Cubic Spherical Neutrosophic Sets

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Abstract

This paper introduces the concept of cubic spherical neutrosophic sets (CSNSs), a geometric representation of neutrosophic sets, as well as a specification of its operational principles. In CSNs, two aggregation operators are investigated. The shape of CSNSs represents the evaluation values of alternatives with respect to criteria in a MCDM strategy based on the two aggregation operators and cosine distance for CSNSs. The cosine distance between an alternative and the ideal alternative is used to rank them, and the best alternative(s) can be selected. A numerical example concludes by demonstrating the use of the suggested method.

Keywords: neutrosophic set; cubic spherical neutrosophic set; cubic spherical neutrosophic aggregation operators; multi criteria decision making.

1 Introduction and Preliminaries

The neutrosophic set (NS) presented initially by Smarandache[9,10] generalizes an IFS from philosophical point of view. Truth, falsity-membership, and indeterminacy are NS attributes that are independent in nature functions. Numerous sets are introduced as a generalization of NSs, and their properties and applications are studied. These sets include interval valued NSs,[16] single valued NSs,[15] bipolar NSs,[4] multi-valued NSs,[6] simplified NSs,[7] Type-2 NSs,[5] Possibilistic NSs,[11] Trapezoidal NSs,[17] Linear Diophantine NSs,[3] Quadripartitioned single-valued NSs[2] Pentapartitioned NSs,[8] Interval Valued Pentapartitioned NSs[13]and refined single-valued NSs.[14] Additionally, the neutrosophic theory extends to include number theory, operations research, algebra, topology, graphs, probability, and numerical measures. Furthermore, statistical terms like level of significance, confidence interval, and central limit theorem. The field neutrosophic set theory is applied in many filed which includes logic in image processing, large-scale image and multimedia processing, machine learning, big data analytics, deep learning, data visualization, feature learning, classification, regression, and clustering, virtual reality, heterogeneous data mining multimodal sensor data, wireless sensor networks, astronomy, space sciences, boinformatics and medical analytics, retrieval of medical images, brain-machine interfaces and medical signal analysis and large-scale health care data.

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Table 1: The acronyms used in the current research have been listed below:

Acronyms	Expansion
NS	Neutroshopic set
CSNS	Cubic spherical neutrosophic set
CSNWAAO	cubic spherical neutrosophic weighted aritheoremetic average operator
CSNWGAO	cubic spherical neutrosophic weighted geometric average operator
MCDM	multi criteria decision making
DM	Decision Making
MINI	Minimum
MAXI	Maximum

2 CUBIC SPHERICAL NEUTROSOPHIC SETS

Let's assume a fixed universe X and its subset csA. The set

$$csA_\rho = \{< x, cs\mu(x), cs\nu(x), cs\eta(x); \rho >: x \in X\}$$

where $cs\mu, cs\nu, cs\eta : X \to [0,1]$ are functions such that $cs\mu_A + cs\nu_A + cs\eta_A \leq 3$ and $\rho \in [0,1]$. The radius ρ of the sphere with center $(cs\mu(x), cs\nu(x), cs\eta(x))$ inside the cube or cube inside the sphere is called cubic spherical neutrosophic set (CSNS) csA_ρ. This sphere represents the membership degree, indeterminacy degree and non-membership degree of $x \in X$.

Let $\{< cs\mu_{i,1}, cs\nu_{i,1}, cs\eta_{i,1} >, < cs\mu_{i,2}, cs\nu_{i,2}, cs\eta_{i,2} >, ..., < cs\mu_{i,k_i}, cs\nu_{i,k_i}, cs\eta_{i,k_i} >\}$ be a collection of NSs assigned for any x_i in X. We construct the center of the sphere by

$$< cs\mu(x_i), cs\nu(x_i), cs\eta(x_i) >=< \frac{\sum_{j=1}^{k_i} cs\mu_{i,j}}{k_i}, \frac{\sum_{j=1}^{k_i} cs\nu_{i,j}}{k_i}, \frac{\sum_{j=1}^{k_i} cs\eta_{i,j}}{k_i} > \quad (1)$$

and the radius using

$$\rho_i = \min\left\{ \max_{1\leq j\leq k_i} \sqrt{(cs\mu(x_i) - cs\mu_{i,j})^2 + (cs\nu(x_i) - cs\nu_{i,j})^2 + (cs\eta(x_i) - cs\eta_{i,j})^2}, 1 \right\}. \quad (2)$$

Then the spheres inside the cube or cube inside the sphere is

$$csA_\rho = \{< x_i, cs\mu(x_i), cs\nu(x_i), cs\eta(x_i); \rho >: x_i \in \mathbb{X}\}. \quad (3)$$

The possible spheres inside the cube or cube inside the sphere is represented in the following figure:

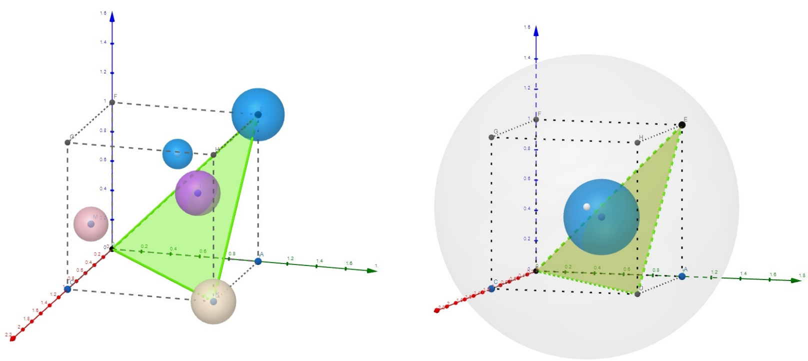

Figure 1: Geometric representation of CSNSs

Example 2.1. Let $\{< 0.9, 0.1, 0.1 >, < 0.8, 0.2, 0.15 >< 0.65, 0.35, 0.3 >< 0.5, 0.5, 0.5 >\}$ and $\{< 1, 0, 0 >, < 0, 1, 0 >, < 0, 0, 1 >, < 1, 1, 1 >\}$, be the collection of NSs. Then the $CSNSs$ are $csA_{\rho_1} =< 0.71, 0.29, 0.26; 0.38 >$ and $csA_{\rho_2} =< 0.5, 0.5, 0.5; 0.87 >\}$.

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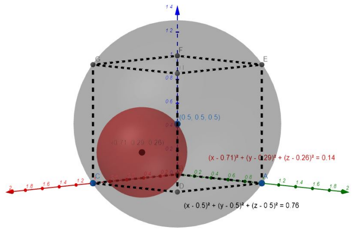

Figure 2: cubic spherical neutrosophic sets

Definition 2.2. Let $\lambda = < cs\mu_\lambda, cs\nu_\lambda, cs\eta_\lambda; \rho_\lambda >$, $\lambda_1 = < cs\mu_{\lambda_1}, cs\nu_{\lambda_1}, cs\eta_{\lambda_1}; \rho_{\lambda_1} >$ and $\lambda_2 = < cs\mu_{\lambda_2}, cs\nu_{\lambda_2}, cs\eta_{\lambda_2}; \rho_{\lambda_2} >$ be three CSNS over the universal set X, $\gamma \in \{MINI, MAXI\}$ and $\alpha > 0$. Then the following operations are defined as follows

1. $\lambda_1 \cup_\gamma \lambda_2 = \langle MAXI\{cs\mu_{\lambda_1}, cs\mu_{\lambda_2}\}, MINI\{cs\nu_{\lambda_1}, cs\nu_{\lambda_2}\}, MINI\{cs\eta_{\lambda_1}, cs\eta_{\lambda_2}\}; \gamma\{\rho_{\lambda_1}, \rho_{\lambda_2}\}\rangle$.

2. $\lambda_1 \cup_\gamma \lambda_2 = \langle MAXI\{cs\mu_{\lambda_1}, cs\mu_{\lambda_2}\}, MAXI\{cs\nu_{\lambda_1}, cs\nu_{\lambda_2}\}, MINI\{cs\eta_{\lambda_1}, cs\eta_{\lambda_2}\}; \gamma\{\rho_{\lambda_1}, \rho_{\lambda_2}\}\rangle$.

3. $\lambda_1 \cup_\gamma \lambda_2 = \langle MAXI\{cs\mu_{\lambda_1}, cs\mu_{\lambda_2}\}, MAXI\{cs\nu_{\lambda_1}, cs\nu_{\lambda_2}\}, MAXI\{cs\eta_{\lambda_1}, cs\eta_{\lambda_2}\}; \gamma\{\rho_{\lambda_1}, \rho_{\lambda_2}\}\rangle$.

4. $\lambda_1 \cup_\gamma \lambda_2 = \left\langle MAXI\{cs\mu_{\lambda_1}, cs\mu_{\lambda_2}\}, \frac{cs\nu_{\lambda_1} + cs\nu_{\lambda_2}}{2}, MINI\{cs\eta_{\lambda_1}, cs\eta_{\lambda_2}\}; \gamma\{\rho_{\lambda_1}, \rho_{\lambda_2}\}\right\rangle$.

5. $\lambda_1 \cup_\gamma \lambda_2 = \left\langle MAXI\{cs\mu_{\lambda_1}, cs\mu_{\lambda_2}\}, 1 - \frac{cs\nu_{\lambda_1} + cs\nu_{\lambda_2}}{2}, MINI\{cs\eta_{\lambda_1}, cs\eta_{\lambda_2}\}; \gamma\{\rho_{\lambda_1}, \rho_{\lambda_2}\}\right\rangle$.

6. $\lambda_1 \cup_\gamma \lambda_2 = \langle MAXI\{cs\mu_{\lambda_1}, cs\mu_{\lambda_2}\}, |cs\nu_{\lambda_1} - cs\nu_{\lambda_2}|, MAXI\{cs\eta_{\lambda_1}, cs\eta_{\lambda_2}\}; \gamma\{\rho_{\lambda_1}, \rho_{\lambda_2}\}\rangle$.

7. $\lambda_1 \cap_\gamma \lambda_2 = \langle MINI\{cs\mu_{\lambda_1}, cs\mu_{\lambda_2}\}, MAXI\{cs\nu_{\lambda_1}, cs\nu_{\lambda_2}\}, MAXI\{cs\eta_{\lambda_1}, cs\eta_{\lambda_2}\}; \gamma\{\rho_{\lambda_1}, \rho_{\lambda_2}\}\rangle$.

8. $\lambda_1 \cap_\gamma \lambda_2 = \langle MINI\{cs\mu_{\lambda_1}, cs\mu_{\lambda_2}\}, MINI\{cs\nu_{\lambda_1}, cs\nu_{\lambda_2}\}, MAXI\{cs\eta_{\lambda_1}, cs\eta_{\lambda_2}\}; \gamma\{\rho_{\lambda_1}, \rho_{\lambda_2}\}\rangle$.

9. $\lambda_1 \cap_\gamma \lambda_2 = \langle MINI\{cs\mu_{\lambda_1}, cs\mu_{\lambda_2}\}, MINI\{cs\nu_{\lambda_1}, cs\nu_{\lambda_2}\}, MINI\{cs\eta_{\lambda_1}, cs\eta_{\lambda_2}\}; \gamma\{\rho_{\lambda_1}, \rho_{\lambda_2}\}\rangle$.

10. $\lambda_1 \cap_\gamma \lambda_2 = \left\langle MINI\{cs\mu_{\lambda_1}, cs\mu_{\lambda_2}\}, \frac{cs\nu_{\lambda_1} + cs\nu_{\lambda_2}}{2}, MAXI\{cs\eta_{\lambda_1}, cs\eta_{\lambda_2}\}; \gamma\{\rho_{\lambda_1}, \rho_{\lambda_2}\}\right\rangle$.

11. $\lambda_1 \cap_\gamma \lambda_2 = \left\langle MINI\{cs\mu_{\lambda_1}, cs\mu_{\lambda_2}\}, 1 - \frac{cs\nu_{\lambda_1} + cs\nu_{\lambda_2}}{2}, MAXI\{cs\eta_{\lambda_1}, cs\eta_{\lambda_2}\}; \gamma\{\rho_{\lambda_1}, \rho_{\lambda_2}\}\right\rangle$.

12. $\lambda_1 \cap_\gamma \lambda_2 = \langle MINI\{cs\mu_{\lambda_1}, cs\mu_{\lambda_2}\}, |cs\nu_{\lambda_1} - cs\nu_{\lambda_2}|, MAXI\{cs\eta_{\lambda_1}, cs\eta_{\lambda_2}\}; \gamma\{\rho_{\lambda_1}, \rho_{\lambda_2}\}\rangle$.

13. $\lambda_1 = \lambda_2$. iff $\rho_{\lambda_1} = \rho_{\lambda_2}$ and $cs\mu_{\lambda_1} = cs\mu_{\lambda_2}, cs\nu_{\lambda_1} = cs\nu_{\lambda_2}, cs\eta_{\lambda_1} = cs\eta_{\lambda_2}$.

14. $\lambda_1 \subset \lambda_2$. iff $\rho_{\lambda_1} \subset \rho_{\lambda_2}$ and $cs\mu_{\lambda_1} \subset cs\mu_{\lambda_2}, cs\nu_{\lambda_1} \subset cs\nu_{\lambda_2}, cs\eta_{\lambda_1} \supset cs\eta_{\lambda_2}$.

15. $\lambda_1 \subset \lambda_2$. iff $\rho_{\lambda_1} \subset \rho_{\lambda_2}$ and $cs\mu_{\lambda_1} \subset cs\mu_{\lambda_2}, cs\nu_{\lambda_1} \supset cs\nu_{\lambda_2}, cs\eta_{\lambda_1} \supset cs\eta_{\lambda_2}$.

16. $\lambda_1 \subseteq \lambda_2$. iff $\rho_{\lambda_1} \subseteq \rho_{\lambda_2}$ and $cs\mu_{\lambda_1} \subseteq cs\mu_{\lambda_2}, cs\nu_{\lambda_1} \subseteq cs\nu_{\lambda_2}, cs\eta_{\lambda_1} \supseteq cs\eta_{\lambda_2}$.

17. $\lambda_1 \subseteq \lambda_2$. iff $\rho_{\lambda_1} \subseteq \rho_{\lambda_2}$ and $cs\mu_{\lambda_1} \subseteq cs\mu_{\lambda_2}, cs\nu_{\lambda_1} \supseteq cs\nu_{\lambda_2}, cs\eta_{\lambda_1} \supseteq cs\eta_{\lambda_2}$.

18. $\lambda_1 \subseteq \lambda_2$. iff $\rho_{\lambda_1} \subseteq \rho_{\lambda_2}$ and $cs\mu_{\lambda_1} \subseteq cs\mu_{\lambda_2}, cs\nu_{\lambda_1} \subset cs\nu_{\lambda_2}, cs\eta_{\lambda_1} \supset cs\eta_{\lambda_2}$.

19. $\lambda_1 \subseteq \lambda_2$. iff $\rho_{\lambda_1} \subseteq \rho_{\lambda_2}$ and $cs\mu_{\lambda_1} \subset cs\mu_{\lambda_2}, cs\nu_{\lambda_1} \supseteq cs\nu_{\lambda_2}, cs\eta_{\lambda_1} \supseteq cs\eta_{\lambda_2}$.

20. $\lambda_1 \supset \lambda_2$. iff $\rho_{\lambda_1} \supset \rho_{\lambda_2}$ and $cs\mu_{\lambda_1} \supset cs\mu_{\lambda_2}, cs\nu_{\lambda_1} \supset cs\nu_{\lambda_2}, cs\eta_{\lambda_1} \subset cs\eta_{\lambda_2}$.

21. $\lambda_1 \supset \lambda_2$. iff $\rho_{\lambda_1} \supset \rho_{\lambda_2}$ and $cs\mu_{\lambda_1} \supset cs\mu_{\lambda_2}, cs\nu_{\lambda_1} \subset cs\nu_{\lambda_2}, cs\eta_{\lambda_1} \subset cs\eta_{\lambda_2}$.

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22. $\lambda_1 \supseteq \lambda_2$. iff $\rho_{\lambda_1} \supseteq \rho_{\lambda_2}$ and $cs\mu_{\lambda_1} \supseteq cs\mu_{\lambda_2}, cs\nu_{\lambda_1} \supseteq cs\nu_{\lambda_2}, cs\eta_{\lambda_1} \subseteq cs\eta_{\lambda_2}$.

23. $\lambda_1 \supseteq \lambda_2$. iff $\rho_{\lambda_1} \supseteq \rho_{\lambda_2}$ and $cs\mu_{\lambda_1} \supseteq cs\mu_{\lambda_2}, cs\nu_{\lambda_1} \subseteq cs\nu_{\lambda_2}, cs\eta_{\lambda_1} \subseteq cs\eta_{\lambda_2}$.

24. $\lambda_1 \supseteq \lambda_2$. iff $\rho_{\lambda_1} \supseteq \rho_{\lambda_2}$ and $cs\mu_{\lambda_1} \supseteq cs\mu_{\lambda_2}, cs\nu_{\lambda_1} \supset cs\nu_{\lambda_2}, cs\eta_{\lambda_1} \subset cs\eta_{\lambda_2}$.

25. $\lambda_1 \supseteq \lambda_2$. iff $\rho_{\lambda_1} \supseteq \rho_{\lambda_2}$ and $cs\mu_{\lambda_1} \supset cs\mu_{\lambda_2}, cs\nu_{\lambda_1} \subseteq cs\nu_{\lambda_2}, cs\eta_{\lambda_1} \subseteq cs\eta_{\lambda_2}$.

26. $\lambda_1 \oplus \lambda_2 = \langle cs\mu_{\lambda_1} + cs\mu_{\lambda_2} - cs\mu_{\lambda_1} cs\mu_{\lambda_2}, cs\nu_{\lambda_1} + cs\nu_{\lambda_2} - cs\nu_{\lambda_1} cs\nu_{\lambda_2}, cs\eta_{\lambda_1} + cs\eta_{\lambda_2} - cs\eta_{\lambda_1} cs\eta_{\lambda_2}; \rho_{\lambda_1} + \rho_{\lambda_2} - \rho_{\lambda_1}\rho_{\lambda_2} \rangle$

27. $\lambda_1 \otimes \lambda_2 = \langle cs\mu_{\lambda_1} cs\mu_{\lambda_2}, cs\nu_{\lambda_1} cs\nu_{\lambda_2}, cs\eta_{\lambda_1} cs\eta_{\lambda_2}; \rho_{\lambda_1}\rho_{\lambda_2} \rangle$.

28. $\alpha\lambda = \langle 1 - (1 - cs\mu_\lambda)^\alpha, 1 - (1 - cs\nu_\lambda)^\alpha, 1 - (1 - cs\eta_\lambda)^\alpha; 1 - (1 - \rho_\lambda)^\alpha \rangle$.

29. $\lambda^\alpha = \langle cs\mu_\lambda^\alpha, cs\nu_\lambda^\alpha, cs\eta_\lambda^\alpha; \rho_\lambda^\alpha \rangle$.

30. $\neg\lambda = \langle cs\eta_\lambda, cs\nu_\lambda, cs\mu_p; \rho_\lambda \rangle$.

31. $\neg\lambda = \langle cs\eta_\lambda, 1 - nu\eta_\lambda, cs\mu_\lambda; \rho_\lambda \rangle$.

32. $\neg\lambda = \langle 1 - cs\mu_\lambda, 1 - cs\nu_\lambda, 1 - cs\eta_\lambda; \rho_\lambda \rangle$.

33. $\neg\lambda = \langle 1 - cs\mu_\lambda, cs\nu_\lambda, 1 - cs\eta_\lambda; \rho_\lambda \rangle$.

Proposition 2.3. *For any three $CSNSs$ $\lambda_1, \lambda_2, \lambda_3$ and $\gamma \in \{MINI, MAXI\}$ the following results are valid,*

1. $\lambda_1 \cap_\gamma \lambda_2 = \lambda_2 \cap_\gamma \lambda_1$.
2. $\lambda_1 \cup_\gamma \lambda_2 = \lambda_2 \cup_\gamma \lambda_1$.
3. $\lambda_1 \oplus \lambda_2 = \lambda_2 \oplus \lambda_1$.
4. $\lambda_1 \otimes \lambda_2 = \lambda_2 \otimes \lambda_1$.
5. $(\lambda_1 \cap_\gamma \lambda_2) \cap \lambda_3 = \lambda_1 \cap_\gamma (\lambda_2 \cap_\gamma \lambda_3)$.
6. $(\lambda_1 \cup_\gamma \lambda_2) \cup \lambda_3 = \lambda_1 \cup_\gamma (\lambda_2 \cup_\gamma \lambda_3)$.
7. $(\lambda_1 \oplus \lambda_2) \oplus \lambda_3 = \lambda_1 \oplus (\lambda_2 \oplus \lambda_3)$.
8. $(\lambda_1 \otimes \lambda_2) \otimes \lambda_3 = \lambda_1 \otimes (\lambda_2 \otimes \lambda_3)$.
9. $(\lambda_1 \cap_\gamma \lambda_2) \cap \lambda_3 = (\lambda_1 \cap_\gamma \lambda_3) \cap (\lambda_2 \cap_\gamma \lambda_3)$.
10. $(\lambda_1 \cap_\gamma \lambda_2) \cup \lambda_3 = (\lambda_1 \cup_\gamma \lambda_3) \cap (\lambda_2 \cup_\gamma \lambda_3)$.
11. $(\lambda_1 \cap_\gamma \lambda_2) \oplus \lambda_3 = (\lambda_1 \oplus \lambda_3) \cap (\lambda_2 \oplus \lambda_3)$.
12. $(\lambda_1 \cap_\gamma \lambda_2) \otimes \lambda_3 = (\lambda_1 \otimes \lambda_3) \cap (\lambda_2 \otimes \lambda_3)$.
13. $(\lambda_1 \cup_\gamma \lambda_2) \cap \lambda_3 = (\lambda_1 \cap_\gamma \lambda_3) \cup (\lambda_2 \cap_\gamma \lambda_3)$.
14. $(\lambda_1 \cup_\gamma \lambda_2) \cup \lambda_3 = (\lambda_1 \cup_\gamma \lambda_3) \cup (\lambda_2 \cup_\gamma \lambda_3)$.
15. $(\lambda_1 \cup_\gamma \lambda_2) \oplus \lambda_3 = (\lambda_1 \oplus \lambda_3) \cup (\lambda_2 \oplus \lambda_3)$.
16. $(\lambda_1 \cup_\gamma \lambda_2) \otimes \lambda_3 = (\lambda_1 \otimes \lambda_3) \cup (\lambda_2 \otimes \lambda_3)$.
17. $\lambda_1 \cap_\gamma \lambda_1 = \lambda_1$.
18. $\lambda_1 \cup_\gamma \lambda_1 = \lambda_1$.
19. $\neg(\neg\lambda_1 \cap_\gamma \neg\lambda_2) = \lambda_1 \cup_\gamma \lambda_2$.
20. $\neg(\neg\lambda_1 \cup_\gamma \neg\lambda_2) = \lambda_1 \cap_\gamma \lambda_2$.
21. $\neg(\neg\lambda_1 \oplus \neg\lambda_2) = \lambda_1 \otimes \lambda_2$.
22. $\neg(\neg\lambda_1 \otimes \neg\lambda_2) = \lambda_1 \oplus \lambda_2$.

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3 CUBIC SPHERICAL NEUTROSOPHIC AGGREGATION OPERATOR

Definition 3.1. Let λ_β $(\beta = 1, 2, ..., \theta)$ be a CSNS. Then the cubic spherical neutrosophic weighted arithmetic average operator is defined as $CSNWAAO_\omega(\lambda_1, \lambda_2, ...\lambda_\theta) = \sum_{\beta=1}^{\theta} \omega_\beta \lambda_\beta$, where $\omega = (\omega_1, \omega_2, ..., \omega_\theta)^T$ is the weight vector of $\lambda_\beta(\beta = 1, 2, ..., \theta)$, $\omega_\beta \in [0, 1]$ and $\sum_{\beta=1}^{\theta} \omega_\beta = 1$.

Definition 3.2. Let λ_β $(\beta = 1, 2, ..., \theta)$ be a CSNS. Then the cubic spherical neutrosophic weighted geometric average operator is is defined as $CSNWGAO_\omega(\lambda_1, \lambda_2, ...\lambda_\theta) = \prod_{\beta=1}^{\theta} \omega_\beta \lambda_\beta$, where $\omega = (\omega_1, \omega_2, ..., \omega_\theta)^T$ is the weight vector of $\lambda_\beta(\beta = 1, 2, ..., \theta)$, $\omega_\beta \in [0, 1]$ and $\sum_{\beta=1}^{\theta} \omega_\beta = 1$.

Theorem 3.3. *For a CSNS $\lambda_\beta(\beta = 1, 2, ..., \theta)$, we have the following result:*
$CSNWAAO_\omega(\lambda_1, \lambda_2, ...\lambda_\theta) =$

$$\left\langle 1 - \prod_{\beta=1}^{\theta}(1 - cs\mu_{\lambda_\beta})^{\omega_\beta}, 1 - \prod_{\beta=1}^{\theta}(1 - cs\nu_{\lambda_\beta})^{\omega_\beta}, 1 - \prod_{\beta=1}^{\theta}(1 - cs\eta_{\lambda_\beta})^{\omega_\beta};\ \ 1 - \prod_{\beta=1}^{\theta}(1 - \rho_{\lambda_\beta})^{\omega_\beta} \right\rangle \tag{4}$$

where $\omega = (\omega_1, \omega_2, ..., \omega_\theta)^T$ is the weight vector of $\lambda_\beta(\beta = 1, 2, ..., \theta)$, $\omega_\beta \in [0, 1]$ and $\sum_{\beta=1}^{\theta} \omega_\beta = 1$.

Proof: *Mathematical induction can be used to prove the Theorem.*

Case 1: when $\theta = 2$, then
$\omega_1\lambda_1 = < 1 - (1 - cs\mu_{\lambda_1})^{\omega_1}, 1 - (1 - cs\nu_{\lambda_1})^{\omega_1}, 1 - (1 - cs\eta_{\lambda_1})^{\omega_1}; 1 - (1 - \rho_{\lambda_1})^{\omega_1} >$,
$\omega_2\lambda_2 = < 1 - (1 - cs\mu_{\lambda_2})^{\omega_2}, 1 - (1 - cs\nu_{\lambda_2})^{\omega_2}, 1 - (1 - cs\eta_{\lambda_2})^{\omega_2}; 1 - (1 - \rho_{\lambda_2})^{\omega_2} >$,
Thus, $CSNWAAO_\omega(\lambda_1, \lambda_2) = \omega_1\lambda_1 + \omega_2\lambda_2$

$$\begin{aligned}
&= \langle 2 - (1 - cs\mu_{\lambda_1})^{\omega_1} - (1 - cs\mu_{\lambda_2})^{\omega_2} - (1 - (1 - cs\mu_{\lambda_1})^{\omega_1})(1 - (1 - cs\mu_{\lambda_2})^{\omega_2}),\\
&\quad 2 - (1 - cs\nu_{\lambda_1})^{\omega_1} - (1 - cs\nu_{\lambda_1})^{\omega_2} - (1 - (1 - cs\nu_{\lambda_1})^{\omega_1})(1 - (1 - cs\nu_{\lambda_1})^{\omega_2}),\\
&\quad 2 - (1 - cs\eta_{\lambda_1})^{\omega_1} - (1 - cs\eta_{\lambda_1})^{\omega_2} - (1 - (1 - cs\eta_{\lambda_1})^{\omega_1})(1 - (1 - cs\eta_{\lambda_1})^{\omega_2});\\
&\quad 2 - (1 - \rho_{\lambda_1})^{\omega_1} - (1 - \rho_{\lambda_2})^{\omega_2} - (1 - (1 - \rho_{\lambda_1})^{\omega_1})(1 - (1 - \rho_{\lambda_2})^{\omega_2})\rangle\\
&= \langle 1 - (1 - cs\mu_{\lambda_1})^{\omega_1}(1 - cs\mu_{\lambda_2})^{\omega_2}, 1 - (1 - cs\nu_{\lambda_1})^{\omega_1}(1 - cs\nu_{\lambda_1})^{\omega_2}, 1 - (1 - cs\eta_{\lambda_1})^{\omega_1}(1 - cs\eta_{\lambda_1})^{\omega_2};\\
&\quad 1 - (1 - \rho_{\lambda_1})^{\omega_1}(1 - \rho_{\lambda_2})^{\omega_2}\rangle
\end{aligned}$$

Case 2: when $\theta = u$, then $CSNWAAO_\omega(\lambda_1, \lambda_2, ...\lambda_u) =$
$$\left\langle 1 - \prod_{\beta=1}^{u}(1 - cs\mu_{\lambda_\beta})^{\omega_\beta}, 1 - \prod_{\beta=1}^{u}(1 - cs\nu_{\lambda_\beta})^{\omega_\beta}, 1 - \prod_{\beta=1}^{u}(1 - cs\eta_{\lambda_\beta})^{\omega_\beta}; 1 - \prod_{\beta=1}^{u}(1 - \rho_{\lambda_\beta})^{\omega_\beta} \right\rangle$$

Case 3: when $\theta = u + 1$, then $CSNWAAO_\omega(\lambda_1, \lambda_2, ...\lambda_{u+1})$

$$\begin{aligned}
&= \Big\langle 1 - \prod_{\beta=1}^{u}(1 - cs\mu_{\lambda_\beta})^{\omega_\beta} + (1 - (1 - cs\mu_{\lambda_{u+1}})^{\omega_{u+1}}) - (1 - \prod_{\beta=1}^{u}(1 - cs\mu_{\lambda_\beta})^{\omega_\beta})(1 - (1 - cs\mu_{\lambda_\beta})^{\omega_{u+1}}),\\
&\quad 1 - \prod_{\beta=1}^{u}(1 - cs\nu_{\lambda_\beta})^{\omega_\beta} + (1 - (1 - cs\nu_{\lambda_{u+1}})^{\omega_{u+1}}) - (1 - \prod_{\beta=1}^{u}(1 - cs\nu_{\lambda_\beta})^{\omega_\beta})(1 - (1 - cs\nu_{\lambda_\beta})^{\omega_{u+1}}),\\
&\quad 1 - \prod_{\beta=1}^{u}(1 - cs\eta_{\lambda_\beta})^{\omega_\beta} + (1 - (1 - cs\eta_{\lambda_{u+1}})^{\omega_{u+1}}) - (1 - \prod_{\beta=1}^{u}(1 - cs\eta_{\lambda_\beta})^{\omega_\beta})(1 - (1 - cs\eta_{\lambda_\beta})^{\omega_{u+1}});\\
&\quad 1 - \prod_{\beta=1}^{u}(1 - \rho_{\lambda_\beta})^{\omega_\beta} + (1 - (1 - \rho_{\lambda_{u+1}})^{\omega_{u+1}}) - (1 - \prod_{\beta=1}^{u}(1 - \rho_{\lambda_\beta})^{\omega_\beta}) - (1 - (1 - \rho_{\lambda_\beta})^{\omega_{u+1}})\Big\rangle\\
&= \Big\langle 1 - \prod_{\beta=1}^{u+1}(1 - cs\mu_{\lambda_\beta})^{\omega_\beta}, 1 - \prod_{\beta=1}^{u+1}(1 - cs\nu_{\lambda_\beta})^{\omega_\beta}, 1 - \prod_{\beta=1}^{u+1}(1 - cs\eta_{\lambda_\beta})^{\omega_\beta}; 1 - \prod_{\beta=1}^{u+1}(1 - \rho_{\lambda_\beta})^{\omega_\beta}\Big\rangle
\end{aligned}$$

Therefore considering the above results, we have equation (4) for any θ, This completes the proof.

It is obvious that the $CSNWAAO$ operator has the following properties:

(1) Idempotency : Let λ_β $(\beta = 1, 2, ...\theta)$ be a collection of CSNSs. Ifλ_β $(\beta = 1, 2, ...\theta)$ is equal, that is λ_β = λ for $\beta = 1, 2, ..., \theta$, then $CSNWAAO_\omega(\lambda_1, \lambda_2, ...\lambda_\theta)$ = λ.

(2) Boundedness : Let λ_β $(\beta = 1, 2, ...\theta)$ be a collection of CSNSs,
λ^- = $< \min_\beta cs\mu_{\lambda_\beta}, \max_\beta cs\nu_{\lambda_\beta}, \max_\beta cs\eta_{\lambda_\beta}; \min_\beta \rho_{\lambda_\beta} >$ and
λ^+ = $< \max_\beta cs\mu_{\lambda_\beta}, \min_\beta cs\nu_{\lambda_\beta}, \min_\beta cs\eta_{\lambda_\beta}; \max_\beta \rho_{\lambda_\beta} >$
for $(\beta = 1, 2, ...\theta)$, then $\lambda^- \subseteq CSNWAAO \subseteq \lambda^+$.

(3) Monotonity : Let λ_β $(\beta = 1, 2, ...\theta)$ be a collection of CSNSs. If $\lambda_\beta \subseteq \lambda^*_\beta$ for $\beta = 1, 2, ..., \theta$, then $CSNWAAO_\omega(\lambda_1, \lambda_2, ...\lambda_\theta) \subseteq CSNWAAO_\omega(\lambda^*_1, \lambda^*_2, ...\lambda^*_\theta)$.

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Theorem 3.4. *For a CSNS $\lambda_\beta(\beta = 1, 2, ..., \theta)$, we have the following result:*

$$CSNWGAO_\omega(\lambda_1, \lambda_2, ...\lambda_\theta) = \left\langle \prod_{\beta=1}^{\theta} cs\mu_{\lambda_\beta}^{\omega_\beta}, \prod_{\beta=1}^{\theta} cs\nu_{\lambda_\beta}^{\omega_\beta}, \prod_{\beta=1}^{\theta} cs\eta_{\lambda_\beta}^{\omega_\beta}; \prod_{\beta=1}^{\theta} \rho_{\lambda_\beta}^{\omega_\beta} \right\rangle \tag{5}$$

where $\omega = (\omega_1, \omega_2, ..., \omega_\theta)^T$ is the weight vector of $\lambda_\beta(\beta = 1, 2, ..., \theta)$, $\omega_\beta \in [0, 1]$ and $\sum_{\beta=1}^{\theta} \omega_\beta = 1$.

By the similar proof manner, we can give the proof of Theorem 2.

(1) Idempotency : Let λ_β $(\beta = 1, 2, ...\theta)$ be a collection of CSNSs. Ifλ_β $(\beta = 1, 2, ...\theta)$ is equal, that is λ_β = λ for $\beta = 1, 2, ..., \theta$, then $CSNWGAO_\omega(\lambda_1, \lambda_2, ...\lambda_\theta)$ = λ.

(2) Boundedness : Let λ_β $(\beta = 1, 2, ...\theta)$ be a collection of CSNSs,
λ^- = $< \min_\beta cs\mu_{\lambda_\beta}, \max_\beta cs\nu_{\lambda_\beta}, \max_\beta cs\eta_{\lambda_\beta}; \min_\beta \rho_{\lambda_\beta} >$ and
λ^+ = $< \max_\beta cs\mu_{\lambda_\beta}, \min_\beta cs\nu_{\lambda_\beta}, \min_\beta cs\eta_{\lambda_\beta}; \max_\beta \rho_{\lambda_\beta} >$
for $(\beta = 1, 2, ...\theta)$, then $\lambda^- \subseteq CSNWGAO \subseteq \lambda^+$.

(3) Monotonity : Let λ_β $(\beta = 1, 2, ...\theta)$ be a collection of CSNSs. If $\lambda_\beta \subseteq \lambda_\beta^*$ for $\beta = 1, 2, ..., \theta$, then $CSNWGAO_\omega(\lambda_1, \lambda_2, ...\lambda_\theta) \subseteq CSNWGAO_\omega(\lambda_1^*, \lambda_2^*, ...\lambda_\theta^*)$.

4 Decision-making method based on the cubic spherical neutrosophic weighted aggregation operators

The two aggregation operators and the cosine distance in cubic spherical neutrosophic environment are used to handle MCDM situations in this section.

The DM process for the suggested methods involves the following steps:

(1) Based on the experts' opinion, create the individual neutrosophic numbers decision matrix. Each expert in the decision group is asked to rate the importance of each choice using linguistic criteria. Then, to translate these language phrases into neutrosophic numbers, the corresponding link between linguistic variables and neutrosophic numbers is supplied.

Let ψ= $\{\psi_1, \psi_2...\psi_\theta\}$ be a set of alternatives and let ξ = $\{\xi_1, \xi_2...\xi_\theta\}$ be a set of criteria. Assume that the weight of the criterion ξ_β $(\beta = 1, 2, 3..., \theta)$, entered by the decision-maker, is ω_β, $\omega_\beta \in [0, 1]$ and $\sum_{\beta=1}^{\theta} \omega_\beta = 1$.

$(\delta_{\alpha\beta})_{m\times n} = (cs\mu_{\lambda_\alpha}(\xi_\beta), cs\nu_{\lambda_\alpha}(\xi_\beta), cs\eta_{\lambda_\alpha}(\xi_\beta))_{m\times n}$

$$(\delta_{\alpha\beta})_{m\times n} = \begin{bmatrix} < \mu_{11}, \nu_{11}, \eta_{11} > & < \mu_{12}, \nu_{12}, \eta_{12} > & ... & < \mu_{1\kappa}, \nu_{1\kappa}, \eta_{1\kappa} > \\ < \mu_{21}, \nu_{21}, \eta_{21} > & < \mu_{22}, \nu_{22}, \eta_{22}; > & ... & < \mu_{2\kappa}, \nu_{2\kappa}, \eta_{2\kappa} > \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ < \mu_{\theta1}, \nu_{\theta1}, \eta_{\theta1} > & < \mu_{\theta2}, \nu_{\theta2}, \eta_{\theta2} > & ... & < \mu_{\theta\kappa}, \nu_{\theta\kappa}, \eta_{\theta\kappa} > \end{bmatrix} \tag{6}$$

(2) Calculate the cubic spherical neutrosophic sets by using Equation (1), (2) and (3). The form of a CSNS is used in the DM process to describe the evaluation data of the alternative λ_α on the criteria:

$$\lambda_\alpha = \{< \xi_\beta, cs\mu_{\lambda_\alpha}(\xi_\beta), cs\nu_{\lambda_\alpha}(\xi_\beta), cs\eta_{\lambda_\alpha}(\xi_\beta); \rho_{\lambda_\alpha}(\xi_\beta) > \xi_\beta \in \xi\} \tag{7}$$

where $0 \leq cs\mu_{\lambda_\alpha}(\xi_\beta) + cs\nu_{\lambda_\alpha}(\xi_\beta) + cs\eta_{\lambda_\alpha}(\xi_\beta) \leq 3$, $cs\mu_{\lambda_\alpha}(\xi_\beta) \geq 0$, $cs\nu_{\lambda_\alpha}(\xi_\beta) \geq 0$, $cs\eta_{\lambda_\alpha}(\xi_\beta) \geq 0$, $\rho_{\lambda_\alpha}(\xi_\beta) \geq 0$, β = $1, 2, ...\theta$, and α = $1, 2, ..., \kappa$. The value of CSNSs is indicated for convenience by $(\delta_{\alpha\beta})_{m\times n} =< \mu_{\alpha\beta}, \nu_{\alpha\beta}, \eta_{\alpha\beta}; \rho_{\alpha\beta} >$. We thus obtain a cubic spherical neutrosophic decision matrix.

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$(\delta_{\alpha\beta})_{m\times n} = (cs\mu_{\lambda_\alpha}(\xi_\beta), cs\nu_{\lambda_\alpha}(\xi_\beta), cs\eta_{\lambda_\alpha}(\xi_\beta); \rho_{\lambda_\alpha}(\xi_\beta))_{m\times n}$

$$(\delta_{\alpha\beta})_{m\times n} = \begin{bmatrix} <\mu_{11},\nu_{11},\eta_{11};\rho_{11}> & <\mu_{12},\nu_{12},\eta_{12};\rho_{12}> & ... & <\mu_{1\kappa},\nu_{1\kappa},\eta_{1\kappa};\rho_{1\kappa}> \\ <\mu_{21},\nu_{21},\eta_{21};\rho_{21}> & <\mu_{22},\nu_{22},\eta_{22};\rho_{22}> & ... & <\mu_{2\kappa},\nu_{2\kappa},\eta_{2\kappa};\rho_{2\kappa}> \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ <\mu_{\theta 1},\nu_{\theta 1},\eta_{\theta 1};\rho_{\theta 1}> & <\mu_{\theta 2},\nu_{\theta 2},\eta_{\theta 2};\rho_{\theta 2}> & ... & <\mu_{\theta\kappa},\nu_{\theta\kappa},\eta_{\theta\kappa};\rho_{\theta\kappa}> \end{bmatrix} \quad (8)$$

(3) Calculate the cubic spherical neutrosophic weighted arithematic values using Equation (4) and the cubic spherical neutrosophic weighted geometric average values by using Equation (5). Determine the order of the alternatives from the obtained measure values. The aggregating cubic spherical neutrosophic value (δ_α) for λ_α $(\alpha = 1, 2, ..., \kappa)$ is $\delta_\alpha = <\mu_\alpha, \nu_\alpha, \eta_\alpha; \rho_\alpha>$ = $CSNWAAO_\omega(\delta_{\alpha 1}, \delta_{\alpha 2}, ...\delta_{\alpha\kappa})$ and $\delta_\alpha = <\mu_\alpha, \nu_\alpha, \eta_\alpha; \rho_\alpha>$ = $CSNWGAO_\omega(\delta_{\alpha 1}, \delta_{\alpha 2}, ...\delta_{\alpha\theta})$ is obtained from the Equations (4) and (5) according to each row in the cubic spherical neutrosophic decision matrix $(\delta_{\alpha\beta})_{m\times n}$. To rank alternatives in the decision-making process, the ideal alternative is defined as $\delta^*_\alpha(0, 1, 1; 1)$.

(4) Using the formula $C(A, B) = 1 - \frac{1}{n}\left[\frac{\mu_A\mu_B+\nu_A\nu_B+\eta_A\eta_B}{\sqrt{\mu_A^2+\nu_A^2+\eta_A^2}\sqrt{\mu_B^2+\nu_B^2+\eta_B^2}} + 1 - |\rho_A - \rho_B|\right]$, compute the cosine distance between each alternative and the ideal alternative. Then, the bigger the measure value $C_\alpha(\delta_\alpha, \delta^*_\alpha)(\alpha = 1, 2, ..., \theta)$, the better the alternative δ_α, because it is close to the ideal alternative $\delta^*_\alpha(0, 1, 1; 1)$. The cosine distance between each option and the ideal alternative can be used to establish the ranking order of all alternatives and simply identify the best one.

4.1 Numerical Example

The use of the suggested DM process in a practical setting, as well as its applicability and efficacy, are demonstrated in this section using an example for a MCDM problem involving engineering options. In recent years, there has been increased interest in maximizing the choice of green suppliers as a result of the escalating environmental issues and the growing environmental consciousness. Given the circumstances, choosing eco-friendly providers has received a lot of attention. In order to meet the expanding need for environmental protection, many managers want to implement green supplier selection in their businesses either reactively or proactively; they do this in an effort to outperform the competition. Because of this, it's essential to choose green suppliers wisely among other things. As a result, this issue is the attention of certain studies. Here we consider the DM problem adapted from.[12] Table 5[12] lists the evaluations of each expert according to the six evaluation criteria. By converting the linguistic phrases to their corresponding neutrosophic values, we apply Equations (1), (2), (3) and obtained the following CSNS decision matrix.

$$(\delta_{\alpha\beta})_{m\times n} = \begin{bmatrix} \psi & \xi_1 & \xi_2 & \xi_3 \\ \psi_1 & (0.73, 0.27, 0.24; 0.42) & (0.7, 0.3, 0.28; 0.36) & (0.68, 0.32, 0.28; 0.34) \\ \psi_2 & (0.76, 0.24, 0.21; 0.4) & (0.68, 0.32, 0.28; 0.34) & (0.81, 0.19, 0.16; 0.27) \\ \psi_3 & (0.71, 0.29, 0.24; 0.16) & (0.82, 0.18, 0.14; 0.12) & (0.71, 0.29, 0.25; 0.39) \\ \psi_4 & (0.79, 0.21, 0.17; 0.24) & (0.68, 0.32, 0.28; 0.34) & (0.78, 0.22, 0.2; 0.50) \\ \psi & \xi_4 & \xi_5 & \xi_6 \\ \psi_1 & (0.74, 0.26, 0.21; 0.16) & (0.76, 0.24, 0.2; 0.22) & (0.7, 0.3, 0.27; 0.36) \\ \psi_2 & (0.74, 0.26, 0.21; 0.16) & (0.7, 0.3, 0.27; 0.36) & (0.73, 0.27, 0.24; 0.42) \\ \psi_3 & (0.73, 0.27, 0.24; 0.42) & (0.76, 0.24, 0.2; 0.22) & (0.73, 0.27, 0.24; 0.42) \\ \psi_4 & (0.68, 0.32, 0.28; 0.34) & (0.65, 0.35, 0.32; 0.28) & (0.71, 0.29, 0.25; 0.39) \end{bmatrix}$$

Aggregated cubic spherical neutrosophic decision matrix for each aggregation operators is obtained using Equations (4), (5) and shown in the following matrix

$$(\delta_{\alpha\beta})_{m\times n} = \begin{bmatrix} \psi & CSNWAAO & CSNWGAO \\ \psi_1 & (0.72, 0.28, 0.25; 0.30) & (0.72, 0.28, 0.24; 0.28) \\ \psi_2 & (0.74, 0.27, 0.23; 0.32) & (0.73, 0.26, 0.22; 0.30) \\ \psi_3 & (0.75, 0.26, 0.22; 0.31) & (0.74, 0.25, 0.21; 0.27) \\ \psi_4 & (0.71, 0.29, 0.26; 0.36) & (0.71, 0.29, 0.25; 0.35) \end{bmatrix}$$

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The aggregated CSNVs and the ideal alternative $\psi^*(0,1,1;1)$ are compared or compared to each other using the cosine distance. The following table displays the findings for the comparison of ideal alternatives $\psi^*(0,1,1;1)$ to alternatives ψ.

Table 2: The results of cosine distance between ideal alternative and alternatives

Aggrigation Operator	$C(\psi_1,\psi^*)$	$C(\psi_2,\psi^*)$	$C(\psi_3,\psi^*)$	$C(\psi_4,\psi^*)$
CSNWAAO	0.6203	0.6242	0.6374	0.5800
CSNWGAO	0.6326	0.6399	0.6620	0.5894

To further validate the effectiveness of our method, we conducted comparisons with the methods of Biswas et al.,[1] Ye,[18] and Sun and Cai[12] in the forms of NSs context. Table 3 shows that the ranking results achieved by Biswas et al.'s[1] technique, Ye's[18] method, Sun and Cai's[12] approach, and proposed method are identical. ψ_3 is still the best provider, while ψ_4 is still the worst.

Table 3: Comparative Analysis of Ranking

Method		Ranking	Best Supplier	Worst supplier
Biswas et. al.[1]		$\psi_3 > \psi_2 > \psi_1 > \psi_4$	ψ_3	ψ_4
Ye's[18]		$\psi_3 > \psi_1 > \psi_2 > \psi_4$	ψ_3	ψ_4
	$\alpha = 0.5$	$\psi_3 > \psi_2 > \psi_1 > \psi_4$	ψ_3	ψ_4
Sun and Cai[12]	$\alpha < 0.2$	$\psi_1 > \psi_3 > \psi_2 > \psi_4$	ψ_1	ψ_4
	$\alpha = 0.2$	$\psi_3 > \psi_1 > \psi_2 > \psi_4$	ψ_3	ψ_4
Proposed Method	CSNWAAO-CD	$\psi_3 > \psi_2 > \psi_1 > \psi_4$	ψ_3	ψ_4
	CSNWGAO-CD	$\psi_3 > \psi_2 > \psi_1 > \psi_4$	ψ_3	ψ_4

In summary, we can say that our method offers a workable and adaptable solution to the issue of choosing a green provider. According to the ranking order's sensitivity to CSNWAAO and CSWGAO weights, the suggested strategy is adaptable to deal with dynamic DM problems. The simultaneous consideration of experts' inconsistent uncertainty, group decision-making, and dynamic DM is another benefit.

5 Conclusion

The idea of CSNSs, a subclass of NSs was proposed in this study, along with some of their operational principles. A CSNWAAO and a CSNWGAO are the two aggregation operators we then suggested for CSNSs. The cubic spherical neutrosophic environment, in which criterion values with respect to alternatives are evaluated by the form of CSN values and the criterion weights are known information, was used to apply the two aggregation operators to MCDM problems. To rank the alternatives and choose the best one(s) based on the measure values, we used the cosine distance between an alternative and the ideal alternative. A numerical example is then given to show how the developed approach is applied. Because it can handle not only incomplete information but also the indeterminate information and inconsistent information that frequently exist in real situations, the proposed CSN MCDM method is more suitable for real scientific and engineering applications. The methods suggested in this paper can give DM more useful options. We will address group DM issues involving incomplete decision contexts and preference relations in the selection process in the future. We will also apply CSN aggregation operators to resolve real-world problems in other domains, such as expert systems, information fusion systems, and medical diagnoses.

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