# Path Coloring Numbers of Neutrosophic Graphs Based on Shared Edges and Neutrosophic Cardinality of Edges With Some Applications from Real-World Problems

**Preprint** · May 2022

DOI: 10.13140/RG.2.2.30105.70244



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# Path Coloring Numbers of Neutrosophic Graphs Based on Shared Edges and Neutrosophic Cardinality of Edges With Some Applications from Real-World Problems

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## Abstract

New setting is introduced to study path-coloring number and neutrosophic path-coloring number arising from different types of paths based on shared edges amid them in neutrosophic graphs assigned to neutrosophic graphs. Consider two vertices. Minimum number of shared edges based on those vertices in the formations of all paths with those vertices as their starts and their ends to compare with other paths, is a number which is representative based on those vertices. Minimum neutrosophic number of shared edges amid neutrosophic cardinality of all sets of shared edges is called neutrosophic path-coloring number. Forming sets from special paths to figure out different types of number of paths having smallest number of colors from shared edges from two vertices are given in the terms of minimum number of paths to get minimum number to assign to neutrosophic graphs is key type of approach to have these notions namely path-coloring number and neutrosophic path-coloring number arising from different types of paths based on shared edges amid them in neutrosophic graphs assigned to neutrosophic graphs. Two numbers and one set are assigned to a neutrosophic graph, are obtained but now both settings lead to approach is on demand which is to compute and to find representatives of paths having smallest number of colors from shared edges from two vertices are given forming different types of sets of paths in the terms of minimum number and minimum neutrosophic number forming it to get minimum number to assign to a neutrosophic graph. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then for given two vertices, $x$ and $y$, there are some paths from $x$ to $y$. If two paths from $x$ to $y$ share one edge, then they're assigned to different colors. The set of colors in this process is called path-coloring set from $x$ to $y$. The minimum cardinality between all path-coloring sets from two given vertices is called path-coloring number and it's denoted by $\mathcal{L}(NTG)$; for given two vertices, $x$ and $y$, there are some paths from $x$ to $y$. If two paths from $x$ to $y$ share one edge, then they're assigned to different colors. The set $S$ of shared edges in this process is called path-coloring set from $x$ to $y$. The minimum neutrosophic cardinality, $\sum_{e\in S}\sum_{i=1}^{3}\mu_i(e)$, between all path-coloring sets, $S$s, is called neutrosophic path-coloring number and it's denoted by $\mathcal{L}_n(NTG)$. As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs, complete-t-partite-neutrosophic graphs and wheel-neutrosophic graphs. The clarifications are also presented in both sections

"Setting of path-coloring number," and "Setting of neutrosophic path-coloring number," for introduced results and used classes. This approach facilitates identifying paths which form path-coloring number and neutrosophic path-coloring number arising from different types of paths based on shared edges amid them in neutrosophic graphs assigned to neutrosophic graphs. In both settings, some classes of well-known neutrosophic graphs are studied. Some clarifications for each result and each definition are provided. The cardinality of set of shared edges and neutrosophic cardinality of set of shared edges have eligibility to define path-coloring number and neutrosophic path-coloring number but different types of shared edges have eligibility to define path-coloring sets. Some results get more frameworks and perspective about these definitions. The way in that, different types of shared edges having smallest number from all paths from two vertices are given forming different types of sets in the terms of minimum number of shared edges having smallest number of different paths from two vertices are given and smallest number of shared edges having smallest number of paths from two vertices are given forming it to get minimum number to assign to neutrosophic graphs or in other words, the way in that, consider two vertices. Minimum number of shared edges based on those vertices in the formations of all paths with those vertices as their starts and their ends to compare with other paths, is a number which is representative based on those vertices; Minimum neutrosophic number of shared edges amid neutrosophic cardinality of all sets of shared edges is called neutrosophic path-coloring number, opens the way to do some approaches. These notions are applied into neutrosophic graphs as individuals but not family of them as drawbacks for these notions. Finding special neutrosophic graphs which are well-known, is an open way to pursue this study. Neutrosophic path-coloring notion is applied to different settings and classes of neutrosophic graphs. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.

**Keywords:** Path-Coloring Number, Neutrosophic Path-Coloring number, Classes of Neutrosophic Graphs

**AMS Subject Classification:** 05C17, 05C22, 05E45

# 1 Background

Fuzzy set in **Ref.** [17], intuitionistic fuzzy sets in **Ref.** [2], a first step to a theory of the intuitionistic fuzzy graphs in **Ref.** [14], a unifying field in logics neutrosophy: neutrosophic probability, set and logic, rehoboth in **Ref.** [15], single-valued neutrosophic graphs in **Ref.** [3], operations on single-valued neutrosophic graphs in **Ref.** [1], neutrosophic soft graphs in **Ref.** [13], the complexity of path coloring and call scheduling in **Ref.** [7], collision-free path coloring with application to minimum-delay gathering in sensor networks in **Ref.** [8], pack: path coloring based k-connectivity detection algorithm for wireless sensor networks in **Ref.** [5], a 2-approximation algorithm for path coloring on a restricted class of trees of rings in **Ref.** [6], the permutation-path coloring problem on trees in **Ref.** [4], dimension and coloring alongside domination in neutrosophic hypergraphs in **Ref.** [10], three types of neutrosophic alliances based on connectedness and (strong) edges in **Ref.** [12], properties of SuperHyperGraph and neutrosophic SuperHyperGraph in **Ref.** [11], are studied. Also, some studies and researches about neutrosophic graphs, are proposed as a book in **Ref.** [9].

In this section, I use two subsections to illustrate a perspective about the background of this study.

## 1.1 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

**Question 1.1.** *Is it possible to use mixed versions of ideas concerning "path-coloring number", "neutrosophic path-coloring number" and "Neutrosophic Graph" to define some notions which are applied to neutrosophic graphs?*

It's motivation to find notions to use in any classes of neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Having connection amid two paths have key roles to assign path-coloring number and neutrosophic path-coloring number arising from different types of paths based on shared edges amid them in neutrosophic graphs assigned to neutrosophic graphs. Thus they're used to define new ideas which conclude to the structure of path-coloring number and neutrosophic path-coloring number arising from different types of paths based on shared edges amid them in neutrosophic graphs assigned to neutrosophic graphs. The concept of having smallest number of paths with shared edge from two vertices are given inspires us to study the behavior of all paths in the way that, some types of numbers, path-coloring number and neutrosophic path-coloring number arising from different types of paths based on shared edges amid them in neutrosophic graphs assigned to neutrosophic graphs, are the cases of study in the setting of individuals. In both settings, a corresponded numbers concludes the discussion. Also, there are some avenues to extend these notions.

The framework of this study is as follows. In the beginning, I introduce basic definitions to clarify about preliminaries. In subsection "Preliminaries", new notions of path-coloring number and neutrosophic path-coloring number arising from different types of paths based on shared edges amid them in neutrosophic graphs assigned to neutrosophic graphs, are highlighted, are introduced and are clarified as individuals. In section "Preliminaries", different types of paths and minimum numbers of shared edges amid them from two vertices are neighbors forming different types of sets in the terms of minimum numbers and minimal sets forming it to get minimum number to assign to neutrosophic graphs, have the key role in this way. General results are obtained and also, the results about the basic notions of path-coloring number and neutrosophic path-coloring number arising from different types of paths based on shared edges amid them in neutrosophic graphs assigned to neutrosophic graphs, are elicited. Some classes of neutrosophic graphs are studied in the terms of path-coloring number and neutrosophic path-coloring number arising from different types of paths based on shared edges amid them in neutrosophic graphs assigned to neutrosophic graphs, in section "Setting of path-coloring number," as individuals. In section "Setting of path-coloring number," path-coloring number is applied into individuals. As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs, complete-t-partite-neutrosophic graphs and wheel-neutrosophic graphs. The clarifications are also presented in both sections "Setting of path-coloring number," and "Setting of neutrosophic path-coloring number," for introduced results and used classes. In section "Applications in Time Table and Scheduling", two applications are posed for quasi-complete and complete notions, namely complete-neutrosophic graphs and complete-t-partite-neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects and the mentioned models are considered as individual. In section "Open Problems", some problems and questions for further studies are proposed. In section "Conclusion and Closing Remarks", gentle discussion about results and applications is featured. In section "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions is formed.

## 1.2 Preliminaries

In this subsection, basic material which is used in this article, is presented. Also, new ideas and their clarifications are elicited.

Basic idea is about the model which is used. First definition introduces basic model.

**Definition 1.2.** (Graph).

$G = (V, E)$ is called a **graph** if $V$ is a set of objects and $E$ is a subset of $V \times V$ ($E$ is a set of 2-subsets of $V$) where $V$ is called **vertex set** and $E$ is called **edge set**. Every two vertices have been corresponded to at most one edge.

Neutrosophic graph is the foundation of results in this paper which is defined as follows. Also, some related notions are demonstrated.

**Definition 1.3.** (Neutrosophic Graph And Its Special Case).

$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, and $\mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_i v_j \in E$,

$$\mu(v_i v_j) \le \sigma(v_i) \wedge \sigma(v_j).$$

$(i)$ : $\sigma$ is called **neutrosophic vertex set**.

$(ii)$ : $\mu$ is called **neutrosophic edge set**.

$(iii)$ : $|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$.

$(iv)$ : $\sum_{v \in V} \sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

$(v)$ : $|E|$ is called **size** of NTG and it's denoted by $\mathcal{L}(NTG)$.

$(vi)$ : $\sum_{e \in E} \sum_{i=1}^{3} \mu_i(e)$ is called **neutrosophic size** of NTG and it's denoted by $\mathcal{S}_n(NTG)$.

Some classes of well-known neutrosophic graphs are defined. These classes of neutrosophic graphs are used to form this study and the most results are about them.

**Definition 1.4.** Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

$(i)$ : a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is called **path** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, \mathcal{O}(NTG) - 1$;

$(ii)$ : **strength** of path $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is $\bigwedge_{i=0,\cdots,\mathcal{O}(NTG)-1} \mu(x_i x_{i+1})$;

$(iii)$ : **connectedness** amid vertices $x_0$ and $x_t$ is

$$\mu^{\infty}(x_0, x_t) = \bigvee_{P : x_0, x_1, \cdots, x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1});$$

$(iv)$ : a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}, x_0$ is called **cycle** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, \mathcal{O}(NTG) - 1,\ x_{\mathcal{O}(NTG)} x_0 \in E$ and there are two edges $xy$ and $uv$ such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$;

$(v)$ : it's **t-partite** where $V$ is partitioned to $t$ parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge $xy$ implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's complete, then it's denoted by $K_{\sigma_1, \sigma_2, \cdots, \sigma_t}$ where $\sigma_i$ is $\sigma$ on $V_i^{s_i}$ instead $V$ which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$;

$(vi):$ t-partite is **complete bipartite** if $t = 2$, and it's denoted by $K_{\sigma_1,\sigma_2}$;

$(vii):$ complete bipartite is **star** if $|V_1| = 1$, and it's denoted by $S_{1,\sigma_2}$;

$(viii):$ a vertex in $V$ is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by $W_{1,\sigma_2}$;

$(ix):$ it's **complete** where $\forall uv \in V,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$;

$(x):$ it's **strong** where $\forall uv \in E,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$.

To make them concrete, I bring preliminaries of this article in two upcoming definitions in other ways.

**Definition 1.5.** (Neutrosophic Graph And Its Special Case).
$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, and $\mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_iv_j \in E$,

$$\mu(v_iv_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

$|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$. $\Sigma_{v\in V}\sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

**Definition 1.6.** Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then it's **complete** and denoted by $CMT_\sigma$ if $\forall x, y \in V, xy \in E$ and $\mu(xy) = \sigma(x) \wedge \sigma(y)$; a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is called **path** and it's denoted by $PTH$ where $x_ix_{i+1} \in E,\ i = 0, 1, \cdots, n-1$; a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}, x_0$ is called **cycle** and denoted by $CYC$ where $x_ix_{i+1} \in E,\ i = 0, 1, \cdots, n-1,\ x_{\mathcal{O}(NTG)}x_0 \in E$ and there are two edges $xy$ and $uv$ such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_iv_{i+1})$; it's **t-partite** where $V$ is partitioned to $t$ parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge $xy$ implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's **complete**, then it's denoted by $CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}$ where $\sigma_i$ is $\sigma$ on $V_i^{s_i}$ instead $V$ which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$; t-partite is **complete bipartite** if $t = 2$, and it's denoted by $CMT_{\sigma_1,\sigma_2}$; complete bipartite is **star** if $|V_1| = 1$, and it's denoted by $STR_{1,\sigma_2}$; a vertex in $V$ is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by $WHL_{1,\sigma_2}$.

*Remark* 1.7. Using notations which is mixed with literatures, are reviewed.

1. $NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3)),\ \mathcal{O}(NTG)$, and $\mathcal{O}_n(NTG)$;
2. $CMT_\sigma, PTH, CYC, STR_{1,\sigma_2}, CMT_{\sigma_1,\sigma_2}, CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}$, and $WHL_{1,\sigma_2}$.

**Definition 1.8.** (path-coloring numbers).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

$(i)$ for given two vertices, $x$ and $y$, there are some paths from $x$ to $y$. If two paths from $x$ to $y$ share one edge, then they're assigned to different colors. The set of colors in this process is called **path-coloring set** from $x$ to $y$. The minimum cardinality between all path-coloring sets from two given vertices is called **path-coloring number** and it's denoted by $\mathcal{L}(NTG)$;

$(ii)$ for given two vertices, $x$ and $y$, there are some paths from $x$ to $y$. If two paths from $x$ to $y$ share one edge, then they're assigned to different colors. The set $S$ of shared edges in this process is called **path-coloring set** from $x$ to $y$. The minimum neutrosophic cardinality, $\sum_{e\in S}\sum_{i=1}^{3}\mu_i(e)$, between all path-coloring sets, $S$s, is called **neutrosophic path-coloring number** and it's denoted by $\mathcal{L}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

**Example 1.9.** In Figure (1), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. New viewpoint implies different kinds of definitions to get more scrutiny and more discernment.

$(i)$ Consider two vertices $n_1$ and $n_2$. All paths are as follow:

$$\begin{aligned} P_1 &: n_1, n_2 \rightarrow \text{red} \\ P_2 &: n_1, n_3, n_2 \rightarrow \text{red} \\ P_3 &: n_1, n_4, n_2 \rightarrow \text{red} \\ P_4 &: n_1, n_3, n_4, n_2 \rightarrow \text{blue} \\ P_5 &: n_1, n_4, n_3, n_2 \rightarrow \text{yellow} \end{aligned}$$

The paths $P_1$, $P_2$ and $P_3$ has no shared edge so they've been colored the same as red. The path $P_4$ has shared edge $n_1n_3$ with $P_2$ and shared edge $n_4n_2$ with $P_3$ thus it's been colored the different color as blue in comparison to them. The path $P_5$ has shared edge $n_1n_4$ with $P_3$ and shared edge $n_3n_4$ with $P_4$ thus it's been colored the different color as yellow in comparison to different paths in the terms of different colors. Thus $S = \{\text{red}, \text{blue}, \text{yellow}\}$ is path-coloring set and its cardinality, 3, is path-coloring number. To sum them up, for given two vertices, $x$ and $y$, there are some paths from $x$ to $y$. If two paths from $x$ to $y$ share one edge, then they're assigned to different colors. The set of colors, $S = \{\text{red}, \text{blue}, \text{yellow}\}$, in this process is called path-coloring set from $x$ to $y$. The minimum cardinality between all path-coloring sets from two given vertices, 3, is called path-coloring number and it's denoted by $\mathcal{L}(NTG) = 3$;

$(ii)$ all vertices have same positions in the matter of creating paths. So for every two given vertices, the number and the behaviors of paths are the same;

$(iii)$ there are three different paths which have no shared edges. So they've been assigned to same color;

$(iv)$ shared edges form a set of representatives of colors. Each color is corresponded to an edge which has minimum neutrosophic cardinality;

$(v)$ every color in $S$ is corresponded to an edge has minimum neutrosophic cardinality. Minimum neutrosophic cardinality is obtained in this way but other way is to use all shared edges to form $S$ and after that minimum neutrosophic cardinality is optimal;

$(vi)$ two edges $n_1n_3$ and $n_4n_2$ are shared with $P_4$ by $P_3$ and $P_2$. The minimum neutrosophic cardinality is 0.6 corresponded to $n_4n_2$. Other corresponded color has only one shared edge $n_3n_4$ and minimum neutrosophic cardinality is 0.9. Thus minimum neutrosophic cardinality is 1.5. And corresponded set is $S = \{n_4n_2, n_3n_4\}$. To sum them up, for given two vertices, $x$ and $y$, there are some paths from $x$ to $y$. If two paths from $x$ to $y$ share one edge, then they're assigned to different colors. The set $S = \{n_4n_2, n_3n_4\}$ of shared edges in this process is called path-coloring set from $x$ to $y$. The minimum neutrosophic cardinality, $\sum_{e \in S} \sum_{i=1}^{3} \mu_i(e)$, between all path-coloring sets, $S$s, is called neutrosophic path-coloring number and it's denoted by $\mathcal{L}_n(NTG) = 1.5$.

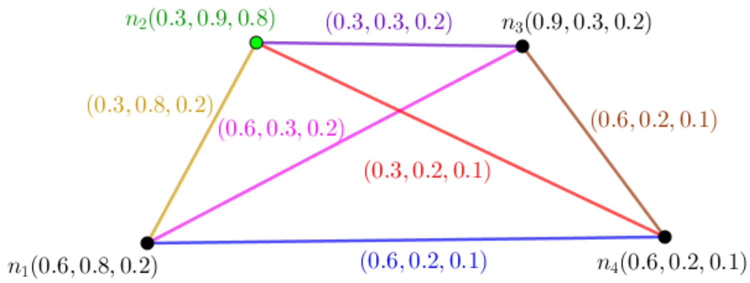


**Figure 1.** A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number.

## 2 Setting of path-coloring number

In this section, I provide some results in the setting of path-coloring number. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, star-neutrosophic graph, bipartite-neutrosophic graph, t-partite-neutrosophic graph, and wheel-neutrosophic graph, are both of cases of study and classes which the results are about them.

**Proposition 2.1.** *Let* $NTG : (V, E, \sigma, \mu)$ *be a complete-neutrosophic graph. Then*

$$\mathcal{L}(CMT_\sigma) = \min_S |S|.$$

*Proof.* Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. The number of vertices is $\mathcal{O}(CMT_\sigma)$. For given two vertices, $x$ and $y$, there are some paths from $x$ to $y$. If two paths from $x$ to $y$ share one edge, then they're assigned to different colors. The set of colors in this process is called path-coloring set from $x$ to $y$. The minimum cardinality between all path-coloring sets from two given vertices is called path-coloring number and it's denoted by $\mathcal{L}(CMT_\sigma)$. Thus

$$\mathcal{L}(CMT_\sigma) = \min_S |S|.$$

□

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

**Example 2.2.** In Figure (2), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. New viewpoint implies different kinds of definitions to get more scrutiny and more discernment.

(*i*) Consider two vertices $n_1$ and $n_2$. All paths are as follow:

$$P_1 : n_1, n_2 \to \text{red}$$
$$P_2 : n_1, n_3, n_2 \to \text{red}$$
$$P_3 : n_1, n_4, n_2 \to \text{red}$$
$$P_4 : n_1, n_3, n_4, n_2 \to \text{blue}$$
$$P_5 : n_1, n_4, n_3, n_2 \to \text{yellow}$$

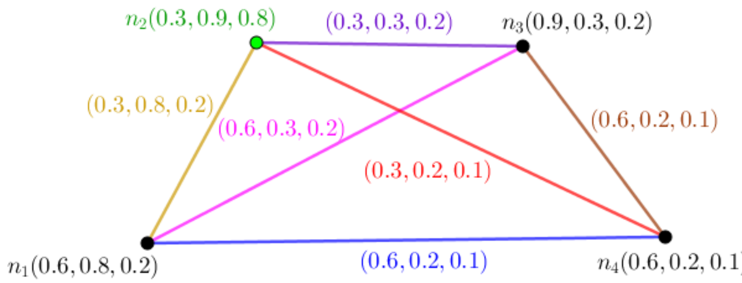


**Figure 2.** A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number.

The paths $P_1,\ P_2$ and $P_3$ has no shared edge so they've been colored the same as red. The path $P_4$ has shared edge $n_1n_3$ with $P_2$ and shared edge $n_4n_2$ with $P_3$ thus it's been colored the different color as blue in comparison to them. The path $P_5$ has shared edge $n_1n_4$ with $P_3$ and shared edge $n_3n_4$ with $P_4$ thus it's been colored the different color as yellow in comparison to different paths in the terms of different colors. Thus $S = \{\text{red}, \text{blue}, \text{yellow}\}$ is path-coloring set and its cardinality, 3, is path-coloring number. To sum them up, for given two vertices, $x$ and $y$, there are some paths from $x$ to $y$. If two paths from $x$ to $y$ share one edge, then they're assigned to different colors. The set of colors, $S = \{\text{red}, \text{blue}, \text{yellow}\}$, in this process is called path-coloring set from $x$ to $y$. The minimum cardinality between all path-coloring sets from two given vertices, 3, is called path-coloring number and it's denoted by $\mathcal{L}(CMT_\sigma) = 3$;

$(ii)$ all vertices have same positions in the matter of creating paths. So for every two given vertices, the number and the behaviors of paths are the same;

$(iii)$ there are three different paths which have no shared edges. So they've been assigned to same color;

$(iv)$ shared edges form a set of representatives of colors. Each color is corresponded to an edge which has minimum neutrosophic cardinality;

$(v)$ every color in $S$ is corresponded to an edge has minimum neutrosophic cardinality. Minimum neutrosophic cardinality is obtained in this way but other way is to use all shared edges to form $S$ and after that minimum neutrosophic cardinality is optimal;

$(vi)$ two edges $n_1n_3$ and $n_4n_2$ are shared with $P_4$ by $P_3$ and $P_2$. The minimum neutrosophic cardinality is 0.6 corresponded to $n_4n_2$. Other corresponded color has only one shared edge $n_3n_4$ and minimum neutrosophic cardinality is 0.9. Thus minimum neutrosophic cardinality is 1.5. And corresponded set is $S = \{n_4n_2, n_3n_4\}$. To sum them up, for given two vertices, $x$ and $y$, there are some paths from $x$ to $y$. If two paths from $x$ to $y$ share one edge, then they're assigned to different colors. The set $S = \{n_4n_2, n_3n_4\}$ of shared edges in this process is called path-coloring set from $x$ to $y$. The minimum neutrosophic cardinality, $\sum_{e \in S} \sum_{i=1}^{3} \mu_i(e)$, between all path-coloring sets, $S$s, is called neutrosophic path-coloring number and it's denoted by $\mathcal{L}_n(CMT_\sigma) = 1.5$.

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

**Proposition 2.3.** *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Then*

$$\mathcal{L}(PTH) = 1.$$

*Proof.* Suppose $PTH : (V, E, \sigma, \mu)$ is a path-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(PTH)}$ be a path-neutrosophic graph. For given two vertices, $x$ and $y$, there's one paths from $x$ to $y$. If two paths from $x$ to $y$ share one edge, then they're assigned to different colors but there's only one path with certain start and end. The set of colors,{red}, in this process is called path-coloring set from $x$ to $y$. The minimum cardinality between all path-coloring sets, 1, from two given vertices is called path-coloring number and it's denoted by $\mathcal{L}(PTH)$. Thus

$$\mathcal{L}(PTH) = 1.$$

□

**Example 2.4.** There are two sections for clarifications.

($a$) In Figure (3), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. New viewpoint implies different kinds of definitions to get more scrutiny and more discernment.

($i$) All paths are as follows.

$$\begin{aligned}
n_1, n_2 &\rightarrow \text{red}\\
n_1, n_2, n_3 &\rightarrow \text{blue}\\
n_1, n_2, n_3, n_4 &\rightarrow \text{yellow}\\
n_1, n_2, n_3, n_4, n_5 &\rightarrow \text{green}\\
n_2, n_3 &\rightarrow \text{red}\\
n_2, n_3, n_4 &\rightarrow \text{pink}\\
n_2, n_3, n_4, n_5 &\rightarrow \text{brown}\\
n_3, n_4 &\rightarrow \text{red}\\
n_3, n_4, n_5 &\rightarrow \text{blue}\\
n_4, n_5 &\rightarrow \text{red}\\
&\text{The number is } 6;
\end{aligned}$$

($ii$) 1-paths have same color;

($iii$) $\mathcal{L}(PTH) = 6$;

($iv$) the position of given vertices could be different in the terms of creating path and the behaviors in path;

($v$) every color is corresponded to some shared edges. Minimum neutrosophic cardinality of edges corresponded to specific color is a representative for that color. Thus every color is corresponded one neutrosophic cardinality of some edges since edges could have same neutrosophic cardinality with exception of initial color. So the summation of 5 numbers is neutrosophic path-coloring number. every color is compared with its previous color. The way is a consecutive procedure;

(*vi*) all paths are as follows.

$$n_1, n_2 \to \text{red}$$
$$n_1, n_2, n_3 \to \text{blue} \to n_1n_2 \to \ 0.8$$
$$n_1, n_2, n_3, n_4 \to \text{yellow} \to n_1n_2, n_2n_3 \to \ 1.3$$
$$n_1, n_2, n_3, n_4, n_5 \to \text{green} \to n_1n_2, n_2n_3, n_3n_4 \to \ 2$$
$$n_2, n_3 \to \text{red}$$
$$n_2, n_3, n_4 \to \text{pink} \to n_2n_3, n_3n_4 \to \ 1.3$$
$$n_2, n_3, n_4, n_5 \to \text{brown} \to n_2n_3, n_3n_4 \to \ 1.3$$
$$n_3, n_4 \to \text{red}$$
$$n_3, n_4, n_5 \to \text{blue} \to n_3n_4 \to \ 0.7$$
$$n_4, n_5 \to \text{red}$$
$$\mathcal{L}_n(PTH) \text{ is } 5.6.$$

(*b*) In Figure (4), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. New definition is applied in this section.

(*i*) All paths are as follows.

$$n_1, n_2 \to \text{red}$$
$$n_1, n_2, n_3 \to \text{red}$$
$$n_1, n_2, n_3, n_4 \to \text{red}$$
$$n_1, n_2, n_3, n_4, n_5 \to \text{red}$$
$$n_1, n_2, n_3, n_4, n_5, n_6 \to \text{red}$$
$$n_2, n_3 \to \text{red}$$
$$n_2, n_3, n_4 \to \text{red}$$
$$n_2, n_3, n_4, n_5 \to \text{red}$$
$$n_2, n_3, n_4, n_5, n_6 \to \text{red}$$
$$n_3, n_4 \to \text{red}$$
$$n_3, n_4, n_5 \to \text{red}$$
$$n_4, n_5 \to \text{red}$$
$$\text{The number is } 1;$$

(*ii*) 1-paths have same color;

(*iii*) $\mathcal{L}(PTH) = 1$;

(*iv*) the position of given vertices could be different in the terms of creating path and the behaviors in path;

(*v*) there's only one path. It implies that there's no shared edge;

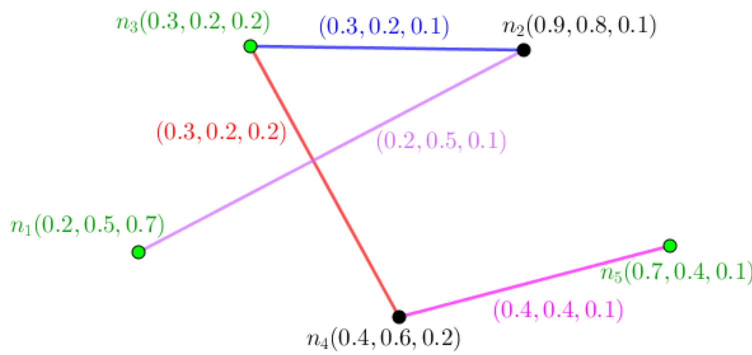


**Figure 3.** A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number.

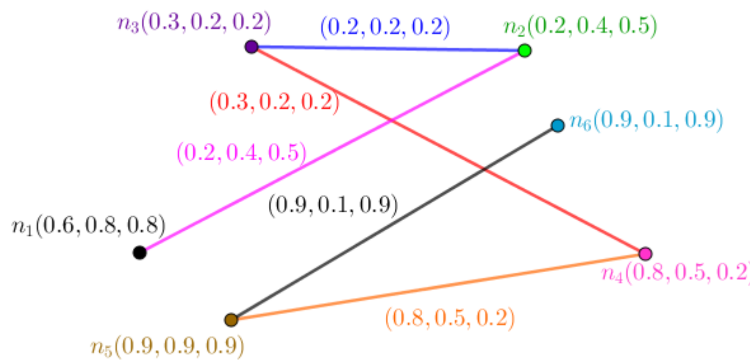


**Figure 4.** A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number.

$(vi)$ all paths are as follows.

$$n_1, n_2 \to \text{red}$$
$$n_1, n_2, n_3 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$n_1, n_2, n_3, n_4 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$n_1, n_2, n_3, n_4, n_5 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$n_2, n_3 \to \text{red}$$
$$n_2, n_3, n_4 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$n_2, n_3, n_4, n_5 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$n_3, n_4 \to \text{red}$$
$$n_3, n_4, n_5 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$n_4, n_5 \to \text{red}$$
$$\mathcal{L}_n(PTH) \text{ is } 0.$$

**Proposition 2.5.** *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*
$$\mathcal{L}(CYC) = 1.$$

*Proof.* Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, $x$ and $y$, there are only two paths from $x$ to $y$. If two paths from $x$ to $y$ share one edge, then they're assigned to different colors but these two paths don't share one edge. The set of colors, {red}, in this process is called path-coloring set from $x$ to $y$. The minimum cardinality between all path-coloring sets from two given vertices, 1, is called path-coloring number and it's denoted by $\mathcal{L}(CYC)$. Thus

$$\mathcal{L}(CYC) = 1.$$

□

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

**Example 2.6.** There are two sections for clarifications.

$(a)$ In Figure (5), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

$(i)$ All paths are as follows.

$$\begin{aligned}
P_1 : n_1, n_2 \ \&\ P_2 : n_1, n_6, n_5, n_4, n_3, n_2 &\to \text{red}\\
P_1 : n_1, n_2, n_3 \ \&\ P_2 : n_1, n_6, n_5, n_4, n_3 &\to \text{red}\\
P_1 : n_1, n_2, n_3, n_4 \ \&\ P_2 : n_1, n_6, n_5, n_4 &\to \text{red}\\
P_1 : n_1, n_2, n_3, n_4, n_5 \ \&\ P_2 : n_1, n_6, n_5 &\to \text{red}\\
P_1 : n_1, n_2, n_3, n_4, n_5, n_6 \ \&\ P_2 : n_1, n_6 &\to \text{red}\\
\text{The number is } 1;
\end{aligned}$$

$(ii)$ 1-paths have same color;

$(iii)$ $\mathcal{L}(CYC) = 1$;

$(iv)$ the position of given vertices could be different in the terms of creating path and the behaviors in path;

$(v)$ there are only two paths but there's no shared edge;

$(vi)$ all paths are as follows.

$$\begin{aligned}
P_1 : n_1, n_2 \ \&\ P_2 : n_1, n_6, n_5, n_4, n_3, n_2 \to \text{red} \to \text{no shared edge} \to &\ 0\\
P_1 : n_1, n_2, n_3 \ \&\ P_2 : n_1, n_6, n_5, n_4, n_3 \to \text{red} \to \text{no shared edge} \to &\ 0\\
P_1 : n_1, n_2, n_3, n_4 \ \&\ P_2 : n_1, n_6, n_5, n_4 \to \text{red} \to \text{no shared edge} \to &\ 0\\
P_1 : n_1, n_2, n_3, n_4, n_5 \ \&\ P_2 : n_1, n_6, n_5 \to \text{red} \to \text{no shared edge} \to &\ 0\\
P_1 : n_1, n_2, n_3, n_4, n_5, n_6 \ \&\ P_2 : n_1, n_6 \to \text{red} \to \text{no shared edge} \to &\ 0\\
\mathcal{L}_n(CYC) \text{ is } 0.
\end{aligned}$$

$(b)$ In Figure (6), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

$(i)$ All paths are as follows.

$$\begin{aligned}
P_1 : n_1, n_2 \ \&\ P_2 : n_1, n_5, n_4, n_3, n_2 &\to \text{red}\\
P_1 : n_1, n_2, n_3 \ \&\ P_2 : n_1, n_5, n_4, n_3 &\to \text{red}\\
P_1 : n_1, n_2, n_3, n_4 \ \&\ P_2 : n_1 n_5, n_4 &\to \text{red}\\
P_1 : n_1, n_2, n_3, n_4, n_5 \ \&\ P_2 : n_1, n_5 &\to \text{red}\\
\text{The number is } 1;
\end{aligned}$$

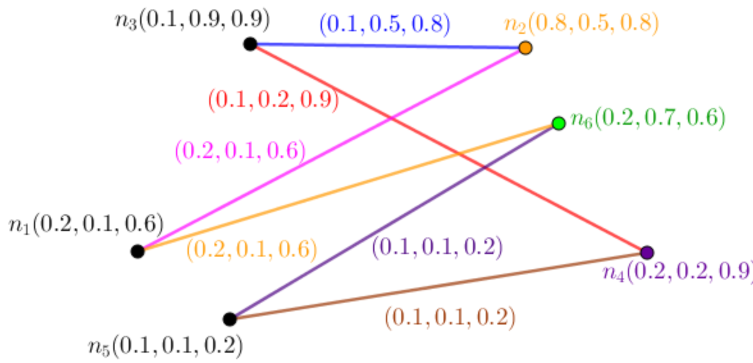


**Figure 5.** A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number.

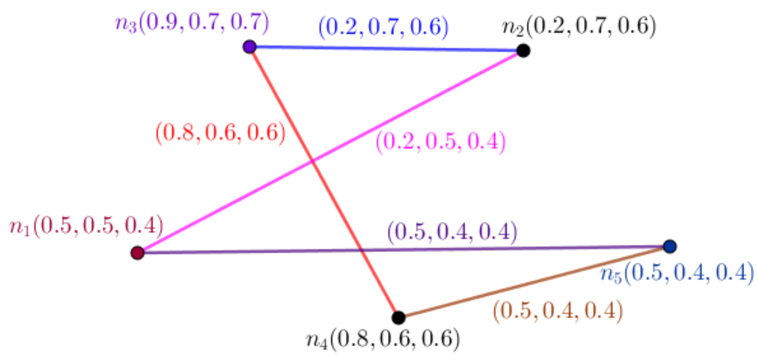


**Figure 6.** A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number.

*(ii)* 1-paths have same color;

*(iii)* $\mathcal{L}(CYC) = 1$;

*(iv)* the position of given vertices could be different in the terms of creating path and the behaviors in path;

*(v)* there are only two paths but there's no shared edge;

*(vi)* all paths are as follows.

$$P_1 : n_1, n_2 \ \& \ P_2 : n_1, n_5, n_4, n_3, n_2 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$P_1 : n_1, n_2, n_3 \ \& \ P_2 : n_1, n_5, n_4, n_3 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$P_1 : n_1, n_2, n_3, n_4 \ \& \ P_2 : n_1, n_5, n_4 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$P_1 : n_1, n_2, n_3, n_4, n_5 \ \& \ P_2 : n_1, n_5 \to \text{red} \to \text{no shared edge} \to \ 0$$
$$\mathcal{L}_n(CYC) \text{ is } 0.$$

**Proposition 2.7.** *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center $c$. Then*

$$\mathcal{L}(STR_{1,\sigma_2}) = 1.$$

*Proof.* Suppose $STR_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. An edge always has center as one of its endpoints. All paths have one as their lengths, forever. So for given vertex, $y$, there's only one path from center to $y$. If two paths from center to $y$ share one edge, then they're assigned to different colors but there's only one path and indeed, there's only one edge. The set of colors, {red}, in this process is called path-coloring set from $x$ to $y$. The minimum cardinality between all path-coloring sets from two given vertices, 1, is called path-coloring number and it's denoted by $\mathcal{L}(STR_{1,\sigma_2})$. Thus

$$\mathcal{L}(STR_{1,\sigma_2}) = 1.$$

□

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

**Example 2.8.** There is one section for clarifications. In Figure (7), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) All paths are as follows.

$$\begin{aligned} P &: n_1, n_2 \to \text{red} \\ P &: n_1, n_3 \to \text{red} \\ P &: n_1, n_4 \to \text{red} \\ P &: n_1, n_5 \to \text{red} \\ &\text{The number is 1;} \end{aligned}$$

(*ii*) 1-paths have same color;

(*iii*) $\mathcal{L}(STR_{1,\sigma_2}) = 1$;

(*iv*) the position of given vertices are the same with only exception the center $n_1$;

(*v*) there's only one path between the vertex $n_1$ and other vertices. There's no shared edge;

(*vi*) all paths are as follows.

$$\begin{aligned} P &: n_1, n_2 \to \text{red} \to \text{no shared edge} \to \ 0 \\ P &: n_1, n_3 \to \text{red} \to \text{no shared edge} \to \ 0 \\ P &: n_1, n_4 \to \text{red} \to \text{no shared edge} \to \ 0 \\ P &: n_1, n_5 \to \text{red} \to \text{no shared edge} \to \ 0 \\ &\mathcal{L}_n(STR_{1,\sigma_2}) \text{ is } 0. \end{aligned}$$

**Proposition 2.9.** *Let $NTG : (V, E, \sigma, \mu)$ be a complete-bipartite-neutrosophic graph. Then*

$$\mathcal{L}(CMC_{\sigma_1,\sigma_2}) = \min_S |S|.$$

*Proof.* Suppose $CMC_{\sigma_1,\sigma_2} : (V, E, \sigma, \mu)$ is a complete-bipartite-neutrosophic graph. For given two vertices, $x$ and $y$, there are some paths from $x$ to $y$. If two paths from $x$ to $y$ share one edge, then they're assigned to different colors. The set of colors in this process is called path-coloring set from $x$ to $y$. The minimum cardinality between all path-coloring sets from two given vertices is called path-coloring number and it's denoted by $\mathcal{L}(CMC_{\sigma_1,\sigma_2})$. Thus

$$\mathcal{L}(CMC_{\sigma_1,\sigma_2}) = \min_S |S|.$$

□

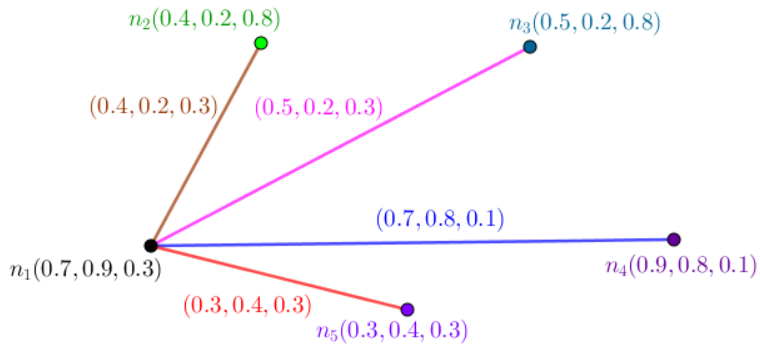


**Figure 7.** A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number.

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

**Example 2.10.** There is one section for clarifications. In Figure (8), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

$(i)$ All paths are as follows.

$$\begin{aligned}
&P_1 : n_1, n_2 \ \& \ P_2 : n_1, n_3, n_4, n_2 \to \text{red} \\
&P_1 : n_1, n_3 \ \& \ P_2 : n_1, n_2, n_4, n_3 \to \text{red} \\
&P_1 : n_1, n_3, n_4 \ \& \ P_2 : n_1, n_2, n_4 \to \text{red} \\
&P_1 : n_2, n_4, n_3 \ \& \ P_2 : n_2, n_1, n_3 \to \text{red} \\
&P_1 : n_2, n_4 \ \& \ P_2 : n_2, n_1, n_3, n_4 \to \text{red} \\
&P_1 : n_3, n_4 \ \& \ P_2 : n_3, n_1, n_2, n_4 \to \text{red} \\
&\text{The number is } 1;
\end{aligned}$$

$(ii)$ 1-paths have same color;

$(iii)$ $\mathcal{L}(CMC_{\sigma_1,\sigma_2}) = 1$;

$(iv)$ the position of given vertices could be different in the terms of creating path and the behaviors in path;

$(v)$ there are only two paths but there's no shared edge;

$(vi)$ all paths are as follows.

$$\begin{aligned}
&P_1 : n_1, n_2 \ \& \ P_2 : n_1, n_3, n_4, n_2 \to \text{red} \to \text{no shared edge} \to \ 0 \\
&P_1 : n_1, n_3 \ \& \ P_2 : n_1, n_2, n_4, n_3 \to \text{red} \to \text{no shared edge} \to \ 0 \\
&P_1 : n_1, n_3, n_4 \ \& \ P_2 : n_1, n_2, n_4 \to \text{red} \to \text{no shared edge} \to \ 0 \\
&P_1 : n_2, n_4, n_3 \ \& \ P_2 : n_2, n_1, n_3 \to \text{red} \to \text{no shared edge} \to \ 0 \\
&P_1 : n_2, n_4 \ \& \ P_2 : n_2, n_1, n_3, n_4 \to \text{red} \to \text{no shared edge} \to \ 0 \\
&P_1 : n_3, n_4 \ \& \ P_2 : n_3, n_1, n_2, n_4 \to \text{red} \to \text{no shared edge} \to \ 0 \\
&\mathcal{L}_n(CMC_{\sigma_1,\sigma_2}) \text{ is } 0.
\end{aligned}$$

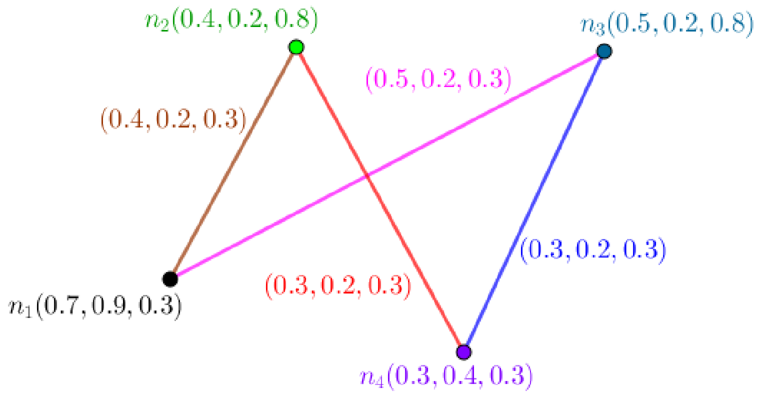


**Figure 8.** A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number.

**Proposition 2.11.** *Let* $NTG : (V, E, \sigma, \mu)$ *be a complete-t-partite-neutrosophic graph where* $t \geq 3$*. Then*

$$\mathcal{L}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \min_S |S|.$$

*Proof.* Suppose $CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t} : (V, E, \sigma, \mu)$ is a complete-t-partite-neutrosophic graph. For given two vertices, $x$ and $y$, there are some paths from $x$ to $y$. If two paths from $x$ to $y$ share one edge, then they're assigned to different colors. The set of colors in this process is called path-coloring set from $x$ to $y$. The minimum cardinality between all path-coloring sets from two given vertices is called path-coloring number and it's denoted by $\mathcal{L}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t})$. Thus

$$\mathcal{L}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \min_S |S|.$$

□

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

**Example 2.12.** There is one section for clarifications. In Figure (9), a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

$(i)$ All paths are as follows.

$$P_1 : n_1, n_2 \ \& \ P_2 : n_1, n_3, n_4, n_2 \ \& \ P_3 : n_1, n_5, n_4, n_2 \to \text{red\&blue}$$
$$P_1 : n_1, n_3 \ \& \ P_2 : n_1, n_2, n_4, n_3 \ \& \ P_3 : n_1, n_5, n_4, n_3 \to \text{red\&blue}$$
$$P_1 : n_1, n_3, n_4 \ \& \ P_2 : n_1, n_2, n_4 \ \& \ P_3 : n_1, n_5, n_4 \to \text{red}$$
$$P_1 : n_1, n_5 \ \& \ P_2 : n_1, n_3, n_4, n_5 \ \& \ P_3 : n_1, n_2, n_4, n_5 \to \text{red\&blue}$$
$$P_1 : n_2, n_4 \ \& \ P_2 : n_2, n_1, n_3, n_4 \ \& \ P_3 : n_2, n_1, n_5, n_4 \to \text{red\&blue}$$
$$P_1 : n_3, n_4 \ \& \ P_2 : n_3, n_1, n_2, n_4 \ \& \ P_3 : n_3, n_1, n_5, n_4 \to \text{red\&blue}$$
$$\text{The number is 1;}$$

$(ii)$ 1-paths have same color;

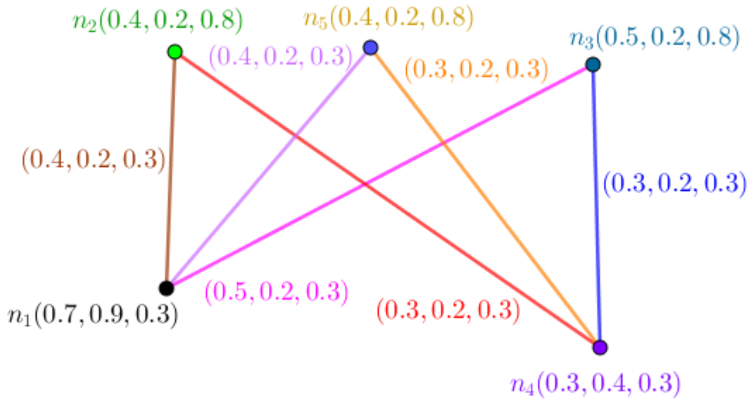


**Figure 9.** A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number.

$(iii)$ $\mathcal{L}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 1$;

$(iv)$ the position of given vertices could be different in the terms of creating path and the behaviors in path;

$(v)$ there are only two paths but there's no shared edge;

$(vi)$ all paths are as follows.

$$P_1 : n_1, n_2 \ \& \ P_2 : n_1, n_3, n_4, n_2 \ \& \ P_3 : n_1, n_5, n_4, n_2 \rightarrow \text{shared edge:} n_4 n_2 \rightarrow \ 0.8$$
$$P_1 : n_1, n_3 \ \& \ P_2 : n_1, n_2, n_4, n_3 \ \& \ P_3 : n_1, n_5, n_4, n_3 \rightarrow \text{shared edge:} n_4 n_3 \rightarrow \ 0.8$$
$$P_1 : n_1, n_3, n_4 \ \& \ P_2 : n_1, n_2, n_4 \ \& \ P_3 : n_1, n_5, n_4 \rightarrow \text{no shared edge} \rightarrow \ 0$$
$$P_1 : n_1, n_5 \ \& \ P_2 : n_1, n_3, n_4, n_5 \ \& \ P_3 : n_1, n_2, n_4, n_5 \rightarrow \text{shared edge:} n_4 n_5 \rightarrow \ 0.8$$
$$P_1 : n_2, n_4 \ \& \ P_2 : n_2, n_1, n_3, n_4 \ \& \ P_3 : n_2, n_1, n_5, n_4 \rightarrow \text{shared edge:} n_2 n_1 \rightarrow \ 0.9$$
$$P_1 : n_3, n_4 \ \& \ P_2 : n_3, n_1, n_2, n_4 \ \& \ P_3 : n_3, n_1, n_5, n_4 \rightarrow \text{shared edge:} n_3 n_1 \rightarrow \ 1$$
$$\mathcal{L}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) \text{ is } 0.$$

**Proposition 2.13.** *Let* $NTG : (V, E, \sigma, \mu)$ *be a wheel-neutrosophic graph. Then*

$$\mathcal{L}(WHL_{1,\sigma_2}) = \min_S |S|.$$

*Proof.* Suppose $WHL_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a wheel-neutrosophic graph. The argument is elementary. All vertices of a cycle join to one vertex. For given two vertices, $x$ and $y$, there are some paths from $x$ to $y$. If two paths from $x$ to $y$ share one edge, then they're assigned to different colors. The set of colors in this process is called path-coloring set from $x$ to $y$. The minimum cardinality between all path-coloring sets from two given vertices is called path-coloring number and it's denoted by $\mathcal{L}(WHL_{1,\sigma_2})$. Thus

$$\mathcal{L}(WHL_{1,\sigma_2}) = \min_S |S|.$$

□

The clarifications about results are in progress as follows. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

**Example 2.14.** There is one section for clarifications. In Figure (10), a wheel-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) All paths are as follows.

$$P_1 : n_1, n_2 \text{ \& } P_2 : n_1, n_3, n_2 \text{ \& } P_3 : n_1, n_4, n_3, n_2 \text{ \& } P_4 : n_1, n_4, n_5, n_2$$
$$\text{\& } P_5 : n_1, n_5, n_2 \to \text{red\&blue}$$
$$P_1 : n_1, n_3 \text{ \& } P_2 : n_1, n_2, n_3 \text{ \& } P_3 : n_1, n_5, n_2, n_3 \text{ \& } P_4 : n_1, n_5, n_4, n_3$$
$$\text{\& } P_5 : n_1, n_4, n_3 \to \text{red\&blue}$$
$$P_1 : n_1, n_4 \text{ \& } P_2 : n_1, n_3, n_4 \text{ \& } P_3 : n_1, n_2, n_3, n_4 \text{ \& } P_4 : n_1, n_2, n_5, n_4$$
$$\text{\& } P_5 : n_1, n_5, n_4 \to \text{red\&blue}$$
$$P_1 : n_1, n_5 \text{ \& } P_2 : n_1, n_2, n_5 \text{ \& } P_3 : n_1, n_3, n_2, n_5 \text{ \& } P_4 : n_1, n_3, n_4, n_5$$
$$\text{\& } P_5 : n_1, n_4, n_5 \to \text{red\&blue}$$
$$P_1 : n_2, n_3 \text{ \& } P_2 : n_2, n_1, n_3 \text{ \& } P_3 : n_2, n_1, n_4, n_3 \text{ \& } P_4 : n_2, n_1, n_5, n_4, n_3$$
$$\text{\& } P_5 : n_2, n_5, n_1, n_3 \text{ \& } P_6 : n_2, n_5, n_1, n_4, n_3 \text{ \& } P_7 : n_2, n_5, n_4, n_3$$
$$\to \text{red\&blue\&pink\&purple}$$
$$P_1 : n_2, n_1, n_4 \text{ \& } P_2 : n_2, n_1, n_3, n_4 \text{ \& } P_3 : n_2, n_1, n_5, n_4 \text{ \& } P_4 : n_2, n_3, n_4$$
$$\text{\& } P_5 : n_2, n_3, n_1, n_4 \text{ \& } P_6 : n_2, n_3, n_1, n_5, n_4 \text{ \& } P_7 : n_2, n_5, n_4$$
$$\text{\& } P_8 : n_2, n_5, n_1, n_4 \text{ \& } P_9 : n_2, n_5, n_1, n_3, n_4 \to \text{red\&blue\&pink\&purple}$$
$$P_1 : n_2, n_5 \text{ \& } P_2 : n_2, n_1, n_5 \text{ \& } P_3 : n_2, n_1, n_4, n_5 \text{ \& } P_4 : n_2, n_1, n_3, n_4, n_5$$
$$\text{\& } P_5 : n_2, n_3, n_1, n_5 \text{ \& } P_6 : n_2, n_3, n_1, n_4, n_5 \text{ \& } P_7 : n_2, n_3, n_4, n_5$$
$$\to \text{red\&blue\&pink\&purple}$$
$$P_1 : n_3, n_4 \text{ \& } P_2 : n_3, n_1, n_4 \text{ \& } P_3 : n_3, n_1, n_5, n_4 \text{ \& } P_4 : n_3, n_1, n_2, n_5, n_4$$
$$\text{\& } P_5 : n_3, n_2, n_1, n_4 \text{ \& } P_6 : n_3, n_2, n_1, n_5, n_4 \text{ \& } P_7 : n_3, n_2, n_5, n_4$$
$$\to \text{red\&blue\&pink\&purple}$$
$$P_1 : n_3, n_1, n_5 \text{ \& } P_2 : n_3, n_1, n_2, n_5 \text{ \& } P_3 : n_3, n_1, n_4, n_5 \text{ \& } P_4 : n_3, n_2, n_5$$
$$\text{\& } P_5 : n_3, n_2, n_1, n_5 \text{ \& } P_6 : n_3, n_2, n_1, n_4, n_5 \text{ \& } P_7 : n_3, n_4, n_5$$
$$\text{\& } P_8 : n_3, n_4, n_1, n_5 \text{ \& } P_9 : n_3, n_4, n_1, n_2, n_5 \to \text{red\&blue\&pink\&purple}$$
$$P_1 : n_4, n_5 \text{ \& } P_2 : n_4, n_1, n_5 \text{ \& } P_3 : n_4, n_1, n_2, n_5 \text{ \& } P_4 : n_4, n_1, n_3, n_2, n_5$$
$$\text{\& } P_5 : n_4, n_3, n_1, n_5 \text{ \& } P_6 : n_4, n_3, n_1, n_2, n_5 \text{ \& } P_7 : n_4, n_3, n_2, n_5$$
$$\to \text{red\&blue\&pink\&purple}$$
$$\text{The number is 2;}$$

(*ii*) 1-paths have same color;

(*iii*) $\mathcal{L}(WHL_{1,\sigma_2}) = 2$;

(*iv*) the position of given vertices are different in the terms of creating path and the behaviors in path. There are three different cases in the terms of paths;

(*v*) there are either five or seven paths but there are two or four shared edge;

$(vi)$ all paths are as follows.

$$P_1 : n_1, n_2 \ \& \ P_2 : n_1, n_3, n_2 \ \& \ P_3 : n_1, n_4, n_3, n_2 \ \& \ P_4 : n_1, n_4, n_5, n_2$$
$$\& \ P_5 : n_1, n_5, n_2 \rightarrow \text{shared edges: } n_3n_2, n_1n_4 \rightarrow 1.6;$$
$$P_1 : n_1, n_3 \ \& \ P_2 : n_1, n_2, n_3 \ \& \ P_3 : n_1, n_5, n_2, n_3 \ \& \ P_4 : n_1, n_5, n_4, n_3$$
$$\& \ P_5 : n_1, n_4, n_3 \rightarrow \text{shared edges: } n_2n_3, n_1n_5 \rightarrow 1.7;$$
$$P_1 : n_1, n_4 \ \& \ P_2 : n_1, n_3, n_4 \ \& \ P_3 : n_1, n_2, n_3, n_4 \ \& \ P_4 : n_1, n_2, n_5, n_4$$
$$\& \ P_5 : n_1, n_5, n_4 \rightarrow \text{shared edges: } n_3n_4, n_1n_2 \rightarrow 1.7;$$
$$P_1 : n_1, n_5 \ \& \ P_2 : n_1, n_2, n_5 \ \& \ P_3 : n_1, n_3, n_2, n_5 \ \& \ P_4 : n_1, n_3, n_4, n_5$$
$$\& \ P_5 : n_1, n_4, n_5 \rightarrow \text{shared edges: } n_2n_5, n_1n_3 \rightarrow 1.7;$$
$$P_1 : n_2, n_3 \ \& \ P_2 : n_2, n_1, n_3 \ \& \ P_3 : n_2, n_1, n_4, n_3 \ \& \ P_4 : n_2, n_1, n_5, n_4, n_3$$
$$\& \ P_5 : n_2, n_5, n_1, n_3 \ \& \ P_6 : n_2, n_5, n_1, n_4, n_3 \ \& \ P_7 : n_2, n_5, n_4, n_3$$
$$\rightarrow \text{shared edges: } n_4n_3, n_2n_5, n_2n_1 \rightarrow 2.5;$$
$$P_1 : n_2, n_1, n_4 \ \& \ P_2 : n_2, n_1, n_3, n_4 \ \& \ P_3 : n_2, n_1, n_5, n_4 \ \& \ P_4 : n_2, n_3, n_4$$
$$\& \ P_5 : n_2, n_3, n_1, n_4 \ \& \ P_6 : n_2, n_3, n_1, n_5, n_4 \ \& \ P_7 : n_2, n_5, n_4$$
$$\& \ P_8 : n_2, n_5, n_1, n_4 \ \& \ P_9 : n_2, n_5, n_1, n_3, n_4$$
$$\rightarrow \text{shared edges: } n_2n_1, n_2n_3, n_3n_1, n_4n_5 \rightarrow 3.4;$$
$$P_1 : n_2, n_5 \ \& \ P_2 : n_2, n_1, n_5 \ \& \ P_3 : n_2, n_1, n_4, n_5 \ \& \ P_4 : n_2, n_1, n_3, n_4, n_5$$
$$\& \ P_5 : n_2, n_3, n_1, n_5 \ \& \ P_6 : n_2, n_3, n_1, n_4, n_5 \ \& \ P_7 : n_2, n_3, n_4, n_5$$
$$\rightarrow \text{shared edges: } n_2n_1, n_2n_3, n_4n_5 \rightarrow 2.5;$$
$$P_1 : n_3, n_4 \ \& \ P_2 : n_3, n_1, n_4 \ \& \ P_3 : n_3, n_1, n_5, n_4 \ \& \ P_4 : n_3, n_1, n_2, n_5, n_4$$
$$\& \ P_5 : n_3, n_2, n_1, n_4 \ \& \ P_6 : n_3, n_2, n_1, n_5, n_4 \ \& \ P_7 : n_3, n_2, n_5, n_4$$
$$\rightarrow \text{shared edges: } n_3n_1, n_3n_2, n_2n_5 \rightarrow 2.5;$$
$$P_1 : n_3, n_1, n_5 \ \& \ P_2 : n_3, n_1, n_2, n_5 \ \& \ P_3 : n_3, n_1, n_4, n_5 \ \& \ P_4 : n_3, n_2, n_5$$
$$\& \ P_5 : n_3, n_2, n_1, n_5 \ \& \ P_6 : n_3, n_2, n_1, n_4, n_5 \ \& \ P_7 : n_3, n_4, n_5$$
$$\& \ P_8 : n_3, n_4, n_1, n_5 \ \& \ P_9 : n_3, n_4, n_1, n_2, n_5$$
$$\rightarrow \text{shared edges: } n_3n_1, n_3n_2, n_3n_4, n_4n_5 \rightarrow 3.3;$$
$$P_1 : n_4, n_5 \ \& \ P_2 : n_4, n_1, n_5 \ \& \ P_3 : n_4, n_1, n_2, n_5 \ \& \ P_4 : n_4, n_1, n_3, n_2, n_5$$
$$\& \ P_5 : n_4, n_3, n_1, n_5 \ \& \ P_6 : n_4, n_3, n_1, n_2, n_5 \ \& \ P_7 : n_4, n_3, n_2, n_5$$
$$\rightarrow \text{shared edges: } n_4n_3, n_2n_5, n_4n_1 \rightarrow 2.4;$$

$$\mathcal{L}_n(WHL_{1,\sigma_2}) \text{ is } 1.6.$$

## 3 Setting of neutrosophic path-coloring number

In this section, I provide some results in the setting of neutrosophic path-coloring number. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, star-neutrosophic graph, bipartite-neutrosophic graph, t-partite-neutrosophic graph, and wheel-neutrosophic graph, are both of cases of study and classes which the results are about them.

**Proposition 3.1.** *Let* $NTG : (V, E, \sigma, \mu)$ *be a complete-neutrosophic graph. Then*

$$\mathcal{L}_n(CMT_\sigma) = \min_S \sum_{e \in S} \sum_{i=1}^{3} \mu_i(e).$$

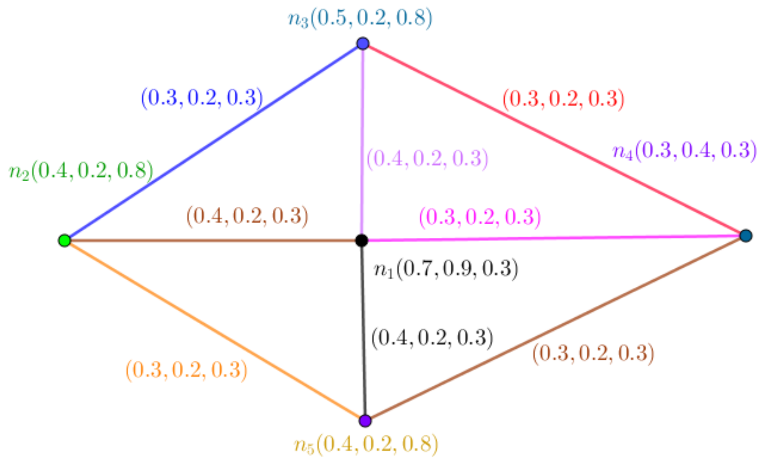


**Figure 10.** A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number.

*Proof.* Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. The number of vertices is $\mathcal{O}(CMT_\sigma)$. For given two vertices, $x$ and $y$, there are some paths from $x$ to $y$. If two paths from $x$ to $y$ share one edge, then they're assigned to different colors. The set $S$ of shared edges in this process is called path-coloring set from $x$ to $y$. The minimum neutrosophic cardinality, $\sum_{e\in S}\sum_{i=1}^{3}\mu_i(e)$, between all path-coloring sets, $S$s, is called neutrosophic path-coloring number and it's denoted by $\mathcal{L}_n(CMT_\sigma)$.Thus

$$\mathcal{L}_n(CMT_\sigma) = \min_S \sum_{e\in S}\sum_{i=1}^{3}\mu_i(e).$$

□

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

**Example 3.2.** In Figure (11), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. New viewpoint implies different kinds of definitions to get more scrutiny and more discernment.

$(i)$ Consider two vertices $n_1$ and $n_2$. All paths are as follow:

$$\begin{aligned} P_1 &: n_1, n_2 \to \text{red} \\ P_2 &: n_1, n_3, n_2 \to \text{red} \\ P_3 &: n_1, n_4, n_2 \to \text{red} \\ P_4 &: n_1, n_3, n_4, n_2 \to \text{blue} \\ P_5 &: n_1, n_4, n_3, n_2 \to \text{yellow} \end{aligned}$$

The paths $P_1$, $P_2$ and $P_3$ has no shared edge so they've been colored the same as red. The path $P_4$ has shared edge $n_1n_3$ with $P_2$ and shared edge $n_4n_2$ with $P_3$

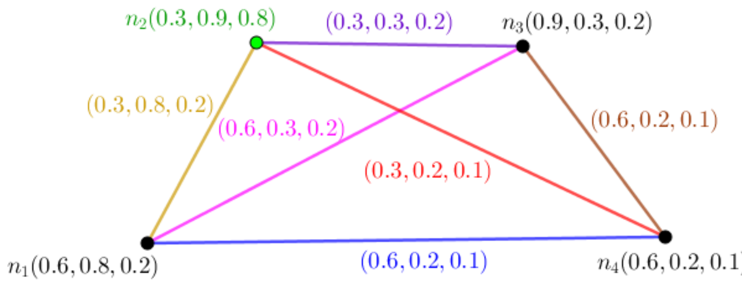


**Figure 11.** A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number.

thus it's been colored the different color as blue in comparison to them. The path $P_5$ has shared edge $n_1n_4$ with $P_3$ and shared edge $n_3n_4$ with $P_4$ thus it's been colored the different color as yellow in comparison to different paths in the terms of different colors. Thus $S = \{\text{red}, \text{blue}, \text{yellow}\}$ is path-coloring set and its cardinality, 3, is path-coloring number. To sum them up, for given two vertices, $x$ and $y$, there are some paths from $x$ to $y$. If two paths from $x$ to $y$ share one edge, then they're assigned to different colors. The set of colors, $S = \{\text{red}, \text{blue}, \text{yellow}\}$, in this process is called path-coloring set from $x$ to $y$. The minimum cardinality between all path-coloring sets from two given vertices, 3, is called path-coloring number and it's denoted by $\mathcal{L}(CMT_\sigma) = 3$;

$(ii)$ all vertices have same positions in the matter of creating paths. So for every two given vertices, the number and the behaviors of paths are the same;

$(iii)$ there are three different paths which have no shared edges. So they've been assigned to same color;

$(iv)$ shared edges form a set of representatives of colors. Each color is corresponded to an edge which has minimum neutrosophic cardinality;

$(v)$ every color in $S$ is corresponded to an edge has minimum neutrosophic cardinality. Minimum neutrosophic cardinality is obtained in this way but other way is to use all shared edges to form $S$ and after that minimum neutrosophic cardinality is optimal;

$(vi)$ two edges $n_1n_3$ and $n_4n_2$ are shared with $P_4$ by $P_3$ and $P_2$. The minimum neutrosophic cardinality is 0.6 corresponded to $n_4n_2$. Other corresponded color has only one shared edge $n_3n_4$ and minimum neutrosophic cardinality is 0.9. Thus minimum neutrosophic cardinality is 1.5. And corresponded set is $S = \{n_4n_2, n_3n_4\}$. To sum them up, for given two vertices, $x$ and $y$, there are some paths from $x$ to $y$. If two paths from $x$ to $y$ share one edge, then they're assigned to different colors. The set $S = \{n_4n_2, n_3n_4\}$ of shared edges in this process is called path-coloring set from $x$ to $y$. The minimum neutrosophic cardinality, $\sum_{e \in S} \sum_{i=1}^{3} \mu_i(e)$, between all path-coloring sets, $S$s, is called neutrosophic path-coloring number and it's denoted by $\mathcal{L}_n(CMT_\sigma) = 1.5$.

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

**Proposition 3.3.** *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Then*

$$\mathcal{L}_n(PTH) = \min_{e \in S} \sum_{i=1}^{3} \mu_i(e).$$

*Proof.* Suppose $PTH : (V, E, \sigma, \mu)$ is a path-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(PTH)}$ be a path-neutrosophic graph. For given two vertices, $x$ and $y$, there's only one path from $x$ to $y$. If two paths from $x$ to $y$ share one edge, then they're assigned to different colors but there's only one path with certain start and end. The set $S$ of shared edges in this process is called path-coloring set from $x$ to $y$. The minimum neutrosophic cardinality, $\sum_{e \in S} \sum_{i=1}^{3} \mu_i(e)$, between all path-coloring sets, $S$s, is called neutrosophic path-coloring number and it's denoted by $\mathcal{L}_n(PTH)$. Thus

$$\mathcal{L}_n(PTH) = \min_{e \in S} \sum_{i=1}^{3} \mu_i(e).$$

□

**Example 3.4.** There are two sections for clarifications.

($a$) In Figure (12), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. New viewpoint implies different kinds of definitions to get more scrutiny and more discernment.

($i$) All paths are as follows.

$$\begin{aligned}
n_1, n_2 &\to \text{red}\\
n_1, n_2, n_3 &\to \text{blue}\\
n_1, n_2, n_3, n_4 &\to \text{yellow}\\
n_1, n_2, n_3, n_4, n_5 &\to \text{green}\\
n_2, n_3 &\to \text{red}\\
n_2, n_3, n_4 &\to \text{pink}\\
n_2, n_3, n_4, n_5 &\to \text{brown}\\
n_3, n_4 &\to \text{red}\\
n_3, n_4, n_5 &\to \text{blue}\\
n_4, n_5 &\to \text{red}\\
&\text{The number is 6;}
\end{aligned}$$

($ii$) 1-paths have same color;

($iii$) $\mathcal{L}(PTH) = 6$;

($iv$) the position of given vertices could be different in the terms of creating path and the behaviors in path;

($v$) every color is corresponded to some shared edges. Minimum neutrosophic cardinality of edges corresponded to specific color is a representative for that color. Thus every color is corresponded one neutrosophic cardinality of some edges since edges could have same neutrosophic cardinality with exception of initial color. So the summation of 5 numbers is neutrosophic path-coloring number. every color is compared with its previous color. The way is a consecutive procedure;

(*vi*) all paths are as follows.

$$n_1, n_2 \to \text{red}$$
$$n_1, n_2, n_3 \to \text{blue} \to n_1n_2 \to \ 0.8$$
$$n_1, n_2, n_3, n_4 \to \text{yellow} \to n_1n_2, n_2n_3 \to \ 1.3$$
$$n_1, n_2, n_3, n_4, n_5 \to \text{green} \to n_1n_2, n_2n_3, n_3n_4 \to \ 2$$
$$n_2, n_3 \to \text{red}$$
$$n_2, n_3, n_4 \to \text{pink} \to n_2n_3, n_3n_4 \to \ 1.3$$
$$n_2, n_3, n_4, n_5 \to \text{brown} \to n_2n_3, n_3n_4 \to \ 1.3$$
$$n_3, n_4 \to \text{red}$$
$$n_3, n_4, n_5 \to \text{blue} \to n_3n_4 \to \ 0.7$$
$$n_4, n_5 \to \text{red}$$
$$\mathcal{L}_n(PTH) \text{ is } 5.6.$$

(*b*) In Figure (13), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. New definition is applied in this section.

(*i*) All paths are as follows.

$$n_1, n_2 \to \text{red}$$
$$n_1, n_2, n_3 \to \text{red}$$
$$n_1, n_2, n_3, n_4 \to \text{red}$$
$$n_1, n_2, n_3, n_4, n_5 \to \text{red}$$
$$n_1, n_2, n_3, n_4, n_5, n_6 \to \text{red}$$
$$n_2, n_3 \to \text{red}$$
$$n_2, n_3, n_4 \to \text{red}$$
$$n_2, n_3, n_4, n_5 \to \text{red}$$
$$n_2, n_3, n_4, n_5, n_6 \to \text{red}$$
$$n_3, n_4 \to \text{red}$$
$$n_3, n_4, n_5 \to \text{red}$$
$$n_4, n_5 \to \text{red}$$
$$\text{The number is } 1;$$

(*ii*) 1-paths have same color;

(*iii*) $\mathcal{L}(PTH) = 1$;

(*iv*) the position of given vertices could be different in the terms of creating path and the behaviors in path;

(*v*) there's only one path. It implies that there's no shared edge;

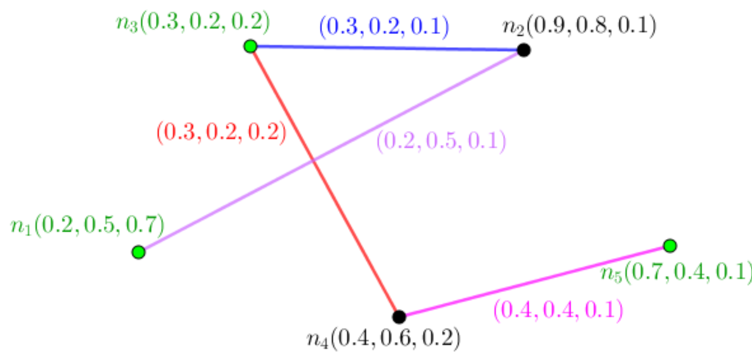


**Figure 12.** A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number.

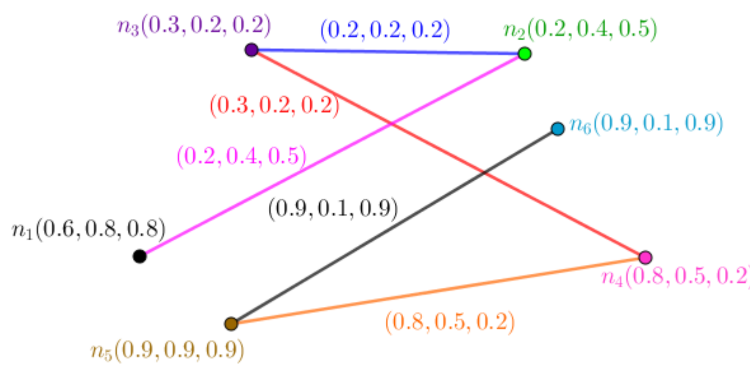


**Figure 13.** A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number.

$(vi)$ all paths are as follows.

$$n_1, n_2 \to \text{red}$$
$$n_1, n_2, n_3 \to \text{red} \to \text{no shared edge} \to 0$$
$$n_1, n_2, n_3, n_4 \to \text{red} \to \text{no shared edge} \to 0$$
$$n_1, n_2, n_3, n_4, n_5 \to \text{red} \to \text{no shared edge} \to 0$$
$$n_2, n_3 \to \text{red}$$
$$n_2, n_3, n_4 \to \text{red} \to \text{no shared edge} \to 0$$
$$n_2, n_3, n_4, n_5 \to \text{red} \to \text{no shared edge} \to 0$$
$$n_3, n_4 \to \text{red}$$
$$n_3, n_4, n_5 \to \text{red} \to \text{no shared edge} \to 0$$
$$n_4, n_5 \to \text{red}$$
$$\mathcal{L}_n(PTH) \text{ is } 0.$$

**Proposition 3.5.** *Let* $NTG : (V, E, \sigma, \mu)$ *be a cycle-neutrosophic graph where* $\mathcal{O}(CYC) \geq 3$. *Then*

$$\mathcal{L}_n(CYC) = \min_{e \in S} \sum_{i=1}^{3} \mu_i(e).$$

*Proof.* Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, $x$ and $y$, there are only two paths from $x$ to $y$. If two paths from $x$ to $y$ share one edge, then they're assigned to different colors but these two paths don't share one edge. The set $S$ of shared edges in this process is called path-coloring set from $x$ to $y$. The minimum neutrosophic cardinality, $\sum_{e\in S}\sum_{i=1}^{3}\mu_i(e)$, between all path-coloring sets, $S$s, is called neutrosophic path-coloring number and it's denoted by

$$\mathcal{L}_n(CYC) = \min_{e\in S}\sum_{i=1}^{3}\mu_i(e).$$

□

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

**Example 3.6.** There are two sections for clarifications.

($a$) In Figure (14), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- ($i$) All paths are as follows.

$$\begin{aligned}
&P_1 : n_1, n_2 \ \&\ P_2 : n_1, n_6, n_5, n_4, n_3, n_2 \to \text{red}\\
&P_1 : n_1, n_2, n_3 \ \&\ P_2 : n_1, n_6, n_5, n_4, n_3 \to \text{red}\\
&P_1 : n_1, n_2, n_3, n_4 \ \&\ P_2 : n_1, n_6, n_5, n_4 \to \text{red}\\
&P_1 : n_1, n_2, n_3, n_4, n_5 \ \&\ P_2 : n_1, n_6, n_5 \to \text{red}\\
&P_1 : n_1, n_2, n_3, n_4, n_5, n_6 \ \&\ P_2 : n_1, n_6 \to \text{red}\\
&\text{The number is 1;}
\end{aligned}$$

- ($ii$) 1-paths have same color;
- ($iii$) $\mathcal{L}(CYC) = 1$;
- ($iv$) the position of given vertices could be different in the terms of creating path and the behaviors in path;
- ($v$) there are only two paths but there's no shared edge;
- ($vi$) all paths are as follows.

$$\begin{aligned}
&P_1 : n_1, n_2 \ \&\ P_2 : n_1, n_6, n_5, n_4, n_3, n_2 \to \text{red} \to \text{no shared edge} \to\ 0\\
&P_1 : n_1, n_2, n_3 \ \&\ P_2 : n_1, n_6, n_5, n_4, n_3 \to \text{red} \to \text{no shared edge} \to\ 0\\
&P_1 : n_1, n_2, n_3, n_4 \ \&\ P_2 : n_1, n_6, n_5, n_4 \to \text{red} \to \text{no shared edge} \to\ 0\\
&P_1 : n_1, n_2, n_3, n_4, n_5 \ \&\ P_2 : n_1, n_6, n_5 \to \text{red} \to \text{no shared edge} \to\ 0\\
&P_1 : n_1, n_2, n_3, n_4, n_5, n_6 \ \&\ P_2 : n_1, n_6 \to \text{red} \to \text{no shared edge} \to\ 0\\
&\mathcal{L}_n(CYC) \text{ is } 0.
\end{aligned}$$

($b$) In Figure (15), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

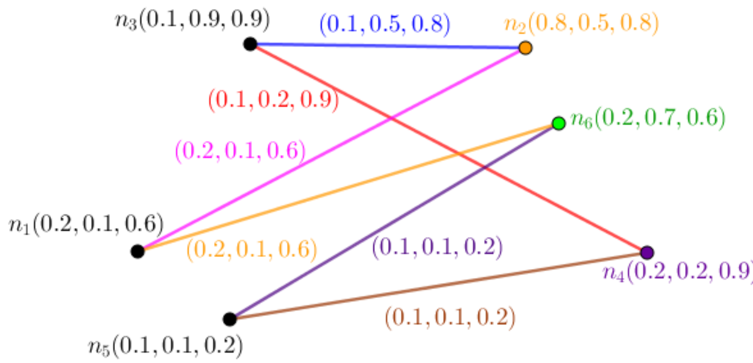


**Figure 14.** A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number.

$(i)$ All paths are as follows.

$$\begin{aligned}
P_1 : n_1, n_2 \ \&\ P_2 : n_1, n_5, n_4, n_3, n_2 &\to \text{red} \\
P_1 : n_1, n_2, n_3 \ \&\ P_2 : n_1, n_5, n_4, n_3 &\to \text{red} \\
P_1 : n_1, n_2, n_3, n_4 \ \&\ P_2 : n_1 n_5, n_4 &\to \text{red} \\
P_1 : n_1, n_2, n_3, n_4, n_5 \ \&\ P_2 : n_1, n_5 &\to \text{red} \\
\text{The number is } 1;
\end{aligned}$$

$(ii)$ 1-paths have same color;

$(iii)$ $\mathcal{L}(CYC) = 1$;

$(iv)$ the position of given vertices could be different in the terms of creating path and the behaviors in path;

$(v)$ there are only two paths but there's no shared edge;

$(vi)$ all paths are as follows.

$$\begin{aligned}
P_1 : n_1, n_2 \ \&\ P_2 : n_1, n_5, n_4, n_3, n_2 \to \text{red} \to \text{no shared edge} &\to \ 0 \\
P_1 : n_1, n_2, n_3 \ \&\ P_2 : n_1, n_5, n_4, n_3 \to \text{red} \to \text{no shared edge} &\to \ 0 \\
P_1 : n_1, n_2, n_3, n_4 \ \&\ P_2 : n_1, n_5, n_4 \to \text{red} \to \text{no shared edge} &\to \ 0 \\
P_1 : n_1, n_2, n_3, n_4, n_5 \ \&\ P_2 : n_1, n_5 \to \text{red} \to \text{no shared edge} &\to \ 0 \\
\mathcal{L}_n(CYC) \text{ is } 0.
\end{aligned}$$

**Proposition 3.7.** *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center $c$. Then*

$$\mathcal{L}_n(STR_{1,\sigma_2}) = \min_{e \in S} \sum_{i=1}^{3} \mu_i(e).$$

*Proof.* Suppose $STR_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. An edge always has center as one of its endpoints. All paths have one as their lengths, forever. So for given vertex, $y$, there's only one path from center to $y$. If two paths from center to $y$ share one edge, then they're assigned to different colors but there's only one path and indeed, there's only one edge. The set $S$ of shared edges in this process is called path-coloring set from $x$ to $y$. The minimum neutrosophic cardinality, $\sum_{e \in S} \sum_{i=1}^{3} \mu_i(e)$, between all path-coloring sets, $S$s, is called neutrosophic path-coloring number and it's denoted by $\mathcal{L}_n(STR_{1,\sigma_2})$. Thus

$$\mathcal{L}_n(STR_{1,\sigma_2}) = \min_{e \in S} \sum_{i=1}^{3} \mu_i(e).$$

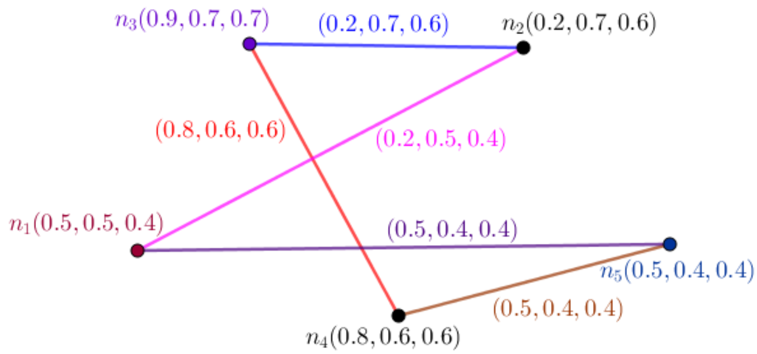


**Figure 15.** A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number.

□

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

**Example 3.8.** There is one section for clarifications. In Figure (16), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

$(i)$ All paths are as follows.

$$\begin{aligned} P &: n_1, n_2 \to \text{red} \\ P &: n_1, n_3 \to \text{red} \\ P &: n_1, n_4 \to \text{red} \\ P &: n_1, n_5 \to \text{red} \\ &\text{The number is } 1; \end{aligned}$$

$(ii)$ 1-paths have same color;

$(iii)$ $\mathcal{L}(STR_{1,\sigma_2}) = 1$;

$(iv)$ the position of given vertices are the same with only exception the center $n_1$;

$(v)$ there's only one path between the vertex $n_1$ and other vertices. There's no shared edge;

$(vi)$ all paths are as follows.

$$\begin{aligned} P &: n_1, n_2 \to \text{red} \to \text{no shared edge} \to \ 0 \\ P &: n_1, n_3 \to \text{red} \to \text{no shared edge} \to \ 0 \\ P &: n_1, n_4 \to \text{red} \to \text{no shared edge} \to \ 0 \\ P &: n_1, n_5 \to \text{red} \to \text{no shared edge} \to \ 0 \\ &\mathcal{L}_n(STR_{1,\sigma_2}) \text{ is } 0. \end{aligned}$$

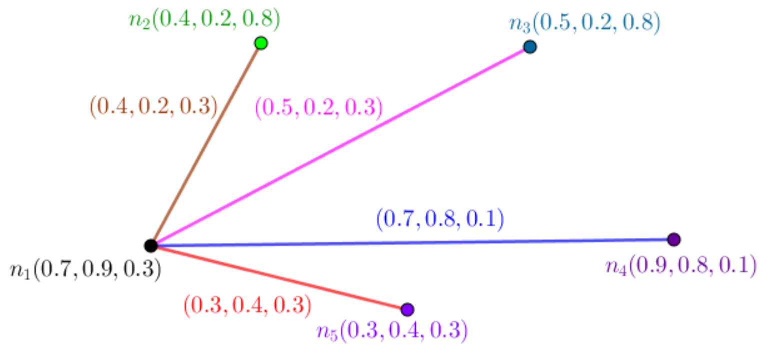


**Figure 16.** A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number.

**Proposition 3.9.** *Let $NTG : (V, E, \sigma, \mu)$ be a complete-bipartite-neutrosophic graph. Then*

$$\mathcal{L}_n(CMC_{\sigma_1,\sigma_2}) = \min_S \sum_{e \in S} \sum_{i=1}^{3} \mu_i(e).$$

*Proof.* Suppose $CMC_{\sigma_1,\sigma_2} : (V, E, \sigma, \mu)$ is a complete-bipartite-neutrosophic graph. For given two vertices, $x$ and $y$, there are some paths from $x$ to $y$. If two paths from $x$ to $y$ share one edge, then they're assigned to different colors. The set $S$ of shared edges in this process is called path-coloring set from $x$ to $y$. The minimum neutrosophic cardinality, $\sum_{e \in S} \sum_{i=1}^{3} \mu_i(e)$, between all path-coloring sets, $S$s, is called neutrosophic path-coloring number and it's denoted by $\mathcal{L}_n(CMC_{\sigma_1,\sigma_2})$. Thus

$$\mathcal{L}_n(CMC_{\sigma_1,\sigma_2}) = \min_S \sum_{e \in S} \sum_{i=1}^{3} \mu_i(e).$$

□

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

**Example 3.10.** There is one section for clarifications. In Figure (17), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) All paths are as follows.

$$P_1 : n_1, n_2 \text{ \& } P_2 : n_1, n_3, n_4, n_2 \rightarrow \text{red}$$
$$P_1 : n_1, n_3 \text{ \& } P_2 : n_1, n_2, n_4, n_3 \rightarrow \text{red}$$
$$P_1 : n_1, n_3, n_4 \text{ \& } P_2 : n_1, n_2, n_4 \rightarrow \text{red}$$
$$P_1 : n_2, n_4, n_3 \text{ \& } P_2 : n_2, n_1, n_3 \rightarrow \text{red}$$
$$P_1 : n_2, n_4 \text{ \& } P_2 : n_2, n_1, n_3, n_4 \rightarrow \text{red}$$
$$P_1 : n_3, n_4 \text{ \& } P_2 : n_3, n_1, n_2, n_4 \rightarrow \text{red}$$
$$\text{The number is 1;}$$

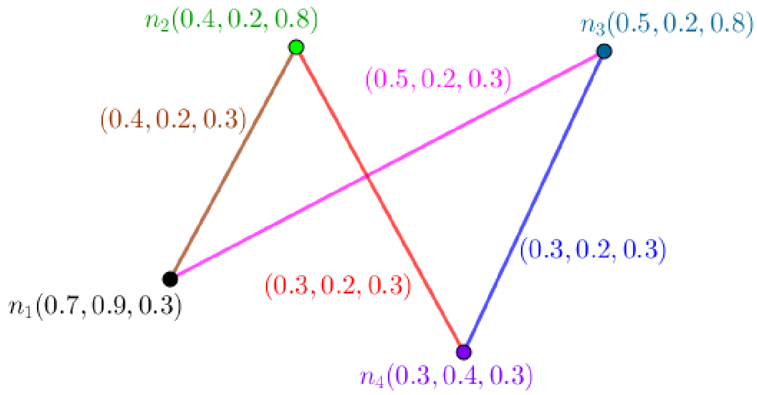


**Figure 17.** A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number.

$(ii)$ 1-paths have same color;

$(iii)$ $\mathcal{L}(CMC_{\sigma_1,\sigma_2}) = 1$;

$(iv)$ the position of given vertices could be different in the terms of creating path and the behaviors in path;

$(v)$ there are only two paths but there's no shared edge;

$(vi)$ all paths are as follows.

$$
\begin{aligned}
&P_1 : n_1, n_2 \ \& \ P_2 : n_1, n_3, n_4, n_2 \to \text{red} \to \text{no shared edge} \to \ 0\\
&P_1 : n_1, n_3 \ \& \ P_2 : n_1, n_2, n_4, n_3 \to \text{red} \to \text{no shared edge} \to \ 0\\
&P_1 : n_1, n_3, n_4 \ \& \ P_2 : n_1, n_2, n_4 \to \text{red} \to \text{no shared edge} \to \ 0\\
&P_1 : n_2, n_4, n_3 \ \& \ P_2 : n_2, n_1, n_3 \to \text{red} \to \text{no shared edge} \to \ 0\\
&P_1 : n_2, n_4 \ \& \ P_2 : n_2, n_1, n_3, n_4 \to \text{red} \to \text{no shared edge} \to \ 0\\
&P_1 : n_3, n_4 \ \& \ P_2 : n_3, n_1, n_2, n_4 \to \text{red} \to \text{no shared edge} \to \ 0\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \mathcal{L}_n(CMC_{\sigma_1,\sigma_2}) \text{ is } 0.
\end{aligned}
$$

**Proposition 3.11.** *Let $NTG : (V, E, \sigma, \mu)$ be a complete-t-partite-neutrosophic graph where $t \geq 3$. Then*

$$\mathcal{L}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \min_S \sum_{e \in S} \sum_{i=1}^{3} \mu_i(e).$$

*Proof.* Suppose $CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t} : (V, E, \sigma, \mu)$ is a complete-t-partite-neutrosophic graph. For given two vertices, $x$ and $y$, there are some paths from $x$ to $y$. If two paths from $x$ to $y$ share one edge, then they're assigned to different colors. The set $S$ of shared edges in this process is called path-coloring set from $x$ to $y$. The minimum neutrosophic cardinality, $\sum_{e \in S} \sum_{i=1}^{3} \mu_i(e)$, between all path-coloring sets, $S$s, is called neutrosophic path-coloring number and it's denoted by $\mathcal{L}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t})$. Thus

$$\mathcal{L}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \min_S \sum_{e \in S} \sum_{i=1}^{3} \mu_i(e).$$

□

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

**Example 3.12.** There is one section for clarifications. In Figure (18), a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

$(i)$ All paths are as follows.

$$
\begin{aligned}
P_1 : n_1, n_2 \ \&\ P_2 : n_1, n_3, n_4, n_2 \ \&\ P_3 : n_1, n_5, n_4, n_2 &\rightarrow \text{red\&blue}\\
P_1 : n_1, n_3 \ \&\ P_2 : n_1, n_2, n_4, n_3 \ \&\ P_3 : n_1, n_5, n_4, n_3 &\rightarrow \text{red\&blue}\\
P_1 : n_1, n_3, n_4 \ \&\ P_2 : n_1, n_2, n_4 \ \&\ P_3 : n_1, n_5, n_4 &\rightarrow \text{red}\\
P_1 : n_1, n_5 \ \&\ P_2 : n_1, n_3, n_4, n_5 \ \&\ P_3 : n_1, n_2, n_4, n_5 &\rightarrow \text{red\&blue}\\
P_1 : n_2, n_4 \ \&\ P_2 : n_2, n_1, n_3, n_4 \ \&\ P_3 : n_2, n_1, n_5, n_4 &\rightarrow \text{red\&blue}\\
P_1 : n_3, n_4 \ \&\ P_2 : n_3, n_1, n_2, n_4 \ \&\ P_3 : n_3, n_1, n_5, n_4 &\rightarrow \text{red\&blue}\\
&\text{The number is 1;}
\end{aligned}
$$

$(ii)$ 1-paths have same color;

$(iii)$ $\mathcal{L}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 1$;

$(iv)$ the position of given vertices could be different in the terms of creating path and the behaviors in path;

$(v)$ there are only two paths but there's no shared edge;

$(vi)$ all paths are as follows.

$$
\begin{aligned}
P_1 : n_1, n_2 \ \&\ P_2 : n_1, n_3, n_4, n_2 \ \&\ P_3 : n_1, n_5, n_4, n_2 &\rightarrow \text{shared edge:} n_4n_2 \rightarrow\ 0.8\\
P_1 : n_1, n_3 \ \&\ P_2 : n_1, n_2, n_4, n_3 \ \&\ P_3 : n_1, n_5, n_4, n_3 &\rightarrow \text{shared edge:} n_4n_3 \rightarrow\ 0.8\\
P_1 : n_1, n_3, n_4 \ \&\ P_2 : n_1, n_2, n_4 \ \&\ P_3 : n_1, n_5, n_4 &\rightarrow \text{no shared edge} \rightarrow\ 0\\
P_1 : n_1, n_5 \ \&\ P_2 : n_1, n_3, n_4, n_5 \ \&\ P_3 : n_1, n_2, n_4, n_5 &\rightarrow \text{shared edge:} n_4n_5 \rightarrow\ 0.8\\
P_1 : n_2, n_4 \ \&\ P_2 : n_2, n_1, n_3, n_4 \ \&\ P_3 : n_2, n_1, n_5, n_4 &\rightarrow \text{shared edge:} n_2n_1 \rightarrow\ 0.9\\
P_1 : n_3, n_4 \ \&\ P_2 : n_3, n_1, n_2, n_4 \ \&\ P_3 : n_3, n_1, n_5, n_4 &\rightarrow \text{shared edge:} n_3n_1 \rightarrow\ 1\\
&\mathcal{L}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) \text{ is } 0.
\end{aligned}
$$

**Proposition 3.13.** *Let $NTG : (V, E, \sigma, \mu)$ be a wheel-neutrosophic graph. Then*

$$
\mathcal{L}_n(WHL_{1,\sigma_2}) = \min_S \sum_{e \in S} \sum_{i=1}^{3} \mu_i(e).
$$

*Proof.* Suppose $WHL_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a wheel-neutrosophic graph. The argument is elementary. All vertices of a cycle join to one vertex. For given two vertices, $x$ and $y$, there are some paths from $x$ to $y$. If two paths from $x$ to $y$ share one edge, then they're assigned to different colors. The set $S$ of shared edges in this process is called path-coloring set from $x$ to $y$. The minimum neutrosophic cardinality,

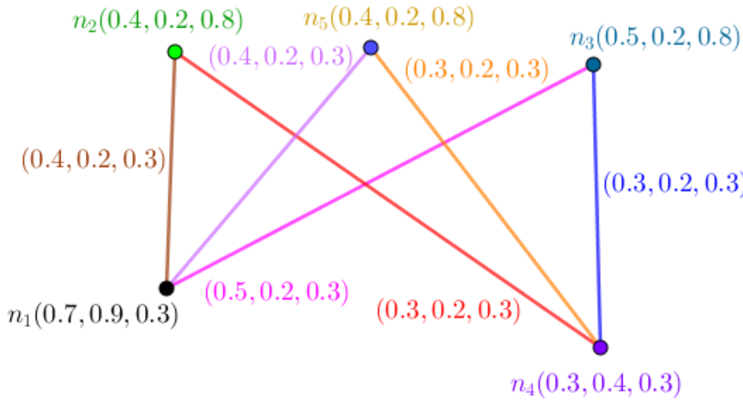


**Figure 18.** A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number.

$\sum_{e\in S}\sum_{i=1}^{3}\mu_i(e)$, between all path-coloring sets, $S$s, is called neutrosophic path-coloring number and it's denoted by $\mathcal{L}_n(WHL_{1,\sigma_2})$. Thus

$$\mathcal{L}_n(WHL_{1,\sigma_2}) = \min_{S}\sum_{e\in S}\sum_{i=1}^{3}\mu_i(e).$$

□

The clarifications about results are in progress as follows. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

**Example 3.14.** There is one section for clarifications. In Figure (19), a wheel-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

$(i)$ All paths are as follows.

$$P_1 : n_1, n_2 \;\&\; P_2 : n_1, n_3, n_2 \;\&\; P_3 : n_1, n_4, n_3, n_2 \;\&\; P_4 : n_1, n_4, n_5, n_2$$
$$\&\; P_5 : n_1, n_5, n_2 \rightarrow \text{red\&blue}$$
$$P_1 : n_1, n_3 \;\&\; P_2 : n_1, n_2, n_3 \;\&\; P_3 : n_1, n_5, n_2, n_3 \;\&\; P_4 : n_1, n_5, n_4, n_3$$
$$\&\; P_5 : n_1, n_4, n_3 \rightarrow \text{red\&blue}$$
$$P_1 : n_1, n_4 \;\&\; P_2 : n_1, n_3, n_4 \;\&\; P_3 : n_1, n_2, n_3, n_4 \;\&\; P_4 : n_1, n_2, n_5, n_4$$
$$\&\; P_5 : n_1, n_5, n_4 \rightarrow \text{red\&blue}$$
$$P_1 : n_1, n_5 \;\&\; P_2 : n_1, n_2, n_5 \;\&\; P_3 : n_1, n_3, n_2, n_5 \;\&\; P_4 : n_1, n_3, n_4, n_5$$
$$\&\; P_5 : n_1, n_4, n_5 \rightarrow \text{red\&blue}$$
$$P_1 : n_2, n_3 \;\&\; P_2 : n_2, n_1, n_3 \;\&\; P_3 : n_2, n_1, n_4, n_3 \;\&\; P_4 : n_2, n_1, n_5, n_4, n_3$$
$$\&\; P_5 : n_2, n_5, n_1, n_3 \;\&\; P_6 : n_2, n_5, n_1, n_4, n_3 \;\&\; P_7 : n_2, n_5, n_4, n_3$$
$$\rightarrow \text{red\&blue\&pink\&purple}$$
$$P_1 : n_2, n_1, n_4 \;\&\; P_2 : n_2, n_1, n_3, n_4 \;\&\; P_3 : n_2, n_1, n_5, n_4 \;\&\; P_4 : n_2, n_3, n_4$$
$$\&\; P_5 : n_2, n_3, n_1, n_4 \;\&\; P_6 : n_2, n_3, n_1, n_5, n_4 \;\&\; P_7 : n_2, n_5, n_4$$
$$\&\; P_8 : n_2, n_5, n_1, n_4 \;\&\; P_9 : n_2, n_5, n_1, n_3, n_4 \rightarrow \text{red\&blue\&pink\&purple}$$
$$P_1 : n_2, n_5 \;\&\; P_2 : n_2, n_1, n_5 \;\&\; P_3 : n_2, n_1, n_4, n_5 \;\&\; P_4 : n_2, n_1, n_3, n_4, n_5$$
$$\&\; P_5 : n_2, n_3, n_1, n_5 \;\&\; P_6 : n_2, n_3, n_1, n_4, n_5 \;\&\; P_7 : n_2, n_3, n_4, n_5$$
$$\rightarrow \text{red\&blue\&pink\&purple}$$
$$P_1 : n_3, n_4 \;\&\; P_2 : n_3, n_1, n_4 \;\&\; P_3 : n_3, n_1, n_5, n_4 \;\&\; P_4 : n_3, n_1, n_2, n_5, n_4$$
$$\&\; P_5 : n_3, n_2, n_1, n_4 \;\&\; P_6 : n_3, n_2, n_1, n_5, n_4 \;\&\; P_7 : n_3, n_2, n_5, n_4$$
$$\rightarrow \text{red\&blue\&pink\&purple}$$
$$P_1 : n_3, n_1, n_5 \;\&\; P_2 : n_3, n_1, n_2, n_5 \;\&\; P_3 : n_3, n_1, n_4, n_5 \;\&\; P_4 : n_3, n_2, n_5$$
$$\&\; P_5 : n_3, n_2, n_1, n_5 \;\&\; P_6 : n_3, n_2, n_1, n_4, n_5 \;\&\; P_7 : n_3, n_4, n_5$$
$$\&\; P_8 : n_3, n_4, n_1, n_5 \;\&\; P_9 : n_3, n_4, n_1, n_2, n_5 \rightarrow \text{red\&blue\&pink\&purple}$$
$$P_1 : n_4, n_5 \;\&\; P_2 : n_4, n_1, n_5 \;\&\; P_3 : n_4, n_1, n_2, n_5 \;\&\; P_4 : n_4, n_1, n_3, n_2, n_5$$
$$\&\; P_5 : n_4, n_3, n_1, n_5 \;\&\; P_6 : n_4, n_3, n_1, n_2, n_5 \;\&\; P_7 : n_4, n_3, n_2, n_5$$
$$\rightarrow \text{red\&blue\&pink\&purple}$$
$$\text{The number is } 2;$$

$(ii)$ 1-paths have same color;

$(iii)$ $\mathcal{L}(WHL_{1,\sigma_2}) = 2$;

$(iv)$ the position of given vertices are different in the terms of creating path and the behaviors in path. There are three different cases in the terms of paths;

$(v)$ there are either five or seven paths but there are two or four shared edge;

$(vi)$ all paths are as follows.

$$P_1 : n_1, n_2 \ \& \ P_2 : n_1, n_3, n_2 \ \& \ P_3 : n_1, n_4, n_3, n_2 \ \& \ P_4 : n_1, n_4, n_5, n_2$$
$$\& \ P_5 : n_1, n_5, n_2 \to \text{shared edges: } n_3n_2, n_1n_4 \to 1.6;$$
$$P_1 : n_1, n_3 \ \& \ P_2 : n_1, n_2, n_3 \ \& \ P_3 : n_1, n_5, n_2, n_3 \ \& \ P_4 : n_1, n_5, n_4, n_3$$
$$\& \ P_5 : n_1, n_4, n_3 \to \text{shared edges: } n_2n_3, n_1n_5 \to 1.7;$$
$$P_1 : n_1, n_4 \ \& \ P_2 : n_1, n_3, n_4 \ \& \ P_3 : n_1, n_2, n_3, n_4 \ \& \ P_4 : n_1, n_2, n_5, n_4$$
$$\& \ P_5 : n_1, n_5, n_4 \to \text{shared edges: } n_3n_4, n_1n_2 \to 1.7;$$
$$P_1 : n_1, n_5 \ \& \ P_2 : n_1, n_2, n_5 \ \& \ P_3 : n_1, n_3, n_2, n_5 \ \& \ P_4 : n_1, n_3, n_4, n_5$$
$$\& \ P_5 : n_1, n_4, n_5 \to \text{shared edges: } n_2n_5, n_1n_3 \to 1.7;$$
$$P_1 : n_2, n_3 \ \& \ P_2 : n_2, n_1, n_3 \ \& \ P_3 : n_2, n_1, n_4, n_3 \ \& \ P_4 : n_2, n_1, n_5, n_4, n_3$$
$$\& \ P_5 : n_2, n_5, n_1, n_3 \ \& \ P_6 : n_2, n_5, n_1, n_4, n_3 \ \& \ P_7 : n_2, n_5, n_4, n_3$$
$$\to \text{shared edges: } n_4n_3, n_2n_5, n_2n_1 \to 2.5;$$
$$P_1 : n_2, n_1, n_4 \ \& \ P_2 : n_2, n_1, n_3, n_4 \ \& \ P_3 : n_2, n_1, n_5, n_4 \ \& \ P_4 : n_2, n_3, n_4$$
$$\& \ P_5 : n_2, n_3, n_1, n_4 \ \& \ P_6 : n_2, n_3, n_1, n_5, n_4 \ \& \ P_7 : n_2, n_5, n_4$$
$$\& \ P_8 : n_2, n_5, n_1, n_4 \ \& \ P_9 : n_2, n_5, n_1, n_3, n_4$$
$$\to \text{shared edges: } n_2n_1, n_2n_3, n_3n_1, n_4n_5 \to 3.4;$$
$$P_1 : n_2, n_5 \ \& \ P_2 : n_2, n_1, n_5 \ \& \ P_3 : n_2, n_1, n_4, n_5 \ \& \ P_4 : n_2, n_1, n_3, n_4, n_5$$
$$\& \ P_5 : n_2, n_3, n_1, n_5 \ \& \ P_6 : n_2, n_3, n_1, n_4, n_5 \ \& \ P_7 : n_2, n_3, n_4, n_5$$
$$\to \text{shared edges: } n_2n_1, n_2n_3, n_4n_5 \to 2.5;$$
$$P_1 : n_3, n_4 \ \& \ P_2 : n_3, n_1, n_4 \ \& \ P_3 : n_3, n_1, n_5, n_4 \ \& \ P_4 : n_3, n_1, n_2, n_5, n_4$$
$$\& \ P_5 : n_3, n_2, n_1, n_4 \ \& \ P_6 : n_3, n_2, n_1, n_5, n_4 \ \& \ P_7 : n_3, n_2, n_5, n_4$$
$$\to \text{shared edges: } n_3n_1, n_3n_2, n_2n_5 \to 2.5;$$
$$P_1 : n_3, n_1, n_5 \ \& \ P_2 : n_3, n_1, n_2, n_5 \ \& \ P_3 : n_3, n_1, n_4, n_5 \ \& \ P_4 : n_3, n_2, n_5$$
$$\& \ P_5 : n_3, n_2, n_1, n_5 \ \& \ P_6 : n_3, n_2, n_1, n_4, n_5 \ \& \ P_7 : n_3, n_4, n_5$$
$$\& \ P_8 : n_3, n_4, n_1, n_5 \ \& \ P_9 : n_3, n_4, n_1, n_2, n_5$$
$$\to \text{shared edges: } n_3n_1, n_3n_2, n_3n_4, n_4n_5 \to 3.3;$$
$$P_1 : n_4, n_5 \ \& \ P_2 : n_4, n_1, n_5 \ \& \ P_3 : n_4, n_1, n_2, n_5 \ \& \ P_4 : n_4, n_1, n_3, n_2, n_5$$
$$\& \ P_5 : n_4, n_3, n_1, n_5 \ \& \ P_6 : n_4, n_3, n_1, n_2, n_5 \ \& \ P_7 : n_4, n_3, n_2, n_5$$
$$\to \text{shared edges: } n_4n_3, n_2n_5, n_4n_1 \to 2.4;$$

$\mathcal{L}_n(WHL_{1,\sigma_2})$ is 1.6.

# 4 Applications in Time Table and Scheduling

In this section, two applications for time table and scheduling are provided where the models are either complete models which mean complete connections are formed as individual and family of complete models with common neutrosophic vertex set or quasi-complete models which mean quasi-complete connections are formed as individual and family of quasi-complete models with common neutrosophic vertex set.

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has importance to avoid mixing up.

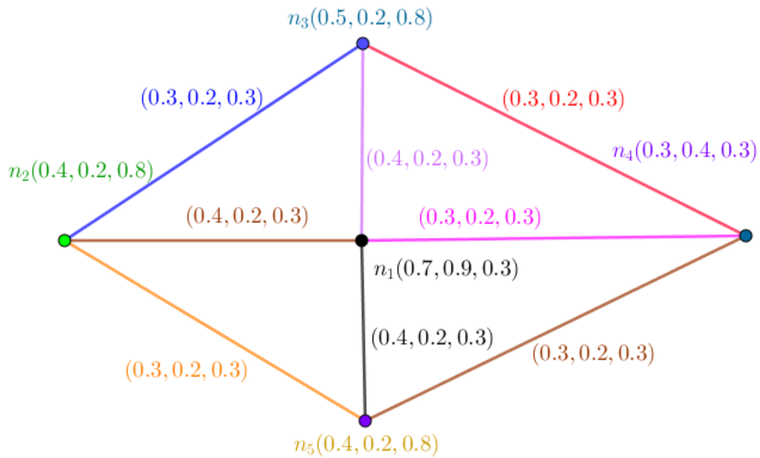


**Figure 19.** A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number.

**Step 1. (Definition)** Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

**Step 2. (Issue)** Scheduling of program has faced with difficulties to differ amid consecutive sections. Beyond that, sometimes sections are not the same.

**Step 3. (Model)** The situation is designed as a model. The model uses data to assign every section and to assign to relation amid sections, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relations amid them. Table (1), clarifies about the assigned numbers to these situations.

**Table 1.** Scheduling concerns its Subjects and its Connections as a neutrosophic graph in a Model.

| Sections of $NTG$ | $n_1$ | $n_2 \cdots$ | $n_5$ |
|---|---|---|---|
| Values | $(0.7, 0.9, 0.3)$ | $(0.4, 0.2, 0.8) \cdots$ | $(0.4, 0.2, 0.8)$ |
| Connections of $NTG$ | $E_1$ | $E_2 \cdots$ | $E_6$ |
| Values | $(0.4, 0.2, 0.3)$ | $(0.5, 0.2, 0.3) \cdots$ | $(0.3, 0.2, 0.3)$ |

### 4.1 Case 1: Complete-t-partite Model alongside its path-coloring number and its neutrosophic path-coloring number

**Step 4. (Solution)** The neutrosophic graph alongside its path-coloring number and its neutrosophic path-coloring number as model, propose to use specific number. Every subject has connection with some subjects. Thus the connection is applied as possible and the model demonstrates quasi-full connections as quasi-possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is star, the number is different. Also, it holds for other types such that complete, wheel,

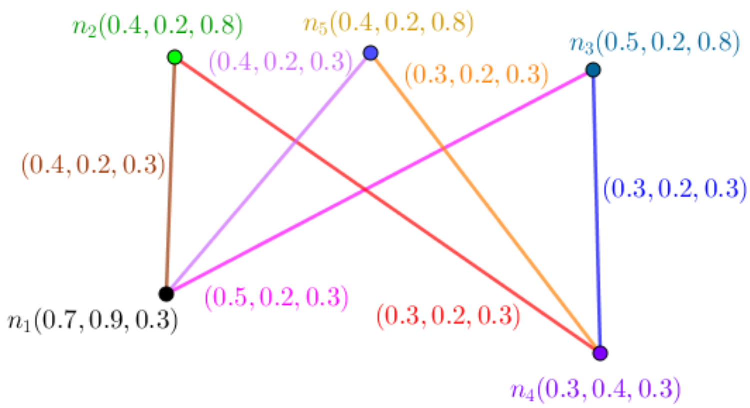


**Figure 20.** A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number

path, and cycle. The collection of situations is another application of its path-coloring number and its neutrosophic path-coloring number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are five subjects which are represented as Figure (20). This model is strong and even more it's quasi-complete. And the study proposes using specific number which is called its path-coloring number and its neutrosophic path-coloring number. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to this model and situation to compare them with same situations to get more precise. Consider Figure (20). In Figure (20), an complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

$(i)$ All paths are as follows.

$$\begin{aligned} P_1 : n_1, n_2 \ \&\ P_2 : n_1, n_3, n_4, n_2 \ \&\ P_3 : n_1, n_5, n_4, n_2 &\to \text{red\&blue} \\ P_1 : n_1, n_3 \ \&\ P_2 : n_1, n_2, n_4, n_3 \ \&\ P_3 : n_1, n_5, n_4, n_3 &\to \text{red\&blue} \\ P_1 : n_1, n_3, n_4 \ \&\ P_2 : n_1, n_2, n_4 \ \&\ P_3 : n_1, n_5, n_4 &\to \text{red} \\ P_1 : n_1, n_5 \ \&\ P_2 : n_1, n_3, n_4, n_5 \ \&\ P_3 : n_1, n_2, n_4, n_5 &\to \text{red\&blue} \\ P_1 : n_2, n_4 \ \&\ P_2 : n_2, n_1, n_3, n_4 \ \&\ P_3 : n_2, n_1, n_5, n_4 &\to \text{red\&blue} \\ P_1 : n_3, n_4 \ \&\ P_2 : n_3, n_1, n_2, n_4 \ \&\ P_3 : n_3, n_1, n_5, n_4 &\to \text{red\&blue} \\ &\text{The number is 1;} \end{aligned}$$

$(ii)$ 1-paths have same color;

$(iii)$ $\mathcal{L}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 1$;

$(iv)$ the position of given vertices could be different in the terms of creating path and the behaviors in path;

$(v)$ there are only two paths but there's no shared edge;

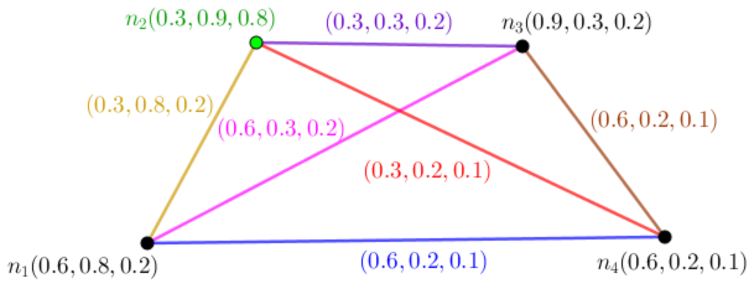


**Figure 21.** A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number

$(vi)$ all paths are as follows.

$$P_1 : n_1, n_2 \ \&\ P_2 : n_1, n_3, n_4, n_2 \ \&\ P_3 : n_1, n_5, n_4, n_2 \rightarrow \text{shared edge:} n_4n_2 \rightarrow \ 0.8$$
$$P_1 : n_1, n_3 \ \&\ P_2 : n_1, n_2, n_4, n_3 \ \&\ P_3 : n_1, n_5, n_4, n_3 \rightarrow \text{shared edge:} n_4n_3 \rightarrow \ 0.8$$
$$P_1 : n_1, n_3, n_4 \ \&\ P_2 : n_1, n_2, n_4 \ \&\ P_3 : n_1, n_5, n_4 \rightarrow \text{no shared edge} \rightarrow \ 0$$
$$P_1 : n_1, n_5 \ \&\ P_2 : n_1, n_3, n_4, n_5 \ \&\ P_3 : n_1, n_2, n_4, n_5 \rightarrow \text{shared edge:} n_4n_5 \rightarrow \ 0.8$$
$$P_1 : n_2, n_4 \ \&\ P_2 : n_2, n_1, n_3, n_4 \ \&\ P_3 : n_2, n_1, n_5, n_4 \rightarrow \text{shared edge:} n_2n_1 \rightarrow \ 0.9$$
$$P_1 : n_3, n_4 \ \&\ P_2 : n_3, n_1, n_2, n_4 \ \&\ P_3 : n_3, n_1, n_5, n_4 \rightarrow \text{shared edge:} n_3n_1 \rightarrow \ 1$$
$$\mathcal{L}_n(CMC_{\sigma_1, \sigma_2, \cdots, \sigma_t}) \text{ is } 0.$$

### 4.2 Case 2: Complete Model alongside its A Neutrosophic Graph in the Viewpoint of its path-coloring number and its neutrosophic path-coloring number

**Step 4. (Solution)** The neutrosophic graph alongside its path-coloring number and its neutrosophic path-coloring number as model, propose to use specific number. Every subject has connection with every given subject in deemed way. Thus the connection applied as possible and the model demonstrates full connections as possible between parts but with different view where symmetry amid vertices and edges are the matters. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete multipartite, the number is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application of its path-coloring number and its neutrosophic path-coloring number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are four subjects which are represented in the formation of one model as Figure (21). This model is neutrosophic strong as individual and even more it's complete. And the study proposes using specific number which is called its path-coloring number and its neutrosophic path-coloring number for this model. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to these models as individual. A model as a collection of situations to compare them with another model as a collection of situations to get more precise. Consider Figure (21). There is one section for clarifications.

($i$) Consider two vertices $n_1$ and $n_2$. All paths are as follow:

$$\begin{aligned} P_1 &: n_1, n_2 \to \text{red} \\ P_2 &: n_1, n_3, n_2 \to \text{red} \\ P_3 &: n_1, n_4, n_2 \to \text{red} \\ P_4 &: n_1, n_3, n_4, n_2 \to \text{blue} \\ P_5 &: n_1, n_4, n_3, n_2 \to \text{yellow} \end{aligned}$$

The paths $P_1,\ P_2$ and $P_3$ has no shared edge so they've been colored the same as red. The path $P_4$ has shared edge $n_1n_3$ with $P_2$ and shared edge $n_4n_2$ with $P_3$ thus it's been colored the different color as blue in comparison to them. The path $P_5$ has shared edge $n_1n_4$ with $P_3$ and shared edge $n_3n_4$ with $P_4$ thus it's been colored the different color as yellow in comparison to different paths in the terms of different colors. Thus $S = \{\text{red}, \text{blue}, \text{yellow}\}$ is path-coloring set and its cardinality, 3, is path-coloring number. To sum them up, for given two vertices, $x$ and $y$, there are some paths from $x$ to $y$. If two paths from $x$ to $y$ share one edge, then they're assigned to different colors. The set of colors, $S = \{\text{red}, \text{blue}, \text{yellow}\}$, in this process is called path-coloring set from $x$ to $y$. The minimum cardinality between all path-coloring sets from two given vertices, 3, is called path-coloring number and it's denoted by $\mathcal{L}(CMT_\sigma) = 3$;

($ii$) all vertices have same positions in the matter of creating paths. So for every two given vertices, the number and the behaviors of paths are the same;

($iii$) there are three different paths which have no shared edges. So they've been assigned to same color;

($iv$) shared edges form a set of representatives of colors. Each color is corresponded to an edge which has minimum neutrosophic cardinality;

($v$) every color in $S$ is corresponded to an edge has minimum neutrosophic cardinality. Minimum neutrosophic cardinality is obtained in this way but other way is to use all shared edges to form $S$ and after that minimum neutrosophic cardinality is optimal;

($vi$) two edges $n_1n_3$ and $n_4n_2$ are shared with $P_4$ by $P_3$ and $P_2$. The minimum neutrosophic cardinality is 0.6 corresponded to $n_4n_2$. Other corresponded color has only one shared edge $n_3n_4$ and minimum neutrosophic cardinality is 0.9. Thus minimum neutrosophic cardinality is 1.5. And corresponded set is $S = \{n_4n_2, n_3n_4\}$. To sum them up, for given two vertices, $x$ and $y$, there are some paths from $x$ to $y$. If two paths from $x$ to $y$ share one edge, then they're assigned to different colors. The set $S = \{n_4n_2, n_3n_4\}$ of shared edges in this process is called path-coloring set from $x$ to $y$. The minimum neutrosophic cardinality, $\sum_{e \in S} \sum_{i=1}^{3} \mu_i(e)$, between all path-coloring sets, $S$s, is called neutrosophic path-coloring number and it's denoted by $\mathcal{L}_n(CMT_\sigma) = 1.5$.

# 5 Open Problems

In this section, some questions and problems are proposed to give some avenues to pursue this study. The structures of the definitions and results give some ideas to make new settings which are eligible to extend and to create new study.

Notion concerning its path-coloring number and its neutrosophic path-coloring number are defined in neutrosophic graphs. Thus,

**Question 5.1.** *Is it possible to use other types of its path-coloring number and its neutrosophic path-coloring number?*

**Question 5.2.** *Are existed some connections amid different types of its path-coloring number and its neutrosophic path-coloring number in neutrosophic graphs?*

**Question 5.3.** *Is it possible to construct some classes of neutrosophic graphs which have "nice" behavior?*

**Question 5.4.** *Which mathematical notions do make an independent study to apply these types in neutrosophic graphs?*

**Problem 5.5.** *Which parameters are related to this parameter?*

**Problem 5.6.** *Which approaches do work to construct applications to create independent study?*

**Problem 5.7.** *Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?*

# 6 Conclusion and Closing Remarks

In this section, concluding remarks and closing remarks are represented. The drawbacks of this article are illustrated. Some benefits and advantages of this study are highlighted.

This study uses two definitions concerning path-coloring number and neutrosophic path-coloring number arising from different types of paths based on shared edges amid them in neutrosophic graphs assigned to neutrosophic graphs. Consider two vertices. Minimum number of shared edges based on those vertices in the formations of all paths with those vertices as their starts and their ends to compare with other paths, is a number which is representative based on those vertices. Minimum neutrosophic number of shared edges amid neutrosophic cardinality of all sets of shared edges is called neutrosophic path-coloring number. The connections of paths which aren't clarified by a common edge differ them from each other and put them in different categories to

**Table 2.** A Brief Overview about Advantages and Limitations of this Study

| Advantages | Limitations |
|---|---|
| 1. path-coloring number of Model | 1. Connections amid Classes |
| 2. neutrosophic path-coloring number of Model | |
| 3. Minimal path-coloring sets | 2. Study on Families |
| 4. Shared Edges amid all Paths | |
| 5. Acting on All Paths | 3. Same Models in Family |

represent a number which is called path-coloring number and neutrosophic path-coloring number arising from different types of paths based on shared edges amid them in neutrosophic graphs assigned to neutrosophic graphs. Further studies could be about changes in the settings to compare these notions amid different settings of neutrosophic graphs theory. One way is finding some relations amid all definitions of notions to make sensible definitions. In Table (2), some limitations and advantages of this study are pointed out.

# References


1. M. Akram, and G. Shahzadi, "*Operations on Single-Valued Neutrosophic Graphs*", Journal of uncertain systems 11 (1) (2017) 1-26.

2. K. Atanassov, "*Intuitionistic fuzzy sets*", Fuzzy Sets Syst. 20 (1986) 87-96.

3. S. Broumi et al., "*Single-valued neutrosophic graphs*", Journal of New Theory 10 (2016) 86-101.

4. S. Corteel et al., "*The permutation-path coloring problem on trees*", Theoretical Computer Science 297 (1-3) (2003) 119-143.

5. O. Dagdeviren, and V. Khalilpour Akram, "*Pack: Path coloring based k-connectivity detection algorithm for wireless sensor networks*", Ad Hoc Networks 64 (2017) 41-52.

6. X. Deng et al., "*A 2-approximation algorithm for path coloring on a restricted class of trees of rings*", Journal of Algorithms 47 (1) (2003) 1-13.

7. T. Erlebach, and Klaus Jansen, "*The complexity of path coloring and call scheduling*", Theoretical Computer Science 255 (1-2) (2001) 33-50.

8. L. Gargano, and A.R. Rescigno, "*Collision-free path coloring with application to minimum-delay gathering in sensor networks*", Discrete Applied Mathematics 157 (8) (2009) 1858-1872.

9. Henry Garrett, (2022). "*Beyond Neutrosophic Graphs*", Ohio: E-publishing: Educational Publisher 1091 West 1st Ave Grandview Heights, Ohio 43212 United States. ISBN: 978-1-59973-725-6 (http://fs.unm.edu/BeyondNeutrosophicGraphs.pdf).

10. Henry Garrett, "*Dimension and Coloring alongside Domination in Neutrosophic Hypergraphs*", Preprints 2021, 2021120448 (doi: 10.20944/preprints202112.0448.v1).

11. Henry Garrett, "*Properties of SuperHyperGraph and Neutrosophic SuperHyperGraph*", Neutrosophic Sets and Systems 49 (2022) 531-561 (doi: 10.5281/zenodo.6456413). (http://fs.unm.edu/NSS/NeutrosophicSuperHyperGraph34.pdf). (https://digitalrepository.unm.edu/nss_journal/vol49/iss1/34).

12. Henry Garrett, "*Three Types of Neutrosophic Alliances based on Connectedness and (Strong) Edges*", Preprints 2022, 2022010239 (doi: 10.20944/preprints202201.0239.v1).

13. N. Shah, and A. Hussain, "*Neutrosophic soft graphs*", Neutrosophic Set and Systems 11 (2016) 31-44.

14. A. Shannon and K.T. Atanassov, "*A first step to a theory of the intuitionistic fuzzy graphs*", Proceeding of FUBEST (Lakov, D., Ed.) Sofia (1994) 59-61.

15. F. Smarandache, "*A Unifying field in logics neutrosophy: Neutrosophic probability, set and logic, Rehoboth:* " American Research Press (1998).

16. H. Wang et al., "*Single-valued neutrosophic sets*", Multispace and Multistructure 4 (2010) 410-413.

17. L. A. Zadeh, "*Fuzzy sets*", Information and Control 8 (1965) 338-354.