][\frac{E}{c}I^{(3)} + \vec{p}\cdot\vec{S}]\vec{\Psi} - \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}(\vec{p}\cdot\vec{\Psi}) = 0, \tag{9}$$

Where $\vec{S}$is a spin one vector matrix with components [4]:

$$S_x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & -i & 0 \end{pmatrix}, \tag{10}$$

$$S_y = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \tag{11}$$

$$S_z = \begin{pmatrix} 0 & -i & 0 \\ -i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{12}$$

And with the properties:

$$[S_x, S_y] = iS_z, \quad [S_x, S_z] = iS_y, [S_y, S_z] = iS_x, \vec{S}^2 = 2I^{(3)}. \tag{13}$$

Gersten asserts that equation (22) will be satisfied if the two equations [4][5]:

$$[\frac{E}{c}I^{(3)} + \vec{p}\cdot\vec{S}]\vec{\Psi} = 0, \tag{14}$$

$$\vec{p}\cdot\vec{\Psi} = 0, \tag{15}$$

are simultaneously satisfied. The Maxwell equations [9] will be obtained by substitution of E and p with the ordinary quantum operators (see for instance, H. Bethe, *Field Theory*):

$$E \rightarrow i\hbar\frac{\partial}{\partial t} \quad , \tag{16}$$

and

$$p \rightarrow -ih\nabla, \tag{17}$$

And the wavefunction substitution:

$$\vec{\Psi} = \vec{E} - i\vec{B}, \tag{18}$$

Where E and B are electric and magnetic fields, respectively. With the identity:

$$(\vec{p}\cdot\vec{S})\vec{\Psi} = \hbar\nabla\times\vec{\Psi}. \tag{19}$$

Then from equation (17)-(19) one will obtain:

$$i\frac{\hbar}{c}\frac{\partial(\vec{E} - i\vec{B})}{\partial t} = -\hbar\nabla\times(\vec{E} - i\vec{B}), \tag{20}$$

$$\nabla\cdot(\vec{E} - i\vec{B}) = 0, \tag{21}$$

Which are the Maxwell equations if the electric and magnetic fields are real [4][5].

We can remark here that the combination of E and B as introduced in eq. (18) is quite well known in literature [6][7]. For instance, if we use positive signature in (21), then it is known as Bateman representation of Maxwell equations ($div\vec{\varepsilon} = 0; rot\vec{\varepsilon} = \frac{1}{c}\frac{\partial\vec{\varepsilon}}{\partial t}; \varepsilon = \vec{E} + i\vec{B}$). But the equation (21) with negative signature represents the *complex nature* of Electromagnetic fields [6], which indicates that these fields can also be represented in quaternion form.

Now if we represent in other form $\vec{\varepsilon} = \vec{E} - i\vec{B}$ as more conventional notation, then equation (20) and (21) will get a quite simple form:

$$i\frac{\hbar}{c}\frac{\partial\vec{\varepsilon}}{\partial t} = -\hbar\nabla\times\vec{\varepsilon}, \tag{22}$$

$$\nabla\cdot\vec{\varepsilon} = 0. \tag{23}$$

Now to consider quaternionic expression of the above results from Gersten [4], one can begin with the same linearization procedure just as in equation (3):

$$dz = (dx_k + idt_k)\, q_k,$$

Which can be viewed as the quaternionic square root of the metric interval dz:

$$dz^2 = dx^2 - dt^2. \tag{24}$$

Now consider the relativistic energy condition (for massless photon [5]) similar to equation (4):

$$E^2 = p^2c^2 \Rightarrow (\frac{E^2}{c^2} - \vec{p}^2) = k^2, \tag{25}$$

It is obvious that equation (25) has the same form with (8), therefore we may find its quaternionic square root too, then we find:

$$k = (E_{qk} + i\vec{p}_{qk})\, q_k, \tag{26}$$

Where q represents the quaternion unit matrix. Therefore the linearized quaternion root decomposition of equation (11) can be written as follows [4]:

$$\begin{array}{l} [\frac{E_{qk}q_k}{c} I^{(3)} + i\vec{p}_{qk}q_k \cdot \vec{S}][\frac{E_{qk}q_k}{c} I^{(3)} + i\vec{p}_{qk}q_k \cdot \vec{S}]\vec{\Psi} \\ -\begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} (i\vec{p}_{qk}q_k \cdot \vec{\Psi}) = 0 \end{array} \quad . \quad . \tag{27}$$

Accordingly, equation (40) will be satisfied if the two equations:

$$[\frac{E_{qk}q_k}{c} I^{(3)} + i\vec{p}_{qk}q_k \cdot \vec{S}]\vec{\Psi}_k = 0, \tag{28}$$

$$i\vec{p}_{qk}q_k \cdot \vec{\Psi}_k = 0, \tag{29}$$

are simultaneously satisfied. Now we introduce similar wavefunction substitution, but this time in quaternion form:

$$\vec{\Psi}_{qk} = \vec{E}_{qk} - i\vec{B}_{qk} = \vec{\varepsilon}_{qk}. \tag{30}$$

And with the identity:

$$(\vec{p}_{qk}q_k \cdot \vec{S})\vec{\Psi}_k = \hbar\nabla_k \times \vec{\Psi}_k \quad . \tag{31}$$

Then from equation (30) and (31) one will obtain *the Maxwell equations in Quaternion-space* as follows:

$$i\frac{\hbar}{c}\frac{\partial \vec{\varepsilon}_q k}{\partial t} = -\hbar\nabla_k \times \vec{\varepsilon}_{qk} \quad , \tag{32}$$

$$\nabla_k \cdot \vec{\varepsilon}_{qk} = 0. \tag{33}$$

Now the remaining question is to define quaternion differential operator in the right hand side of (32) and (33).

In this regards one can choose some definitions of quaternion differential operator, for instance the so-called "Moisil-Theodoresco operator"[8] :

$$D[\varphi] = grad\varphi = \sum_{k=1}^{3} i_k\partial_k\varphi = i_1\partial_1\varphi + i_2\partial_2\varphi + i_3\partial_3\varphi, \tag{34}$$

Where we can define here that $i_1 = i, i_2 = j, i_3 = k$ to represent 2x2 quaternion unit matrix, for instance.

Therefore the differential of equation (33) now can be expressed in similar notation of (34) :

$$\begin{array}{l} D[\vec{\Psi}] = D[\vec{\varepsilon}] = i_1\partial_1 E_1 + i_2\partial_2 E_2 + i_3\partial_3 E_3 \\ -i\,(i_1\partial_1 B_1 + i_2\partial_2 B_2 + i_3\partial_3 B_3) \end{array} \quad . \tag{35}$$

This expression indicates that both electric and magnetic fields can be represented in unified manner in a biquaternion form.

Then we define quaternion differential operator in the right-hand-side of equation (32) by an extension of the conventional definition of curl:

$$\nabla \times A_{qk} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}, \tag{36}$$

To become its quaternion counterpart, where i,j,k represents quaternion matrix as described above. This quaternionic extension of curl operator is based on the known relation of multiplication of two arbitrary complex quaternions q and b as follows:

$$q \cdot b = q_0 b_0 - \left\langle \vec{q}, \vec{b} \right\rangle + [\vec{q} \times \vec{b}] + q_0\vec{b} + b_0\vec{q}, \tag{37}$$

Where

$$\left\langle \vec{q}, \vec{b} \right\rangle := \sum_{k=1}^{3} q_k b_k \in C, \tag{38}$$

And

$$\left[\vec{q} \times \vec{b}\right] := \begin{vmatrix} i & j & k \\ q_1 & q_2 & q_3 \\ b_1 & b_2 & b_3 \end{vmatrix}. \tag{39}$$

We can note here that there could be more rigorous approach to define such a quaternionic curl operator.[7]

In the present paper we only discuss derivation of Maxwell equations in Quaternion Space using the decomposition method described by Gersten [4][5]. Further extension to Proca equations in Quaternion Space seems possible too using the same method [5], but it will not be discussed here.

In the next section we will discuss some physical implications of this new derivation of Maxwell equations in Quaternion Space.

4 Discussion: other solution in real Euclidean 3D Space, and possible Helical solutions of Maxwell equations; and limitations of this paper.

In the foregoing section we derived a consistent description of Maxwell equations in Q-Space by virtue of Dirac-Gersten's decomposition. Actually there is also one more plausible approach in the context of Geometric algebra, where Dirac equation can be put into real Euclidean 3D space (Cl3), which can be linked to rotation in Euclidean space (SO3). Miroslav Josipovic wrote: "First, the Cl3 formulation of the Dirac theory is simple and the derivation of the Dirac's formula is straightforward. Second, it is relatively easy to show that gamma matrices are not the only possibility in linearizing the Klein-Gordon equation (we even do not need it in Cl3). ...the fact that it is possible to use the same mathematical (3D) formalism for classical mechanics..." Nonetheless, we don't explore in more detailed on such a possibility, except that such a Cl3 method has been utilised to analyse graphene crystal. (cf. M. Josipovic, Dirac theory in Euclidean 3D Geometric algebra (Cl3), chapter in Geometric Multiplication of Vectors: An Introduction to Geometric Algebra in Physics, Birkhäuser, 2019; also A. Dargys. Quantum flatland and monolayer graphene... ACTA PHYSICA POLONICA A, Vol. 124 (2013); G. Coddens. The Geometrical Meaning of Spinors Lights the Way to Make Sense of Quantum Mechanics. Symmetry 2021, 13, 659 (MDPI), 45 pp.)

Now we are going to discuss some plausible implications of the new proposition of plausible one-to-one correspondence and even isomorphism between Maxwell-Dirac equations: (*) First, in accordance with Gersten, we submit the viewpoint that the Maxwell equations yield wavefunctions which can be used as guideline for interpretation of quantum mechanics [4][5]. We shall emphasize here: "The one-to-one correspondence between classical and quantum wave interpretation actually can be expected not only in the context of Feynman's derivation of Maxwell equations from Lorentz force, but also from known exact correspondence between commutation relation and Poisson bracket." See also [3][4].

(**) Secondly, the above expressions of Maxwelll equations in Q-space are still missing the A term. Provided we include A term as defined by R.W. Bass [2], as follows:

$A = \nabla x(\psi u) + (1/\lambda)\nabla x(\nabla x(\psi u))$

then we got helical solutions of Maxwell eq., which are more consistent with many experimental results, instead of the well-known "sinusoidal" solutions.[2]

A few more remark deserves further attention, as follows: "This helical form of EM ties into the KH electron, during photon capture events, The captured photon causes an energetic imbalance in the desired and required stability of the electron, which causes the photon to be ejected in a short while. We're still trying to discover what is the cause of the form of the electron being as stable as it is, and why it has such a strong surface boundary." (KH=Kelvin-Helmholtz)

Further implications of this new proposition of helical solutions of Maxwell eq. require further study, and therefore they are excluded from the present paper.

Before we close this paper, allow us to point out two limitations of the procedure as outlined above: (a) It begins with Gersten decomposition and Dirac equation, which assumes that mass-energy equivalence holds true. Whenever one can show that such an equivalence does not hold true, then our procedure should be revised. (b) Our model assumes orthogonality. Some authors have argued that orthogonality of Dirac equation prevents it to predict fractional states of hydrogen (hydrino, cf. R.L. Mills).[15] However, the majority of interactions and appearances in Nature are not orthogonal, but are comprised of acute and oblique angles, usually far from orthogonality. This fact is covered by dot products and cross products, as required by engineering applications of the Maxwell equations. Quaternion solutions are not capable of doing dot products, to my knowledge.

5 Concluding remarks

In the present paper we review our previous paper on derivation of a consistent description of Maxwell equations in Q-space. First, we present another method to derive Maxwell equations by virtue of Dirac decomposition, introduced by Gersten (1999).

In accordance with Gersten, we submit the viewpoint that the Maxwell equations yield wavefunctions which can be used as guideline for interpretation of quantum mechanics. The one-to-one correspondence between classical and quantum wave interpretation asserted here actually can be expected not only in the context of Feynman's derivation of Maxwell equations from Lorentz force, but also from known exact correspondence between commutation relation and Poisson bracket [2][4].

A somewhat unique implication obtained from the above results of Maxwell equations in Quaternion Space, is that it suggests that the helical solutions, especially if we consider A vector from R. Bass. Further implications, however, are beyond the scope of the present paper. Another plausible new approach to get to Maxwell-Dirac isomorphism is through Cl3 / Geometric Algebra formalism in Real Euclidean 3D space.

This proposition, however, deserves further theoretical considerations. Further observation is of course recommended in order to refute or verify some implications of this result.

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