Inventory System with Generalized Triangular Neutrosophic Cost Pattern incorporating Maximum Lifetime based Deterioration and Novel Demand through PSO

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Abstract

At the manufacturing plant or while the products are being transferred from one supply layer to another, there is a considerable possibility of receiving damaged or faulty items mixed in with non-defective commodities. This research focuses on the non-defective and defective products that are shipped to retailers by their suppliers. The retailer reworks faulty items to make them non-defective, and the retailer receives a discount on the cost of purchasing defective items. The presented inventory system addresses the uncertainty in inventory costs and also considers the deterioration of items with prioritized maximum product life. In this study, our aim is to minimize the total inventory cost when demand rate as a function of quality and power pattern of time under crisp and generalized triangular neutrosophic environments. Based on the payment deal, interest charges are imposed only when the payment delay has passed a particular allowable time limit. The neutrosophic number, which provides three different types of membership functions representing truth, hesitation, and falseness, is used in the inventory model to handle the cost pattern's uncertainty. A particle swarm optimization approach is used to analyze the proposed inventory model, and the results are validated using a numerical example and sensitivity analysis for various parameters.

Keywords: Deterioration; Triangular Neutrosophic set; quality; Delay in Payment; Holding cost; Maximum lifetime.

1 Introduction

Regarding the wide range of different inventory models, researchers need to emphasise a few essential elements, including degradation, demand, defective goods, the quality of the products, and so on. It is abundantly clear that degradation is a time-dependent component; furthermore, it worsens with time, which results in a reduction in the amount of interest in the product. There are a variety of challenges that arise when products are kept in warehouses, including variations in quality, volume, and degradation of commodities. These challenges are specific to each type of commodity. As a consequence, the holding costs have a considerable influence on the value of the quantity that is being stored. The quality of items that have been held for an extended period decreases due to deterioration. The rate of degradation is relatively low in long-lasting products such as those made of iron or steel, as well as toys, consumer electronics, furniture, tools, jewellery, vehicles, and other types of durable goods. Concurrently, the pace of degradation faces quick changes in semi-durable things such as food items, pharmaceuticals, apparel, and cosmetics, amongst other examples. As a result, the study of the deterioration of goods in inventory systems plays a vital role since the economic order quantity (EOQ) model accounts for various degradation patterns. When a product is being manufactured or when it is being shipped to retailers, faulty items are discovered most frequently. In the case of some goods, such as clothing, footwear, and furniture, defective products significantly impact the total profit generated by the inventory system. For this reason, when modelling the inventory system, the researchers need to concentrate on defected products. In recent years, a significant number of models have been created, each of which takes into account the deterioration rate and defective rate. Several researchers ([1], [2], [3], [4], [5], [6], [7]) has considered deterioration and defective items ([8], [9], [10]) for the study of different inventory models. **Demand plays an important role in inventory management, the study of inventory systems can not be anticipated without addressing demand rates. Due to various facts of fluctuating demand in different situations, the inventory systems have been developed by considering demand depending on: price and stock dependent** ([11],[12], [13], [14]), quadratic form ([15]), simply constant demand ([16]), etc.

The mathematical structures for dealing with various practical implications of uncertainties are interval numbers, fuzzy numbers, intuitionistic fuzzy numbers, and neutrosophic numbers. Since three independent membership functions represent the feature of neutrosophic numbers

[17] to deal with uncertainties, such as truth, indeterminacy, and falsity membership function, it has practical advantages: a generalization of interval, fuzzy, and intuitionistic fuzzy numbers. Researchers can make use of neutrosophic numbers in order to account for the fact that majority of the metrics in a real market are not accurate. In recent studies, a large number of researchers have been building their models in a variety of different uncertain contexts. De et al. [18] developed an EPQ model for the non-random uncertain environment using the neutrosophic fuzzy approach. Mariagrazia et al. [19] introduced fuzzy technique for supply chain network under quantity discounts. Haripriya et al. [20] developed an inventory model under the cloudy-fuzzy uncertain environment. Bonilla et al. [21] proposed a supply chain model by taking uncertain demand.

This paper considers an inventory system which imposes an interest after a specific maximum length of grace time to pay the payment. The model of the inventory that has been developed takes into consideration the rate of deterioration, which depends on the maximum lifetime of the product. The demand rate of this inventory model is depends on quality and power pattern time. These scenarios of the inventory system have been developed by considering the impreciseness of cost parameters, and the optimal total inventory cost is evaluated via particle swarm optimization (PSO) algorithm. The following is how this article is structured. Section 2 offers a review of the literature that highlights the reason for the study as well as the research gap. The inventory problem is mathematically formulated in Section 3. Section 4 develops the model formulation in a generalized triangular neutrosophic environment and proposed new ranking method. Section 5 contains the proposed inventory model using the PSO algorithm. In Section 6, a numerical example is explained, and then a sensitivity analysis of the optimal inventory policy for the system input parameters is presented, along with some significant managerial insights obtained from the results. Section 7 concludes with some findings and research directions for the future.

2 Literature review

Researchers consider various types of deterioration rates to develop their models based on the nature of the products. Most of the item's deterioration rate increases with time, such items are fruits, vegetables, flowers, food items, etc., and a few items have constant deterioration rates, such items are electronic goods, toys, plastic items, etc. Skouri et al. [22] proposed an inventory model of time-dependent deterioration rate. Pandu and Tadikamalla [23] introduced an EOQ

model using gamma distribution for representing the constant, increasing, and decreasing rates of deterioration with time. Pal et al. [24] proposed an EPQ model for deteriorating items with two-parameter Weibull distribution. Wang and Lin [25] developed the optimal replenishment strategy with deterioration, market demand, and price changes. An inventory model for deteriorating items with maximum lifetime proposed by Wang et al. [26]. Pervin et al. [27] developed an inventory model by considering stochastic deterioration. Sarkar et al. [28] proposed an inventory model with a trade-credit policy with variable deterioration for fixed lifetime products. In actual life, if a product has a maximum life duration, the degradation rate of the product decreases. This model takes into account the rate of degradation over the product's maximum life-time.

In the actual market, the demand for the majority of items is affected by their quality, such as footwear, apparel, household items, fruits and vegetables, etc. For each item, the demand rate is different. The demand rate for some items rises at the start of the cycle. The demand rate for some items remains constant throughout the cycle, whereas the demand rate for others increases at the end. The demand rate for cooked items such as bread, sweets, cakes, etc., increases at the beginning of the cycle because customers love just-made goods. Due to the expiration date, the demand rate decreases at the end of the cycle for goods such as fish, vegetables, fruits, etc. On the contrary, others have an increasing demand rate at the end of the cycle, such as household goods such as oil, sugar, milk, etc., and the demand rate is constant for furniture, electrical goods, etc. Khedlekar and Sukhla [29] developed a dynamic pricing model for logarithmic demand. Smaila et al. [30] introduced an EOQ model with quadratic demand trends and quasi partial backlogging. Dutta et al. [31] presented an inventory model by taking two components demand. Prasad and Mukherjee [32] developed an inventory model on stock and time-dependent demand. Wu et al. [33] developed an inventory policy for trapezoidal-type demand patterns and maximum lifetime under trade credits. Mahapatra et al. [34] presented an EPQ model by taking limited available intuitionistic fuzzy type storage space. Shaikh and Mishra [35] developed an EOQ model for price-sensitive quadratic demand and inflationary conditions. **Pervin et al. [10] developed an inventory system considering demand rate as a quadratic decreasing function of time. Al-Aminkhan et al. [36] presented an inventory model by considering advertisement and selling price dependent demand. By considering above mentioned all scenarios the demand rate of this model depends on quality and power pattern of time dependent demand.**

A typical phenomenon always arises in the inventory management system is a lag in the payment

process. A delay in payment is offered by the supplier, where the retailer's purchase cost is paid at a later date without an interest charge. Liao et al. [37] proposed an inventory model with deteriorating items under inflation and permissible delay in payment. Several researchers ([38], [39], [40], [41], [42], [43], [44], [45]) developed their inventory models under the condition of delay in payment. Teng et al. [46] presented an inventory model under progressive payment strategy and also studied an EOQ model under a trade credit financing scheme [47]. **Pervin et al. [48] presented an inventory model under trade-credit policy. In this paper, we consider a delay in payment for two different situations based on time.**

Neutrosophic numbers explain the impreciseness of the systems. In real life, most of the parameters are uncertain, so in this situation, neutrosophic numbers play a crucial role in overcoming uncertainty. Many models are developed under neutrosophic environment. Mullai and Surya [49] developed a price break EOQ model with neutrosophic demand, and purchasing cost as triangular neutrosophic numbers. In the present paper, we consider the cost pattern of the inventory system as a generalized triangular neutrosophic numbers.

In general, classic direct optimization methods are used to solve inventory problems; however, one of the disadvantages of these approaches is that these techniques frequently become stuck on the local optimal solution. PSO is a precious tool for finding inventory control solutions since it helps avoid some of the flaws of global optimization. Biuki et al. [50] presented an inventory problem in optimizing through two hybrid metaheuristics as parallel and series combinations of genetic algorithm and PSO. Alejo-Rees et al. [51] introduced inventory model for supplier selection and order quantity allocation by using metaheuristic algorithms. Rau et al. [52] proposed a multi-objective green cyclic inventory routing problem via discrete multi-swarm PSO method. Khazraji et al. [53] applied multi-objective PSO to optimise the production inventory control systems. Patne et al. [54] presented a closed-loop supply chain network configuration model by using game-theoretic PSO. Dabiri et al. [55] presented a bi-objective inventory routing problem with a step cost function by using multi-objective PSO. Kundu et al. [56] presented EPQ model with fuzzy demand using compared hybrid particle swarm-genetic algorithm. Manatkar et al. [57] presented an integrated inventory distribution optimization model for multiple products by using a novel hybrid multi-objective self-learning PSO. Srinivasan et al. [58] applied PSO for optimizing a mathematical model with defective goods. A multi-item EPQ model with a production capacity

restriction PSO algorithm developed by Pirayesh and Poormoaied [59]. Masud et al. [60] presented an inventory model with the quality of the production process through PSO.

Based on the literature review on the topics considered for this study as mentioned above scenarios, this inventory management work has been presented exclusively based on a comparison of contribution with the existing articles explored in Table-1.

Table 1: Exclusive contributions centred comparison of this article to previous research

Articles	Demand rate	Deterioration	Inspection cost	Defective items	Rework cost	Imprecise parameters	Nature of impreciseness	Delay payment
Wee et al. [8] (2007)	Constant	×	✓	✓	×	×	×	×
Skouri et al. [22] (2011)	Ramp type	Time	×	×	×	×	×	×
Teng et al. [46] (2011)	Stock	Constant	×	×	×	×	×	×
Pal et al. [24] (2014)	Ramp type	Weibull	×	×	×	Cost parameters	Triangular fuzzy	×
Chen and Teng [45] (2014)	Constant	Product's maximum life time	×	×	×	×	×	✓
Sarkar et al. [38] (2014)	Mixture of normal distribution	×	✓	✓	✓	×	×	✓
Pal et al. [61] (2015)	Ramp type	Weibull	×	×	×	Cost parameters & inflation rate	Triangular fuzzy	No
Prasad and Mukherjee [32] (2016)	Stock & time	Weibull	×	×	×	×	×	×
Wu et al.[62] (2016)	Constant	Maximum life time of product	×	×	×	×	×	×
Sonia at al. [63] (2016)	Fuzzy	Fuzzy	×	×	×	Demand rate and deterioration rate	Fuzzy	✓
Sanjose et al. [64] (2017)	Power pattern of time	×	×	×	×	×	×	×
Naoufel et al. [65] (2018)	Constant	×	✓	✓	×	×	×	×
Chakraborty et al.[66] (2018)	Ramp type	Weibull	×	×	×	×	×	✓
Bhaula et al. [40] (2019)	Price & time	×	×	×	×	×	×	✓
Bardhan et al. [67] (2019)	Stock	Constant	×	×	×	×	×	×
Mahapatra et al. [68] (2019)	Price, stock, reliability & advertisement	Weibull	×	×	×	Cost parameters	Triangular fuzzy	✓
Roy et al. [69] (2020)	Variable & time	Time	×	✓	×	×	×	×
Lin et al. [70] (2021)	Constant	×	✓	✓	✓	×	×	×
Karakatsoulis & Skouri [71] (2021)	Constant	×	✓	✓	×	×	×	×
Mondal et al. [72] (2021)	Time varying logistic	Weibull	×	×	×	Cost parameters	Triangular neutrosophic	✓
Sepehri and Gholamian[73] (2022)	Price	Constant	✓	✓	×	×	×	×
This paper	Quality & power pattern of time	Maximum life time of product	✓	✓	✓	Cost parameters	Generalized triangular neutrosophic	✓

3 Mathematical Development of the Proposed Inventory System

In the context of a generalized triangular neutrosophic cost pattern, the contribution of this work is to determine the optimal cycle time in order to cut down on the overall cost of the inventory management system. To the best of our knowledge, this study explores all of the concepts listed, which have never been studied together in the literature earlier:

1. **The demand rate of the item follows a power demand pattern on time and product quality.**
2. **The retailer's product quality depends on the supplier's product quality, i.e.,** $q(r) = (1 - e^{-ar})$**, where** $a > 0$ **and** $0 < r < 1$**. The function** $q(r)$ **is continuously differentiable with the conditions** $q^{'}(r) > 0$, $q^{''}(r) < 0$.
3. **Deterioration of this inventory system reduces with product's maximum life time.**
4. **After receiving products from the supplier, the retailer distinguishes faulty and non-defective items through screening, and the defective items are sent to rework. In addition, the retailer gets a discount on the purchase price of faulty products.**
5. **Because of allowing for a delay in payment, this deal is beneficial to both the retailer and the supplier during the payment process.**
6. **The impreciseness of cost parameters are taken as generalized triangular neutrosophic numbers, and we proposed a new ranking method to change neutrosophic numbers to crisp numbers.**
7. **The goal is to minimize the total inventory cost under neutrosophic environment.**

The simultaneous review of the assumptions considered in this study enables us to portray a more realistic inventory model that can be used in various circumstances that occur in real life.

Problem Definition:

In this study, we presented the mathematical modelling of a real-world phenomenon called the growing rate of deterioration with time. The study assumes that an item's maximum lifetime is already known to its providers. In the proposed inventory management system, shortages are not permitted, and the maximum life period is taken into consideration for the item in each

replenishment cycle. In the process of modelling the inventory system, the demand is the most crucial aspect. Although the inventory system is dependent on a number of factors, quality is one of the essential attributes. This study considers the demand rate depends on the quality of the product and the power pattern of time, i.e., $D(q(r),t) = \frac{xg(1-e^{-ra})t^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}$.

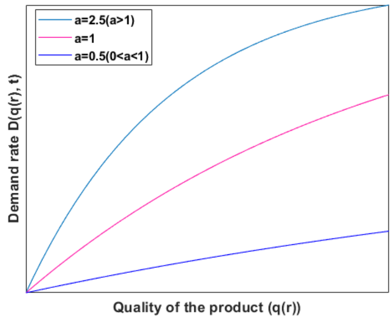
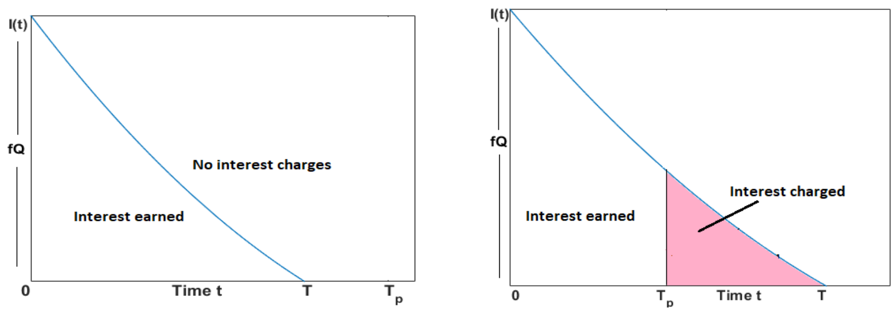


Figure 1: Graph of Demand rate vs quality

The deterioration rate of the inventory system depends upon the maximum lifetime of the product, i.e., $\theta(t) = \frac{1}{m-t}$ for $0 \leq t \leq T < m$. When the credit period is less than or equal to T_p, the inventory system considers payment delays that are permissible. During the credit period, the buyer is not required to pay any interest to the retailer; however, interest will be charged for any period beyond the period T_p as shown in figure 2. In order to construct a realistic inventory system, generalized triangular neutrosophic numbers are considered for the inventory cost parameters rather than set crisp values.



(a) Case I: $T \leq T_P$ (b) Case II: $T > T_P$

Figure 2: Graphical representation of the inventory model under delay in payment scenarios

Mathematical Formulation:

Based on these considerations, the inventory system can be represented by the differential equation in the time interval [0,T] as given below:

$$\frac{dI(t)}{dt} + \frac{1}{m-t}I(t) = -\frac{dg(1-e^{-ra})t^{\frac{1}{n}-1}}{nT^{\frac{1}{n}-1}}, \quad for \quad 0 \leq t \leq T, \tag{1}$$

subject to boundary conditions: $I(T) = 0$ *and* $I(0) = fQ$, where Q is the initial inventory per each cycle.

Lemma 1. *The product's quality $q(r)$ and demand rate $D(q(r),t)$ is increasing with supplier product's quality r for $a > 0$.*

Proof. Since, the product's quality $q(r)$ of the retailer is given by $q(r) = (1 - e^{-ra})$ for $a > 0$.

Then

$$\frac{dq(r)}{dr} = ae^{-ra} > 0, \quad \forall a > 0.$$

Thus, the product's quality $q(r)$ increases with the supplier product's quality r.

Also the demand rate of the given model is $D(q(r),t) = \frac{dg(1-e^{-ra})}{n}\left(\frac{t}{T}\right)^{\frac{1}{n}-1}$ and then

$$\frac{dD(q(r),t)}{dr} = \frac{dgae^{-ra}}{n}\left(\frac{t}{T}\right)^{\frac{1}{n}-1} > 0, \quad \forall a > 0.$$

Therefore, the demand rate $D(q(r),t)$ is increasing with supplier product's quality r for $a > 0$.

We can see this in figure 1.

Hence the proof. □

Lemma 2. *The demand rate $D(q(r),t)$ is decreasing with time for $n > 1$, increasing with time for $0 < n < 1$ and constant with time for $n = 1$.*

Proof. Since, demand rate of the given model is $D(q(r),t) = \frac{dg(1-e^{-ra})}{n}\left(\frac{t}{T}\right)^{\frac{1}{n}-1}$. Now for the time t, when $0 < n < 1$, we have

$$\frac{dD(q(r),t)}{dt} = \frac{dg(1-e^{-ra})(1-n)}{n^2T^{\frac{1}{n}-1}}(t)^{\frac{1}{n}-2} > 0.$$

It shows that the demand rate $D(q(r),t)$ increasing with time for all t.

When $n = 1$, we have

$$\frac{dD(q(r),t)}{dt} = \frac{dg(1-e^{-ra})(1-n)}{n^2T^{\frac{1}{n}-1}}(t)^{\frac{1}{n}-2} = 0.$$

It shows that the demand rate $D(q(r),t)$ constant with time for all t.

When $n>1$, we have

$$\frac{dD(q(r),t)}{dt} = \frac{dg(1-e^{-ra})(1-n)}{n^2 T^{\frac{1}{n}-1}}(t)^{\frac{1}{n}-2} < 0.$$

It shows that the demand rate $D(q(r),t)$ decreases with time for all t.

Hence the proof. □

By using the boundary conditions $I(T)=0$ *and* $I(0)=fQ$, then the solution of the differential equation (1), we obtain the inventory level of the proposed inventory model during the time interval $0 \le t \le T$ as given follows:

$$I(t) = \frac{dg(1-e^{-ra})(m-t)}{mT^{\frac{1}{n}-1}}\left[\left[T^{\frac{1}{n}} - t^{\frac{1}{n}}\right] + \frac{1}{m(n+1)}\left[T^{\frac{1}{n}+1} - t^{\frac{1}{n}+1}\right]\right]. \tag{2}$$

Again using boundary condition $I(0)=fQ$, we get

$$Q = \frac{dg(1-e^{-ra})T}{f}\left[1+\frac{T}{m(n+1)}\right]. \tag{3}$$

Now using the equations (2) and (3), the holding cost (HC), deterioration cost (DC), the purchasing cost (PC) and the ordering cost (OC) of the proposed inventory model can be obtained as given in the succeeding equations. The total holding cost of non-defective and defective items given by

$$\begin{aligned} HC = C_1\left[\int_0^T I(t)dt + (1-f)Q\right] &= \frac{nC_1T^2dg(1-e^{-ra})}{m}\left[\frac{m}{n+1} + \frac{T}{2(2n+1)} - \frac{T^2}{2m(3n+1)}\right] \\ &\quad +(1-f)\frac{C_1dg(1-e^{-ra})T}{f}\left[1+\frac{T}{m(n+1)}\right]. \end{aligned} \tag{4}$$

The deterioration cost (DC) of decaying goods in inventory system is given by

$$DC = C_2\left[fQ - \int_0^T D(q(r),t)dt\right] = \frac{C_2dg(1-e^{-ra})T^2}{m(n+1)}. \tag{5}$$

The purchase cost of the inventory system consider per item and require to purchase the item for inventory

$$PC = C_3Q = \frac{C_3dg(1-e^{-ra})T}{f}\left[1+\frac{T}{m(n+1)}\right]. \tag{6}$$

The cost of ordering is fixed for each cycle of unit inventory model

$$OC = C_0. \tag{7}$$

Damaged or defective commodities may qualify for a reduction from the supplier in the amount of γ off the total cost of the purchase. Therefore, the reduction in the cost of purchasing defective products is:

$$DPC = \gamma C_3(1-f)Q = \frac{C_3(1-f)\gamma dg(1-e^{-ra})T}{f}\left[1+\frac{T}{m(n+1)}\right]. \tag{8}$$

The inspection cost of the inventory system is given by

$$SC = C_4Q = \frac{C_4dg(1-e^{-ra})T}{f}\left[1+\frac{T}{m(n+1)}\right]. \tag{9}$$

Rework cost of the defective items of the inventory system is given by

$$RC = C_5(1-f)Q = \frac{C_5(1-f)dg(1-e^{-ra})T}{f}\left[1+\frac{T}{m(n+1)}\right]. \tag{10}$$

This inventory system allows a delay in payment conditionally for the buyer under certain circumstances. Because of this, the two possible outcomes are that either the delay period is longer than the cycle time or the delay period is shorter than the cycle time. Now, we will analyze the interests due to delayed payments for both situations in order to determine the total inventory costs for the two cases that follow:

Case I: ($T \leq T_p$) Delay period is greater than the cycle time:

The retailer earns interest at a return rate I_e per cycle if $T \leq T_p$, then the annual interest earned is given by

$$\begin{aligned} IE_1 &= SI_e\left[\int_0^T D(q(r),t)tdt + (T_p - T)\int_0^T D(q(r),t)dt\right] \\ &= \frac{STI_edg(1-e^{-ra})}{(n+1)}\big[n(T_p-T)+T_p\big]. \end{aligned} \tag{11}$$

The total inventory cost per unit time is obtained as follows:

$$\begin{aligned} IC_1 &= \frac{1}{T}\big(HC+DC+PC+OC+SC+RC-DPC-IE_1\big) \qquad (12)\\ &= \frac{dg(1-e^{-ra})}{f}\left(1+\frac{T}{m(n+1)}\right)\big(C_3+C_4+(C_1-\gamma C_3+C_5)(1-f)\big)+\frac{C_0}{T}+\frac{C_2dg(1-e^{-ra})T}{m(n+1)} \\ &\quad +\frac{C_1Tdg(1-e^{-ra})}{m}\left(\frac{m}{n+1}+\frac{T}{2(2n+1)}-\frac{T^2}{2m(3n+1)}\right)-\frac{SI_edg(1-e^{-ra})}{(n+1)}\big(n(T_p-T)+T_p\big). \end{aligned}$$

The total inventory cost under the delay period is greater than the cycle time IC_1 is to be optimized to the optimal cycle T^* using PSO.

Case II: ($T_p < T$) Delay period is less than the cycle time:

The interest charged by the retailer per cycle is obtained by

$$\begin{aligned} IC &= C_3I_c\int_{T_p}^T I(t)dt \\ &= \frac{C_3TI_cdg(1-e^{-ra})}{m}\left[\frac{mT}{n+1}+\frac{T^2}{2(2n+1)}-\frac{T^3}{2m(3n+1)}-\frac{T_p(2m-T_p)}{2}\left(1+\frac{T}{m(n+1)}\right)\right. \\ &\quad \left.+\frac{nT_p^{\frac{1}{n}+1}}{T^{\frac{1}{n}}}\left(\frac{m}{(n+1)}-\frac{nT_p}{(2n+1)(n+1)}-\frac{T_p^2}{m(n+1)(3n+1)}\right)\right]. \end{aligned} \tag{13}$$

The interest earned during the time 0 to T_p is given by

$$IE_2 = SI_e\int_0^{T_p} D(q(r),t)tdt = \frac{SI_eT_p^{\frac{1}{n}+1}dg(1-e^{-ra})}{(n+1)T^{\frac{1}{n}-1}}. \tag{14}$$

In the situation $T_p < T$, the total inventory cost per unit time (IC_2) is obtained as:

$$\begin{aligned} IC_2 &= \frac{1}{T}\big(HC + DC + PC + OC + SC + RC - DPC - IE_2 + IC\big) \\ &= \frac{dg(1-e^{-ra})}{f}\left(1+\frac{T}{m(n+1)}\right)\big(C_3 + C_4 + (C_1 - \gamma C_3 + C_5)(1-f)\big) + \frac{C_0}{T} + \frac{C_2 dg(1-e^{-ra})T}{m(n+1)} \\ &+ \frac{C_1 T dg(1-e^{-ra})}{m}\left(\frac{m}{n+1} + \frac{T}{2(2n+1)} - \frac{T^2}{2m(3n+1)}\right) - \frac{SI_e T_p^{\frac{1}{n}+1} dg(1-e^{-ra})}{(n+1)T^{\frac{1}{n}}} \\ &+ \frac{nC_3 I_c dg(1-e^{-ra})}{m}\Bigg[\frac{mT}{n+1} + \frac{T^2}{2(2n+1)} - \frac{T^3}{2m(3n+1)} - \frac{T_p(2m-T_p)}{2}\left(1+\frac{T}{m(n+1)}\right) \\ &+ \frac{nT_p^{\frac{1}{n}+1}}{T^{\frac{1}{n}}}\left(\frac{m}{(n+1)} - \frac{nT_p}{(2n+1)(n+1)} - \frac{T_p^2}{m(3n+1)(n+1)}\right)\Bigg]. \end{aligned} \tag{15}$$

The total inventory cost under the delay period is less than the cycle time is to be optimized (IC_2) to the optimal cycle T^* using PSO.

4 Generalized triangular Neutosophic Number and its De-neutrosophic

In real-life scenarios, most of the parameters are imprecise, which means inexact, invalid, or inaccurate, to overcome this type of impreciseness. The generalized triangular neutrosophic numbers (GTNN) can explain the truth, hesitation, and falsity of the given parameters.

Definition 3. ***Single-Valued Neutrosophic set***: *A single-valued Neutrosophic set* $(\tilde{S})$ *of a single-valued independent variable (x) is defined by* $\tilde{S} = \{\langle x; [\ \pi_{\tilde{S}}(x), \theta_{\tilde{S}}(x), \eta_{\tilde{S}}(x)]\ \rangle : x \in X\}$, *where* $\pi_{\tilde{S}}(x), \theta_{\tilde{S}}(x), \eta_{\tilde{S}}(x)$ *represents the concept of truth, hesitation and falsity membership functions, respectively. Here,* $\pi_{\tilde{S}} : \mathbf{R} \rightarrow [0,1]$ *is the truth membership function,* $\theta_{\tilde{S}} : \mathbf{R} \rightarrow [0,1]$ *is the hesitation membership function, and the falsity membership function is* $\eta_{\tilde{S}} : \mathbf{R} \rightarrow [0,1]$.

Neutro-normal: Let us consider three points p, q, r for which, $\pi_{\tilde{S}}(p) = 1, \theta_{\tilde{S}}(q) = 1, \eta_{\tilde{S}}(r) = 1$, then the $\tilde{S}$ is defined as neutro-normal.

Definition 4. ***(α, β, γ) cut:*** *(α, β, γ) cut of a neutrosophic number is defined as* $\tilde{S}_c = \{x \in X | \pi_{\tilde{S}}(x) \geq \alpha, \theta_{\tilde{S}}(x) \leq \beta, \eta_{\tilde{S}}(x) \leq \gamma\}$.

Definition 5. ***Neutro-convex***: *A neutrosophic set* $\tilde{S}$ *is called neutro-convex if the following condition holds: (i).* $\pi_{\tilde{S}}(\lambda\alpha + (1-\lambda)\beta) \geq min(\pi_{\tilde{S}}(\alpha), \pi_{\tilde{S}}(\beta))$, *(ii).* $\theta_{\tilde{S}}(\lambda\alpha + (1-\lambda)\beta) \leq max(\theta_{\tilde{S}}(\alpha), \theta_{\tilde{S}}(\beta))$, *(iii).* $\eta_{\tilde{S}}(\lambda\alpha + (1-\lambda)\beta) \leq max(\eta_{\tilde{S}}(\alpha), \eta_{\tilde{S}}(\beta))$, *where* $\alpha, \beta \in R$, *and* $\lambda \in [0,1]$.

Definition 6. ***Generalized Triangular Neutrosophic Number***: *A generalized triangular Neutrosophic number* $(\tilde{N})$ *is defined as* $\tilde{N} = \langle (m_1, m_2, m_3; \mu), (n_1, n_2, n_3; \nu), (p_1, p_2, p_3; \zeta) \rangle$, *where* $\mu, \nu, \zeta \in [0,1]$. *Here,*

$\pi_{\tilde{N}} : \mathbf{R} \to [0, \mu]$ is the truth membership function, $\theta_{\tilde{N}} : \mathbf{R} \to [\nu, 1]$ is the hesitation membership function, and the falsity membership function is $\eta_{\tilde{N}} : \mathbf{R} \to [\zeta, 1]$, where the membership functions are mathematically defined as follows.

$$\pi_{\tilde{N}}(x) = \begin{cases} \frac{(x-m_1)}{(m_2-m_1)}\mu \ if \ m_1 \le x < m_2 \\ \mu \quad if x = m_2 \\ \frac{(m_3-x)}{(m_3-m_2)}\mu \ if \ m_2 < x \le m_3 \\ 0 \ otherwise \end{cases}, \ \theta_{\tilde{N}}(x) = \begin{cases} \frac{(n_2-x)+\nu(x-n_1)}{(n_2-n_1)} \ if \ n_1 \le x < n_2 \\ \nu \quad if x = n_2 \\ \frac{(x-n_2)+\nu(n_3-x)}{(n_3-n_2)} \ if \ n_2 < x \le n_3 \\ 1 \ otherwise \end{cases}, \ and$$

$$\eta_{\tilde{N}}(x) = \begin{cases} \frac{(p_2-x)+\zeta(x-p_1)}{(p_2-p_1)}, \ for \ p_1 \le x < p_2 \\ \zeta, \ for \ x = p_2 \\ \frac{(x-p_2)+\zeta(p_3-x)}{(p_3-p_2)}, \ for \ p_2 < x \le p_3 \\ 1 \ otherwise \end{cases}$$

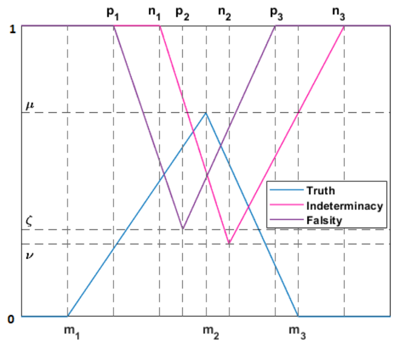


Figure 3: Graph of Generalized Triangular Neutrosophic number

Sevaral methods are exist for defuzzification of fuzzy numbers in literature, this paper proposed new defuzzification technique for GTNN by using Rouben's [74] ranking function which is given as follows

$$R_r(\mu_{\tilde{A}}) = \frac{1}{2}\int_0^1 \left(inf\tilde{A}_\alpha + sup\tilde{A}_\alpha\right)d\alpha \tag{16}$$

for any fuzzy number $\tilde{A}$ with membership function $\mu_{\tilde{A}}$.

Definition 7. *The defuzzification of neutrosophic number $\tilde{S}$ with truth membership function $\pi_{\tilde{S}}$, indeterminacy membership function $\theta_{\tilde{S}}$ and falsity membership function $\eta_{\tilde{S}}$ is defined by*

$$R(\tilde{S}) = \frac{1}{3}[R_r(\pi_{\tilde{S}}) + R_r(\theta_{\tilde{S}}) + R_r(\eta_{\tilde{S}})], \tag{17}$$

where R_r is the Rouben's ranking function given in (16). *Based on equation* (17) *If $\tilde{N} = \langle(m_1, m_2, m_3; \mu), (n_1, n_2, n_3; \nu), (p_1, p_2$*

is a GTNN, then

$$R(\tilde{N}) = \frac{1}{3}\bigg(\frac{1}{4\mu}\big(2m_2+(2\mu-1)(m_1+m_3)\big)+\frac{1}{4(1-\nu)}\big(2n_2+(1-2\nu)(n_1+n_3)\big) + \frac{1}{4(1-\zeta)}\big(2p_2+(1-2\zeta)(p_1+p_3)\big)\bigg). \quad (18)$$

5 Inventory System under Generalized Neutrosophic Environment

In a real market, the cost parameters are unpredictable, and the decision-maker is faced with a dilemma. As a result, we attempt to express the inventory system by introducing a generalized neutrosophic set to represent the proposed inventory system's various inventory charges and rates. For this study, we have considered ordering cost (C_0), holding cost (C_1), deterioration cost (C_2), purchase cost (C_3), inspection cost (C_4) and rework cost (C_5) as a GTNNs. The representation of GTNNs $\tilde{C_0}$, $\tilde{C_1}$, $\tilde{C_2}$, $\tilde{C_3}$, (C_4) and $\tilde{C_5}$ are as follows:
$\tilde{C_0} =< (a_{11}, a_{12}, a_{13}; \mu), (a_{21}, a_{22}, a_{23}; \nu), (a_{31}, a_{32}, a_{33}; \zeta) >$, $\tilde{C_1} =< (b_{11}, b_{12}, b_{13}; \mu), (b_{21}, b_{22}, b_{23}; \nu), ba_{31}, b_{32}, b_{33}; \zeta) >$,
$\tilde{C_2} =< (c_{11}, c_{12}, c_{13}; \mu), (c_{21}, c_{22}, c_{23}; \nu), (c_{31}, c_{32}, c_{33}; \zeta) >$, $\tilde{C_3} =< (d_{11}, d_{12}, d_{13}; \mu), (d_{21}, d_{22}, d_{23}; \nu), (d_{31}, d_{32}, d_{33}; \zeta) >$,
$\tilde{C_4} =< (e_{11}, e_{12}, e_{13}; \mu), (e_{21}, e_{22}, e_{23}; \nu), (e_{31}, e_{32}, e_{33}; \zeta) >$ and $\tilde{C_5} =< (f_{11}, f_{12}, f_{13}; \mu), (f_{21}, f_{22}, f_{23}; \nu), (f_{31}, f_{32}, f_{33}; \zeta) >$.
Using equation (18), the cost parameters of GTNNs are converted into a deneutrosophic value such as $\tilde{C}_{0(D)}$, $\tilde{C}_{1(D)}$, $\tilde{C}_{2(D)}$, $\tilde{C}_{3(D)}$, $\tilde{C}_{4(D)}$ and $\tilde{C}_{5(D)}$ To obtain the total cost in the generalized neutrosophic domain ($\widetilde{IC_1}, \widetilde{IC_2}$), substituting the values of the deneutrosophic values $\tilde{C}_{0(D)}$, $\tilde{C}_{1(D)}$, $\tilde{C}_{2(D)}$, $\tilde{C}_{3(D)}$, $\tilde{C}_{4(D)}$ and $\tilde{C}_{5(D)}$ in the total cost of both cases in equation (12) and equation (15), we get the total cost of both cases in equation (12) and equation (15) as follows:

$$\widetilde{IC_1} = \frac{dg(1-e^{-ra})}{f}\left(1+\frac{T}{m(n+1)}\right)\big(\tilde{C}_{3(D)}+\tilde{C}_{4(D)}+(\tilde{C}_{1(D)}-\gamma\tilde{C}_{3(D)}+\tilde{C}_{5(D)})(1-f)\big)+\frac{\tilde{C}_{0(D)}}{T} + \frac{\tilde{C}_{1(D)}Tdg(1-e^{-ra})}{m}\left(\frac{m}{n+1}+\frac{T}{2(2n+1)}-\frac{T^2}{2m(3n+1)}\right)-\frac{SI_e dg(1-e^{-ra})}{(n+1)}\big(n(T_p-T)+T_p\big) + \frac{\tilde{C}_{2(D)}dg(1-e^{-ra})T}{m(n+1)}, \quad (19)$$

$$\widetilde{IC_2} = \frac{dg(1-e^{-ra})}{f}\left(1+\frac{T}{m(n+1)}\right)\big(\tilde{C}_{3(D)}+\tilde{C}_{4(D)}+(\tilde{C}_{1(D)}-\gamma\tilde{C}_{3(D)}+\tilde{C}_{5(D)})(1-f)\big)+\frac{\tilde{C}_{2(D)}dg(1-e^{-ra})T}{m(n+1)} + \frac{\tilde{C}_{1(D)}Tdg(1-e^{-ra})}{m}\left(\frac{m}{n+1}+\frac{T}{2(2n+1)}-\frac{T^2}{2m(3n+1)}\right)-\frac{SI_eT_p^{\frac{1}{n}+1}dg(1-e^{-ra})}{(n+1)T^{\frac{1}{n}}}+\frac{\tilde{C}_{0(D)}}{T} + \frac{n\tilde{C}_{3(D)}I_c dg(1-e^{-ra})}{m}\bigg[\frac{mT}{n+1}+\frac{T^2}{2(2n+1)}-\frac{T^3}{2m(3n+1)}-\frac{T_p(2m-T_p)}{2}\left(1+\frac{T}{m(n+1)}\right) + \frac{nT_p^{\frac{1}{n}+1}}{T^{\frac{1}{n}}}\left(\frac{m}{(n+1)}-\frac{nT_p}{(2n+1)(n+1)}-\frac{T_p^2}{m(3n+1)(n+1)}\right)\bigg]. \quad (20)$$

Now, we will find the optimal total cost in a generalized neutrosophic environment from equations (19) and (20) through PSO.

6 Optimization of Proposed Inventory Model using PSO

PSO is a nature-inspired optimization technique that can be used to solve even the most challenging optimization issues. The movement and intelligence of swarms are impacted in PSO, which was developed by Kennedy and Eberhart [75]. The swarms are considered as vector points in the domain space where the optimum value of a given objective function lies. The four critical vectors in PSO that: (i) x vector: records the current position, (ii) p-vector: records the personal best, (iii) v- vector: control the velocity of the moving particle at each instance, and (iv) g-vector: records the direction towards the global best position. Four essential parameters control the movement of particles; a parameter representing the coefficient of inertia (‘w') along with a damping inertial coefficient ‘wdic’, and two constants, c_1 representing the acceleration of individual swarms, and c_2 representing the social acceleration. These parameters are varied according to the choice of the optimization problem. To find the optimal total cost of the proposed inventory model, we present the algorithm 1 of the variant of the PSO technique as follows:

Algorithm 1 : PSO Algorithm for generalized neutrosophic inventory model

Step I. Read: Swarm Population (SwPop), Maximum Iteration (MaxIt), particle(i).Velocity (V_i), particle(i).Position (P_i), particle(i).Best.Position ($P_{BP,i}$), Global Best.Position ($P_{BP,G}$), particle(i).Cost (C_i), particle(i).Best.Cost ($C_{BC,i}$), Global Best.Cost ($C_{BC,G}$),

Step II. Set the objective function:

(a) $ObjFunction = @(T,P)\ IC_1fcn(T,P)$; (Optimum for IC_1),

(b) $ObjFunction = @(T,P)\ IC_2fcn(T,P)$; (Optimum for IC_2)

Step III. Set the parameters of PSO for Inventory system:

(a) $VarSize = 1$, $lowerbound = 0.0001$, $upperbound = 4$, $MaxI = 10$, $SwPop = 50$

(b) Set coefficient of inertia, $w = 1$; the damping ratio of inertial coefficient $wdic = 0.99$, $c_1 = 2$, and $c_2 = 0.2$

(c) Set de-neutrosophic inventory parameters as $P = \left[\tilde{C_0}_D\ \tilde{C_1}_D\ \tilde{C_2}_D\ \tilde{C_3}_D\ \tilde{C_4}_D\ \tilde{C_5}_D\ m\ n\ d\ r\ a\ f\ S\ T_p\ I_e\ g\ I_c\right]$

Step IV. Define the structure of void particles:

(a) set $void\ particle.Position = [\,]$, $void\ particle.Velocity = [\,]$, $void\ particle.Cost = [\,]$, $void\ P_{Best} = [\,]$, and $void\ C_{Best} = [\,]$

(b) Create vector of void particles of size equal to SwPop

(c) Set initial $C_{BC,G} = \infty$

Step V. Initialize the population member:
For $i = 1$ to SwPop initiate $P_i = unifrnd(VarMin,\ VarMax,\ VarSize)$ and $V_i = zeros(VarSize)$

Step VI. Estimate and evaluate:

(a) Set $T = P_i$, find $C_i = ObjFunction(T,P)$, $P_{BP,i} = P_i$ and $C_{BC,i} = C_i$

(b) $if\ C_{Best,i} < C_{BC,G}$ & $P_{BP,i} > 0$ then $GlobalBest = P_{BP,i}$

(c) $Best_{Costs} = zeros(MaxIt, 1)$

Step VII. Optimize and update for every iteration:

(a) for $iT = 1$ to MaxIt, and for $i = 1$ to SwPop update the velocity and position:

(b) $V_i = w * V_i + c_1 * rand(VarSize) \cdot *(P_{BP,i} - P_i) + c2 * rand(VarSize) \cdot *(P_{BP,G} - P_i)$

(c) $P_i = P_i + V_i; T = P_i;\ and\ C_i = ObjFunction(T, P2)$

(d) if $C_i < C_{BC,i}$ & $C_i > 0$ then set $P_{BP,i} = P_i\ and\ C_{BC,i} = C_i$;

(e) if $C_{BC,i} < C_{BC,G}$ & $C_{Best,i} > 0$ then set $GlobalBest = P_{BP,i}$;

Step VIII. Set up the global best cost:

(a) $Best.Costs(iT) = C_{BC,G}$

(b) Set $w = w * wdic$ and continue the iteration by going back to 6.

Step IX. Stop

7 Numerical Solution

A numerical example of the presented inventory system is shown to assist the analytical derivation with the following configuration: assume that the ordering cost is \$250 per order, the cost of raw material \$16 per unit, the cost for holding the item is \$3 per item per unit time, the cost of deteriorating per item is \$2.5, and the selling price of each item is \$25. Because the system allows both defective and perfect things, let's assume the percentage of non-defective items is 0.9, and the item's maximum lifetime is three years. Let the rework cost of the defective items is \$1.8 per unit, the inspection cost is \$2.2 per unit, and the faulty item's purchase cost be discounted by 3%. Allow the supplier to accept a two-year payment period during which the customer earns 5% interest, but after the permissible delay period assuming the supplier's annual interest rate on stocks is 8%. Let us further consider the parameters settings for the PSO: $m = 3$ years, T_p=1 year(case I), T_p=6 months (case II), $I_e = 0.05$, $I_c = 0.08$, $f = 0.9$, $n = 1$, $d = 55$, $r = 0.95$, $\gamma = 0.3$, $g = 1.2$ and $d = 2.5$.

We use a generalized triangular neutrosophic set to numerically demonstrate the influence of the cost parameters' imprecision on the proposed inventory system.

$\tilde{C_0} =< (170, 250, 310; 0.75), (190, 260, 310 : 0.25), (180, 250, 300; 0.25) >$, $\tilde{C_1} =< (1, 3, 4; 0.7), (2, 3.5, 5; 0.4), (1.5, 2.5, 4.5 : 0.3) >$, $\tilde{C_2} =< (0.5, 2.5, 4; 0.7), (1.5, 3, 4.5 : 0.4), (1, 2, 3.5; 0.3) >$, $\tilde{C_3} =< (12, 15, 18; 0.8), (14, 17, 20; 0.3), (13, 16, 19; 0.2) >$, $\tilde{C_4} =< (1, 1.8, 2.4; 0.7), (1.4, 2, 2.6; 0.3), (1.2, 1.6, 2.2; 0.3) >$ and $\tilde{C_5} =< (1.4, 2, 2.6; 0.7), (1.8, 2.4, 3.2; 0.2), (1.6, 2.2, 2.6; 0.2) >$.

Thus, de-neutrosophication of the GTNNs by using above ranking method, we obtain the values of the cost parameters as $\tilde{C}_{0(D)} = \$250$, $\tilde{C}_{1(D)} = \$3$, $\tilde{C}_{2(D)} = \$2.5$, $\tilde{C}_{3(D)} = \$16$, $\tilde{C}_{4(D)} = \$1.8$ and $\tilde{C}_{5(D)} = \$2.2$

Table 2 shows the optimal solution of the suggested inventory model in a generalized neutrosophic environment that was obtained using the PSO for the inventory system.

Table 2: Optimal inventory costs

n value	T^*	$\widetilde{IC_1}(T^*)$	T^*	$\widetilde{IC_2}(T^*)$
0.5	0.685142	2021.62	0.662316	2075.08
0.75	0.730700	1977.04	0.720712	2024.97
1	0.770615	1941.63	0.770257	1985.31
1.5	0.840094	1888.56	0.853733	1925.82
2	0.898355	1850.39	0.923258	1882.85

The PSO is implemented to find the optimal total costs $\widetilde{IC_1}$ and $\widetilde{IC_2}$ incorporating the generalized neutrosophic cost parameters, the parameters of the PSO algorithm are $w = 1$, $wdic = 0.99$, $c_1 = 2$, $c_2 = 0.2$. From Table 2, we can observe that the total cost is higher at $n = 0.75 (0 < n < 1)$ since the demand rate increases with time for $0 < n < 1$, so the total cost of inventory systems increases. The total inventory cost is less at $n = 2 (n > 1)$ as the demand rate decreases over time for $n > 1$, so the total cost of inventory systems decreases. Decision makers face difficulty in estimating the values of the cost parameters in real-world market situations. The decision-maker can determine optimal values for cost parameters based on market conditions using neutrosophic figures. Neutrosophic numbers have membership functions for truth, hesitation, and falsity, which are helpful for overcoming imprecision in cost parameters. The decision-maker can determine values for cost parameters based on market conditions using neutrosophic numbers. Table 2 shows that $\widetilde{IC_1} < \widetilde{IC_2}$ for all values on n, i.e., the total inventory cost of case I is less than the total inventory cost of case II since the delay period of case I is greater than the cycle time (T), which helps to conclude that the total cost decreases with increasing delay period.

7.1 Sensitivity Analysis

We undertake a sensitivity analysis based on the numerical example of the inventory system described in the preceding section by modifying the parameters by a specific percentage depending on the parameter threshold. We consider one parameter at a time for the sensitivity analysis and keep the other parameter fixed.

Table 3: Sensitivity analysis for the different inventory related cost

Parameter	% change	T^* (year)	% change in T^*	$\widetilde{IC_1}(T^*)$	% change in $\widetilde{IC_1}(T^*)$	T^*(years)	% change in T^*	$\widetilde{IC_2}(T^*)$	% change in $\widetilde{IC_2}(T^*)$
$\tilde{C}_{0(D)}$	-20	0.689955	-10.47	1873.16	-3.53	0.689780	-10.45	1916.82	-3.45
	-10	0.731416	-5.08	1908.34	-1.71	0.731152	-5.07	1952.01	-1.68
	10	0.807888	4.84	1973.30	1.63	0.807432	4.83	2017.01	1.60
	20	0.843496	9.46	2003.58	3.19	0.842942	9.44	2047.30	3.12
$\tilde{C}_{1(D)}$	-20	0.793668	2.99	1918.84	-1.17	0.793224	2.98	1962.53	-1.15
	-10	0.781877	1.46	1930.30	-0.58	0.781478	1.46	1973.99	-0.57
	10	0.759842	-1.39	1952.82	0.58	0.759520	-1.93	1996.50	0.56
	20	0.749524	-2.74	1963.88	1.15	0.749235	-2.73	2007.56	1.12
$\tilde{C}_{2(D)}$	-20	0.775949	0.69	1937.13	-0.23	0.775572	0.69	1980.82	-0.23
	-10	0.773268	0.34	1939.38	-0.12	0.772900	0.34	1983.07	-0.11
	10	0.767989	-0.34	1943.87	0.12	0.767639	-0.34	1987.55	0.11
	20	0.765389	-0.68	1946.10	0.23	0.765049	-0.68	1989.78	0.23
$\tilde{C}_{3(D)}$	-20	0.809817	5.09	1669.06	-14.04	0.815180	5.83	1711.71	-13.78
	-10	0.789491	2.45	1805.54	-7.01	0.791668	2.78	1848.76	-6.88
	10	0.753026	-2.28	2077.35	6.99	0.750654	-2.55	2121.40	6.85
	20	0.736583	-4.42	2212.73	13.96	0.732624	-4.89	2257.06	13.69
$\tilde{C}_{4(D)}$	-20	0.774873	0.55	1910.10	-1.02	0.774500	0.55	1953.78	-1.59
	-10	0.772735	0.26	1925.86	-0.81	0.772370	0.27	1969.55	-0.79
	10	0.768512	-0.27	1957.39	0.81	0.768161	-0.27	2001.07	0.79
	20	0.766426	-0.54	1973.14	1.62	0.766082	-0.54	2016.83	1.59
$\tilde{C}_{5(D)}$	-20	0.771132	0.07	1937.78	-0.20	0.770771	0.07	1981.46	-0.19
	-10	0.770873	0.03	1939.70	-0.10	0.770514	0.03	1983.39	-0.10
	10	0.770357	-0.03	1943.55	0.10	0.769999	-0.03	1987.24	0.10
	20	0.770099	-0.07	1945.48	0.20	0.769743	-0.07	1989.17	0.19

Observations from result of Table-3:

1. **Total inventory cost functions $\widetilde{IC_1}$ and $\widetilde{IC_2}$ are highly sensitive to the purchasing cost $(\tilde{C}_{3(D)})$. If the purchase cost $(\tilde{C}_{3(D)})$ increases, the total cost of the inventory system increases dramatically and the cycle duration (T) decreases moderately. The cycle length (T) decreases in the medium (Figure-4) if the purchase cost $(\tilde{C}_{3(D)})$ increases, because the retailer reduces the percentage of purchase items.**

2. **The total inventory cost functions $\widetilde{IC_1}$ and $\widetilde{IC_2}$ are moderately sensitive to the holding cost $(\tilde{C}_{1(D)})$. The holding cost increases by 20%, then the total inventory cost functions $\widetilde{IC_1}$ and $\widetilde{IC_2}$ increase in an analogous manner obtained from figure 4. If the holding cost $(\tilde{C}_{1(D)})$ increases in**

this model, then the cycle length (T) of the inventory system moderately decreases. Because if holding costs increase, retailers try to reduce stock or send inventory as much as possible to customers, so that the cycle length (T) decreases.

3. **The ordering cost $(\tilde{C}_{0(D)})$ is moderately sensitive to total inventory costs $(\widetilde{IC_1}, \widetilde{IC_2})$ and highly sensitive to cycle length (T). Due to an increase in the ordering cost $(\tilde{C}_{0(D)})$, the total inventory cost increases moderately and the cycle length increases (T) much. The cycle length (T) is highly sensitive to the ordering cost in both cases. Because if $(\tilde{C}_{0(D)})$ increases, the retailer orders a bulk amount of goods at a time that causes the cycle length (T) automatically increases.**

4. **The total inventory costs are moderately sensitive to $(\tilde{C}_{4(D)})$, and the cycle duration (T) is less sensitive to inspection cost $(\tilde{C}_{4(D)})$. Due to the increase in $(\tilde{C}_{4(D)})$, the cycle length decreases and the total cost increases in this model (Figure 4).**

5. **Furthermore, $\widetilde{IC_1}$, $\widetilde{IC_2}$ and T are less sensitive to the deterioration cost $(\tilde{C}_{2(D)})$ and the rework cost $(\tilde{C}_{5(D)})$. Due to the increase in these costs, the total cost increases and the cycle length decreases, but it will not show much impact on the inventory system.**

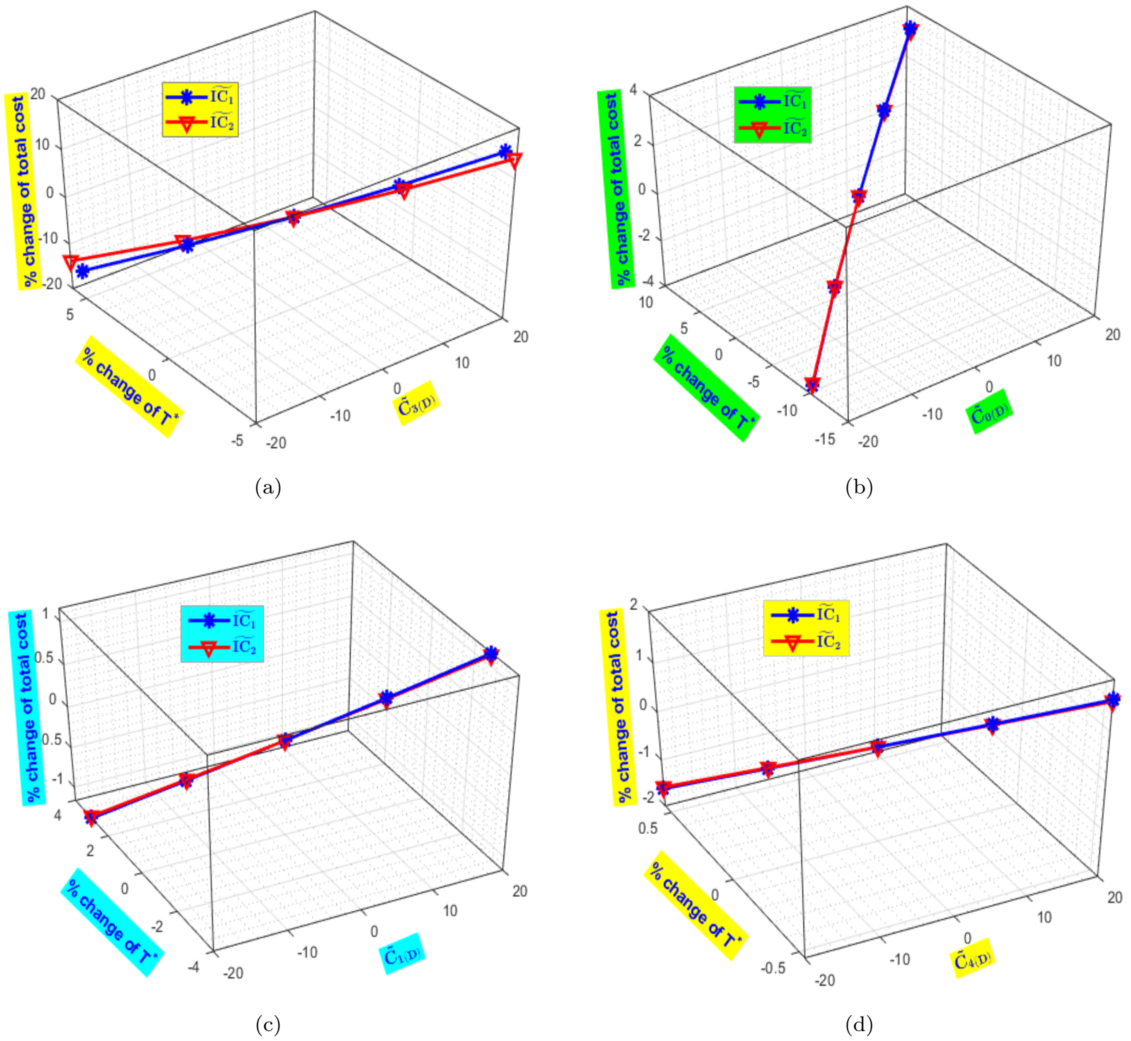


Figure 4: Sensitive analysis on optimal total inventory cost for inventory costs

Table 4: Sensitivity analysis for the different inventory related parameters

Parameter	% change	T^* (year)	% change in T^*	$\widetilde{IC_1}(T^*)$	% change in $\widetilde{IC_1}(T^*)$	T^*(years)	% change in T^*	$\widetilde{IC_2}(T^*)$	% change in $\widetilde{IC_2}(T^*)$
γ	-20	0.769491	-0.15	1950.03	0.43	0.769137	-0.15	1993.72	0.42
	-10	0.770052	-0.07	1945.83	0.22	0.769696	-0.07	1989.52	0.21
	10	0.771179	0.07	1937.43	-0.22	0.770818	0.07	1981.11	-0.21
	20	0.771744	0.15	1933.22	-0.43	0.771381	0.15	1976.91	-0.42
m	-20	0.716460	-7.03	1990.80	2.53	0.716568	-6.97	2034.41	2.47
	-10	0.745036	-3.32	1963.95	1.15	0.744931	-3.29	2007.60	1.12
	10	0.793691	2.99	1992.75	-0.97	0.793101	2.96	1966.47	-0.95
	20	0.814647	5.72	1906.56	-1.81	0.813834	5.66	1950.31	-1.76
d	-20	0.860741	11.70	1641.60	-16.84	0.860138	11.67	1649.58	-16.91
	-10	0.811923	5.36	1779.06	-8.37	0.811456	5.35	1818.39	-8.41
	10	0.735067	-4.61	2102.58	8.29	0.734795	-4.60	2150.62	8.33
	20	0.704051	-8.64	2262.14	16.51	0.703848	-8.62	2314.54	16.58
f	-10	0.747929	-2.94	2118.40	9.10	0.747644	-2.94	2162.08	8.90
	-5	0.759616	-1.43	2025.44	4.32	0.759295	-1.42	2069.12	4.22
	5	0.780987	1.35	1865.68	-3.91	0.780592	1.34	1909.37	-3.83
	10	0.790787	2.62	1796.54	-7.47	0.790355	2.61	1840.23	-7.31
r	-5	0.775601	0.65	1920.70	-1.08	0.775230	0.65	1963.83	-1.08
	-2	0.772527	0.25	1933.56	-0.42	0.772164	0.25	1977.03	-0.42
	2	0.768805	-0.23	1949.32	0.40	0.768451	-0.23	1993.21	0.40
	5	0.766267	-0.56	1960.19	0.96	0.765920	-0.56	2004.37	0.96

Observations from the result of Table-4:

1. **The total inventory cost functions $\widetilde{IC_1}$ and $\widetilde{IC_2}$ are highly sensitive to the demand parameter d. If the average demand for the inventory system increases in this inventory model, the total cost increases and the cycle length (T) decreases dramatically. Because if the demand increases, then item sales increase because of that the stock gets over soon so that the cycle length (T) decreases, as shown in figure 5.**

2. **The non-defective rate of item f is highly sensitive to total inventory cost functions ($\widetilde{IC_1}$, $\widetilde{IC_2}$) and moderately sensitive to cycle length (T). If the non-defective rate of items in the inventory system increases, then the total cost decreases and cycle length (T) increases because of the relaxation of the rework cost. Table 4 shows that the sensitivity analysis has taken changes of f from -10% to 10% because if a percentage of f increases by more than 10%, then f exceeds its permissible value.**

3. **The maximum lifetime of the product m is moderately sensitive to total inventory costs and**

highly sensitive to cycle length (T). Due to an increase in maximum lifetime, total inventory costs decrease moderately and the cycle length increases highly in this model.

4. **The total inventory costs and cycle duration are moderately sensitive to the quality of the supplier's product r. Due to the increase in supplier product quality, the cycle length (T) decreases and the total cost increases, as shown in figure 5. If the supplier's product quality increases, then the retailer's product quality also increases, and hence the cycle length (T) decreases due to the increase in demand. Table-4 shows that the sensitivity analysis has taken changes of r from -5% to 5% because if a percentage of r increases by more than 5%, then r exceeds its permissible value.**

5. **The total inventory costs, and cycle length are less sensitive to discount on the purchase cost of defective items γ. If the discount on defective items increases from the supplier, then the total cost of the inventory system decreases in both cases, but does not have much impact on the total cost.**

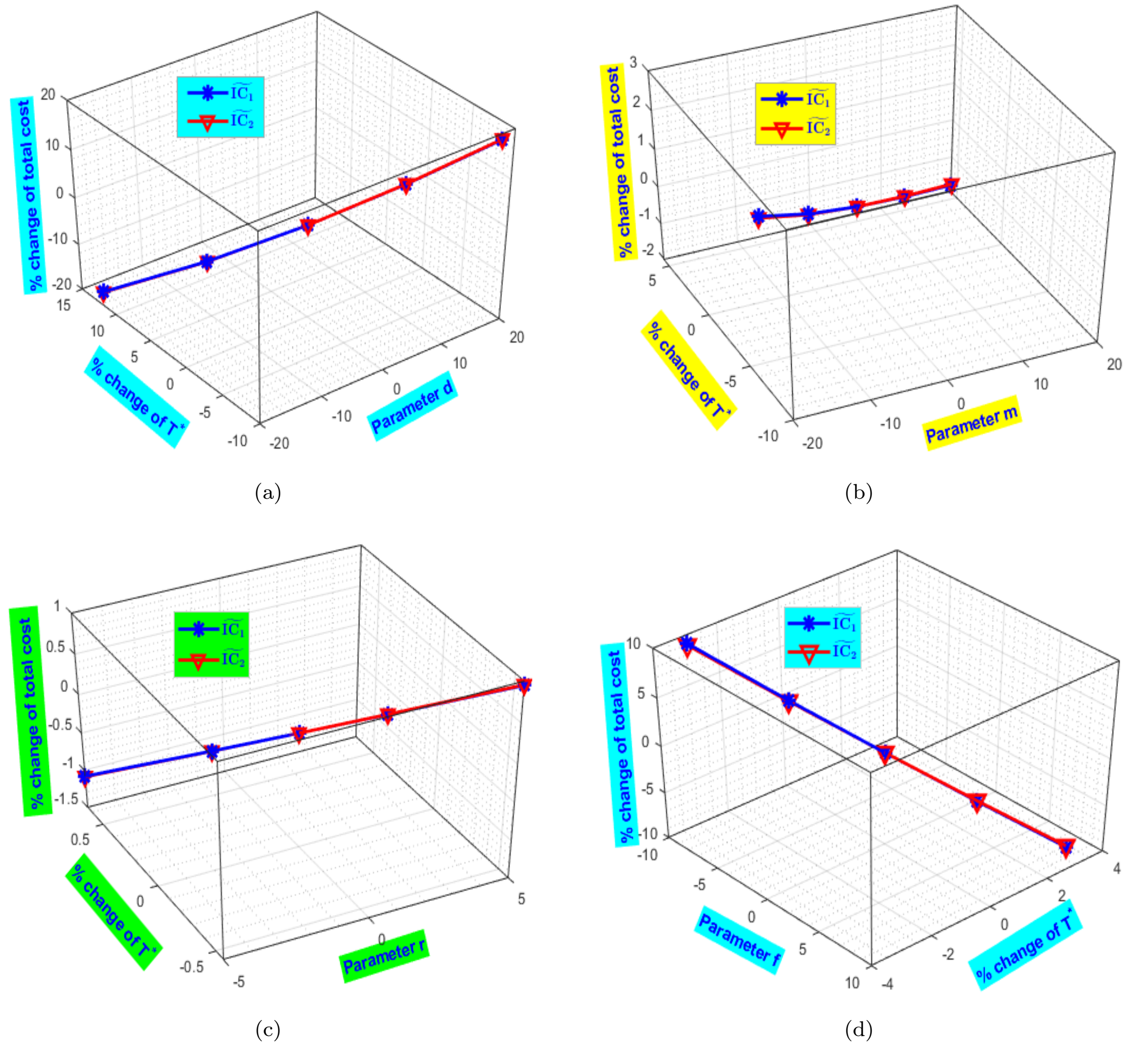


Figure 5: Sensitivity analysis of optimal costs versus inventory associated parameters

7.2 Managerial perspective

Any product's value is usually time-dependent and the cost of such things will rise over time, whereas the price of a few select products will fall. Because most food items are perishable, their value will depreciate over time as the storage period is extended. A few select products' prices will rise as the storage period lengthens, similar to wine. This study persuaded decision-makers that perishable food should be stored in a healthier atmosphere to prolong shelf life and slow deterioration.

From a managerial standpoint, this research increases the quality of the product. The demand for a well-maintained product will be stronger, resulting in more earnings during sales. Furthermore,

management may have to invest higher holding costs to keep such a product. This indicates that we must strike a balance between these aspects in order to keep our total inventory cost low.

The inventory system's flow has preserved the efficiency of decaying items, cost trends and quality criteria. As a result, this research can be used to manage the inventory system regarding quality and deterioration rates.

As a result, the current inventory model was designed to represent these realistic qualities that can be used to forecast the components of the system. This model is vulnerable to the purchase cost, non-defective rate f, and demand parameter d, as shown in the numerical analysis section, demonstrating the model's fundamental character. As a result, the decision-maker should focus on choosing these parameters.

The study of generalized neutrosophic cost parameters helps the decision-maker decide the appropriate value for the uncertain cost parameter. The decision-maker can deal with items with varying demand rates from this innovative demand rate.

The total inventory cost is susceptible to the non-defective rate f, as shown in the sensitivity table 3. Because the quality of the product acquired from the supplier affects the demand rate in this model, the decision-maker should have reasonable concern about the product quality.

8 Conclusion and Future direction

An optimum order quantity inventory model without shortages in selecting quality goods, the time-dependent power pattern, and the permissible time delay is presented in this article. The inventory system is improved, the nature of objects for time-dependent deterioration factors is addressed, and the cost pattern is approximated for a real-world scenario. The proposed inventory management system includes various sophisticated and vital features to order and store products with varying uncertainties. The main benefit of this research work is that it can handle a wide range of product demand rates. This research on inventory management sending out defective goods for rework reduces the number of defective products. The environment benefits from the reduction of damaged items and waste. The merchant will undoubtedly focus on product quality to ensure that clients receive high-quality goods by considering the quality-dependent demand. The concept of impreciseness in the cost pattern is based on generalized triangular neutrosophic numbers. The inventory model with quality-dependent demand was addressed and formulated

in this work, and the solutions were analyzed by changing various parameter values. A PSO algorithm is presented to determine the optimal total cost in two different time delay scenarios. In the generalized neutrosophic environment, PSO is utilized to find the solution and analyze the sensitivity of the inventory parameters.

This model does not permit shortages to occur through it frequently occur in the inventory system. This is the disadvantage of the proposed inventory system which can be expanded in due course of time. Different inventory management models can be presented for multi-items with varying parameters based on this proposed study. More studies on inventory models can be performed in a probabilistic or probabilistic environment using various types of uncertainty, such as intuitionistic fuzzy, Hesitant fuzzy, Fermatean fuzzy, interval type-2 fuzzy, and many more.

Acknowledgments: "The authors express their gratitude to the editors and referees for their encouragement and constructive comments on this article."

Compliance with Ethical Standards

1. *"Funding: This study did not receive any funding or grants from any sources / company / agencies".*
2. *"Conflict of Interest: The authors declare that we have no conflict of interest.*
3. *"Ethical approval: This article does not contain any studies with human participants or animals performed by any of the authors".*

Appendix-I:

This inventory management study is developed using the following notation:

C_0	:	Ordering cost per order (\$/order).
S	:	Selling price (\$/unit).
C_1	:	Holding cost per item (\$/unit/unit time).
C_2	:	Deterioration cost per item (\$/unit/unit time).
C_3	:	Purchasing cost per unit item (\$/unit).
C_4	:	Inspection cost per unit item (\$/unit).
C_5	:	Rework cost per unit item (\$/unit).
I_c	:	Rate of interest charged per year in stocks by suppliers.
I_e	:	Rate of interest earned by investment per year.
$\theta(t)$	:	Deterioration rate of items.
m	:	Maximum life time in years of item, ($m > 2$).
T_p	:	Supplier permissible delay period.
$q(r)$	:	Retailer product's quality.
T	:	Cycle time in per cycle.
f	:	Non-defective rate of items ($0 < f < 1$).
r	:	Supplier's product quality ($0 < r < 1$).
γ	:	Reduction percentage of purchasing cost of defective items.
d	:	Average demand per cycle ($d = \frac{x}{T} > 0$) .
x	:	Total demand per cycle.
n	:	Demand Pattern index ($n > 0$).

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