New Similarity and Entropy Measures of Temporal Complex Neutrosophic Set and Its application in Multi-Criteria Decision Making

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Abstract

The Temporal complex neutrosophic set (TCNS) is an effective structure for resolving information that has uncertain, indeterminate, and time-related factors in decision-making problems. In the last decade, many researchers focused on estimating the vagueness and ambiguity in knowledge using the theory of neutrosophic sets or extensions of neutrosophic sets. Similarity and entropy measures are valuable tools to measure information with the aim of dealing with real-life multi-criteria decision-making (MCDM) problems. However, the existing method did not care or interest in the information measures of TCNS. This paper proposes several new similarity and entropy measures under the TCNS environment. The suggested measurements have been confirmed and proven to concede with the manifest definition of the similarity and entropy measure for the TCNS. Finally, a numerical example related to deciding on a tourist terminus in Vietnam is given to confirm the practical applicability of the proposed measures. The numerical example proves that the proposed similarity and entropy measures of TCNS can produce accurate and reasonable results for decision-making problems in the real world.

Keywords: MCDM; Complex Neutrosophic Set; Temporal complex neutrosophic set; Similarity measure; Entropy measure

1 Introduction

In 1999, Smarandache [1] introduced the notion of a neutrosophic set (NS) that each part of this set has a degree of truth, indeterminacy, and falsity, respectively. It's also an essential and powerful tool to deal with incomplete, indeterminate, and inconsistent information in some real-life problems. Due to its ability to deal with the uncertainty factor, it has many applications in the areas (theoretical as well as practical) related to engineering, arts, humanities, computer science, health sciences, life sciences, etc. Although NS theory is very successful in handling problems arising from vagueness and uncertainty, it cannot address the periodicity occurring in some uncertain information. Because of the indeterminate information and the complexity of the periodic form of decision-making problems, it is challenging to explain attributes in terms of crisp sets, fuzzy sets, and neutrosophic sets.

To better design and model real-life applications, the "complex" feature deals simultaneously with uncertainty, inconsistency, and periodicity data. To solve this issue, Ramot et al. [2] proposed the theory of the complex fuzzy set (CFS) that supplements a phase component to gain information regarding a particular higher dimensional periodic problem. Also, with the aim of adding the "complex" part to make NS more flexible and adaptable to occasional vague information, Ali and Smarandache [3] proposed the definition of the complex neutrosophic set (CNS) that considers a combined version of the CFS and NS. Three membership degree functions characterize a CNS tackle uncertainty, inconsistency, and indeterminacy. These functions have complex-valued and the range in the unit circle in the complex space. The constraint of the CNS is that the sum of positive, abstinence, and negative grades is less than or equal to three. After its introduction, CNS proved its significance in describing data with indeterminate, uncertain, inconsistent, and periodic factors.

The notion of similarity plays a fundamental role in solving assorted complex, indeterminate matters in human life. The similarity and entropy measures exhibited the degree of similarity between two objects in an uncertain, indeterminate environment. Hence, many researchers have concentrated on measuring uncertainty and fuzziness in information using neutrosophic sets or extensions. Various similarity and entropy measures have been introduced. They have successfully applied in solving problems in real-life applications, such as medical diagnosis [4–6], pattern recognition [7, 8], decision making [9–11], and clustering analysis [12, 13].

In the neutrosophic environment, Broumi and Smarandache [14] introduced some similarities based on Hausdorff distance for NS. Majumdar et al. [15] proposed some measures of similarity and entropy of a single-valued NS. Jun Ye [16] presented the similarity measures that construct based on the Hamming and Euclidean distances for interval neutrosophic sets. Following that, Mukherjee and Sarkar [17] defined some similarity measures based on Harmming and Euclidean distances between two interval neutrosophic soft sets with an application in pattern recognition. Abu Qamar and Hassan [18]introduced similarity and entropy tools to the Q-Neutrosophic Soft set and examined their effectiveness for medical diagnosis problems. Rıdvan Sahin [19] proposed two novel distance measures for SVNHFNs based on single-valued neutrosophic hesitant fuzzy numbers. Besides, similarity and entropy are widely used as critical measures for solving multi-criteria decision-making problems in real life [7–10, 17, 20].

As two critical topics in the CNS, entropy and similarity measures have been extensively studied by many studies from different perspectives. Researchers made significant additions to the literature on similarity measures domains by using hybridized models to manage the uncertainty of periodic data, where time plays a vital role in depicting it. Al-Qudah and Hassan [21] introduced axiomatic definitions of entropy and similarity measure for Complex Multi Fuzzy Soft Sets. Ganeshsree Selvachandran et al. [22] established the measures of distance and similarity based on a complex vague soft set with the aim of solving pattern recognition problems involving digital images. Mondal et al. [23] proposed three complex neutrosophic similarity measures, such as cosine, Jaccard, and Dice. A numerical example of stream selection of students after secondary examination to select an appropriate educational stream for higher secondary education is given to demonstrate the proposed strategies for multi-attribute decision-making problems. Ali et al. [24] proposed novel dice similarity measures using CNS. Then, the authors applied the presented criteria to the pattern recognition model to examine the reliability and superiority of the established approaches. Faisal et al. [20] introduced some similarity measures of interval complex neutrosophic soft sets based on the distance measures: Hamming distance and Euclidean distance for solving decision-making problems in medical diagnosis.

For the purpose of modeling data with the time factor and dynamic factor of some real-world decision-making problems, recently, Lan et al. [25] introduced the definition of a Temporal Complex neutrosophic set (TCNS). TCNS is an extension of CNS. TCNS concerns decision-making problems that have uncertain, temporal, and periodical factors. An application for tourist destination choice problems illustrated the effectiveness and flexibility of TCNS in multi-criteria decision-making. It is clear that TCNS is a new opening with many real-life applications when it considers the time cycle and the time factor. However, previous studies were still not yet taken an interest in similarity and entropy measures in the temporal complex neutrosophic environments. Therefore, the paper presents new similarity and entropy measures of the TCNS.

Specifically, the contributions of this paper are highlighted as follows: (1) Define new similarity and entropy measures of the Temporal complex neutrosophic set. (2) Construct and develop the decision-making model based on new similarity and entropy measures of TCNS. (3) Demonstrate the feasibility and rationality of solving the decision-making problem of the proposed model via an actual world-life case study of choosing a tourist destination in Vietnam and comparative analysis.

The rest of this paper is organized as follows. In Section 2, we first review some fundamental concepts of the TCNS. Section 3 defines new similarity and entropy measures based on the TCNS theory. Section 4 presents the developing multi-criteria decision-making model that uses proposed similarity and entropy measures. Section 5 describes a practical application related to a chosen tourist destination and demonstrates our method's effectiveness via a comparison experiment. Finally, the conclusions are presented in Section 6.

2 PRELIMINARIES

In this section, we recapitulate the basic notion of TCNS and their operations, namely complement, union, and intersection, which will be used in our paper.

Definition 1 ([25]) Suppose there are a universal set X and time periods $\tilde{\tau} = \{\tau_1, \tau_2, ..., \tau_{n_\tau}\}$. A Temporal complex neutrosophic set TCNS on X is denoted by

$$\text{TCNS}(x, \tilde{\tau}) = \{x, \langle T(x, \tilde{\tau}), I(x, \tilde{\tau}), F(x, \tilde{\tau}) \mid x \in X\} \tag{1}$$

Where $T(x, \tilde{\tau}) = p(x, \tau_l) . e^{j\mu(\tau_l)}$; $I(x, \tilde{\tau}_x) = q(\tau_l) . e^{j\nu(x,\tau_l)}$;

$F(x, \tilde{\tau}) = r(x, \tau_l) . e^{j\eta(\tau_l)}$; $l = 1, 2, ..., n_\tau$; for ease of use, we'll call $\tilde{n}_E$ is a Temporal Complex Neutrosophic Element (TCNE) of element $x \in X$ to the set E and $\tilde{n}_E$ is given by the Equation (2)

$$\tilde{n}_E = \left\langle p_x(\tau_l) . e^{j\mu_x(\tau_l)}, q_x(\tau_l) . e^{j\nu_x(\tau_l)}, r_x(\tau_l) . e^{j\eta_x(\tau_l)} \right\rangle \tag{2}$$

Definition 2 ([25]) Suppose there are two TCNSs described by their temporal complex neutrosophic membership functions $\Psi_1(x, \tilde{\tau})$ and $\Psi_2(x, \tilde{\tau})$ respectively.

$$\Psi_1(x, \tilde{\tau}) = \left\{x, \left\langle p_{\Psi_1}(x, \tilde{\tau}) e^{j\mu_{\Psi_1}(x,\tilde{\tau})}, q_{\Psi_1}(x, \tilde{\tau}) e^{j\nu_{\Psi_1}(x,\tilde{\tau})}, r_{\Psi_1}(x, \tilde{\tau}) e^{j\eta_{\Psi_1}(x,\tilde{\tau})} \right\rangle\right\}$$
$$\Psi_2(x, \tilde{\tau}) = \left\{x, \left\langle p_{\Psi_2}(x, \tilde{\tau}) e^{j\mu_{\Psi_2}(x,\tilde{\tau})}, q_{\Psi_2}(x, \tilde{\tau}) e^{j\nu_{\Psi_2}(x,\tilde{\tau})}, r_{\Psi_2}(x, \tilde{\tau}) e^{j\eta_{\Psi_2}(x,\tilde{\tau})} \right\rangle\right\}$$

The basic operations of TCNS are given as follows:
Complement:

$$C\left(\Psi_1\left(x,\tilde{\tau}\right)\right)=\left\{x,\left\langle\begin{array}{l} r_{\Psi_1}\left(x,\tilde{\tau}\right)e^{j\left(2\pi-\eta_{\Psi_1}(x,\tilde{\tau})\right)},\\ \left(1-q_{\Psi_1}\left(x,\tilde{\tau}\right)\right)e^{j\left(2\pi-v_{\Psi_1}(x,\tilde{\tau})\right)},\\ p_{\Psi_1}\left(x,\tilde{\tau}\right)e^{j\left(2\pi-\mu_{\Psi_1}(x,\tilde{\tau})\right)}\end{array}\right\rangle \mid x\in X\right\} \tag{3}$$

Union:

$$\Psi_1\left(x,\tilde{\tau}\right)\cup\Psi_2\left(x,\tilde{\tau}\right)=\left\{x,\left\langle\begin{array}{l}\left(p_{\Psi_1}\left(x,\tilde{\tau}\right)\vee p_{\Psi_2}\left(x,\tilde{\tau}\right)\right)e^{j\mu_{\Psi_1\cup\Psi_2}(x,\tilde{\tau})},\\ \left(q_{\Psi_1}\left(x,\tilde{\tau}\right)\wedge q_{\Psi_2}\left(x,\tilde{\tau}\right)\right)e^{j\nu_{\Psi_1\cup\Psi_2}(x,\tilde{\tau})},\\ \left(r_{\Psi_1}\left(x,\tilde{\tau}\right)\wedge r_{\Psi_2}\left(x,\tilde{\tau}\right)\right)e^{j\eta_{\Psi_1\cup\Psi_2}(x,\tilde{\tau})}\end{array}\right\rangle \mid x\in X\right\} \tag{4}$$

Intersect:

$$\Psi_1\left(x,\tilde{\tau}\right)\cap\Psi_2\left(x,\tilde{\tau}\right)=\left\{x,\left\langle\begin{array}{l}\left(p_{\Psi_1}\left(x,\tilde{\tau}\right)\wedge p_{\Psi_2}\left(x,\tilde{\tau}\right)\right)e^{j\mu_{\Psi_1\cap\Psi_2}(x,\tilde{\tau})},\\ \left(q_{\Psi_1}\left(x,\tilde{\tau}\right)\vee q_{\Psi_2}\left(x,\tilde{\tau}\right)\right)e^{j\nu_{\Psi_1\cap\Psi_2}(x,\tilde{\tau})},\\ \left(r_{\Psi_1}\left(x,\tilde{\tau}\right)\vee r_{\Psi_2}\left(x,\tilde{\tau}\right)\right)e^{j\eta_{\Psi_1\cap\Psi_2}(x,\tilde{\tau})}\end{array}\right\rangle \mid x\in X\right\} \tag{5}$$

3 New Similarity and Entropy of Temporal complex neutrosophic set

In this section, the paper introduces some similarity measures of TCNS, such as Dice, Jaccard, Cosine, and Cotangent similarity measures. Furthermore, an entropy measurement of TCNS is also proposed as the baseline for decision-making models in temporal complex neutrosophic environments.

3.1 Dice similarity measure of TCNS

Definition 3 Assume that
$\Psi_1 = \left\langle p_{\Psi_1}\left(x_i,\tau_l\right)e^{j\mu_{\Psi_1}(x_i,\tau_l)}, q_{\Psi_1}\left(x_i,\tau_l\right)e^{j\nu_{\Psi_1}(x_i,\tau_l)}, r_{\Psi_1}\left(x_i,\tau_l\right)e^{j\eta_{\Psi_1}(x_i,\tau_l)}\right\rangle$
and $\Psi_2 = \left\langle p_{\Psi_2}\left(x_i,\tau_l\right)e^{j\mu_{\Psi_2}(x_i,\tau_l)}, q_{\Psi_2}\left(x_i,\tau_l\right)e^{j\nu_{\Psi_2}(x_i,\tau_l)}, r_{\Psi_2}\left(x_i,\tau_l\right)e^{j\eta_{\Psi_2}(x_i,\tau_l)}\right\rangle$
be are an TCNSs on universe of discourse X for all $x_i\left(1,2,3,...,n_X\right)$ and $\tau_l\left(1,2,3,...,n_\tau\right)$.
A temporal complex Dice similarity measure between TCNSs Ψ_1 and Ψ_2 (denoted as $S_{Dice}\left(\Psi_1,\Psi_2\right)$) is defined as follows:

$$S_{Dice}(\Psi_1,\Psi_2) = \frac{1}{n_X * n_\tau}\sum_{i=1}^{n_X}\sum_{l=1}^{n_\tau}\frac{2*\begin{bmatrix}(p_{\Psi_1}(x_i,\tau_l)+\mu_{\Psi_1}(x_i,\tau_l))(p_{\Psi_2}(x_i,\tau_l)+\mu_{\Psi_2}(x_i,\tau_l))\\+(q_{\Psi_1}(x_i,\tau_l)+\nu_{\Psi_1}(x_i,\tau_l))(q_{\Psi_2}(\tau_l)+\nu_{\Psi_2}(x_i,\tau_l))\\+(r_{\Psi_1}(x_i,\tau_l)+\eta_{\Psi_1}(x_i,\tau_l))(r_{\Psi_2}(x_i,\tau_l)+\eta_{\Psi_2}(x_i,\tau_l))\end{bmatrix}}{\begin{pmatrix}(p_{\Psi_1}(x_i,\tau_l)+\mu_{\Psi_1}(x_i,\tau_l))^2+(q_{\Psi_1}(x_i,\tau_l)+\nu_{\Psi_1}(x_i,\tau_l))^2\\+(r_{\Psi_1}(x_i,\tau_l)+\eta_{\Psi_1}(x_i,\tau_l))^2+(p_{\Psi_2}(x_i,\tau_l)+\mu_{\Psi_2}(x_i,\tau_l))^2\\+(q_{\Psi_2}(x_i,\tau_l)+\nu_{\Psi_2}(x_i,\tau_l))^2+(r_{\Psi_2}(x_i,\tau_l)+\eta_{\Psi_2}(x_i,\tau_l))^2\end{pmatrix}} \tag{6}$$

Example 1 Let $X = \{x_1, x_2\}$ be a universe of discourse; $\tilde{\tau} = \{\tau_1, \tau_2, \tau_3\}$ be time period; Ψ_1 and Ψ_2 be two TCNS in X

$$\Psi_1(x_1) = \left\{\begin{array}{l}\left\langle 0.1e^{j0.2}, 0.6e^{j0.45}, 0.3e^{j0.56}\right\rangle, \left\langle 0.8e^{j0.77}, 0.2e^{j0.86}, 0.4e^{j0.65}\right\rangle,\\ \left\langle 0.6e^{j0.52}, 0.2e^{j0.2}, 0.36e^{j0.2}\right\rangle\end{array}\right\}$$

$$\Psi_1(x_2) = \left\{\begin{array}{l}\left\langle 0.5e^{j0.12}, 0.2e^{j0.65}, 0.5e^{j0.85}\right\rangle, \left\langle 0.3e^{j0.8}, 0.7e^{j0.23}, 0.1e^{j0.32}\right\rangle,\\ \left\langle 0.74e^{j0.6}, 0.5e^{j0.15}, 0.66e^{j0.41}\right\rangle\end{array}\right\}$$

$$\Psi_2(x_1) = \left\{\begin{array}{l}\left\langle 0.6e^{j0.8}, 0.9e^{j0.25}, 0.4e^{j0.52}\right\rangle, \left\langle 0.6e^{j0.62}, 0.5e^{j0.24}, 0.3e^{j0.18}\right\rangle,\\ \left\langle 0.3e^{j0.82}, 0.3e^{j0.6}, 0.4e^{j0.71}\right\rangle\end{array}\right\}$$

$$\Psi_2(x_2) = \left\{\begin{array}{l}\left\langle 0.3e^{j0.22}, 0.4e^{j0.56}, 0.6e^{j0.26}\right\rangle, \left\langle 0.1e^{j0.38}, 0.5e^{j0.43}, 0.5e^{j0.24}\right\rangle,\\ \left\langle 0.2e^{j0.29}, 0.4e^{j0.72}, 0.5e^{j0.65}\right\rangle\end{array}\right\}$$

Then, by using Equations (6), we get Temporal complex Dice similarity measure between Ψ_1 and Ψ_2, denoted as $S_{Dice}(\Psi_1, \Psi_2)$ as follows:
$S_{Dice}(\Psi_1, \Psi_2) = 0.879416$

Theorem 1 *Let Ψ_1 and Ψ_2 be two TCNSs then,*
(1) $0 \le S_{Dice}(\Psi_1, \Psi_2) \le 1$
(2) $S_{Dice}(\Psi_1, \Psi_2) = S_{Dice}(\Psi_2, \Psi_1)$
(3) $S_{Dice}(\Psi_1, \Psi_2) = 1$ *, if and only if* $\Psi_1 = \Psi_2$
(4) if Ψ_3 is a TCNS in x and $\Psi_1 \subseteq \Psi_2 \subseteq \Psi_3$ then $S_{Dice}(\Psi_1, \Psi_3) \le S_{Dice}(\Psi_1, \Psi_2)$ *and* $S_{Dice}(\Psi_1, \Psi_3) \le S_{Dice}(\Psi_1, \Psi_2)$

Proof

(1) We can have,

$$2 * \begin{bmatrix} (p_{\Psi_1}(x_i,\tau_l)+\mu_{\Psi_1}(x_i,\tau_l))(p_{\Psi_2}(x_i,\tau_l)+\mu_{\Psi_2}(x_i,\tau_l)) \\ +(q_{\Psi_1}(x_i,\tau_l)+\nu_{\Psi_1}(x_i,\tau_l))(q_{\Psi_2}(x_i,\tau_l)+\nu_{\Psi_2}(x_i,\tau_l)) \\ +(r_{\Psi_1}(x_i,\tau_l)+\eta_{\Psi_1}(x_i,\tau_l))(r_{\Psi_2}(x_i,\tau_l)+\eta_{\Psi_2}(x_i,\tau_l)) \end{bmatrix}$$
$$\leq \begin{pmatrix} (p_{\Psi_1}(x_i,\tau_l)+\mu_{\Psi_1}(x_i,\tau_l))^2+(q_{\Psi_1}(x_i,\tau_l)+\nu_{\Psi_1}(x_i,\tau_l))^2 \\ +(r_{\Psi_1}(x_i,\tau_l)+\eta_{\Psi_1}(x_i,\tau_l))^2+(p_{\Psi_2}(x_i,\tau_l)+\mu_{\Psi_2}(x_i,\tau_l))^2 \\ +(q_{\Psi_2}(x_i,\tau_l)+\nu_{\Psi_2}(x_i,\tau_l))^2+(r_{\Psi_2}(x_i,\tau_l)+\eta_{\Psi_2}(x_i,\tau_l))^2 \end{pmatrix}$$

Hence, $0 \leq S_{Dice}(\Psi_1,\Psi_2) \leq 1$. So, it proves the first inequality.

(2) It is obvious that the Theorem 1 is true. This proves the second inequality.

(3) When $\Psi_1 = \Psi_2$, then it implies that $S_{Dice}(\Psi_1,\Psi_2) = 1$. On the other hand, if $S_{Dice}(\Psi_1,\Psi_2) = 1$ then,

$p_{\Psi_1}(x_i,\tau_l) = p_{\Psi_2}(x_i,\tau_l)$; $\mu_{\Psi_1}(x_i,\tau_l) = \mu_{\Psi_2}(x_i,\tau_l)$; $q_{\Psi_1}(x_i,\tau_l) = q_{\Psi_2}(x_i,\tau_l)$; $\nu_{\Psi_1}(x_i,\tau_l) = \nu_{\Psi_2}(x_i,\tau_l)$; $r_{\Psi_1}(x_i,\tau_l) = r_{\Psi_2}(x_i,\tau_l)$; $\eta_{\Psi_1}(x_i,\tau_l) = \eta_{\Psi_2}(x_i,\tau_l)$;

This implies that $\Psi_1 = \Psi_2$. So, the third inequality is proved.

(4) When $\Psi_1 \subseteq \Psi_2 \subseteq \Psi_3$, we can have

$p_{\Psi_1}(x_i,\tau_l)+\mu_{\Psi_1}(x_i,\tau_l) \leq p_{\Psi_2}(x_i,\tau_l)+\mu_{\Psi_2}(x_i,\tau_l) \leq p_{\Psi_3}(x_i,\tau_l)+\mu_{\Psi_3}(x_i,\tau_l)$;

$q_{\Psi_1}(x_i,\tau_l)+\nu_{\Psi_1}(x_i,\tau_l) \geq q_{\Psi_2}(x_i,\tau_l)+\nu_{\Psi_2}(x_i,\tau_l) \geq q_{\Psi_3}(x_i,\tau_l)+\nu_{\Psi_3}(x_i,\tau_l)$;

$r_{\Psi_1}(x_i,\tau_l)+\eta_{\Psi_1}(x_i,\tau_l) \geq r_{\Psi_2}(x_i,\tau_l)+\eta_{\Psi_2}(x_i,\tau_l) \geq r_{\Psi_3}(x_i,\tau_l)+\eta_{\Psi_3}(x_i,\tau_l)$;

So, $S_{Dice}(\Psi_1,\Psi_3) \leq S_{Dice}(\Psi_1,\Psi_2)$ and $S_{Dice}(\Psi_1,\Psi_3) \leq S_{Dice}(\Psi_2,\Psi_3)$. This proves the fourth inequality.

Finally, all the equalities in Theorem 1 are proved.

3.2 Jaccard similarity measure of TCNE

Definition 4 Let two TCNSs
$\Psi_1 = \left\langle p_{\Psi_1}(x_i,\tau_l)e^{j\mu_{\Psi_1}(x_i,\tau_l)}, q_{\Psi_1}(x_i,\tau_l)e^{j\nu_{\Psi_1}(x_i,\tau_l)}, r_{\Psi_1}(x_i,\tau_l)e^{j\eta_{\Psi_1}(x_i,\tau_l)} \right\rangle$
and $\Psi_2 = \left\langle p_{\Psi_2}(x_i,\tau_l)e^{j\mu_{\Psi_2}(x_i,\tau_l)}, q_{\Psi_2}(x_i,\tau_l)e^{j\nu_{\Psi_2}(x_i,\tau_l)}, r_{\Psi_2}(x_i,\tau_l)e^{j\eta_{\Psi_2}(x_i,\tau_l)} \right\rangle$
for all $x_i\,(1,2,3,...,n_X)$ belong to X and time period $\tau_l\,(1,2,3,...,n_\tau)$.
A temporal complex Jaccard similarity measure between TCNSs Ψ_1 and Ψ_2 (denoted

as $S_J(\Psi_1,\Psi_2))$ is defined as follows:

$$S_J(\Psi_1,\Psi_2)=$$

$$\frac{1}{n_X*n_\tau}\sum_{i=1}^{n_X}\sum_{l=1}^{n_\tau}\frac{\begin{array}{c}(p_{\Psi_1}(x_i,\tau_l)+\mu_{\Psi_1}(x_i,\tau_l))(p_{\Psi_2}(x_i,\tau_l)+\mu_{\Psi_2}(x_i,\tau_l))\\+(q_{\Psi_1}(x_i,\tau_l)+\nu_{\Psi_1}(x_i,\tau_l))(q_{\Psi_2}(x_i,\tau_l)+q_{\Psi_2}(x_i,\tau_l))\\+(r_{\Psi_1}(x_i,\tau_l)+r_{\Psi_1}(x_i,\tau_l))(r_{\Psi_2}(x_i,\tau_l)+r_{\Psi_2}(x_i,\tau_l))\end{array}}{\begin{array}{l}(p_{\Psi_1}(x_i,\tau_l)+\mu_{\Psi_1}(x_i,\tau_l))^2+(q_{\Psi_1}(x_i,\tau_l)+\nu_{\Psi_1}(x_i,\tau_l))^2\\+(r_{\Psi_1}(x_i,\tau_l)+r_{\Psi_1}(x_i,\tau_l))^2+(p_{\Psi_2}(x_i,\tau_l)+\mu_{\Psi_2}(x_i,\tau_l))^2\\+(q_{\Psi_2}(x_i,\tau_l)+q_{\Psi_2}(x_i,\tau_l))^2+(r_{\Psi_2}(x_i,\tau_l)+r_{\Psi_2}(x_i,\tau_l))^2\\-\left(\begin{array}{c}(p_{\Psi_1}(x_i,\tau_l)+\mu_{\Psi_1}(x_i,\tau_l))(p_{\Psi_2}(x_i,\tau_l)+\mu_{\Psi_2}(x_i,\tau_l))\\+(q_{\Psi_1}(x_i,\tau_l)+\nu_{\Psi_1}(x_i,\tau_l))(q_{\Psi_2}(x_i,\tau_l)+q_{\Psi_2}(x_i,\tau_l))\\+(r_{\Psi_1}(x_i,\tau_l)+r_{\Psi_1}(x_i,\tau_l))(r_{\Psi_2}(x_i,\tau_l)+r_{\Psi_2}(x_i,\tau_l))\end{array}\right)\end{array}} \tag{7}$$

Example 2 Let $X=\{x_1,x_2\}$;$\tilde{\tau}=\{\tau_1,\tau_2,\tau_3\}$; and two TCNS Ψ_1, Ψ_2 in X.

$$\Psi_1(x_1)=\left\{\begin{array}{l}\left\langle 0.1e^{j0.2},0.6e^{j0.45},0.3e^{j0.56}\right\rangle,\left\langle 0.8e^{j0.77},0.2e^{j0.86},0.4e^{j0.65}\right\rangle,\\\left\langle 0.6e^{j0.52},0.2e^{j0.2},0.36e^{j0.2}\right\rangle\end{array}\right\}$$

$$\Psi_1(x_2)=\left\{\begin{array}{l}\left\langle 0.5e^{j0.12},0.2e^{j0.65},0.5e^{j0.85}\right\rangle,\left\langle 0.3e^{j0.8},0.7e^{j0.23},0.1e^{j0.32}\right\rangle,\\\left\langle 0.74e^{j0.6},0.5e^{j0.15},0.66e^{j0.41}\right\rangle\end{array}\right\}$$

$$\Psi_2(x_1)=\left\{\begin{array}{l}\left\langle 0.6e^{j0.8},0.9e^{j0.25},0.4e^{j0.52}\right\rangle,\left\langle 0.6e^{j0.62},0.5e^{j0.24},0.3e^{j0.18}\right\rangle,\\\left\langle 0.3e^{j0.82},0.3e^{j0.6},0.4e^{j0.71}\right\rangle\end{array}\right\}$$

$$\Psi_2(x_2)=\left\{\begin{array}{l}\left\langle 0.3e^{j0.22},0.4e^{j0.56},0.6e^{j0.26}\right\rangle,\left\langle 0.1e^{j0.38},0.5e^{j0.43},0.5e^{j0.24}\right\rangle,\\\left\langle 0.2e^{j0.29},0.4e^{j0.72},0.5e^{j0.65}\right\rangle\end{array}\right\}$$

The temporal complex Jaccard similarity measure between two TCNS Ψ_1, Ψ_2 can be obtained by applying Definitions 4 as follows: $S_J(\Psi_1,\Psi_2)=0.788072$

Theorem 2 *Let Ψ_1 and Ψ_2 be two TCNSs then,*

(1) $0\leq S_J(\Psi_1,\Psi_2)\leq 1$

(2) $S_J(\Psi_1,\Psi_2)=S_J(\Psi_2,\Psi_1)$

(3) $S_J(\Psi_1,\Psi_2)=1$, *if and only if* $\Psi_1=\Psi_2$

(4) if Ψ_3 is a TCNS in X and $\Psi_1\subseteq\Psi_2\subseteq\Psi_3$then $S_J(\Psi_1,\Psi_3)\leq S_J(\Psi_1,\Psi_2)$ *and* $S_J(\Psi_1,\Psi_3)\leq S_J(\Psi_2,\Psi_3)$

Proof

(1) Since:

$$\left(\begin{array}{l} \left(p_{\Psi_1}\left(x_i,\tau_l\right)+\mu_{\Psi_1}\left(x_i,\tau_l\right)\right)\left(p_{\Psi_2}\left(x_i,\tau_l\right)+\mu_{\Psi_2}\left(x_i,\tau_l\right)\right) \\ +\left(q_{\Psi_1}\left(x_i,\tau_l\right)+\nu_{\Psi_1}\left(x_i,\tau_l\right)\right)\left(q_{\Psi_2}\left(x_i,\tau_l\right)+\nu_{\Psi_2}\left(x_i,\tau_l\right)\right) \\ +\left(r_{\Psi_1}\left(x_i,\tau_l\right)+\eta_{\Psi_1}\left(x_i,\tau_l\right)\right)\left(r_{\Psi_2}\left(x_i,\tau_l\right)+\eta_{\Psi_2}\left(x_i,\tau_l\right)\right) \end{array} \right)$$

$$\leq \left(\begin{array}{l} \left(p_{\Psi_1}\left(x_i,\tau_l\right)+\mu_{\Psi_1}\left(x_i,\tau_l\right)\right)^2+\left(q_{\Psi_1}\left(x_i,\tau_l\right)+\nu_{\Psi_1}\left(x_i,\tau_l\right)\right)^2 \\ +\left(r_{\Psi_1}\left(x_i,\tau_l\right)+\eta_{\Psi_1}\left(x_i,\tau_l\right)\right)^2+\left(p_{\Psi_2}\left(x_i,\tau_l\right)+\mu_{\Psi_2}\left(x_i,\tau_l\right)\right)^2 \\ +\left(q_{\Psi_2}\left(x_i,\tau_l\right)+\nu_{\Psi_2}\left(x_i,\tau_l\right)\right)^2+\left(r_{\Psi_2}\left(x_i,\tau_l\right)+\eta_{\Psi_2}\left(x_i,\tau_l\right)\right)^2 \\ -\left[\begin{array}{l} \left(p_{\Psi_1}\left(x_i,\tau_l\right)+\mu_{\Psi_1}\left(x_i,\tau_l\right)\right)\left(p_{\Psi_2}\left(x_i,\tau_l\right)+\mu_{\Psi_2}\left(x_i,\tau_l\right)\right) \\ +\left(q_{\Psi_1}\left(x_i,\tau_l\right)+\nu_{\Psi_1}\left(x_i,\tau_l\right)\right)\left(q_{\Psi_2}\left(x_i,\tau_l\right)+\nu_{\Psi_2}\left(x_i,\tau_l\right)\right) \\ +\left(r_{\Psi_1}\left(x_i,\tau_l\right)+\eta_{\Psi_1}\left(x_i,\tau_l\right)\right)\left(r_{\Psi_2}\left(x_i,\tau_l\right)+\eta_{\Psi_2}\left(x_i,\tau_l\right)\right) \end{array} \right] \end{array} \right)$$

Hence, we can write $0 \leq S_J\left(\Psi_1,\Psi_2\right) \leq 1$.

(2) It is obvious that the Theorem 2 is true

(3) When $\Psi_1 = \Psi_2$, then obviously $S_J\left(\Psi_1,\Psi_2\right) = 1$. On the other hand, if $S_J\left(\Psi_1,\Psi_2\right) = 1$ then,

$p_{\Psi_1}\left(x_i,\tau_l\right) = p_{\Psi_2}\left(x_i,\tau_l\right)$; $\mu_{\Psi_1}\left(x_i,\tau_l\right) = \mu_{\Psi_2}\left(x_i,\tau_l\right)$; $q_{\Psi_1}\left(x_i,\tau_l\right) = q_{\Psi_2}\left(x_i,\tau_l\right)$; $\nu_{\Psi_1}\left(x_i,\tau_l\right) = \nu_{\Psi_2}\left(x_i,\tau_l\right)$; $r_{\Psi_1}\left(x_i,\tau_l\right) = r_{\Psi_2}\left(x_i,\tau_l\right)$; $\eta_{\Psi_1}\left(x_i,\tau_l\right) = \eta_{\Psi_2}\left(x_i,\tau_l\right)$

This implies that $\Psi_1 = \Psi_2$.

(4) When $\Psi_1 \subseteq \Psi_2 \subseteq \Psi_3$, we can write $p_{\Psi_1}\left(x_i,\tau_l\right)+\mu_{\Psi_1}\left(x_i,\tau_l\right) \leq p_{\Psi_2}\left(x_i,\tau_l\right)+\mu_{\Psi_2}\left(x_i,\tau_l\right) \leq p_{\Psi_3}\left(x_i,\tau_l\right)+\mu_{\Psi_3}\left(x_i,\tau_l\right)$;

$q_{\Psi_1}\left(x_i,\tau_l\right)+\nu_{\Psi_1}\left(x_i,\tau_l\right) \geq q_{\Psi_2}\left(x_i,\tau_l\right)+\nu_{\Psi_2}\left(x_i,\tau_l\right) \geq q_{\Psi_3}\left(x_i,\tau_l\right)+\nu_{\Psi_3}\left(x_i,\tau_l\right)$;

$r_{\Psi_1}\left(x_i,\tau_l\right)+\eta_{\Psi_1}\left(x_i,\tau_l\right) \geq r_{\Psi_2}\left(x_i,\tau_l\right)+\eta_{\Psi_2}\left(x_i,\tau_l\right) \geq r_{\Psi_3}\left(x_i,\tau_l\right)+\eta_{\Psi_3}\left(x_i,\tau_l\right)$;

So, $S_J\left(\Psi_1,\Psi_3\right) \leq S_J\left(\Psi_1,\Psi_2\right)$ and $S_J\left(\Psi_1,\Psi_3\right) \leq S_J\left(\Psi_2,\Psi_3\right)$.

All the equalities in Theorem 2 are proved.

3.3 Cosine Similarity of TCNS

The cosine similarity measure for temporal complex neutrosophic sets is defined as follows:

Definition 5 Let $\Psi_1 = \left\langle p_{\Psi_1}\left(x_i,\tau_l\right)e^{j\mu_{\Psi_1}\left(x_i,\tau_l\right)}, q_{\Psi_1}\left(x_i,\tau_l\right)e^{j\nu_{\Psi_1}\left(x_i,\tau_l\right)}, r_{\Psi_1}\left(x_i,\tau_l\right)e^{j\eta_{\Psi_1}\left(x_i,\tau_l\right)}\right\rangle$ and $\Psi_2 = \left\langle p_{\Psi_2}\left(x_i,\tau_l\right)e^{j\mu_{\Psi_2}\left(x_i,\tau_l\right)}, q_{\Psi_2}\left(x_i,\tau_l\right)e^{j\nu_{\Psi_2}\left(x_i,\tau_l\right)}, r_{\Psi_2}\left(x_i,\tau_l\right)e^{j\eta_{\Psi_2}\left(x_i,\tau_l\right)}\right\rangle$ be two TCNSs for all $x_i\left(1,2,3,...,n_X\right)$ belong to X and $\tau_l\left(1,2,3,...,n_\tau\right)$.

A temporal complex Cosine similarity measure between TCNSs Ψ_1 and Ψ_2 (denoted as $S_{\text{Cos}}(\Psi_1,\Psi_2)$ is proposed as follows:

$$S_{\text{Cos}}(\Psi_1,\Psi_2) = \frac{1}{n_X * n_\tau}\sum_{i=1}^{n_X}\sum_{l=1}^{n_\tau} \frac{\begin{array}{c}(p_{\Psi_1}(x_i,\tau_l)+\mu_{\Psi_1}(x_i,\tau_l))(p_{\Psi_2}(x_i,\tau_l)+\mu_{\Psi_2}(x_i,\tau_l))\\ +(q_{\Psi_1}(x_i,\tau_l)+\nu_{\Psi_1}(x_i,\tau_l))(q_{\Psi_2}(x_i,\tau_l)+q_{\Psi_2}(x_i,\tau_l))\\ +(r_{\Psi_1}(x_i,\tau_l)+r_{\Psi_1}(x_i,\tau_l))(r_{\Psi_2}(x_i,\tau_l)+r_{\Psi_2}(x_i,\tau_l))\end{array}}{\begin{array}{l}\sqrt{\begin{array}{l}(p_{\Psi_1}(x_i,\tau_l)+\mu_{\Psi_1}(x_i,\tau_l))^2+(q_{\Psi_1}(x_i,\tau_l)+\nu_{\Psi_1}(x_i,\tau_l))^2\\ +(r_{\Psi_1}(x_i,\tau_l)+r_{\Psi_1}(x_i,\tau_l))^2\end{array}}\\ \times\sqrt{\begin{array}{l}(p_{\Psi_2}(x_i,\tau_l)+\mu_{\Psi_2}(x_i,\tau_l))^2+(q_{\Psi_2}(x_i,\tau_l)+q_{\Psi_2}(x_i,\tau_l))^2\\ +(r_{\Psi_2}(x_i,\tau_l)+r_{\Psi_2}(x_i,\tau_l))^2\end{array}}\end{array}} \tag{8}$$

Example 3 Let $X=\{x_1,x_2\}$;$\tilde{\tau}=\{\tau_1,\tau_2,\tau_3\}$ and Ψ_1 and Ψ_2 are two TCNS in X

$$\Psi_1(x_1)=\left\{\begin{array}{l}\left\langle 0.1e^{j0.2},0.6e^{j0.45},0.3e^{j0.56}\right\rangle,\left\langle 0.8e^{j0.77},0.2e^{j0.86},0.4e^{j0.65}\right\rangle,\\ \left\langle 0.6e^{j0.52},0.2e^{j0.2},0.36e^{j0.2}\right\rangle\end{array}\right\}$$

$$\Psi_1(x_2)=\left\{\begin{array}{l}\left\langle 0.5e^{j0.12},0.2e^{j0.65},0.5e^{j0.85}\right\rangle,\left\langle 0.3e^{j0.8},0.7e^{j0.23},0.1e^{j0.32}\right\rangle,\\ \left\langle 0.74e^{j0.6},0.5e^{j0.15},0.66e^{j0.41}\right\rangle\end{array}\right\}$$

$$\Psi_2(x_1)=\left\{\begin{array}{l}\left\langle 0.6e^{j0.8},0.9e^{j0.25},0.4e^{j0.52}\right\rangle,\left\langle 0.6e^{j0.62},0.5e^{j0.24},0.3e^{j0.18}\right\rangle,\\ \left\langle 0.3e^{j0.82},0.3e^{j0.6},0.4e^{j0.71}\right\rangle\end{array}\right\}$$

$$\Psi_2(x_2)=\left\{\begin{array}{l}\left\langle 0.3e^{j0.22},0.4e^{j0.56},0.6e^{j0.26}\right\rangle,\left\langle 0.1e^{j0.38},0.5e^{j0.43},0.5e^{j0.24}\right\rangle,\\ \left\langle 0.2e^{j0.29},0.4e^{j0.72},0.5e^{j0.65}\right\rangle\end{array}\right\}$$

Now, by Definition 5, we have the following results:
$S_{\text{Cos}}(\Psi_1,\Psi_2)=0.732025$

Theorem 3 *Let Ψ_1 and Ψ_2 be two TCNSs then,*
(1) $0\le S_{\text{Cos}}(\Psi_1,\Psi_2)\le 1$
(2) $S_{\text{Cos}}(\Psi_1,\Psi_2)=S_{\text{Cos}}(\Psi_2,\Psi_1)$
(3) $S_{\text{Cos}}(\Psi_1,\Psi_2)=1$*, if and only if* $\Psi_1=\Psi_2$
(4) if Ψ_3 *is a TCNS in* X *and* $\Psi_1\subseteq\Psi_2\subseteq\Psi_3$ *then* $S_{\text{Cos}}(\Psi_1,\Psi_3)\le S_{\text{Cos}}(\Psi_1,\Psi_2)$ *and* $S_{\text{Cos}}(\Psi_1,\Psi_3)\le S_{\text{Cos}}(\Psi_2,\Psi_3)$

Proof

(1) We can have,

$$\left(\begin{array}{l}\left(p_{\Psi_1}\left(x_i,\tau_l\right)+\mu_{\Psi_1}\left(x_i,\tau_l\right)\right)\left(p_{\Psi_2}\left(x_i,\tau_l\right)+\mu_{\Psi_2}\left(x_i,\tau_l\right)\right)\\ +\left(q_{\Psi_1}\left(x_i,\tau_l\right)+\nu_{\Psi_1}\left(x_i,\tau_l\right)\right)\left(q_{\Psi_2}\left(x_i,\tau_l\right)+\nu_{\Psi_2}\left(x_i,\tau_l\right)\right)\\ +\left(r_{\Psi_1}\left(x_i,\tau_l\right)+\eta_{\Psi_1}\left(x_i,\tau_l\right)\right)\left(r_{\Psi_2}\left(x_i,\tau_l\right)+\eta_{\Psi_2}\left(x_i,\tau_l\right)\right)\end{array}\right)^2$$

$$\leq\left(\begin{array}{l}\left(p_{\Psi_1}\left(x_i,\tau_l\right)+\mu_{\Psi_1}\left(x_i,\tau_l\right)\right)^2+\left(q_{\Psi_1}\left(x_i,\tau_l\right)+\nu_{\Psi_1}\left(x_i,\tau_l\right)\right)^2\\ +\left(r_{\Psi_1}\left(x_i,\tau_l\right)+\eta_{\Psi_1}\left(x_i,\tau_l\right)\right)^2+\left(p_{\Psi_2}\left(x_i,\tau_l\right)+\mu_{\Psi_2}\left(x_i,\tau_l\right)\right)^2\\ +\left(q_{\Psi_2}\left(x_i,\tau_l\right)+\nu_{\Psi_2}\left(x_i,\tau_l\right)\right)^2+\left(r_{\Psi_2}\left(x_i,\tau_l\right)+\eta_{\Psi_2}\left(x_i,\tau_l\right)\right)^2\end{array}\right)$$

Hence, $0\leq S_{\text{Cos}}\left(\Psi_1,\Psi_2\right)\leq 1$

(2) It is obvious that the Theorem 3 is true

(3) When $\Psi_1=\Psi_2$, then obviously $S_{\text{Cos}}\left(\Psi_1,\Psi_2\right)=1$. On the other hand, if $S_{\text{Cos}}\left(\Psi_1,\Psi_2\right)=1$ then,

$p_{\Psi_1}\left(x_i,\tau_l\right) = p_{\Psi_2}\left(x_i,\tau_l\right)$;$\mu_{\Psi_1}\left(x_i,\tau_l\right) = \mu_{\Psi_2}\left(x_i,\tau_l\right)$; $q_{\Psi_1}\left(x_i,\tau_l\right) = q_{\Psi_2}\left(x_i,\tau_l\right)$.; $\nu_{\Psi_1}\left(x_i,\tau_l\right)=\nu_{\Psi_2}\left(x_i,\tau_l\right)$; $r_{\Psi_1}\left(x_i,\tau_l\right)=r_{\Psi_2}\left(x_i,\tau_l\right)$; $\eta_{\Psi_1}\left(x_i,\tau_l\right)=\eta_{\Psi_2}\left(x_i,\tau_l\right)$;

This implies that $\Psi_1=\Psi_2$

(4) When $\Psi_1\subseteq\Psi_2\subseteq\Psi_3$, we can have

$p_{\Psi_1}\left(x_i,\tau_l\right)+\mu_{\Psi_1}\left(x_i,\tau_l\right)\leq p_{\Psi_2}\left(x_i,\tau_l\right)+\mu_{\Psi_2}\left(x_i,\tau_l\right)\leq p_{\Psi_3}\left(x_i,\tau_l\right)+\mu_{\Psi_3}\left(x_i,\tau_l\right)$;

$q_{\Psi_1}\left(x_i,\tau_l\right)+\nu_{\Psi_1}\left(x_i,\tau_l\right)\geq q_{\Psi_2}\left(x_i,\tau_l\right)+\nu_{\Psi_2}\left(x_i,\tau_l\right)\geq q_{\Psi_3}\left(x_i,\tau_l\right)+\nu_{\Psi_3}\left(x_i,\tau_l\right)$;

$r_{\Psi_1}\left(x_i,\tau_l\right)+\eta_{\Psi_1}\left(x_i,\tau_l\right)\geq r_{\Psi_2}\left(x_i,\tau_l\right)+\eta_{\Psi_2}\left(x_i,\tau_l\right)\geq r_{\Psi_3}\left(x_i,\tau_l\right)+\eta_{\Psi_3}\left(x_i,\tau_l\right)$;

So, $S_J\left(\Psi_1,\Psi_3\right)\leq S_J\left(\Psi_1,\Psi_2\right)$ and $S_J\left(\Psi_1,\Psi_3\right)\leq S_J\left(\Psi_2,\Psi_3\right)$

Hence, all the conditions are proved.

3.4 Cotangent Similarity of TCNS

In this section, we define a Cotangent function to define the similarity measure on the TCNS environment.

Definition 6 Let two TCNSs
$\Psi_1 = \left\langle p_{\Psi_1}\left(x_i,\tau_l\right)e^{j\mu_{\Psi_1}\left(x_i,\tau_l\right)}, q_{\Psi_1}\left(x_i,\tau_l\right)e^{j\nu_{\Psi_1}\left(x_i,\tau_l\right)}, r_{\Psi_1}\left(x_i,\tau_l\right)e^{j\eta_{\Psi_1}\left(x_i,\tau_l\right)}\right\rangle$
and $\Psi_2 = \left\langle p_{\Psi_2}\left(x_i,\tau_l\right)e^{j\mu_{\Psi_2}\left(x_i,\tau_l\right)}, q_{\Psi_2}\left(x_i,\tau_l\right)e^{j\nu_{\Psi_2}\left(x_i,\tau_l\right)}, r_{\Psi_2}\left(x_i,\tau_l\right)e^{j\eta_{\Psi_2}\left(x_i,\tau_l\right)}\right\rangle$
for all $x_i\left(1,2,3,...,n_X\right)$ belong to X and $\tau_l\left(1,2,3,...,n_\tau\right)$.

A temporal complex Cotangent similarity measure between TCNSs Ψ_1 and Ψ_2 (denoted as $S_{\text{Cot\_1}}(\Psi_1,\Psi_2)$) is defined as follows:

$$S_{\text{Cot\_1}}(\Psi_1,\Psi_2)=$$
$$cot\left(\frac{\pi}{4}+\frac{\pi}{8}\times\frac{1}{n_X*n_\tau}\sum_{i=1}^{n_X}\sum_{l=1}^{n_\tau}\left[\begin{array}{l} max\left(\begin{array}{l} \mid p_{\Psi_1}(x_i,\tau_l)-p_{\Psi_2}(x_i,\tau_l)\mid, \\ \mid q_{\Psi_1}(x_i,\tau_l)-q_{\Psi_2}(x_i,\tau_l)\mid, \\ \mid r_{\Psi_1}(x_i,\tau_l)-r_{\Psi_2}(x_i,\tau_l)\mid \end{array}\right) \\ +\frac{1}{2\pi}max\left(\begin{array}{l} \mid \mu_{\Psi_1}(x_i,\tau_l)-\mu_{\Psi_2}(x_i,\tau_l)\mid, \\ \mid \nu_{\Psi_1}(x_i,\tau_l)-\nu_{\Psi_2}(x_i,\tau_l)\mid, \\ \mid \eta_{\Psi_1}(x_i,\tau_l)-\eta_{\Psi_2}(x_i,\tau_l)\mid \end{array}\right)\end{array}\right]\right) \quad (9)$$

$$S_{\text{Cot\_2}}(\Psi_1,\Psi_2)=$$
$$cot\left(\frac{\pi}{4}+\frac{\pi}{24}\times\frac{1}{n_X*n_\tau}\sum_{i=1}^{n_X}\sum_{l=1}^{n_\tau}\left[\begin{array}{l} \left(\begin{array}{l} \mid p_{\Psi_1}(x_i,\tau_l)-p_{\Psi_2}(x_i,\tau_l)\mid \\ +\mid q_{\Psi_1}(x_i,\tau_l)-q_{\Psi_2}(x_i,\tau_l)\mid \\ +\mid r_{\Psi_1}(x_i,\tau_l)-r_{\Psi_2}(x_i,\tau_l)\mid \end{array}\right) \\ +\frac{1}{2\pi}max\left(\begin{array}{l} \mid \mu_{\Psi_1}(x_i,\tau_l)-\mu_{\Psi_2}(x_i,\tau_l)\mid \\ +\mid \nu_{\Psi_1}(x_i,\tau_l)-\nu_{\Psi_2}(x_i,\tau_l)\mid \\ +\mid \eta_{\Psi_1}(x_i,\tau_l)-\eta_{\Psi_2}(x_i,\tau_l)\mid \end{array}\right)\end{array}\right]\right) \quad (10)$$

Example 4 Let $X=\{x_1,x_2\}$; $\tilde{\tau}=\{\tau_1,\tau_2,\tau_3\}$; Ψ_1 and Ψ_2 are two TCNS in X

$$\Psi_1(x_1)=\left\{\begin{array}{l}\left\langle 0.1e^{j0.2},0.6e^{j0.45},0.3e^{j0.56}\right\rangle,\left\langle 0.8e^{j0.77},0.2e^{j0.86},0.4e^{j0.65}\right\rangle, \\ \left\langle 0.6e^{j0.52},0.2e^{j0.2},0.36e^{j0.2}\right\rangle\end{array}\right\}$$

$$\Psi_1(x_2)=\left\{\begin{array}{l}\left\langle 0.5e^{j0.12},0.2e^{j0.65},0.5e^{j0.85}\right\rangle,\left\langle 0.3e^{j0.8},0.7e^{j0.23},0.1e^{j0.32}\right\rangle, \\ \left\langle 0.74e^{j0.6},0.5e^{j0.15},0.66e^{j0.41}\right\rangle\end{array}\right\}$$

$$\Psi_2(x_1)=\left\{\begin{array}{l}\left\langle 0.6e^{j0.8},0.9e^{j0.25},0.4e^{j0.52}\right\rangle,\left\langle 0.6e^{j0.62},0.5e^{j0.24},0.3e^{j0.18}\right\rangle, \\ \left\langle 0.3e^{j0.82},0.3e^{j0.6},0.4e^{j0.71}\right\rangle\end{array}\right\}$$

$$\Psi_2(x_2)=\left\{\begin{array}{l}\left\langle 0.3e^{j0.22},0.4e^{j0.56},0.6e^{j0.26}\right\rangle,\left\langle 0.1e^{j0.38},0.5e^{j0.43},0.5e^{j0.24}\right\rangle, \\ \left\langle 0.2e^{j0.29},0.4e^{j0.72},0.5e^{j0.65}\right\rangle\end{array}\right\}$$

The following results can be obtained by applying Definitions 6
$S_{Cot\_1}(\Psi_1,\Psi_2)=0.690485$; $S_{Cot\_2}(\Psi_1,\Psi_2)=0.822429$

Theorem 4 *Let* Ψ_1 *and* Ψ_2 *be two TCNSs then,*
(1) $0\le S_{Cot}(\Psi_1,\Psi_2)\le 1$
(2) $S_{Cot}(\Psi_1,\Psi_2)=S_{Cot}(\Psi_2,\Psi_1)$

(3) $S_{Cot}(\Psi_1, \Psi_2) = 1$*, if and only if* $\Psi_1 = \Psi_2$

(4) if Ψ_3 *is a TCNS in X and* $\Psi_1 \subseteq \Psi_2 \subseteq \Psi_3$ *then* $S_{Cot}(\Psi_1, \Psi_3) \leq S_{Cot}(\Psi_1, \Psi_2)$ *and* $S_{Cot}(\Psi_1, \Psi_3) \leq S_{Cot}(\Psi_2, \Psi_3)$

Proof

The proof is similar to the proof of Equation (6) and Equation (7).

(1) We have,

Hence, $0 \leq S_{Cot}(\Psi_1, \Psi_2) \leq 1$

(2) It is obvious that the Theorem 4 is true

(3) When $\Psi_1 = \Psi_2$, then obviously$S_{Cot_2}(\Psi_1, \Psi_2) = 1$. On the other hand, if $S_{Cot\_2}(\Psi_1, \Psi_2) = 1$ then,

$p_{\Psi_1}(x_i, \tau_l) = p_{\Psi_2}(x_i, \tau_l)$;$\mu_{\Psi_1}(x_i, \tau_l) = \mu_{\Psi_2}(x_i, \tau_l)$;$q_{\Psi_1}(x_i, \tau_l) = q_{\Psi_2}(x_i, \tau_l)$; $\nu_{\Psi_1}(x_i, \tau_l) = \nu_{\Psi_2}(x_i, \tau_l)$; $r_{\Psi_1}(x_i, \tau_l) = r_{\Psi_2}(x_i, \tau_l)$; $\eta_{\Psi_1}(x_i, \tau_l) = \eta_{\Psi_2}(x_i, \tau_l)$;

This implies that $\Psi_1 = \Psi_2$

(4) When $\Psi_1 \subseteq \Psi_2 \subseteq \Psi_3$, wen can have

$p_{\Psi_1}(x_i, \tau_l) + \mu_{\Psi_1}(x_i, \tau_l) \leq p_{\Psi_2}(x_i, \tau_l) + \mu_{\Psi_2}(x_i, \tau_l) \leq p_{\Psi_3}(x_i, \tau_l) + \mu_{\Psi_3}(x_i, \tau_l)$;

$q_{\Psi_1}(x_i, \tau_l) + \nu_{\Psi_1}(x_i, \tau_l) \geq q_{\Psi_2}(x_i, \tau_l) + \nu_{\Psi_2}(x_i, \tau_l) \geq q_{\Psi_3}(x_i, \tau_l) + \nu_{\Psi_3}(x_i, \tau_l)$;

$r_{\Psi_1}(x_i, \tau_l) + \eta_{\Psi_1}(x_i, \tau_l) \geq r_{\Psi_2}(x_i, \tau_l) + \eta_{\Psi_2}(x_i, \tau_l) \geq r_{\Psi_3}(x_i, \tau_l) + \eta_{\Psi_3}(x_i, \tau_l)$;

So, $S_{Cot\_2}(\Psi_1, \Psi_3) \leq S_{Cot\_2}(\Psi_1, \Psi_2)$ and $S_{Cot\_2}(\Psi_1, \Psi_3) \leq S_{Cot\_2}(\Psi_2, \Psi_3)$.

The proof is completed.

3.5 Entropy measures of TCNE

The entropy measure was first introduced by Zadeh [26] in 1965 with the aim of measuring fuzziness in information. Entropy measure can be considered as a measure of uncertainty and fuzziness involved in a set, whether fuzzy set, complex fuzzy set, neutrosophic set, etc. In this work, the TCNS are also capable of handling uncertain and vague data; therefore, we also care about discovering the entropy of a TCNS.

Definition 7 Let X be a finite set of objects. A function *E: TCNS(X)* $\rightarrow$ [0, 1] is called an entropy measure for a temporal complex neutrosophic set if it satisfies the following properties:

(1) $E(\Psi) = 0$ if Ψ is a crisp set

(2) $E(\Psi) = 1$ if $p_\Psi(x, \tau) = 0.5$;$q_\Psi(x, \tau) = 0.5$;$r_\Psi(x, \tau) = 0.5$ and $\mu_\Psi(x, \tau) = \pi$;$\nu_\Psi(x, \tau) = \pi$;$\eta_\Psi(x, \tau) = \pi$ for $\forall \tau \in \tilde{\tau}$; $\forall x \in X$.

(3) $E(\Psi_1) \leq E(\Psi_2)$ if Ψ_2 is more uncertain that Ψ_1, that is $\mid p_{\Psi_1}(x,\tau) - 0.5 \mid \geq \mid p_{\Psi_2}(x,\tau) - 0.5 \mid$; $\mid q_{\Psi_1}(x,\tau) - 0.5 \mid \geq \mid q_{\Psi_2}(x,\tau) - 0.5 \mid$; $\mid r_{\Psi_1}(x,\tau) - 0.5 \mid \geq \mid r_{\Psi_2}(x,\tau) - 0.5 \mid$ and

$\mid \mu_{\Psi_1}(x,\tau) - \pi \mid \geq \mid \mu_{\Psi_2}(x,\tau) - \pi \mid$; $\mid \nu_{\Psi_1}(x,\tau) - \pi \mid \geq \mid \nu_{\Psi_2}(x,\tau) - \pi \mid$; $\mid \eta_{\Psi_1}(x,\tau) - \pi \mid \geq \mid \eta_{\Psi_2}(x,\tau) - \pi \mid$

(4) $E(\Psi) = E(\Psi^c)$, Ψ^c is a temporal complex neutrosophic complement of Ψ

Theorem 5 *Suppose that Ψ is a TCNS on X. A Temporal complex neutrosophic entropy measure of Ψ is proposed as follows:*

$$E(\Psi) = \tan\left(\frac{\pi}{6 * n_X * n_\tau} \sum_{i=1}^{n_X} \sum_{l=1}^{n_\tau} e_{il}(\Psi)\right) \tag{11}$$

Where

$$e_{il}(\Psi) = \begin{pmatrix} p_\Psi(x_i,\tau_l)(1 - p_\Psi(x_i,\tau_l)) + q_\Psi(x_i,\tau_l)(1 - q_\Psi(x_i,\tau_l)) \\ + r_\Psi(x_i,\tau_l)(1 - r_\Psi(x_i,\tau_l)) + \left(\frac{\mu_\Psi(x_i,\tau_l)}{2\pi}\right)\left(1 - \frac{\mu_\Psi(x_i,\tau_l)}{2\pi}\right) \\ + \left(\frac{\nu_\Psi(x_i,\tau_l)}{2\pi}\right)\left(1 - \frac{\nu_\Psi(x_i,\tau_l)}{2\pi}\right) + \left(\frac{\eta_\Psi(x_i,\tau_l)}{2\pi}\right)\left(1 - \frac{\eta_\Psi(x_i,\tau_l)}{2\pi}\right) \end{pmatrix}$$

Proof.

Consider $E(\Psi)$ be can entropy measure for TCNSs. Then, this measure must satisfy all the conditions in Definition 7.

(1) Suppose $E(\Psi) = 0$, it follows that $p(x_i,\tau_l) = 0$; $q(x_i,\tau_l) = 0$; $r(x_i,\tau_l) = 0$; $\mu(x_i,\tau_l) = 0$; $v(x_i,\tau_l) = 0$; $r(x_i,\tau_l) = 0$or $p(x_i,\tau_l) = 1$; $q(x_i,\tau_l) = 1$; $r(x_i,\tau_l) = 1$; $\mu(x_i,\tau_l) = 2\pi$; $v(x_i,\tau_l) = 2\pi$; $r(x_i,\tau_l) = 2\pi$;

(2) $E(\Psi) = 1$, if and only if $p(x_i,\tau_l) = 0.5$; $q(x_i,\tau_l) = 0.5$; $r(x_i,\tau_l) = 0.5$; $\mu(x_i,\tau_l) = \pi$; $v(x_i,\tau_l) = \pi$; $r(x_i,\tau_l) = \pi$

(3) Assum Ψ_1, Ψ_2 and Ψ_2 more uncertain than that Ψ_2. It follows that, if $p_{\Psi_1}(x,\tau) \leq p_{\Psi_2}(x,\tau) \leq 0.5$; $q_{\Psi_1}(x,\tau) \leq q_{\Psi_2}(x,\tau) \leq 0.5$; $r_{\Psi_1}(x,\tau) \leq r_{\Psi_2}(x,\tau) \leq 0.5$ or $p_{\Psi_1}(x,\tau) \geq p_{\Psi_2}(x,\tau) \geq 0.5$; $q_{\Psi_1}(x,\tau) \geq q_{\Psi_2}(x,\tau) \geq 0.5$; $r_{\Psi_1}(x,\tau) \geq r_{\Psi_2}(x,\tau) \geq 0.5$ for each $x_i \in X$.

Hence,

$p_{\Psi 1}(x_i,\tau_l)(1 - p_{\Psi_1}(x_i,\tau_l)) \leq p_{\Psi 2}(x_i,\tau_l)(1 - p_{\Psi_2}(x_i,\tau_l))$

$q_{\Psi 1}(x_i,\tau_l)(1 - q_{\Psi_1}(x_i,\tau_l)) \leq q_{\Psi 2}(x_i,\tau_l)(1 - q_{\Psi_2}(x_i,\tau_l))$;

$r_{\Psi 1}(x_i,\tau_l)(1 - r_{\Psi_1}(x_i,\tau_l)) \leq r_{\Psi 2}(x_i,\tau_l)(1 - r_{\Psi_2}(x_i,\tau_l))$;

and $\frac{\mu_{\Psi_1}(x_i,\tau_l)}{2\pi}\left(1 - \frac{\mu_{\Psi_1}(x_i,\tau_l)}{2\pi}\right) \leq \frac{\mu_{\Psi_2}(x_i,\tau_l)}{2\pi}\left(1 - \frac{\mu_{\Psi_2}(x_i,\tau_l)}{2\pi}\right)$;

$\frac{\nu_{\Psi 1}(x_i,\tau_l)}{2\pi}\left(1 - \frac{\nu_{\Psi_1}(x_i,\tau_l)}{2\pi}\right) \leq \frac{\nu_{\Psi 2}(x_i,\tau_l)}{2\pi}\left(1 - \frac{\nu_{\Psi_2}(x_i,\tau_l)}{2\pi}\right)$;

$\frac{\eta_{\Psi 1}(x_i,\tau_l)}{2\pi}\left(1-\frac{\eta_{\Psi 1}(x_i,\tau_l)}{2\pi}\right)\leq\frac{\eta_{\Psi_2}(x_i,\tau_l)}{2\pi}\left(1-\frac{\eta_{\Psi 2}(x_i,\tau_l)}{2\pi}\right)$;
which implies $E\left(\Psi_1\right)\leq E\left(\Psi_2\right)$

(4) $E\left(\Psi\right)=E\left(\Psi^c\right)$ is trivial

Example 5 Let $X=\{x_1,x_2\}$;$\tilde{\tau}=\{\tau_1,\tau_2,\tau_3\}$ and Ψ_1 and Ψ_2 are two TCNS in X

$$\Psi_1\left(x_1\right)=\left\{\begin{array}{l}\left\langle 0.1e^{j0.2},0.6e^{j0.45},0.3e^{j0.56}\right\rangle,\left\langle 0.8e^{j0.77},0.2e^{j0.86},0.4e^{j0.65}\right\rangle,\\\left\langle 0.6e^{j0.52},0.2e^{j0.2},0.36e^{j0.2}\right\rangle\end{array}\right\}$$

$$\Psi_1\left(x_2\right)=\left\{\begin{array}{l}\left\langle 0.5e^{j0.12},0.2e^{j0.65},0.5e^{j0.85}\right\rangle,\left\langle 0.3e^{j0.8},0.7e^{j0.23},0.1e^{j0.32}\right\rangle,\\\left\langle 0.74e^{j0.6},0.5e^{j0.15},0.66e^{j0.41}\right\rangle\end{array}\right\}$$

$$\Psi_2\left(x_1\right)=\left\{\begin{array}{l}\left\langle 0.6e^{j0.8},0.9e^{j0.25},0.4e^{j0.52}\right\rangle,\left\langle 0.6e^{j0.62},0.5e^{j0.24},0.3e^{j0.18}\right\rangle,\\\left\langle 0.3e^{j0.82},0.3e^{j0.6},0.4e^{j0.71}\right\rangle\end{array}\right\}$$

$$\Psi_2\left(x_2\right)=\left\{\begin{array}{l}\left\langle 0.3e^{j0.22},0.4e^{j0.56},0.6e^{j0.26}\right\rangle,\left\langle 0.1e^{j0.38},0.5e^{j0.43},0.5e^{j0.24}\right\rangle,\\\left\langle 0.2e^{j0.29},0.4e^{j0.72},0.5e^{j0.65}\right\rangle\end{array}\right\}$$

The following results can be obtained by applying Definitions 7
$E_{\Psi_1}=0.444759$; $E_{\Psi_2}=0.475798$;

4 Multi-Criteria Decision Making

Assume $\tilde{A}=\{A_1,A_2,...,A_{n_A}\}$ and $\tilde{C}=\{C_1,C_2,...,C_{n_C}\}$ and $\tilde{D}=\{D_1,D_2,...,D_{n_D}\}$ are sets of alternatives, criteria and decision makers. For decision maker D_{i_d}; $i_d=1,2,...,n_D$, the evaluation characteristic of an alternatives A_{i_a}; $i_a=1,2,...,n_A$ on an attribute C_{i_c}; $i_c=1,2,...,n_C$ in time period τ_l; $l=1,2,...,n_\tau$ is represented by matrix $X^{i_d}\left(\tau_l\right)=\left(\theta^{i_d}_{i_ai_c}\left(\tau_l\right)\right)_{m\times n}$. where $\theta^{i_d}_{i_ai_c}\left(\tau_l\right)$ taken language label of complex neutrosophic set by time period τ_l.

Let
$x_{i_ai_ci_d}\left(\tau_l\right)=\left\langle p_{i_ai_ci_d}\left(\tau_l\right)e^{j\mu_{i_ai_ci_d}(\tau_l)},q_{i_ai_ci_d}\left(\tau_l\right)e^{j\nu_{i_ai_ci_d}(\tau_l)},r_{i_ai_ci_d}\left(\tau_l\right)e^{j\eta_{i_ai_ci_d}(\tau_l)}\right\rangle$

Step 1. Assuming that the weight information of the criterion is completely unknown. Hence, the weight w_{i_c} of criterion C_{i_c} is calculated as follows:

The averaged rating of decision makers can be evaluated by Equation (12)

$$x_{i_ai_c}=\frac{1}{n_D*n_\tau}\overset{n_\tau}{\underset{l=1}{\oplus}}\overset{n_D}{\underset{i_a=1}{\oplus}}x_{i_ai_ci_d}\left(\tau_l\right)=\left\langle\tilde{T}_{i_ai_c},\tilde{I}_{i_ai_c},\tilde{F}_{i_ai_c}\right\rangle \tag{12}$$

The weight w_{i_c} of criterion C_{i_c} is calculated by Equation (13)

$$w_{i_c} = \frac{\left(1 - E_{C_{i_c}}\right)}{n_c - \sum_{i=1}^{n_c} E_{C_{i_c}}} \tag{13}$$

Where $E_{C_{i_c}} = \frac{1}{n_A} \sum_{i_a=1}^{n_A} E\left(x_{i_a i_c}\right)$; each $E\left(x_{i_a i_c}\right)$ is computed using Equation

(11)

Step 2. The temporal complex neutrosophic positive ideal solution (TCN-PIS) and negative ideal solution (TCN-NIS) are identified as follows:

$$A^+ = \left\{x, \left\langle \max\left(p_{i_a i_c}\left(x, \tilde{\tau}\right)\right) e^{j \max\left(\mu_{i_a i_c}\left(x, \tilde{\tau}\right)\right)}, 0, 0 \right\rangle\right\} \tag{14}$$

$$A^- = \left\{x, \left\langle 0, \min\left(q_{i_a i_c}\left(x, \tilde{\tau}\right)\right) e^{j \min\left(\nu_{i_a i_c}\left(x, \tilde{\tau}\right)\right)}, \min\left(r_{i_a i_c}\left(x, \tilde{\tau}\right)\right) e^{j \min\left(\eta_{i_a i_c}\left(x, \tilde{\tau}\right)\right)} \right\rangle\right\} \tag{15}$$

Step 3. Theorem 1, 2, 3, or 4 is used to compute the weighted similarity measure for alternatives to TCN-PIS and TCN-NIS. The weighted similarity measure $S_{i_a}^+$ and $S_{i_a}^-$ of alternative A_{i_a} $(i_a = 1, 2, ..., n_A)$ is calculated by Equation (16)-(17)

$$S_{i_a}^+ = \bigoplus_{i_a=1}^{n_A} w_{i_c} S\left(x_{i_a i_c}, A^+\right) \tag{16}$$

$$S_{i_a}^- = \bigoplus_{i_a=1}^{n_A} w_{i_c} S\left(x_{i_a i_c}, A^-\right) \tag{17}$$

Step 4. The relative closeness of alternative A_{i_a} to the ideal solution is calculated as follows

$$RC_{i_a} = \frac{S_{i_a}^-}{S_{i_a}^+ + S_{i_a}^-} \tag{18}$$

Step 5. The alternatives are ranked according to their relative closeness and in descending order.

5 Numberical Example

In this section, by considering the real-life case, we present an application of proposed measures to a decision-making problem.

5.1 *Problem*

An example related to choosing a tourist destination in VN given by Lan et al. [25] demonstrates the feasibility and effectiveness of the proposed decision method. In this case, suppose a company wants to find a tourist destination in Vietnam. The company suggested a list of five tourist destinations to choose from, such as A_i; $i = 1, 2, 3, 4, 5$. For a more accurate evaluation of the different locations, there are three group criteria and 20 sub-criteria corresponding to each location to choose the best tourist destination.

The first group of criteria includes 07 sub-criteria related to the quality of services and tourism. These sub-criteria are given below:

C_1 : the service staff is well-trained, helpful, and friendly;
C_2 : This is a value-for-money destination;
C_3 : This is a safe destination for travelers;
C_4 : It has a variety of entertainment/ nightlife activities for travelers;
C_5 : It offers many opportunities for sports and adventurous activities;
C_6 : It offers a variety of souvenirs and duty-free goods for travelers;
C_7 : Easy to move around the place

The second group includes criteria related to the Quality of life in the tourist destination. It includes the following criteria:

C_8 : This place is a country with comfortable living conditions;
C_9 : Easy to get there from home;
C_{10} : This place has good social welfare;
C_{11} : This place has good foods;
C_{12} : Local people are friendly and kindly.

The last group includes criteria related to the natural environment of the tourist destination. It includes 8 sub-criteria as follows:

C_{13} : This place has a good climate;
C_{14} : This is a good place for relaxation;
C_{15} : This place has good tourism infrastructure facilities (e.g., restaurants, accommodations, etc.);
C_{16} : This place is a country with many well-known tourist sites;
C_{17} : This place has magnificent sunny beaches;
C_{18} : The environment in this place is very clean;
C_{19} : This place has fascinating native animals and vegetation;
C_{20} : This place is a country with a vast land area and a relatively small population

To comprehensively evaluate the different locations over four periods, three experts with different experiences and knowledge have been invited to perform the evaluation. The expert's estimates are expressed by temporal complex neutrosophic numbers.

5.2 *An MCDM example using the proposed measures*

TCNS is a tool for describing two-dimensional uncertain information of a periodic nature in our day-to-day lives. In this section, we present a practical example in the TCNS environment to demonstrate that the proposed similarity measures play a significant role in solving real-life problems, such as tourist destinations chosen in multi-criteria decision-making problems.

Table 1 Language variables

Label	Value	Descriptiom
VL	$\langle 0.15e^{j0.55}, 0.65e^{j0.45}, 0.65e^{j0.35}\rangle$	Very low
L	$\langle 0.25e^{j0.65}, 0.55e^{j0.55}, 0.65e^{j0.45}\rangle$	Low
M	$\langle 0.40e^{j0.75}, 0.50e^{j0.65}, 0.45e^{j0.55}\rangle$	Medium
H	$\langle 0.55e^{j0.85}, 0.45e^{j0.75}, 0.35e^{j0.65}\rangle$	High
VH	$\langle 0.65e^{j0.95}, 0.25e^{j0.85}, 0.25e^{j0.75}\rangle$	Very high

Step 1. Determine the weight of the twenty criteria using Equations 12 and 13.

$w_1 = 0.049199;\ w_2 = 0.057871;\ w_3 = 0.052736;\ w_4 = 0.049812;\ w_5 = 0.038422;$
$w_6 = 0.042157;\ w_7 = 0.048045;\ w_8 = 0.055468;\ w_9 = 0.053227;\ w_{10} = 0.04191;$
$w_{11} = 0.049674;\ w_{12} = 0.04392;\ w_{13} = 0.044175;\ w_{14} = 0.049675;\ w_{15} = 0.058608;$
$w_{16} = 0.049995;\ w_{17} = 0.053227;\ w_{18} = 0.059684;\ w_{19} = 0.053973;\ w_{20} = 0.057452;$

Step 2. Estimate the temporal complex neutrosophic positive ideal solution (TCN-PIS) and negative ideal solution (TCN-NIS). The values are obtained as follows:

$$A^+ = \left\{ \begin{array}{l} \left\langle 0.696238.\ \mathrm{e}^{\mathrm{j}0.916667}, 0, 0\right\rangle, \left\langle 0.696238.\mathrm{e}^{\mathrm{j}0.916667}, 0, 0\right\rangle, \\ \left\langle 0.696238.\mathrm{e}^{\mathrm{j}0.916667}, 0, 0\right\rangle, \left\langle 0.696238.\mathrm{e}^{\mathrm{j}0.916667}, 0, 0\right\rangle \end{array} \right\}$$

$$A^- = \left\{ \begin{array}{l} \left\langle 0, 0.30411.\mathrm{e}^{\mathrm{j}0.583333}, 0.279672.\mathrm{e}^{0.483333}\right\rangle, \\ \left\langle 0, 0.30411.\mathrm{e}^{\mathrm{j}0.583333}, 0.279672.\mathrm{e}^{0.483333}\right\rangle, \\ \left\langle 0, 0.30411.\mathrm{e}^{\mathrm{j}0.583333}, 0.279672.\mathrm{e}^{0.483333}\right\rangle, \\ \left\langle 0, 0.30411.\mathrm{e}^{\mathrm{j}0.583333}, 0.279672.\mathrm{e}^{0.483333}\right\rangle \end{array} \right\}$$

Step 3. Calculate the similarity measure of five locations for TCN-PIS and TCN-NIS. They are shown in Table 2

Step 4. Determine the relative closeness of five locations to the ideal solution. They are shown in 3

Step 5. The five locations are ranked based on their relative closeness are shown in Table 3

Table 2 Similarity measures of five locations to TCN-PIS and TCN-NIS

Mesuares	Ideal solution	Location				
		A_1	A_2	A_3	A_4	A_5
Dice	S^+	0.63893	0.639313	0.637272	0.63702	0.634593
	S^-	0.656714	0.65388	0.656766	0.660245	0.660896
Jaccard	S^+	0.469471	0.469888	0.46768	0.467401	0.464794
	S^-	0.488921	0.485795	0.488989	0.492825	0.493544
Cosine	S^+	0.586304	0.588898	0.58585	0.582696	0.58158
	S^-	0.559327	0.559667	0.560673	0.560043	0.562002
Cot_1	S^+	0.641325	0.62623	0.63643	0.662388	0.652045
	S^-	0.595166	0.577427	0.594766	0.621144	0.616897
Cot_2	S^+	0.718926	0.714373	0.717399	0.724657	0.720939
	S^-	0.7867	0.767714	0.784763	0.813698	0.806133

Table 3 The Relative closeness of locations based on similarity measures

Mesuares		Location				
		A_1	A_2	A_3	A_4	A_5
Dice	RC	0.506863	0.505632	0.507532	0.508951	**0.510152**
	Ranking	4	5	3	2	1
Jaccard	RC	0.510147	0.508323	0.511137	0.513239	**0.515**
	Ranking	4	5	3	2	1
Cosine	RC	0.488226	0.487275	0.48902	0.490088	**0.49144**
	Ranking	4	5	3	2	1
Cot_1	RC	0.481335	0.479727	0.48308	0.483933	**0.486151**
	Ranking	4	5	3	2	1
Cot_2	RC	0.522507	0.517995	0.522422	**0.52894**	0.527894
	Ranking	3	5	4	1	2

5.3 A Comparison Analysis

In this section, to demonstrate the effectiveness of the proposed model, we compare the proposed model with those in Ali et al. [24] on temporal complex neutrosophic environments.

The ranking results of five locations at four time periods, $\tau_1, \tau_4, \tau_3, \tau_4$ are appropriately described in Table 4 for choosing a tourist destination problem. Table 4 shows that the rankings offive locations at four times τ_1, τ_4, τ_3 and τ_4 respectively are $A_5 \succ A_4 \succ A_1 \succ A_3 \succ A_2$; $A_5 \succ A_4 \succ A_3 \succ A_1 \succ A_2$; $A_5 \succ A_4 \succ A_3 \succ A_1 \succ A_2$ and $A_5 \succ A_4 \succ A_1 \succ A_3 \succ A_2$ and all 4 time periods are given A_5 as the best location. Meanwhile, the result of the proposed method gives the overall result at four time periods based on Dice, Jaccard, Cosine, and Cotangent similarity measures, respectively, are $A_5 \succ$

Table 4 Generalised-Dice-Similarity-measures of Ali et al for locations at four periods

Mesuares		Location				
		A_1	A_2	A_3	A_4	A_5
τ_1	RC	0.814267	0.808659	0.812204	0.816266	**0.819484**
	Ranking	3	5	4	2	1
τ_2	RC	0.8133	0.810594	0.813347	0.81762	**0.822208**
	Ranking	4	5	3	2	1
τ_3	RC	0.814128	0.808873	0.813478	0.817074	**0.821965**
	Ranking	4	5	3	2	1
τ_4	RC	0.813158	0.808602	0.813087	0.816648	**0.81934**
	Ranking	3	5	4	2	1

$A_4 \succ A_3 \succ A_1 \succ A_2$; $A_5 \succ A_4 \succ A_3 \succ A_1 \succ A_2$; $A_5 \succ A_4 \succ A_3 \succ A_1 \succ A_2$; $A_4 \succ A_5 \succ A_1 \succ A_3 \succ A_2$. The overall result of Dice, Jaccard, and Cosine similarity measures are given A_5 as the best location, and the Cotangent similarity measure is given A_4 as the best location. This result shows that the advantage and the practical applicability of the proposed measures can process the decision-making problems with time-related factors in a temporal complex neutrosophic environment. Moreover, it is more generalized and flexible than Ali et al. [24]'s method in temporal complex neutrosophic environments

6 CONCLUSIONS

In this paper, we have proposed four similarity measures, namely Dice, Jaccard, cosine, and Cotangent, in the TCNS environment. We have also proved some of their basic properties. In addition, we have defined an entropy measure of TCNS for determining unknown attribute weights in MCDM. Next, a new MCDM strategy has also been developed based on proposed similarity and entropy measures in the TCNS environment. Finally, a numerical example of decision-making problems regarding choosing a tourist destination in Vietnam is described under the temporal complex neutrosophic environment and is given to demonstrate the advantage and the practical applicability of the proposed measures. In the numerical example, it is proven that the proposed similarity and entropy measures of TCNS are able to produce reasonable results for decision-making problems in the real world. In future research, it will be interesting to develop new aggregation operators of TCNS and their applications in MADM problems such as logistics center selection, medical diagnosis, personnel selection, etc.

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Compliance with ethical standards

Conflict of interest: The authors declare no conflict of interest.
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