n-Cyclic Refined Neutrosophic Vector Spaces and Matrices

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Abstract: This paper is dedicated to study for the first time the concept of n-cyclic refined neutrosophic vector space as a direct application of n-cyclic refined neutrosophic sets. Also, It presents some elementary properties of these spaces such as homomorphisms and subspaces.

On the other hand, this work defines n-cyclic refined neutrosophic real matrices, and illustrates some examples to clarify these structures.

Keywords: n-cyclic refined neutrosophic vector space, n-cyclic refined neutrosophic ring, n-cyclic refined neutrosophic matrix

1. Introduction

Neutrosophy is a new branch of philosophy which concerns with the indeterminacy in real life actions and sciences.

In the literature, neutrosophy has got many applications in pure mathematics areas such as space theory [1,2], module theory [4,5], matrix theory [31,32,42], and number theory [3,35]. Also, it plays an important role in applied mathematics such as equations [30], special elements [41], and topology [27,29].

n-cyclic refined neutrosophic sets were defined in [39], and used in the study of some related rings and modules. These sets are considered as a new kind of n-refined neutrosophic sets [12], with a similar structure and different operations.

In this work, we define the concept of n-cyclic refined neutrosophic vector spaces and n-cyclic refined neutrosophic matrices. Also, we illustrate many examples to clarify the validity of these concepts, and we list some of related open questions.

2. n-Cyclic Refined neutrosophic vector space.

Definition 2.1 [39]

Let $(R,+,\times)$ be a ring and$I_k; 1 \leq k \leq n$ be n indeterminacies. We define

$R_n(I) = \{a_0 + a_1 I_1 + a_2 I_2 + \cdots + a_n I_n; a_i \in R\}$ to be n-cyclic refined neutrosophic ring.

Operations on $R_n(I)$ are defined as:

$\sum_{i=0}^{n} x_i I_i + \sum_{i=0}^{n} y_i I_i = \sum_{i=0}^{n}(x_i + y_i) I_i$, $\sum_{i=0}^{n} x_i I_i \times \sum_{i=0}^{n} y_i I_i = \sum_{i,j=0}^{n}(x_i \times y_i) I_i I_j = \sum_{i,j=0}^{n}(x_i \times y_i) I_{(i+j mod n)}$

Where $\times$ is the multiplication on the ring R, and $xI_0 = x$, for all $x \in R$.

Definition 2.2 [39]

Let $(K,+,\times)$ be a field, we say that $K_n(I) = K + KI_1 + \cdots + KI_n = \{a_0 + a_1 I_1 + a_2 I_2 + \cdots + a_n I_n; a_i \in K\}$ is a n-cyclic refined neutrosophic field.

Definition 2.3

Let $(V,+,\times)$ be any vector space over a field K. Then we say that $V_n(I) = V + VI_1 + \cdots + VI_n = \{x_0 + x_1 I_1 + \cdots + x_n I_n; x_i \in V\}$ is a weak n-cyclic refined neutrosophic vector space over the field K.Elements of $V_n(I)$ are called n-cyclic refined neutrosophic vectors, elements of K are called scalars.

If we take scalars from the n-cyclic refined neutrosophic field $K_n(I)$, we say that $V_n(I)$ is a strong n-cyclic refined neutrosophic vector space over thea n-cyclic refined neutrosophic field $K_n(I)$. Elements of $K_n(I)$ n-cyclic refined neutrosophic scalars.

Remark 2.1. Multiplication by an n-cyclic refined neutrosophic scalar $m = \sum_{i=0}^{n} m_i I_i \in k_n(I)$ is defined as:

$$\left(\sum_{i=0}^{n} m_i I_i\right) \times \left(\sum_{i=0}^{n} a_i I_i\right) = \sum_{i,j=0}^{n} (m_i a_j) I_i I_j$$

Where $a_i \in V$, $m_i \in K$, $I_i I_j = I_{(i+j mod n)}$.

Definition 2.5

Let $V_n(I)$ be a weak n-cyclic refined neutrosophic vector space over the n-cyclic refined neutrosophic field K; a nonempty $W_n(I)$ is called a weak n-cyclic refined neutrosophic vector subspaceof $V_n(I)$ if $W_n(I)$ is a subspace of $V_n(I)$ itself.

Definition 2.6

Let $V_n(I)$ be a strong n-cyclic refined neutrosophic vector space over then-cyclic refined neutrosophic field $K_n(I)$. A nonempty subset $W_n(I)$ is called a strong n-cyclic refined neutrosophic vector submodule of $V_n(I)$ if $W_n(I)$ is a submodule of $V_n(I)$ itself.

Theorem 2.1

Let $V_n(I)$ be a weak n-cyclic refined neutrosophic vector space over the n-cyclic refined neutrosophic field$K$, $W_n(I)$ be a nonempty subset of $V_n(I)$. Then $W_n(I)$ is a weak n-cyclic refined neutrosophic subspace if only if:

$x + y \in W_n(I), m \times x \in W_n(I)$ for all $x, y \in W_n(I)$, $m \in K$.

proof:
it holds directly from the condition of subspace.

Definition 2.7

Let $V_n(I)$ be a weak n-cyclic refined neutrosophic vector space over the field K, x be an arbitrary element of $V_n(I)$, we say that x is a linear combination of $\{x_1, x_2, \dots, x_m\} \subseteq V_n(I)$ if

$x = (a_1 \times x_1) + (a_2 \times x_2) + \dots + (a_m \times x_m)$: $a_i \in K(I), x_i \in V_n(I)$.

Definition 2.8

Let $V_n(I)$ be a strong n-cyclic refined neutrosophic vector space over the n-cyclic refined neutrosophic field $K_n(I)$, x be an arbitrary element of $V_n(I)$, we say that x is a linear combination of $\{x_1, x_2, \dots, x_m\} \subseteq V_n(I)$if

$x = (a_1 \times x_1) + (a_2 \times x_2) + \dots + (a_m \times x_m)$: $a_i \in K_n(I), x_i \in V_n(I)$.

Definition 2.9

Let $X = \{x_1, x_2, \dots, x_m\}$ be a subset of a weak n-cyclic refined neutrosophic vector space $V_n(I)$ over the field K, X is a weak linearly independent set if $\sum_{i=0}^{n} a_i \times x_i = 0$ implies $a_i = 0$; $a_i \in K$.

Definition 2.10

Let $X = \{x_1, x_2, \dots, x_m\}$ be a subset of a strong n-cyclic refined neutrosophic vector space $V_n(I)$ over the n-cyclic refined neutrosophic field $K_n(I)$, X is a weak linearly independent set if $\sum_{i=0}^{n} a_i \times x_i = 0$ implies $a_i = 0$; $a_i \in K_n(I)$.

Definition 2.11

Let $V_n(I), W_n(I)$ be two strong n-cyclic refined neutrosophic vector space over the n-cyclic refined neutrosophic field $K_n(I)$, let $f:V_n(I) \to U_n(I)$ be a well defined map. It is called a strong n-cyclic refined neutrosophic homomorphism if:

$f((a \times x) + (b \times y)) = a \times f(x) + b \times f(y)$ for all $x, y \in V_n(I), a, b \in K_n(I)$.

A weak n-cyclic refined neutrosophic homomorphism can be defined as the same.

Definition 2.12

Let $f:V_n(I) \to U_n(I)$ be a weak/strong n-cyclic refined neutrosophic homomorphism, we define:

(a) $Ker(f) = \{x \in V_n(I); f(x) = 0\}$.

(b) $Im(f) = \{y \in U_n(I);\ \exists x \in V_n(I) \text{ and } y = f(x)\}$.

Theorem 2.2

Let $f:V_n(I) \to U_n(I)$ be a weak n-cyclic refined neutrosophic homomorphism. Then

(a) $Ker(f)$ is a weak n-cyclic refined neutrosophic subspace of V_n(I).

(b) $Im(f)$ is a weak nn-cyclic refined neutrosophic subspace of $U_n(I)$.

Proof:

(a) f is a vector space homomorphism since $V_n(I), U_n(I)$ are vector spaces, hence $Ker(f)$ is a subspace of the vector space $V_n(I)$, thus $Ker(f)$ is a weak n- cyclic refined neutrosophic subspace of V_n(I).

(b) It hold by similar argument.

Theorem 2.3

Let $f:V_n(I) \to U_n(I)$ be a strong n-cyclic refined neutrosophic homomorphism. Then

(a) $Ker(f)$ is a strong n- cyclic refined neutrosophic subspace of V_n(I).

(b) $Im(f)$ is a strong n- cyclic refined neutrosophic subspace of $U_n(I)$.

Proof:

(a) f is a module homomorphism since $V_n(I), U_n(I)$ are modules over the n-cyclic refined neutrosophic field $K_n(I)$, hence $Ker(f)$ is a submodule of the module $V_n(I)$, thus $Ker(f)$is a strong n- cyclic refined neutrosophic subspace of V_n(I).

(b) Holds by similar argument.

Definition 2.13 n-cyclic refined neutrosophic matrix

Let $A_{m\times n} = \{(a_{ij}) : a_{ij} \in K_n(I)\}$, where $K_n(I)$ is a n-cyclic refined neutrosophic field. We call to be the n-cyclic refined neutrosophic matrix.

Definition 2.14 n-cyclic refined neutrosophic square matrix

Let $A_{m\times n}$ is a neutrosophic matrix. We call to be the n-cyclic refinedneutrosophic square matrix if $m = n$.

Example 2.1

Let $n = 3$, then

$$A = \begin{pmatrix} 1 + I_1 + 2I_2 - I_3 & 2 - I_1 - 2I_3 & -1 + I_1 + I_2 + I_3 \\ I_2 + I_3 & 3 + I_1 + 2I_2 & I_1 + I_2 + I_3 \\ 1 - I_1 - I_3 & 4 - I_1 + I_2 - I_3 & -2 - I_1 + 3I_2 - I_3 \end{pmatrix}$$

A is a 3-cyclic refined neutrosophic square matrix.

A can be written as:

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & 0 \\ 1 & 4 & -2 \end{pmatrix} + \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ -1 & -1 & -1 \end{pmatrix} I_1 + \begin{pmatrix} 2 & 0 & 1 \\ 1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix} I_2 + \begin{pmatrix} -1 & -2 & 1 \\ 1 & 0 & 1 \\ -1 & -1 & -1 \end{pmatrix} I_3$$

Example 2.2

Let $n = 4$, then

$$A = \begin{pmatrix} -I_1 + I_2 - I_4 & 1 + I_1 - I_3 + I_4 & 1 + 2I_1 + I_2 + I_3 - I_4 \\ 3 - I_2 + 2I_3 - 3I_4 & -2 + I_1 + I_2 + I_4 & 2 - I_1 - I_2 + I_3 \\ 1 - I_1 - I_3 - 2I_4 & 5 + 3I_1 - I_2 - I_3 + 2I_4 & I_1 - 3I_2 - I_3 + I_4 \end{pmatrix}$$

A is a 4-cyclic refined neutrosophic square matrix.

A can be written as

$$A=\begin{pmatrix}0&1&1\\3&-2&2\\1&4&-2\end{pmatrix}+\begin{pmatrix}-1&-1&1\\0&1&-1\\-1&3&1\end{pmatrix}I_1+\begin{pmatrix}1&0&1\\-1&1&-1\\0&-1&-3\end{pmatrix}I_2+\begin{pmatrix}0&-1&1\\2&0&1\\-1&-1&-1\end{pmatrix}I_3+\begin{pmatrix}-1&1&-1\\-3&1&0\\-2&2&1\end{pmatrix}I_4$$

Example 2.3: Multiplication of n-cyclic refined neutrosophic square matrix

Let $A=A_0+A_1I_1+A_2I_2+A_3I_3$, $B=B_0+B_1I_1+B_2I_2+B_3I_3$ are two 3-cyclic refined neutrosophic square matrixes, where

$$A=\begin{pmatrix}1&0\\1&1\end{pmatrix}+\begin{pmatrix}-2&1\\3&2\end{pmatrix}I_1+\begin{pmatrix}1&1\\-1&-2\end{pmatrix}I_2+\begin{pmatrix}0&2\\3&0\end{pmatrix}I_3,$$

$$B=\begin{pmatrix}1&1\\0&1\end{pmatrix}+\begin{pmatrix}-1&-2\\3&0\end{pmatrix}I_1+\begin{pmatrix}4&2\\3&0\end{pmatrix}I_2+\begin{pmatrix}2&-2\\1&1\end{pmatrix}I_3$$

Then we have.

$$A\times B=A_0B_0+A_0B_1I_1+A_0B_2I_2+A_0B_3I_3+A_1B_0I_1+A_1B_1I_1+A_1B_2I_1I_2+A_1B_3I_1I_3+A_2B_0I_2+A_2B_1I_2I_1+A_2B_2I_2I_2+A_2B_3I_2I_3+A_3B_0I_3+A_3B_1I_3I_1+A_3B_2I_3I_2+A_3B_3I_3I_3$$

$$\begin{aligned}A\times B=&\begin{pmatrix}1&0\\1&1\end{pmatrix}\begin{pmatrix}1&1\\0&1\end{pmatrix}+\begin{pmatrix}1&0\\1&1\end{pmatrix}\begin{pmatrix}-1&-2\\3&0\end{pmatrix}I_1+\begin{pmatrix}1&0\\1&1\end{pmatrix}\begin{pmatrix}4&2\\3&0\end{pmatrix}I_2+\begin{pmatrix}1&0\\1&1\end{pmatrix}\begin{pmatrix}2&-2\\1&1\end{pmatrix}I_3\\&+\begin{pmatrix}-2&1\\3&2\end{pmatrix}\begin{pmatrix}1&1\\0&1\end{pmatrix}I_1+\begin{pmatrix}-2&1\\3&2\end{pmatrix}\begin{pmatrix}-1&-2\\3&0\end{pmatrix}I_1I_1+\begin{pmatrix}-2&1\\3&2\end{pmatrix}\begin{pmatrix}4&2\\3&0\end{pmatrix}I_1I_2\\&+\begin{pmatrix}-2&1\\3&2\end{pmatrix}\begin{pmatrix}2&-2\\1&1\end{pmatrix}I_1I_3+\begin{pmatrix}1&1\\-1&-2\end{pmatrix}\begin{pmatrix}1&1\\0&1\end{pmatrix}I_2+\begin{pmatrix}1&1\\-1&-2\end{pmatrix}\begin{pmatrix}-1&-2\\3&0\end{pmatrix}I_2I_1\\&+\begin{pmatrix}1&1\\-1&-2\end{pmatrix}\begin{pmatrix}4&2\\3&0\end{pmatrix}I_2I_2+\begin{pmatrix}1&1\\-1&-2\end{pmatrix}\begin{pmatrix}2&-2\\1&1\end{pmatrix}I_2I_3+\begin{pmatrix}0&2\\3&0\end{pmatrix}\begin{pmatrix}1&1\\0&1\end{pmatrix}I_3\\&+\begin{pmatrix}0&2\\3&0\end{pmatrix}\begin{pmatrix}-1&-2\\3&0\end{pmatrix}I_3I_1+\begin{pmatrix}0&2\\3&0\end{pmatrix}\begin{pmatrix}4&2\\3&0\end{pmatrix}I_3I_2+\begin{pmatrix}0&2\\3&0\end{pmatrix}\begin{pmatrix}2&-2\\1&1\end{pmatrix}I_3I_3\end{aligned}$$

Now, we have in 3-cyclic refined neutrosophic ring

$I_1I_1=I_1$, $I_2I_1=I_1I_2=I_3$, $I_1I_3=I_3I_1=I_1$, $I_2I_2=I_1$, $I_2I_3=I_3I_2=I_2$, $I_3I_3=I_3$.

Thus.

$$\begin{aligned}A\times B=&\begin{pmatrix}1&1\\1&2\end{pmatrix}+\begin{pmatrix}-1&-2\\2&-2\end{pmatrix}I_1+\begin{pmatrix}4&2\\7&2\end{pmatrix}I_2+\begin{pmatrix}2&-2\\3&-1\end{pmatrix}I_3+\begin{pmatrix}-2&-1\\3&5\end{pmatrix}I_1+\begin{pmatrix}5&4\\3&-6\end{pmatrix}I_1\\&+\begin{pmatrix}5&-4\\18&6\end{pmatrix}I_3+\begin{pmatrix}-3&5\\8&-4\end{pmatrix}I_1+\begin{pmatrix}1&2\\-1&-3\end{pmatrix}I_2+\begin{pmatrix}2&-2\\-5&2\end{pmatrix}I_3+\begin{pmatrix}7&2\\-10&-2\end{pmatrix}I_1\\&+\begin{pmatrix}1&2\\-4&0\end{pmatrix}I_2+\begin{pmatrix}0&2\\6&-6\end{pmatrix}I_3+\begin{pmatrix}6&0\\-3&-6\end{pmatrix}I_1+\begin{pmatrix}6&0\\12&6\end{pmatrix}I_2+\begin{pmatrix}2&2\\6&-6\end{pmatrix}I_3\end{aligned}$$

$$A \times B = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} + \begin{pmatrix} 12 & -1 \\ 3 & -15 \end{pmatrix} I_1 + \begin{pmatrix} 12 & 6 \\ 14 & 5 \end{pmatrix} I_2 + \begin{pmatrix} 11 & -4 \\ 28 & -5 \end{pmatrix} I_3$$

Example 2.4: Addition on n-cyclic refined neutrosophic rings

Let $A = \begin{pmatrix} 2I_1 - I_2 + 3I_3 & 1 + I_1 + 2I_2 \\ -3 - 2I_1 + 4I_2 + I_3 & I_1 + I_3 \end{pmatrix}$, $B = \begin{pmatrix} I_2 - 2I_3 & -2 + I_1 + I_2 + I_3 \\ 1 + I_1 - I_3 & 1 - I_1 - 4I_2 + I_3 \end{pmatrix}$

Hence,

$$A + B = \begin{pmatrix} 2I_1 - I_2 + 3I_3 & 1 + I_1 + 2I_2 \\ -3 - 2I_1 + 4I_2 + I_3 & I_1 + I_3 \end{pmatrix} + \begin{pmatrix} I_2 - 2I_3 & -2 + I_1 + I_2 + I_3 \\ 1 + I_1 - I_3 & 1 - I_1 - 4I_2 + I_3 \end{pmatrix}$$

$$A + B = \begin{pmatrix} 2I_1 + 3I_3 & -1 + 2I_1 + 3I_2 + I_3 \\ -2 - I_1 + 4I_2 & 1 - 4I_2 + 2I_3 \end{pmatrix}.$$

5. Conclusions

In this paper, we have defined for the first time the concept of n-cyclic refined neutrosophic vector space, and n-cyclic refined neutrosophic real matrices. Also, we have presented some of their elementary properties and illustrated many examples to clarify the validity of our work.

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References

[1] Abobala, M., "AH-Subspaces in Neutrosophic Vector Spaces", International Journal of Neutrosophic Science, Vol. 6 , pp. 80-86. 2020.

[2] Abobala, M.,. "A Study of AH-Substructures in *n*-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science", Vol. 9, pp.74-85. 2020.

[3] Sankari, H., and Abobala, M., "Neutrosophic Linear Diophantine Equations With two Variables", Neutrosophic Sets and Systems, Vol. 38, pp. 22-30, 2020.

[4] Sankari, H., and Abobala, M." *n*-Refined Neutrosophic Modules", Neutrosophic Sets and Systems, Vol. 36, pp. 1-11. 2020.

[5] Alhamido, R., and Abobala, M., "AH-Substructures in Neutrosophic Modules", International Journal of Neutrosophic Science, Vol. 7, pp. 79-86 . 2020.

[6]Smarandache, F., " A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, Neutrosophic Set, Neutrosophic Probability", American Research Press. Rehoboth, 2003.

[7]Suresh, R., and S. Palaniammal,. "Neutrosophic Weakly Generalized open and Closed Sets", Neutrosophic Sets and Systems, Vol. 33, pp. 67-77,. 2020.

[8]Olgun, N., and Hatip, A., "The Effect Of The Neutrosophic Logic On The Decision Making, in Quadruple Neutrosophic Theory And Applications", Belgium, EU, Pons Editions Brussels,pp. 238-253. 2020.

[9]Hatip, A., Alhamido, R., and Abobala, M., "A Contribution to Neutrosophic Groups", International Journal of Neutrosophic Science", Vol. 0, pp. 67-76 . 2019.

[10] Abobala, M., " *n*-Refined Neutrosophic Groups I", International Journal of Neutrosophic Science, Vol. 0, pp. 27-34. 2020.

[11]Abobala, M., "Classical Homomorphisms Between n-refined Neutrosophic Rings", International Journal of Neutrosophic Science", Vol. 7, pp. 74-78. 2020.

[12]Smarandache, F., and Abobala, M., n-Refined neutrosophic Rings, International Journal of Neutrosophic Science, Vol. 5 , pp. 83-90, 2020.

[13]Abobala, M., On Some Special Substructures of Neutrosophic Rings and Their Properties, International Journal of Neutrosophic Science", Vol. 4 , pp. 72-81, 2020.

[14] Abobala, M., "On Some Special Substructures of Refined Neutrosophic Rings*"*, International Journal of Neutrosophic Science, Vol. 5, pp. 59-66. 2020.

[15]Sankari, H., and Abobala, M.," AH-Homomorphisms In neutrosophic Rings and Refined Neutrosophic Rings", Neutrosophic Sets and Systems, Vol. 38, pp. 101-112, 2020.

[16]Smarandache, F., and Kandasamy, V.W.B., " Finite Neutrosophic Complex Numbers",·Source: arXiv. 2011.

[17] Agboola, A.A.A,. Akwu, A.D,. and Oyebo, Y.T., " Neutrosophic Groups and Subgroups", International .J .Math. Combin, Vol. 3, pp. 1-9. 2012.

[18]Smarandache, F., " *n*-Valued Refined Neutrosophic Logic and Its Applications in Physics", Progress in Physics, 143-146, Vol. 4, 2013.

[19]Adeleke, E.O., Agboola, A.A.A.,and Smarandache, F., "Refined Neutrosophic Rings I"**,** International Journal of Neutrosophic Science, Vol. 2(2), pp. 77-81. 2020.

[20]Hatip, A., and Abobala, M., "AH-Substructures In Strong Refined Neutrosophic Modules", International Journal of Neutrosophic Science, Vol. 9, pp. 110-116 . 2020.

[21]Smarandache F., and Abobala, M., *"n*-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science, Vol. 7, pp. 47-54. 2020.

[22]Sankari, H., and Abobala, M., "Solving Three Conjectures About Neutrosophic Quadruple Vector Spaces", Neutrosophic Sets and Systems, Vol. 38, pp. 70-77. 2020.

[23]Adeleke, E.O., Agboola, A.A.A., and Smarandache, F., "Refined Neutrosophic Rings II**",** International Journal of Neutrosophic Science, Vol. 2(2), pp. 89-94. 2020.

[24]Abobala, M., On Refined Neutrosophic Matrices and Their Applications In Refined Neutrosophic Algebraic Equations, Journal Of Mathematics, Hindawi, 2021

[25]Abobala, M., A Study of Maximal and Minimal Ideals of n-Refined Neutrosophic Rings, Journal of Fuzzy Extension and Applications, Vol. 2, pp. 16-22, 2021.

[26]Abobala, M., " Semi Homomorphisms and Algebraic Relations Between Strong Refined Neutrosophic Modules and Strong Neutrosophic Modules", Neutrosophic Sets and Systems, Vol. 39, 2021.

[27]Giorgio, N, Mehmood, A., and Broumi, S.," Single Valued neutrosophic Filter", International Journal of Neutrosophic Science, Vol. 6, 2020.

[28]Chellamani, P., and Ajay, D., "Pythagorean neutrosophic Fuzzy Graphs", International Journal of Neutrosophic Science, Vol. 11, 2021.

[29]Milles, S, Barakat, M, and Latrech, A., " Completeness and Compactness In Standard Single Valued neutrosophic Metric Spaces", International Journal of Neutrosophic Science, Vol.12 , 2021.

[30]Abobala, M., "On Some Neutrosophic Algebraic Equations", Journal of New Theory, Vol. 33, 2020.

[31]Abobala, M., On The Representation of Neutrosophic Matrices by Neutrosophic Linear Transformations, Journal of Mathematics, Hindawi, 2021.

[32] Abobala, M., "On Some Algebraic Properties of n-Refined Neutrosophic Elements and n-Refined Neutrosophic Linear Equations", Mathematical Problems in Engineering, Hindawi, 2021

[33] Kandasamy V, Smarandache F., and Kandasamy I., Special Fuzzy Matrices for Social Scientists . Printed in the United States of America,2007, book, 99 pages.

[34] Khaled, H., and Younus, A., and Mohammad, A., " The Rectangle Neutrosophic Fuzzy Matrices", Faculty of Education Journal Vol. 15, 2019. (Arabic version).

[35]Abobala, M., Partial Foundation of Neutrosophic Number Theory, Neutrosophic Sets and Systems, Vol. 39 , 2021.

[36] F. Smarandache, *Neutrosophic Theory and Applications*, Le Quy Don Technical University, Faculty of Information technology, Hanoi, Vietnam, 17th May 2016.

[37]Ibrahim, M.A., Agboola, A.A.A, Badmus, B.S. and Akinleye, S.A., "On refined Neutrosophic Vector Spaces I", International Journal of Neutrosophic Science, Vol. 7, pp. 97-109. 2020.

[38]Ibrahim, M.A., Agboola, A.A.A, Badmus, B.S., and Akinleye, S.A., "On refined Neutrosophic Vector Spaces II", International Journal of Neutrosophic Science, Vol. 9, pp. 22-36. 2020.

[39]Abobala, M, "*n*-Cyclic Refined Neutrosophic Algebraic Systems Of Sub-Indeterminacies, An Application To Rings and Modules", International Journal of Neutrosophic Science, Vol. 12, pp. 81-95 . 2020.

[40]Smarandache, F., "Neutrosophic Set a Generalization of the Intuitionistic Fuzzy Sets", Inter. J. Pure Appl. Math., pp. 287-297. 2005.

[41] Abobala, M., "On Some Special Elements In Neutrosophic Rings and Refined Neutrosophic Rings", Journal of New Theory, vol. 33, 2020.

[42]Abobala, M., Hatip, A., Olgun, N., Broumi, S., Salama, A,A., and Khaled, E, H., The algebraic creativity In The Neutrosophic Square Matrices, Neutrosophic Sets and Systems, Vol. 40, pp. 1-11, 2021.

[43] Aswad, F, M., " A Study of Neutrosophic Complex Number and Applications", Neutrosophic Knowledge, Vol. 1, 2020.

[44] Aswad. F, M., " A Study of neutrosophic Bi Matrix", Neutrosophic Knowledge, Vol. 2, 2021.

[45] Abobala, M., "Neutrosophic Real Inner Product Spaces", Neutrosophic Sets and Systems, vol. 43, 2021.

[46] Aswad, M., " A Study of The Integration Of Neutrosophic Thick Function", International journal of neutrosophic Science, 2020.

[47] Abobala, M., "A Study Of Nil Ideals and Kothe's Conjecture In Neutrosophic Rings", International Journal of Mathematics and Mathematical Sciences, hindawi, 2021.

[48] Ali, R, "Neutrosophic Matrices and their Properties", Hal- Archives, 2021.

[49] Abobala, M., and Hatip, A., "An Algebraic Approach to Neutrosophic Euclidean Geometry", Neutrosophic Sets and Systems, Vol. 43, 2021.

[50] Aswad, M., " A Study Of neutrosophic Differential Equation By using A Neutrosophic Thick Function", neutrosophic knowledge, Vol. 1, 2020.

[51] Abobala, M., " A Review On Recent Developments of Neutrosophic Matrix Theory and Open Problems", hal- Archives, 2021.

[52] Abobala, M., "n- Refined Neutrosophic Groups II", International journal Of Neutrosophic Science, Vol. 0, 2019.