Some Results in Classes Of Neutrosophic Graphs

Henry Garrett

Independent Researcher

DrHenryGarrett@gmail.com

Twitter's ID: @DrHenryGarrett | ©DrHenryGarrett.wordpress.com

Abstract

New setting is introduced to study co-neighborhood, neutrosophic t-neighborhood, neutrosophic quasi-vertex set, neutrosophic quasi-order, neutrosophic neighborhood, neutrosophic co-t-neighborhood, neutrosophic quasi-edge set, neutrosophic quasi-size, Neutrosophic number, neutrosophic co-neighborhood, co-neutrosophic number, quasi-number and quasi-co-number. Some classes of neutrosophic graphs are investigated.

Keywords: Neutrosophic Quasi-Order, Neutrosophic Quasi-Size, Neutrosophic Quasi-Number, Neutrosophic Quasi-Co-Number, Neutrosophic Co-t-Neighborhood
AMS Subject Classification: 05C17, 05C22, 05E45

1 Background

Fuzzy set in **Ref.** [16], neutrosophic set in **Ref.** [3], related definitions of other sets in **Refs.** [3, 13, 15], graphs and new notions on them in **Refs.** [1, 4, 8–11, 14, 17], neutrosophic graphs in **Ref.** [5], studies on neutrosophic graphs in **Ref.** [2], relevant definitions of other graphs based on fuzzy graphs in **Ref.** [12], related definitions of other graphs based on neutrosophic graphs in **Ref.** [6], are proposed. Also, some studies and researches about neutrosophic graphs, are proposed as a book in **Ref.** [7].

2 Preliminaries

In this subsection, basic material which is used in this article, is presented. Also, new ideas and their clarifications are elicited.

Basic idea is about the model which is used. First definition introduces basic model.

Definition 2.1. (Graph).
$G = (V, E)$ is called a **graph** if V is a set of objects and E is a subset of $V \times V$ (E is a set of 2-subsets of V) where V is called **vertex set** and E is called **edge set**. Every two vertices have been corresponded to at most one edge.

Neutrosophic graph is the foundation of results in this paper which is defined as follows. Also, some related notions are demonstrated.

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Definition 2.2. (Neutrosophic Graph And Its Special Case).

$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, and $\mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_i v_j \in E$,

$$\mu(v_i v_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

(i) : σ is called **neutrosophic vertex set**.

(ii) : μ is called **neutrosophic edge set**.

(iii) : $|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$.

(iv) : $\Sigma_{v \in V} \sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

(v) : $|E|$ is called **size** of NTG and it's denoted by $\mathcal{S}(NTG)$.

(vi) : $\Sigma_{e \in E} \Sigma_{i=1}^{3} \mu_i(e)$ is called **neutrosophic size** of NTG and it's denoted by $\mathcal{S}_n(NTG)$.

Some classes of well-known neutrosophic graphs are defined. These classes of neutrosophic graphs are used to form this study and the most results are about them.

Definition 2.3. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) : a sequence of vertices $P : x_0, x_1, \cdots, x_n$ is called **path** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1$;

(ii) : **strength** of path $P : x_0, x_1, \cdots, x_n$ is $\bigwedge_{i=0,\cdots,n-1} \mu(x_i x_{i+1})$;

(iii) : **connectedness** amid vertices x_0 and x_n is

$$\mu^{\infty}(x, y) = \bigwedge_{P : x_0, x_1, \cdots, x_n} \bigwedge_{i=0,\cdots,n-1} \mu(x_i x_{i+1});$$

(iv) : a sequence of vertices $P : x_0, x_1, \cdots, x_n$ is called **cycle** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$;

(v) : it's **t-partite** where V is partitioned to t parts, $V_1, V_2, \cdots, V_t$ and the edge xy implies $x \in V_i$ and $y \in V_j$ where $i \neq j$. If it's complete, then it's denoted by $K_{\sigma_1, \sigma_2, \cdots, \sigma_t}$ where σ_i is σ on V_i instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$;

(vi) : t-partite is **complete bipartite** if $t = 2$, and it's denoted by K_{σ_1, σ_2};

(vii) : complete bipartite is **star** if $|V_1| = 1$, and it's denoted by S_{1, σ_2};

$(viii)$: a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by W_{1, σ_2};

(ix) : it's **complete** where $\forall uv \in V,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$;

(x) : it's **strong** where $\forall uv \in E,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$.

The notions of neighbor and neighborhood are about some vertices which have one edge with a fixed vertex. These notions present vertices which are close to a fixed vertex as possible. Based on strong edge, it's possible to define different neighborhood as follows.

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Definition 2.4. (Neighborhood).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Suppose $x \in V$. Then

$$N(x) = \{y \in V \mid xy \in E\}.$$

Definition 2.5. (Co-Neighborhood).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph and $t \in \mathbb{N}$. Suppose $x_i \in V,\ i = 1, 2, \cdots, t$. Then

(i)

$$N_t(x_1, x_2, \cdots, x_t) = \{y \in V \mid x_i y \in E,\ i = 1, 2, \cdots, t\}.$$

If $\min_{x_1, x_2, \cdots, x_t \in V} N_t[x_1, x_2, \cdots, x_t] = V$, then **neutrosophic t-neighborhood** is called **neutrosophic quasi-vertex set** and t is called **neutrosophic quasi-order**. Generally, when t isn't fixed, it's called **neutrosophic neighborhood**. **Neutrosophic number** is

$$\Sigma_{\min_{x_1, x_2, \cdots, x_t \in V} N_t[x_1, x_2, \cdots, x_t] = V} \Sigma_{j=1}^{3} \sigma_j(x_i).$$

(ii)

$$N_t(x_1, x_2, \cdots, x_t) = \{e \in E \mid e = x_i y \in E,\ i = 1, 2, \cdots, t\}.$$

If $\min_{x_1, x_2, \cdots, x_t \in V} N_t[x_1, x_2, \cdots, x_t] = E$, then **neutrosophic co-t-neighborhood** is called **neutrosophic quasi-edge set** and t is called **neutrosophic quasi-size**. Generally, when t isn't fixed, it's called **neutrosophic co-neighborhood**. **Co-neutrosophic number** is

$$\Sigma_{\min_{x_1, x_2, \cdots, x_t \in V} N_t[x_1, x_2, \cdots, x_t] = E} \Sigma_{j=1}^{3} \sigma_j(x_i).$$

3 Neutrosophic Quasi-Order

Proposition 3.1. *Let $NTG : (V, E, \sigma, \mu)$ be an odd path. Then $\{x_2, x_4, \cdots, x_{n-1}\}$ is related to quasi-order $\lfloor \frac{n}{2} \rfloor$.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ be an odd path. Thus $NTG : (V, E, \sigma, \mu)$ is $P : x_1, x_2, \cdots, x_n$ where n and 1 has same parity. There are two sets. $\{x_1, x_3, \cdots, x_n\}$ has quasi-order $\lfloor \frac{n}{2} \rfloor + 1$ but $\{x_2, x_4, \cdots, x_{n-1}\}$ has quasi-order $\lfloor \frac{n}{2} \rfloor$ which is minimum number amid these two sets. So $\{x_2, x_4, \cdots, x_{n-1}\}$ is related to quasi-order $\lfloor \frac{n}{2} \rfloor$. $\{x_2, x_4, \cdots, x_{n-1}\}$ is quasi-vertex set which is optimal. □

Proposition 3.2. *Let $NTG : (V, E, \sigma, \mu)$ be an even path. Then $\{x_2, x_4, \cdots, x_n\}$ is related to quasi-order $\frac{n}{2}$.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ be an even path. Thus $NTG : (V, E, \sigma, \mu)$ is $P : x_1, x_2, \cdots, x_n$ where n and 1 has different parity. There are two sets. $\{x_1, x_3, \cdots, x_{n-1}\}$ has quasi-order $\lfloor \frac{n}{2} \rfloor$ and $\{x_2, x_4, \cdots, x_n\}$ has quasi-order $\lfloor \frac{n}{2} \rfloor$ which is minimum number like first set. So $\{x_2, x_4, \cdots, x_n\}$ and $\{x_1, x_3, \cdots, x_{n-1}\}$ are related to quasi-order $\lfloor \frac{n}{2} \rfloor$. $\{x_2, x_4, \cdots, x_n\}$ and $\{x_1, x_3, \cdots, x_{n-1}\}$ are quasi-vertex sets which are optimal. □

Proposition 3.3. *Let $NTG : (V, E, \sigma, \mu)$ be an odd cycle. Then $\{x_2, x_4, \cdots, x_{n-1}\}$ is related to quasi-order $\lfloor \frac{n}{2} \rfloor$.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ be an odd cycle. Thus $NTG : (V, E, \sigma, \mu)$ is $P : x_1, x_2, \cdots, x_n, x_1$ where n and 1 has same parity. There are two sets. $\{x_1, x_3, \cdots, x_n\}$ has quasi-order $\lfloor \frac{n}{2} \rfloor + 1$ but $\{x_2, x_4, \cdots, x_{n-1}\}$ has quasi-order $\lfloor \frac{n}{2} \rfloor$ which is minimum number amid these two sets. So $\{x_2, x_4, \cdots, x_{n-1}\}$ is related to quasi-order $\lfloor \frac{n}{2} \rfloor$. $\{x_2, x_4, \cdots, x_{n-1}\}$ is quasi-vertex set which is optimal. □

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Proposition 3.4. *Let* $NTG : (V, E, \sigma, \mu)$ *be an even cycle. Then* $\{x_2, x_4, \cdots, x_n\}$ *and* $\{x_1, x_3, \cdots, x_{n-1}\}$ *are related to quasi-order* $\frac{n}{2}$.

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ be an even cycle. Thus $NTG : (V, E, \sigma, \mu)$ is $P : x_1, x_2, \cdots, x_n, x_1$ where n and 1 has different parity. There are two sets. $\{x_1, x_3, \cdots, x_{n-1}\}$ has quasi-order $\lfloor \frac{n}{2} \rfloor$ and $\{x_2, x_4, \cdots, x_n\}$ has quasi-order $\lfloor \frac{n}{2} \rfloor$ which is minimum number like first set. So $\{x_2, x_4, \cdots, x_n\}$ and $\{x_1, x_3, \cdots, x_{n-1}\}$ are related to quasi-order $\lfloor \frac{n}{2} \rfloor$. $\{x_2, x_4, \cdots, x_n\}$ and $\{x_1, x_3, \cdots, x_{n-1}\}$ are quasi-vertex sets which are optimal. □

Proposition 3.5. *Let* $NTG : (V, E, \sigma, \mu)$ *be complete. Then* $\{x\}$ *is related to quasi-order* 1.

Proof. Let $NTG : (V, E, \sigma, \mu)$ be complete. Suppose x is a given vertex. Thus

$$N[x] = \min_{x_1, x_2, \cdots, x_t \in V} N_t[x_1, x_2, \cdots, x_t] = V.$$

It implies the set $\{x\}$ which is 1-neighborhood, is quasi-vertex set. Then $\{x\}$ is related to quasi-order 1. □

Proposition 3.6. *Let* $NTG : (V, E, \sigma, \mu)$ *be star. Then* $\{x\}$ *is related to quasi-order* 1.

Proof. Let $NTG : (V, E, \sigma, \mu)$ be star. Suppose c is the center. Thus

$$N[c] = \min_{x_1, x_2, \cdots, x_t \in V} N_t[x_1, x_2, \cdots, x_t] = V.$$

It implies the set $\{c\}$ which is 1-neighborhood, is quasi-vertex set. Then $\{c\}$ is related to quasi-order 1. □

4 Neutrosophic Quasi-Size

Proposition 4.1. *Let* $NTG : (V, E, \sigma, \mu)$ *be an odd path. Then either* $\{x_1, x_3, \cdots, x_n\}$ *or* $\{x_1, x_3, \cdots, x_{n-1}\}$ *are related to quasi-size* $\lfloor \frac{n}{3} \rfloor$.

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ be an odd path. Thus $NTG : (V, E, \sigma, \mu)$ is $P : x_1, x_2, \cdots, x_n$ where n and 1 has same parity. There is two sets. Either $\{x_1, x_3, \cdots, x_n\}$ or $\{x_1, x_3, \cdots, x_{n-1}\}$ has quasi-size $\lfloor \frac{n}{3} \rfloor$ which are minimum number amid these all sets. So either $\{x_1, x_3, \cdots, x_n\}$ or $\{x_1, x_3, \cdots, x_{n-1}\}$ is related to quasi-size$\lfloor \frac{n}{3} \rfloor$. Either $\{x_1, x_3, \cdots, x_n\}$ or $\{x_1, x_3, \cdots, x_{n-1}\}$ are quasi-vertex set which are optimal. □

Proposition 4.2. *Let* $NTG : (V, E, \sigma, \mu)$ *be an even path. Then* $\{x_2, x_4, \cdots, x_n\}$ *is related to quasi-size* $\lfloor \frac{n}{3} \rfloor$.

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ be an even path. Thus $NTG : (V, E, \sigma, \mu)$ is $P : x_1, x_2, \cdots, x_n$ where n and 1 has different parity. There are two sets. Either $\{x_1, x_3, \cdots, x_n\}$ or $\{x_1, x_3, \cdots, x_{n-1}\}$. $\{x_1, x_3, \cdots, x_n\}$ has quasi-size $\lfloor \frac{n}{3} \rfloor$ and $\{x_1, x_3, \cdots, x_{n-1}\}$ has quasi-size $\lfloor \frac{n}{3} \rfloor$ which is minimum number like first set. So either $\{x_1, x_3, \cdots, x_n\}$ or $\{x_1, x_3, \cdots, x_{n-1}\}$ is related to quasi-size $\lfloor \frac{n}{3} \rfloor$. Either $\{x_1, x_3, \cdots, x_n\}$ or $\{x_1, x_3, \cdots, x_{n-1}\}$ is quasi-vertex set which is optimal. □

Proposition 4.3. *Let* $NTG : (V, E, \sigma, \mu)$ *be an odd cycle. Then* $\{x_1, x_4, \cdots, x_{n-3}\}$ *is related to quasi-size* $\lfloor \frac{n}{3} \rfloor$.

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ be an odd cycle. Thus $NTG : (V, E, \sigma, \mu)$ is $P : x_1, x_2, \cdots, x_n, x_1$ where n and 1 has same parity. There are two sets. $\{x_1, x_3, \cdots, x_n\}$ has quasi-size $\lfloor \frac{n}{3} \rfloor$ and $\{x_1, x_3, \cdots, x_{n-1}\}$ has quasi-size $\lfloor \frac{n}{3} \rfloor$ which are minimum numbers amid all sets. So either $\{x_1, x_3, \cdots, x_{n-1}\}$ or $\{x_1, x_3, \cdots, x_n\}$ is related to quasi-size $\lfloor \frac{n}{3} \rfloor$. Either $\{x_1, x_3, \cdots, x_{n-1}\}$ or $\{x_1, x_3, \cdots, x_n\}$ is quasi-edge set which is optimal. □

Proposition 4.4. *Let $NTG : (V, E, \sigma, \mu)$ be an even cycle. Then $\{x_1, x_4, \cdots, x_{n-3}\}$ is related to quasi-size $\lfloor \frac{n}{3} \rfloor$.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ be an even cycle. Thus $NTG : (V, E, \sigma, \mu)$ is $P : x_1, x_2, \cdots, x_n, x_1$ where n and 1 has different parity. There are two sets. $\{x_1, x_3, \cdots, x_{n-1}\}$ has quasi-size $\lfloor \frac{n}{3} \rfloor$ and $\{x_1, x_3, \cdots, x_{n-1}\}$ has quasi-size $\lfloor \frac{n}{3} \rfloor$ which is minimum number like first set. So either $\{x_1, x_3, \cdots, x_n\}$ or $\{x_1, x_3, \cdots, x_{n-1}\}$ are related to quasi-size $\lfloor \frac{n}{3} \rfloor$ which this number is optimal. Either $\{x_1, x_3, \cdots, x_n\}$ or $\{x_1, x_3, \cdots, x_{n-1}\}$ is quasi-edge set which are optimal. □

Proposition 4.5. *Let $NTG : (V, E, \sigma, \mu)$ be complete. Then $\{x_1, x_2, \cdots, x_{n-1}\}$ is related to quasi-size $n - 1$.*

Proof. Let $NTG : (V, E, \sigma, \mu)$ be complete. Suppose x is a given vertex. Thus

$$N[x_1, x_2, \cdots, x_{n-1}] = \min_{x_1, x_2, \cdots, x_{n-1} \in V} N_t[x_1, x_2, \cdots, x_{n-1}] = E.$$

It implies the set $\{x_1, x_2, \cdots, x_{n-1}\}$ which is co-$(n - 1)$-neighborhood, is only quasi-vertex set as optimal set. Then $\{x_1, x_2, \cdots, x_{n-1}\}$ is related to quasi-size $n - 1$. □

Proposition 4.6. *Let $NTG : (V, E, \sigma, \mu)$ be star. Then $\{x\}$ is related to quasi-size* 1.

Proof. Let $NTG : (V, E, \sigma, \mu)$ be star. Suppose c is the center. Thus

$$N[c] = \min_{x_1, x_2, \cdots, x_t \in V} N_t[x_1, x_2, \cdots, x_t] = E.$$

It implies the set $\{c\}$ which is 1-neighborhood, is quasi-vertex set. Then $\{c\}$ is related to quasi-order 1. □

5 Setting of Neutrosophic Quasi-Number and Quasi-Co-Number

Definition 5.1. (Quasi-Number & Quasi-Co-Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **quasi-number** for x_0 is

$$\mathcal{C}(x_0) = \bigvee_{x_t \in V} \bigvee_{P : x_0, x_1, \cdots, x_t} \bigwedge_{i=0, \cdots, t-1} \mu(x_i x_{i+1});$$

(ii) **quasi-co-number** for x_0 is

$$\mathcal{C}_c(x_0) = \{x_z x_{z'} \mid \mu(x_z x_{z'}) = \bigvee_{x_t \in V} \bigvee_{P : x_0, x_1, \cdots, x_t} \bigwedge_{i=0, \cdots, t-1} \mu(x_i x_{i+1})\}.$$

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Proposition 5.2. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{C}(x_0) = \bigvee_{x_t \in V} \bigvee_{P: x_0, x_1, \cdots, x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \bigvee_{i=1}^{\mathcal{O}} \mu(x_0 x_i).$$

Proof. Assume $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph and $x_0 x'_0 \in E$. Then

$$\mathcal{C}(x_0) = \bigvee_{x_t \in V} \bigvee_{P: x_0, x_1, \cdots, x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \bigvee_{i=1}^{\mathcal{O}} \mu(x_0 x_i).$$

Since

$$\bigvee_{P: x_0, x_1, \cdots, x'_0} \bigwedge_{x_i x_{i+1} \in V(P)} \mu(x_i x_{i+1}) = \mu(x_0 x'_0).$$

and

$$\bigvee_{P: x_0, x_1, \cdots, x'_0} \bigwedge_{x_i x_{i+1} \in V(P)} \mu(x_i x_{i+1}) \leq \mu(x_0 x'_0)$$

hold if $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Also,

$$\bigvee_{P: x_0, x_1, \cdots, x'_0} \bigwedge_{x_i x_{i+1} \in V(P)} \mu(x_i x_{i+1}) \geq \mu(x_0 x'_0)$$

holds since $x_0 x'_0$ is a path from x_0 to x'_0. 135

□ 136

Proposition 5.3. *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Suppose x_0 is a leaf and $x_0 z \in E$. Then*

$$\mathcal{C}(x_0) = \bigvee_{x_t \in V} \bigvee_{P: x_0, x_1, \cdots, x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) \leq \mu(x_0 z).$$

Proof. Assume $NTG : (V, E, \sigma, \mu)$ is a path-neutrosophic graph. Suppose x_0 is a leaf and $x_0 z \in E$. An arbitrary path P has consecutive vertices x_0, z as their ends from a given vertex to leaf z. Thus

$$\bigvee_{P: x_0, x_1, \cdots, x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) \leq \mu(x_0 z).$$

It implies for every given vertex $x_t \in V$,

$$\bigvee_{x_t \in V} \bigvee_{P: x_0, x_1, \cdots, x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) \leq \mu(x_0 z).$$

Therefore,

$$\mathcal{C}(x_0) = \bigvee_{x_t \in V} \bigvee_{P: x_0, x_1, \cdots, x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) \leq \mu(x_0 z).$$

□ 137

Proposition 5.4. *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Suppose x_0 is a leaf and $x_0 z \in E$ is weakest edge. Then*

$$\mathcal{C}(x_0) = \bigvee_{x_t \in V} \bigvee_{P: x_0, x_1, \cdots, x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \mu(x_0 z).$$

Proof. Assume $NTG : (V, E, \sigma, \mu)$ is a path-neutrosophic graph. Suppose x_0 is a leaf and $x_0z \in E$ is a weakest edge. An arbitrary path P has consecutive vertices x_0, z as their ends from a given vertex to leaf x_0. Thus

$$\bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_ix_{i+1}) \leq \mu(x_0z).$$

By $x_0z \in E$ is a weakest edge,

$$\bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_ix_{i+1}) = \mu(x_0z).$$

It implies for every given vertex $x_t \in V$,

$$\bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_ix_{i+1}) = \mu(x_0z).$$

Therefore,

$$\mathcal{C}(x_0) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_ix_{i+1}) = \mu(x_0z).$$

□ 138

Proposition 5.5. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Suppose x_0 has a weakest edge $x_0z \in E$. Then*

$$\mathcal{C}(x_0) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_ix_{i+1}) \geq \mu(x_0z).$$

Proof. Assume $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Suppose x_0 has a weakest edge $x_0z \in E$. For every vertex, there are only two ways to form a path. By $x_0z \in E$ is a weakest edge,

$$\mu(x_ix_{i+1}) \geq \mu(x_0z).$$

Thus

$$\bigwedge_{i=0,\cdots,t-1} \mu(x_ix_{i+1}) \geq \mu(x_0z).$$

$P : x_0, z$ is a path from x_0 to z. It implies

$$\bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_ix_{i+1}) \geq \mu(x_0z).$$

Therefore,

$$\bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_ix_{i+1}) \geq \mu(x_0z).$$

It means that

$$\mathcal{C}(x_0) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_ix_{i+1}) \geq \mu(x_0z).$$

□ 139

Proposition 5.6. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Suppose x_0 is incident to one edge $x_0z \in E$ which isn't a weakest edge. Then*

$$\mathcal{C}(x_0) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_ix_{i+1}) \geq \mu(x_0z).$$

Proof. Assume $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Suppose x_0 has an edge $x_0z \in E$ which isn't a weakest edge. For every vertex, there are only two ways to form a path. By $x_0z \in E$ isn't a weakest edge and $P : x_0, z$ is a path from x_0 to z. It implies

$$\bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_ix_{i+1}) \geq \mu(x_0z).$$

Therefore,

$$\bigvee_{x_t\in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_ix_{i+1}) \geq \mu(x_0z).$$

It means that

$$\mathcal{C}(x_0) = \bigvee_{x_t\in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_ix_{i+1}) \geq \mu(x_0z).$$

□ 140

Proposition 5.7. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Suppose x_0 is incident to two edges $x_0z, x_0z' \in E$ which aren't weakest edges. Then*

$$\mathcal{C}(x_0) = \bigvee_{x_t\in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_ix_{i+1}) \leq \mu(x_0z) \vee \mu(x_0z').$$

Furthermore, if $\mu(x_0z) < \mu(x_0z')$, then

$$\mathcal{C}(x_0) \leq \mu(x_0z');$$

if $\mu(x_0z') < \mu(x_0z)$, then

$$\mathcal{C}(x_0) \leq \mu(x_0z);$$

if $\mu(x_0z') = \mu(x_0z)$, then

$$\mathcal{C}(x_0) \leq \mu(x_0z') = \mu(x_0z).$$

Proof. Assume $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Suppose x_0 is incident to two edges $x_0z, x_oz' \in E$ which aren't weakest edges. For every vertex, there are only two ways to form a path which are with edges either x_0z or x_oz'. $P : x_0, z$ and $P' : x_0, z'$ are paths from x_0 to z and z'. It implies

$$\bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_ix_{i+1}) \leq \mu(x_0z) \vee \mu(x_0z').$$

Therefore,

$$\bigvee_{x_t\in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_ix_{i+1}) \leq \mu(x_0z) \vee \mu(x_0z').$$

It means that

$$\mathcal{C}(x_0) = \bigvee_{x_t\in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_ix_{i+1}) \leq \mu(x_0z) \vee \mu(x_0z').$$

Furthermore, if $\mu(x_0z) < \mu(x_0z')$, then

$$\mathcal{C}(x_0) \leq \mu(x_0z');$$

if $\mu(x_0z') < \mu(x_0z)$, then

$$\mathcal{C}(x_0) \leq \mu(x_0z);$$

if $\mu(x_0z') = \mu(x_0z)$, then

$$\mathcal{C}(x_0) \leq \mu(x_0z') = \mu(x_0z).$$

□ 141

Proposition 5.8. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph. Suppose x_0 is center. Then*

$$\mathcal{C}(x_0) = \bigvee_{x_t \in V} \bigvee_{P: x_0, x_1, \cdots, x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \bigvee_{i=1,\cdots,\mathcal{O}} \mu(x_0 x_i).$$

Proof. Consider $NTG : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. Suppose x_0 is center. All paths from x_0 has a form $x_0 x_i$. Thus

$$\bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \mu(x_0 x_i).$$

Hence

$$\bigvee_{P: x_0, x_1, \cdots, x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \bigvee_{i=1,\cdots,\mathcal{O}} \mu(x_0 x_i).$$

So

$$\bigvee_{x_t \in V} \bigvee_{P: x_0, x_1, \cdots, x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \bigvee_{i=1,\cdots,\mathcal{O}} \mu(x_0 x_i).$$

It implies

$$\mathcal{C}(x_0) = \bigvee_{x_t \in V} \bigvee_{P: x_0, x_1, \cdots, x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \bigvee_{i=1,\cdots,\mathcal{O}} \mu(x_0 x_i).$$

□ 142

Proposition 5.9. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph. Suppose x_0 isn't center and c is a center. Then*

$$\mathcal{C}(x_0) = \bigvee_{x_t \in V} \bigvee_{P: x_0, x_1, \cdots, x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \mu(x_0 c).$$

Proof. Consider $NTG : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. Suppose x_0 isn't center and c is a center. There's only one path from x_0 has a form $x_0 c$. Thus

$$\bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \mu(x_0 c).$$

Hence

$$\bigvee_{P: x_0, x_1, \cdots, x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \mu(x_0 c).$$

So

$$\bigvee_{x_t \in V} \bigvee_{P: x_0, x_1, \cdots, x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \mu(x_0 c).$$

It implies

$$\mathcal{C}(x_0) = \bigvee_{x_t \in V} \bigvee_{P: x_0, x_1, \cdots, x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \mu(x_0 c).$$

□ 143

Proposition 5.10. *Let $NTG : (V, E, \sigma, \mu)$ be a star-strong-neutrosophic graph. Suppose x_0 is center such that $\bigwedge_{i=0}^{\mathcal{O}-1} = \sigma(x_0)$. Then*

$$\mathcal{C}(x_0) = \bigvee_{x_t \in V} \bigvee_{P: x_0, x_1, \cdots, x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \sigma(x_0).$$

Proof. Consider $NTG:(V,E,\sigma,\mu)$ is a star-neutrosophic graph. Suppose x_0 is center such that $\bigwedge_{i=0}^{\mathcal{O}-1}=\sigma(x_0)$. All paths from x_0 has a form x_0x_i. Thus

$$\bigwedge_{i=0,\cdots,t-1}\mu(x_ix_{i+1})=\mu(x_0x_i)=\sigma(x_0).$$

Hence

$$\bigvee_{P:x_0,x_1,\cdots,x_t}\bigwedge_{i=0,\cdots,t-1}\mu(x_ix_{i+1})=\bigvee_{i=1,\cdots,\mathcal{O}}\mu(x_0x_i)=\sigma(x_0).$$

So

$$\bigvee_{x_t\in V}\bigvee_{P:x_0,x_1,\cdots,x_t}\bigwedge_{i=0,\cdots,t-1}\mu(x_ix_{i+1})=\bigvee_{i=1,\cdots,\mathcal{O}}\mu(x_0x_i)=\sigma(x_0).$$

It implies

$$\mathcal{C}(x_0)=\bigvee_{x_t\in V}\bigvee_{P:x_0,x_1,\cdots,x_t}\bigwedge_{i=0,\cdots,t-1}\mu(x_ix_{i+1})=\sigma(x_0).$$

□ 144

Proposition 5.11. *Let $NTG:(V,E,\sigma,\mu)$ be a complete-neutrosophic graph. Then* 145

$$\mathcal{C}_c(x_0)=\{x_zx_{z'}\mid\mu(x_zx_{z'})=\bigvee_{x_t\in V}\bigvee_{P:x_0,x_1,\cdots,x_t}\bigwedge_{i=0,\cdots,t-1}\mu(x_ix_{i+1})\}=$$

$$\{x_zx_{z'}\mid\mu(x_zx_{z'})=\bigvee_{i=1}^{\mathcal{O}}\mu(x_0x_i)\}.$$

Proof. Assume $NTG:(V,E,\sigma,\mu)$ is a complete-neutrosophic graph and $x_0x'_0\in E$. Then

$$\mathcal{C}(x_0)=\bigvee_{x_t\in V}\bigvee_{P:x_0,x_1,\cdots,x_t}\bigwedge_{i=0,\cdots,t-1}\mu(x_ix_{i+1})=\bigvee_{i=1}^{\mathcal{O}}\mu(x_0x_i).$$

Since

$$\bigvee_{P:x_0,x_1,\cdots,x'_0}\bigwedge_{x_ix_{i+1}\in V(P)}\mu(x_ix_{i+1})=\mu(x_0x'_0).$$

and

$$\bigvee_{P:x_0,x_1,\cdots,x'_0}\bigwedge_{x_ix_{i+1}\in V(P)}\mu(x_ix_{i+1})\leq\mu(x_0x'_0)$$

hold if $NTG:(V,E,\sigma,\mu)$ is a complete-neutrosophic graph. Also,

$$\bigvee_{P:x_0,x_1,\cdots,x'_0}\bigwedge_{x_ix_{i+1}\in V(P)}\mu(x_ix_{i+1})\geq\mu(x_0x'_0)$$

holds since $x_0x'_0$ is a path from x_0 to x'_0. So

$$\mathcal{C}_c(x_0)=\{x_zx_{z'}\mid\mu(x_zx_{z'})=\bigvee_{x_t\in V}\bigvee_{P:x_0,x_1,\cdots,x_t}\bigwedge_{i=0,\cdots,t-1}\mu(x_ix_{i+1})\}=$$

$$\{x_zx_{z'}\mid\mu(x_zx_{z'})=\bigvee_{i=1}^{\mathcal{O}}\mu(x_0x_i)\}.$$

□ 146

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Proposition 5.12. *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Suppose x_0 is a leaf and $x_0 z \in E$. Then*

$$\mathcal{C}_c(x_0) = \{x_z x_{z'} \mid \mu(x_z x_{z'}) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1})\} =$$

$$\{x_z x_{z'} \mid \mu(x_z x_{z'}) \leq \mu(x_0 z)\}.$$

Proof. Assume $NTG : (V, E, \sigma, \mu)$ is a path-neutrosophic graph. Suppose x_0 is a leaf and $x_0 z \in E$. An arbitrary path P has consecutive vertices x_0, z as their ends from a given vertex to leaf z. Thus

$$\bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) \leq \mu(x_0 z).$$

It implies for every given vertex $x_t \in V$,

$$\bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) \leq \mu(x_0 z).$$

Therefore,

$$\mathcal{C}(x_0) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) \leq \mu(x_0 z).$$

Thus

$$\mathcal{C}_c(x_0) = \{x_z x_{z'} \mid \mu(x_z x_{z'}) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1})\} =$$

$$\{x_z x_{z'} \mid \mu(x_z x_{z'}) \leq \mu(x_0 z)\}.$$

□ 147

Proposition 5.13. *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Suppose x_0 is a leaf and $x_0 z \in E$ is a weakest edge. Then*

$$\mathcal{C}_c(x_0) = \{x_z x_{z'} \mid \mu(x_z x_{z'}) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1})\} =$$

$$\{x_z x_{z'} \mid \mu(x_z x_{z'}) = \mu(x_0 z)\}.$$

Proof. Assume $NTG : (V, E, \sigma, \mu)$ is a path-neutrosophic graph. Suppose x_0 is a leaf and $x_0 z \in E$ is a weakest edge. An arbitrary path P has consecutive vertices x_0, z as their ends from a given vertex to leaf x_0. Thus

$$\bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) \leq \mu(x_0 z).$$

By $x_0 z \in E$ is a weakest edge,

$$\bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \mu(x_0 z).$$

It implies for every given vertex $x_t \in V$,

$$\bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \mu(x_0 z).$$

Therefore,

$$\mathcal{C}(x_0) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \mu(x_0 z).$$

Thus

$$\mathcal{C}_c(x_0) = \{x_z x_{z'} \mid \mu(x_z x_{z'}) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1})\} =$$

$$\{x_z x_{z'} \mid \mu(x_z x_{z'}) = \mu(x_0 z)\}.$$

□ 148

Proposition 5.14. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Suppose x_0 has two weakest edges $x_0 z, x_0 z' \in E$. Then*

$$\mathcal{C}_c(x_0) = \{x_z x_{z'} \mid \mu(x_z x_{z'}) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1})\} =$$

$$\{x_z x_{z'} \mid \mu(x_z x_{z'}) = \mu(x_0 z) \vee \mu(x_0 z') = \mu(x_0 z) = \mu(x_0 z'),\ x_z x_{z'} \in E)\} = \{x_0 z, x_0 z'\}.$$

Proof. Assume $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Suppose x_0 has a weakest edge $x_0 z \in E$. For every vertex, there are only two ways to form a path. By $x_0 z \in E$ is a weakest edge,

$$\mu(x_i x_{i+1}) \geq \mu(x_0 z).$$

Thus

$$\bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) \geq \mu(x_0 z).$$

$P : x_0, z$ is a path from x_0 to z. It implies

$$\bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) \geq \mu(x_0 z).$$

Therefore,

$$\bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) \geq \mu(x_0 z).$$

It means that

$$\mathcal{C}(x_0) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) \geq \mu(x_0 z).$$

By $x_0 z' \in E$ is a weakest edge,

$$\mu(x_i x_{i+1}) \geq \mu(x_0 z).$$

Thus

$$\bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) \geq \mu(x_0 z').$$

$P : x_0, z'$ is a path from x_0 to z'. It implies

$$\bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) \geq \mu(x_0 z').$$

Therefore,

$$\bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) \geq \mu(x_0 z').$$

It means that

$$\mathcal{C}(x_0) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) \geq \mu(x_0 z').$$

By for every vertex, there are only two ways to form a path,

$$\mathcal{C}_c(x_0) = \{x_z x_{z'} \mid \mu(x_z x_{z'}) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1})\} =$$

$$\{x_z x_{z'} \mid \mu(x_z x_{z'}) = \mu(x_0 z) \vee \mu(x_0 z') = \mu(x_0 z) = \mu(x_0 z'),\ x_z x_{z'} \in E)\} = \{x_0 z, x_0 z'\}.$$

□ 149

Proposition 5.15. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Suppose x_0 is incident to one edge $x_0 z \in E$ which isn't a weakest edge. Then*

$$\mathcal{C}_c(x_0) = \{x_z x_{z'} \mid \mu(x_z x_{z'}) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1})\} =$$

$$\{x_z x_{z'} \mid \mu(x_z x_{z'}) \geq \mu(x_0 z),\ x_z x_{z'} \in E\}.$$

Proof. Assume $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Suppose x_0 has an edge $x_0 z \in E$ which isn't a weakest edge. For every vertex, there are only two ways to form a path. By $x_0 z \in E$ isn't a weakest edge and $P : x_0, z$ is a path from x_0 to z. It implies

$$\bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) \geq \mu(x_0 z).$$

Therefore,

$$\bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) \geq \mu(x_0 z).$$

It means that

$$\mathcal{C}(x_0) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) \geq \mu(x_0 z).$$

Thus

$$\mathcal{C}_c(x_0) = \{x_z x_{z'} \mid \mu(x_z x_{z'}) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1})\} =$$

$$\{x_z x_{z'} \mid \mu(x_z x_{z'}) \geq \mu(x_0 z),\ x_z x_{z'} \in E\}.$$

□ 150

Proposition 5.16. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Suppose x_0 is incident to two edges $x_0 z, x_0 z' \in E$ which aren't weakest edges. Then*

$$\mathcal{C}_c(x_0) = \{x_z x_{z'} \mid \mu(x_z x_{z'}) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1})\} =$$

$$\{x_z x_{z'} \mid \mu(x_z x_{z'}) \leq \mu(x_0 z) \vee \mu(x_0 z'),\ x_z x_{z'} \in E\}.$$

Furthermore, if $\mu(x_0 z) < \mu(x_0 z')$, then

$$\mathcal{C}_c(x_0) = \{x_z x_{z'} \mid \mu(x_z x_{z'}) \leq \mu(x_0 z'),\ x_z x_{z'} \in E\};$$

if $\mu(x_0 z') < \mu(x_0 z)$, then

$$\mathcal{C}_c(x_0) = \{x_z x_{z'} \mid \mu(x_z x_{z'}) \leq \mu(x_0 z),\ x_z x_{z'} \in E\};$$

if $\mu(x_0 z') = \mu(x_0 z)$, then

$$\mathcal{C}_c(x_0) = \{x_z x_{z'} \mid \mu(x_z x_{z'}) \leq \mu(x_0 z) = \mu(x_0 z'),\ x_z x_{z'} \in E\}.$$

Proof. Assume $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Suppose x_0 is incident to two edges $x_0z, x_oz' \in E$ which aren't weakest edges. For every vertex, there are only two ways to form a path which are with edges either x_0z or x_oz'. $P : x_0, z$ and $P' : x_0, z'$ are paths from x_0 to z and z'. It implies

$$\bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_ix_{i+1}) \leq \mu(x_0z) \vee \mu(x_0z').$$

Therefore,

$$\bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_ix_{i+1}) \leq \mu(x_0z) \vee \mu(x_0z').$$

It means that

$$\mathcal{C}(x_0) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_ix_{i+1}) \leq \mu(x_0z) \vee \mu(x_0z').$$

Furthermore, if $\mu(x_0z) < \mu(x_0z')$, then

$$\mathcal{C}(x_0) \leq \mu(x_0z');$$

if $\mu(x_0z') < \mu(x_0z)$, then

$$\mathcal{C}(x_0) \leq \mu(x_0z);$$

if $\mu(x_0z') = \mu(x_0z)$, then

$$\mathcal{C}(x_0) \leq \mu(x_0z') = \mu(x_0z).$$

$$\mathcal{C}_c(x_0) = \{x_zx_{z'} \mid \mu(x_zx_{z'}) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_ix_{i+1})\} =$$

$$\{x_zx_{z'} \mid \mu(x_zx_{z'}) \leq \mu(x_0z) \vee \mu(x_0z'),\ x_zx_{z'} \in E\}.$$

Furthermore, if $\mu(x_0z) < \mu(x_0z')$, then

$$\mathcal{C}_c(x_0) = \{x_zx_{z'} \mid \mu(x_zx_{z'}) \leq \mu(x_0z'),\ x_zx_{z'} \in E\};$$

if $\mu(x_0z') < \mu(x_0z)$, then

$$\mathcal{C}_c(x_0) = \{x_zx_{z'} \mid \mu(x_zx_{z'}) \leq \mu(x_0z),\ x_zx_{z'} \in E\};$$

if $\mu(x_0z') = \mu(x_0z)$, then

$$\mathcal{C}_c(x_0) = \{x_zx_{z'} \mid \mu(x_zx_{z'}) \leq \mu(x_0z) = \mu(x_0z'),\ x_zx_{z'} \in E\}.$$

□

Proposition 5.17. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph. Suppose x_0 is center. Then*

$$\mathcal{C}_c(x_0) = \{x_zx_{z'} \mid \mu(x_zx_{z'}) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_ix_{i+1})\} =$$

$$\{x_zx_{z'} \mid \mu(x_zx_{z'}) = \bigvee_{i=1,\cdots,\mathcal{O}} \mu(x_0x_i)\}.$$

Proof. Consider $NTG : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. Suppose x_0 is center. All paths from x_0 has a form $x_0 x_i$. Thus

$$\bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \mu(x_0 x_i).$$

Hence

$$\bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \bigvee_{i=1,\cdots,\mathcal{O}} \mu(x_0 x_i).$$

So

$$\bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \bigvee_{i=1,\cdots,\mathcal{O}} \mu(x_0 x_i).$$

It implies

$$\mathcal{C}(x_0) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \bigvee_{i=1,\cdots,\mathcal{O}} \mu(x_0 x_i).$$

Thus

$$\mathcal{C}_c(x_0) = \{x_z x_{z'} \mid \mu(x_z x_{z'}) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1})\} =$$

$$\{x_z x_{z'} \mid \mu(x_z x_{z'}) = \bigvee_{i=1,\cdots,\mathcal{O}} \mu(x_0 x_i)\}.$$

□ 154

Proposition 5.18. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph. Suppose x_0 isn't center and c is a center. Then*

$$\mathcal{C}_c(x_0) = \{x_z x_{z'} \mid \mu(x_z x_{z'}) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1})\} =$$

$$\{x_z x_{z'} \mid \mu(x_z x_{z'}) = \mu(c x_0)\} = \{c x_0\}.$$

Proof. Consider $NTG : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. Suppose x_0 isn't center and c is a center. There's only one path from x_0 has a form $x_0 c$. Thus

$$\bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \mu(x_0 c).$$

Hence

$$\bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \mu(x_0 c).$$

So

$$\bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \mu(x_0 c).$$

It implies

$$\mathcal{C}(x_0) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \mu(x_0 c).$$

Thus

$$\mathcal{C}_c(x_0) = \{x_z x_{z'} \mid \mu(x_z x_{z'}) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1})\} =$$

$$\{x_z x_{z'} \mid \mu(x_z x_{z'}) = \mu(c x_0)\} = \{c x_0\}.$$

□ 155

Proposition 5.19. *Let* $NTG : (V, E, \sigma, \mu)$ *be a star-strong-neutrosophic graph. Suppose* x_0 *is center such that* $\bigwedge_{i=0}^{\mathcal{O}-1} = \sigma(x_0)$*. And* x *isn't center. Then*

$$\mathcal{C}_c(x_0) = \{x_z x_{z'} \mid \mu(x_z x_{z'}) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1})\} =$$

$$\{x_z x_{z'} \mid \mu(x_z x_{z'}) = \sigma(x_0)\} = \{x_0 x_i\}_{i=1}^{\mathcal{O}-1}.$$

And

$$\mathcal{C}_c(x) = \{x_z x_{z'} \mid \mu(x_z x_{z'}) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1})\} =$$

$$\{x_z x_{z'} \mid \mu(x_z x_{z'}) = \sigma(x_0)\} = \{x_0 x_i\}_{i=1}^{\mathcal{O}-1}.$$

Proof. Consider $NTG : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. Suppose x_0 is center such that $\bigwedge_{i=0}^{\mathcal{O}-1} = \sigma(x_0)$. All paths from x_0 has a form $x_0 x_i$. Thus

$$\bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \mu(x_0 x_i) = \sigma(x_0).$$

Hence

$$\bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \bigvee_{i=1,\cdots,\mathcal{O}} \mu(x_0 x_i) = \sigma(x_0).$$

So

$$\bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \bigvee_{i=1,\cdots,\mathcal{O}} \mu(x_0 x_i) = \sigma(x_0).$$

It implies

$$\mathcal{C}(x_0) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1}) = \sigma(x_0).$$

Thus

$$\mathcal{C}_c(x_0) = \{x_z x_{z'} \mid \mu(x_z x_{z'}) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1})\} =$$

$$\{x_z x_{z'} \mid \mu(x_z x_{z'}) = \sigma(x_0)\} = \{x_0 x_i\}_{i=1}^{\mathcal{O}-1}.$$

And

$$\mathcal{C}_c(x) = \{x_z x_{z'} \mid \mu(x_z x_{z'}) = \bigvee_{x_t \in V} \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1})\} =$$

$$\{x_z x_{z'} \mid \mu(x_z x_{z'}) = \sigma(x_0)\} = \{x_0 x_i\}_{i=1}^{\mathcal{O}-1}.$$

□

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