Refined Neutrosophic Trilplets

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Abstract: This paper solves the imperfect triplets problem in refined neutrosophic rings, where it presents the necessary and sufficient conditions for a triple (x, y, z) to be an imperfect triplet in any refined neutrosophic ring.

Theorem : [38]

Let $x = (x_0, x_1I_1, x_2I_2), y = (y_0, y_1I_1, y_2I_2)$ be any two elements in $R(I_1, I_2)$, then (x, y) is an imperfect refined neutrosophic duplet with y acts as an identity if and only if

are three imperfect duplets in the classical ring $(x_0, y_0), (x_0 + x_2, y_0 + y_2), (x_0 + x_1 + x_2, y_0 + y_1 + y_2)$ R with $y_0, y_0 + y_2, y_0 + y_1 + y_2$ acts as identities.

For the structure of refined neutrosophic imperfect duplets. See [38].

Theorem:

Let $Q(I_1, I_2)$, $R(I_1, I_2)$ be the neutrosophic ring of rationales and reals respectively. Consider the following two forms of imperfect duplets

1-)$\{(0, x_1I_1, 0), (y_0, y_1I_1, y_2I_2);\ y_0 + y_1 + y_2 = 1, x_1 \neq 0\}$,

2-)$\{(0, x_1I_1, -x_1I_2), (y_0, y_1I_1, y_2I_2); y_0 + y_2 = 1\ and\ x_1 \neq 0\}$.

The corresponding imperfect triplets derived from the previous forms are

(a-)$\{(0, x_1I_1, 0), (0, I_1, 0), (0, \frac{1}{x_1}I_1, 0);\ x_1 \neq 0\}$,

(b-) $\{(0, x_1I_1, -x_1I_2), (0, -I_1, I_2), (0, \frac{1}{x_1}I_1, -\frac{1}{x_1}I_2);\ x_1 \neq 0\}$.

Proof:

Suppose that (x, y, z) is an imperfect triplet, then $xy = yx = x, xz = zx = y, yz = zy = z$. It is clear that $(x, y), (y, z)$ are imperfect duplets with y acts as an identity.

From the form (1), we have $x = (0, x_1 I_1, 0)$, $y =(y_0, y_1 I_1, y_2 I_2)$,$z =(0, z_1 I_1, 0)$ with $y_0 + y_1 + y_2 = 1, x_1 \neq 0$. From the equation $xz = y$, we get that $y_0 = y_2 = 0, y_1 = 1$ and $z_1 = \frac{1}{x_1}$, thus the corresponding imperfect triplet is (a).

From the form (2), we have $x =(0, x_1 I_1, -x_1 I_2)$, $y =(y_0, y_1 I_1, y_2 I_2)$, $z =(0, z_1 I_1, -z_1 I_2)$ with $y_0 + y_2 = 1$. From the equation $xz = y$, we get that $y_2 = 1, y_1 = -1, y_0 = 0, z_1 = \frac{1}{x_1}$

And the corresponding imperfect triplet is (b).

Example:

We construct an example about an imperfect triplet with form (b).

Put $x_1 = \frac{1}{3}$. The corresponding triplet is $x = \left(0, \frac{1}{3} I_1, -\frac{1}{3} I_2\right), y = (0, -I_1, I_2), z = (0,3\, I_1, -3I_2)$.

Theorem :

Let $Q(I_1, I_2)$, $R(I_1, I_2)$ be the neutrosophic ring of rationales and reals respectively. Consider the following two forms of imperfect duplets

3-)$\{(0, x_1 I_1, x_2 I_1), (y_0, 0, y_2 I_2); y_0 + y_2 = 1\}$,

4-)$\{(x_0, 0, -x_0 I_2)$,$(1, y_1 I_1, y_2 I_2)\}$,

The corresponding imperfect triplets has the forms

(c)$\{(0, x_1 I_1, x_2 I_2), (0,0, I_2), \left(0, \frac{-x_1}{x_2(x_1+x_2)} I_1, \frac{1}{x_2} I_2\right); x_2, x_1 + x_2 \neq 0\}$

(d) $\{(x_0, 0, -x_0 I_2)$,$(1,0, -I_2)$, $(\frac{1}{x_0}, 0, \frac{-1}{x_0} I_2); x_0 \neq 0\}$.

Proof:

Suppose that (x, y, z) is an imperfect triplet, then $xy = yx = x, xz = zx = y, yz = zy = z$. It is clear that $(x, y), (y, z)$ are imperfect duplets with y acts as an identity.

From the form (3), we have x=$(0, x_1 I_1, x_2 I_1)$, y=$(y_0, 0, y_2 I_2)$, z=$(0, z_1 I_1, z_2 I_1)$ with $y_0 + y_2 = 1$. From the equation $xz = y$, we get:

. Thus the corresponding imperfect triplet is (c) .$y_0 = 0, y_2 = 1, z_2 = \frac{1}{x_2}, z_1 = \frac{-x_1}{x_2(x_1+x_2)}$

From the form (4), we have x=$(x_0, 0, -x_0 I_2)$, y=$(1, y_1 I_1, y_2 I_2)$, z=$(z_0, 0, -z_0 I_2)$. From equation $xz = y$, we get $z_0 = \frac{1}{x_0}, y_2 = -1, y_1 = 0$. Thus we get the imperfect triplet (d).

Example:

We construct an example about an imperfect triplet with form (c).

Put $x_1 = \frac{1}{2}, x_2 = \frac{5}{2}$,

We get the following triplet $x = \left(0, \frac{1}{2} I_1, \frac{5}{2} I_1\right), y = (0,0, I_2), z = \left(0, -\frac{1}{15} I_1, \frac{2}{5} I_1\right)$

Theorem :

Let $Q(I_1, I_2)$, $R(I_1, I_2)$ be the neutrosophic ring of rationales and reals respectively. Consider the following two forms of imperfect duplets

5-)$\{(x_0, x_1 I_1, -x_0 I_2),(1, y_1 I_1, -y_1 I_2); x_0 \neq 0\}$,

6-)$\{(x_0, x_1 I_1, x_2 I_2),(1, y_1 I_1, 0); x_0 + x_1 + x_2 = 0\}$.

The corresponding imperfect triplets are:

(e) $\{(x_0, x_1 I_1, -x_0 I_2),(1, I_1, -I_2), (\frac{1}{x_0}, \frac{1}{x_1} I_1, \frac{-1}{x_0} I_2); x_0, x_1 \neq 0\}$,

(f)$\{(x_0, x_1 I_1, x_2 I_2),(1, -I_1, 0), (\frac{1}{x_0}, z_1 I_1, \frac{-x_2}{x_0(x_0+x_2)} I_2); x_0 + x_1 + x_2 = 0, x_0, x_0 + x_2 \neq 0, z_1 \text{ is arbitrary}\}$.

Proof:

The proof holds by a similar discussion.

Example:

We construct an example about an imperfect triplet with form (e).

Put $x_0 = 1, x_1 = \frac{1}{2}$. Thus we get the following triplet

$$x = \left(1, \frac{1}{2} I_1, -I_2\right), y = (1, I_1, -I_2), z = (1, 2I_1, -I_2).$$

Theorem :

Let $Q(I_1, I_2)$, $R(I_1, I_2)$ be the neutrosophic ring of rationales and reals respectively. Imperfect triplets has the following 6 forms (a), (b), (c), (d), (e), (f).

The proof holds directly from previous Theorems .

Now, we check the existence of imperfect triplets in the refined neutrosophic ring of integers $Z(I_1, I_2)$.

Remark:

A triple (x, y, z) is an imperfect triplet in $Z(I_1, I_2)$ if it can be represented by one of the forms (a-f) in Theorem 3.14 as a result of the inclusion between $Z(I_1, I_2)$ and $Q(I_1, I_2)$.

This means that the all imperfect triplets in $Z(I_1, I_2)$ are:

(a-)$\{(0, x_1 I_1, 0), (0, I_1, 0I_2), \left(0, \frac{1}{x_1} I_1, 0\right);\ x_1 \in \{-1,1\}\},$

(b-) $\{(0, x_1 I_1, -x_1 I_1), (0, -I_1, I_2), (0, \frac{1}{x_1} I_1, -\frac{1}{x_1} I_1);\ x_1 \in \{-1,1\}\}.$

(c)$\{(0, x_1 I_1, x_2 I_1), (0,0, I_2), \left(0, \frac{-x_1}{x_2(x_1+x_2)} I_1, \frac{1}{x_2} I_2\right); x_2 \in \{-1,1\}\ and\ \frac{-x_1}{x_2(x_1+x_2)}\ \in Z\}$

(d) $\{(x_0, 0, -x_0 I_2), (1, 0, -I_2), (\frac{1}{x_0}, 0, \frac{-1}{x_0} I_2); x_0 \in \{-1,1\}\}.$

(e) $\{(x_0, x_1 I_1, -x_0 I_2), (1, I_1, -I_2), (\frac{1}{x_0}, \frac{1}{x_1} I_1, \frac{-1}{x_0} I_2); x_0, x_1 \in \{-1,1\}\},$

(f)$\{(x_0, x_1 I_1, x_2 I_2), (1, -I_1, 0), (\frac{1}{x_0}, z_1 I_1, \frac{-x_2}{x_0(x_0+x_2)} I_2); x_0 + x_1 + x_2 = 0, x_0\ \in$
$\{-1,1\}\ and\ z_1, \frac{-x_2}{x_0(x_0+x_2)} \in Z\}.$

Example:

The forms (a),(b),(d),(e) are clear. We illustrate an example about an imperfect triplet of form (c).

We put $x_2 = 1$, $x_1 = -2$, so that we get the following triplet

.$x = (0, -2I_1, I_2), y = (0,0, I_2), z = (0, -2I_1, I_2)$

Another example about imperfect triplets with form (f).

Put $x_0 = x_1 = 1, x_2 = -2, z_1 = 5$, we get the following triplet

$x = (1, I_1, -2I_2), y = (1, -I_1, 0), z = (1,5I_1, -2I_2)$.

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