Solving An Open Problem About Kothe's Conjecture In Refined Neutrosophic Rings

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Abstract: This paper is dedicated to prove the equivalence between Kothe's conjecture in the classical ring R and the corresponding Refined neutrosophic ring $R(I_1, I_2)$

Main discussion

In [5], authors have proved that $P = (P_0, P_1 I_1, P_2 I_2)$ is a full ideal of $R(I_1, I_2)$ if and only if P_0, P_1, P_2 *are ideals in R with* $P_0 \leq P_2 \leq P_1$, under the condition that R has unity. First of all, we give an example to show that this statement is not true if R has not a unity.

Example :

Consider $2Z(I_1, I_2) = \{(2a, 2bI_1, 2cI_2); a, b, c \in Z\}$ the refined neutrosophic ring of even integers, let $P = (2Z, 4ZI_1, 4ZI_2) = \{(2a, 4bI_1, 4cI_2); a, b, c \in Z\}$ be a subset of it. First of all, we show that P is a full ideal of $2Z(I_1, I_2)$. It is easy to see that (P,+) is a subgroup. Let $x = (2m, 4nI_1, 4tI_2)$ be any element of P, $r = (2a, 2bI_1, 2cI_2)$ be any element of $2Z(I_1, I_2)$, we have:

$rx = (4am, I_1[8an + 4bm + 8bn + 8bt + 8cn], I_2[8at + 8ct + 4cm]) \in P$. Thus P is a full ideal and the inclusion's condition is not available, that is because 2Z is not contained in 4Z.

The following theorem describes the structure of nil ideals in $R(I_1, I_2)$.

Theorem :

Let $R(I_1, I_2)$ be any neutrosophic ring, we have:

(a) (x, yI_1, zI_2) is nilpotent in $R(I_1, I_2)$ if and only if $x, x + z, x + y + z$ are nilpotent elements in R.

(b) If $P = (Q, MI_1, NI_2)$ is an ideal of $R(I_1, I_2)$, then P is nilpotent if and only if $Q, M, N, Q + N, Q + M + N$ are nilpotent.

(c) If $P = (Q, MI_1, NI_2)$ is a right/left ideal of $R(I_1, I_2)$, then P is nil if and only if $Q, M, N, Q + N, Q + M + N$ are nil.

Proof:

(a) First of all, we prove that $(x, yI_1, zI_2)^n = (x^n, I_1[(x + y + z)^n - (x + z)^n], I_2[(x + z)^n - x^n])$, where n is any positive integer.

For n=1 it is clear. We suppose that it is true for $n = k$, we shall prove it for k+1.

$(x, yI_1, zI_2)^{k+1} = (x, yI_1, zI_2)^k (x, yI_1, zI_2) = (x^k, I_1[(x + y + z)^k - (x + z)^k], I_2[(x + z)^k - x^k]).(x, yI_1, zI_2) = (x^{k+1}, I_1[x(x + y + z)^k - x(x + z)^k + y(x + y + z)^k - y(x + z)^k + yx^k - yx^k + y(x + z)^k + z(x + y + z)^k - z(x + z)^k], I_2[x(x + z)^k - xx^k + z(x + z)^k + zx^k - zx^k])$.

Hence $(x, yI_1, zI_2)^{k+1} = (x^{k+1}, I_1[(x + y + z)^{k+1} - (x + z)^{k+1}], I_2[(x + z)^{k+1} - x^{k+1}])$

Thus it is true by induction.

Now, we suppose that (x, yI_1, zI_2) is nilpotent in $R(I_1, I_2)$, hence there is a positive integer n such that

. By the previous statement, we get $x^n = 0$ and $(x+z)^n - x^n = 0$, and $(x+y+(x,yI_1,zI_2)^n = 0$ $z)^n - (x+z)^n$, $thus$ $(x+y+z)^n = (x+z)^n = 0$. Thus $x, x+y+z, x+z$ are nilpotent elements in R. The converse is clear.

(b) Let $P = (Q, MI_1, NI_2)$ be a nilpotent ideal of $R(I_1, I_2)$, then there exists a positive integer n such that $P^n = \{0\}$.

For any element $x \in Q$ we have $(x, 0,0) \in P$, hence $x^n = 0$, and Q is nilpotent.

On the other hand for any element $y \in M$ we have $(0, yI_1, 0) \in (0, MI_1, 0) \leq P$, hence $(0, yI_1, 0)^n = \{0\}$, $thus$ $y^n = 0$, and M is nilpotent . By the same argument, we get that N is nilpotent.

Now, for every $x \in Q, y \in M, z \in N$, we have $A = (x, yI_1, zI_2) \in P$, by the assumption of the nilpotency of P, we get $A^n = (x^n, I_1[(x+y+z)^n - (x+z)^n], I_2[(x+z)^n - x^n]) = 0$, hence $x^n = 0$ and $(x+z)^n - x^n = 0$, and $(x+y+z)^n - (x+z)^n$, $thus$ $(x+y+z)^n = (x+z)^n = 0$, which implies that $Q+N, Q+M+N$ are nilpotent.

The converse is easy and clear.

(c) Let $P = (Q, MI_1, NI_2)$ be a nil ideal of $R(I_1, I_2)$, and $A = (x, yI_1, zI_2)$ be an arbitrary element of P, then there exists a positive integer n such that $A^n = (x^n, I_1[(x+y+z)^n - (x+z)^n], I_2[(x+z)^n - x^n]) = 0$, thus $x^n = 0, (x+y+z)^n = (x+z)^n = 0$, so that $Q, Q+M+N, Q+M$ are nil.

Also, N, M are nil ideals, that is because $M, N \leq Q+M+N$.

For the converse, we assume that $Q, M, N, Q+N, Q+M+N$ are nil ideals in the classical ring R, we must prove that $P = (Q, MI_1, NI_2)$ is a nil ideal of $R(I_1, I_2)$.

Let $A = (x, yI_1, zI_2)$ be an arbitrary element of P, we have $x \in Q, y \in M, z \in N$. There exists three positive integers m, n, k such that $x^n = (x+y+z)^m = (x+z)^k = 0$. Now, we compute

$A^{m+n+K} = (x^{m+n+k}, I_1[(x+y+z)^{m+n+k} - (x+z)^{m+n+k}], I_2[(x+z)^{m+n+k} - x^{m+n+k}]) =$
. This means that P is a nil ideal of the refined neutrosophic ring $R(I_1, I_2)$.$(0,0,0) = 0$

The following theorem shows the equivalence between Kothe's conjecture in the classical ring R and the corresponding refined neutrosophic ring $R(I_1, I_2)$.

Theorem :

Kothe's conjecture is true in the refined neutrosophic ring $R(I_1, I_2)$ if and only if it is true in the corresponding classical ring R.

Proof:

If Kothe's conjecture is true in $R(I_1, I_2)$, then it is true in R, that is because R is a homomorphic image of $R(I_1, I_2)$. See [9].

Now, suppose that Kothe's conjecture is true in R. If $P = (Q, MI_1, NI_2), L = (S, TI_1, GI_2)$ are two nil ideals of $R(I_1, I_2)$, then by theorem , we get $Q, Q+N, Q+M+N, S, S+G, S+T+G$ are nil in R, hence $P+L = (Q+S, [M+T]I_1, [N+G]I_2)$ is a nil ideal in $R(I_1, I_2)$, that is because are nil in R (Since the Kothe's conjecture is true in the $Q+S, Q+S+N+G, Q+S+N+G+M+T$ ring R by the assumption). This implies that Kothe's conjecture is true in the refined neutrosophic ring $R(I_1, I_2)$.

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