TOPSIS Approach for MCGDM based on Intuitionistic Fuzzy Rough Dombi Aggregation Operations

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Abstract: Atanassov presented the dominant notion of intuitionistic fuzzy sets (IFS) which brought revolution in different fields of science since their inception. The aim of this manuscript is to propos IF rough TOPSIS method based on Dombi operations. For this, first we proposed some new operational laws based on Dombi operations to aggregate averaging and geometric aggregation operators. On the proposed concept, we presented IF rough Dombi weighted averaging (IFRDWA), IF rough Dombi ordered weighted averaging (IFRDOWA) and IF rough Dombi hybrid averaging (IFRDHA) operators. Moreover, on the developed concept we presented IF rough Dombi weighted geometric (IFRDWG), IF rough Dombi ordered weighted geometric (IFRDOWG) and IF rough Dombi hybrid geometric (IFRDHG) operators. The basic related properties of the developed operators are presented in detailed. Then the algorithm for MCGDM based on TOPSIS method for IF rough Dombi averaging and geometric operators is presented. By applying accumulated geometric operator, the IF rough numbers are converted into the IF numbers. The massive outbreak of the pandemic COVID-19 promoted the challenging scenario for the world organizations including scientists, laboratories and researchers to conduct special clinical treatment strategies to prevent the people from COVID-19 pandemic. Additionally, an illustrative example is proposed to solve MCGDM problem to diagnose the most severe patient of COVID-19 by applying TOPSIS. Finally, a comparative analysis of the developed model is presented with some existing methods which shows the applicability and superiority of the developed model.

Key words: IFS, Rough sets, Dombi Operations, averaging and geometric aggregation operators, TOPSIS, MCGDM

1. Introduction

The multi criteria group decision making (MCGDM) the most significant and prominent methodology, in which a team of professional specialist evaluate alternatives for the selection of best optimal object based on multiple criteria. Group decision making (DM) has the ability and capability to improve the assessment process by evaluating multiple conflicting criteria based on the performance of alternatives from independent aspects. In DM it's hard to avoid the uncertainty due to the imprecise judgement by the professional specialist. The process of DM has engaged the attention of scholars in diverse directions around the world and gained the fruitful results by applying different approaches [1, 2]. To cope with vague and uncertain data Zadeh [3] originated the prominent concept of fuzzy sets (FS) and this concept has strong description of ambiguous information in MCGDM problems. After the inception of FS, researchers carried out different methods by applying the concept of FS in diverse directions [4, 5]. Atanassov [6] initiated the dominant notion of intuitionistic fuzzy set [IFS], having the property which is incorporated by the membership degree (MeD) and nonmembership degree (NonMeD) such that their sum belongs to [0, 1], which enables better description of the imperfect and imprecise date in DM problems. Thao and Nguyen [7] put forwarded the concept of correlation coefficient and proposed for the same concept to determine the variance and covariance in sense of IFS. Chen et al. [8] presented new fuzzy DM methods based on evidential reasoning strategy. Chen and Chun [9] put forwarded the technique for TOPSIS method similarity measure based on IF date. In DM one of the most serious issue is to aggregate the preferences reports presented by the several professional experts to get a unique result. In this situation aggregation operators (AO) play significant role to aggregate the collective information presented from the different sources. Xu [10], Yager and Xu [11] developed the dominant concepts of IFWA and IFWG aggregation operators and discussed their fundamental properties. Garg [12] built up some improvement in averaging operators and proposed a series IF interactive weighted averaging operators. Li [13] originated idea of the generalized OWAO to aggregate the decision maker's assessment by applying IF information and solved MADM problems on the proposed concept. Wei [14] investigated the concept of IFOWGA operators and interval-valued IFOWGA operators and presented an illustrated example on the proposed model. The concept of Einstein operators was proposed by Wang and Liu [15] by applying IF information. Huang [16] originated the idea of some new Hamacher operators by applying the idea of IFS and then applied the developed concept in DM. Hwang and Yoon [17] initiated the dominant and top most method technique for order preference by similarity to ideal solution (TOPSIS). This model measures the shortest and farthest distance from PIS and NIS. Garg and Kumar [18] initiated the idea of exponential distance measure by applying the technique of TOPSIS method under interval-valued IFS and solve it application in DM. The concept of new distance measure was proposed by Shen et al. [19] and by applying TOPSIS technique under IF environment and studied its desirable properties. Zeng and Xiao [20] originated TOPSIS technique based on averaging distance and initiated it desirable characteristics. Zeng et al. [21] developed a new score function and used VIKOR and TOPSIS for ranking IF numbers. Zulqarnain et al. [22] proposed the model for TOPSIS approach by using interval-valued IF soft set based on correlation coefficient to aggregate the expert's decision by applying soft aggregation operators. By applying the idea of cosine function Ye [23] discussed concept of two similarity measure. Garg and Kumar [24] proposed similarity measure by using set pair analysis theory. By using the concept of direct operation, Song et al. [25] put forward the notion of similarity measure under IF environment. The geometrical interpretation of entropy measure under IFS was proposed by Szmidt and Kacprzyk [26]. A novel approach of entropy and similarity measure was proposed by Meng and Chen [27] which is based on fuzzy measure. Lin and Ren [28] proposed a new approach for entropy measure based on the weight determination. Garg [29] made some improvement in cosine similarity measure. Yager [30] address the shortcoming in IFS and originated the concept of Pythagorean fuzzy sets (PFS) which become a hot research area for scholars. The notions of averaging and geometric operators was proposed by Yager [31]. Peng et al. [32] put forward some result in PFS. Hussain et al. [33, 34] introduced the algebraic structure of PFS in semigroup and further presented its combined studied with soft and rough sets. Zhang [35] proposed TOPSIS for PSF and described its application in DM. In spite of these, the concept of q-rung orthopair fuzzy sets (qROFS) was delivered by Yager [36]. Ali [37] initiated the ideas of orbits and L-fuzzy sets in qROFS. Hussain et al. [38, 39, 40, 41] proposed the combined study of qROFS with rough and soft sets. Ashraf et al. [42] investigated Einstein averaging and geometric operations for qROF rough sets through EDAS method.

In 1982 Dombi [43] investigated Dombi t-norm and Dombi t-conorm based on Dombi operational parameter. The concept of IFWA and IFWG operators based on Dombi operations was proposed by Seikh et al [44]. The idea of Bonferroni mean operations was proposed by Lui et al. [45] to aggregate the multi attributes based on IF aggregation operators and proposed it application in DM. Later, Chen and Ye [46] made an effort to proposed the Dombi operation in neurtrosophic information and constructed its application in DM. We and We [47] initiated the hybrid study of Dombi operation with prioritized aggregation operators. Jana et al. [48] put forward the idea of arithmetic and geometric operations based on bipolar fuzzy Dombi operations.

Pawlak [49] initiated the prominent concept of rough set (RS) and this novel concept generalized the crisp set theory. The developed notion of RS theory handles the uncertainty and vagueness in more accurate way than classical set theory. From the inception, RS theory has been presented in different directions and proposed its applications in both practical and theoretical aspect as well. Dubois and Prade [50] put forward the idea of fuzzy RS based on fuzzy relation. Cornelis et al. [51] developed the combined structure of RS and IFS to get the dominant concept of IF rough set (IFRS). The constrictive and axiomatic study of rough set was presented by Zhou and Wu [52] by utilizing IF rough aggregation operators. Zhou and Wu [53] developed the idea of rough IFS and IFRS by applying crisp and fuzzy relation. Bustince and Burillo [54] developed the notion of IF relation. By applying the generalized IF relation Zhang et al. [55] proposed IFRS instead of IF relation. Moreover, the combine study of RS, soft set and IFS was investigated by Zhang et al. [56] to obtained the novel concepts of soft rough IFS and IF soft RS based on crisp and fuzzy approximation spaces. By applying the IF soft relation, Zhang et al. [57] developed concept of generalized IF soft RS. Chinram et al. [58] presented the concept of IF rough aggregation operators to aggregate the multi assessment of experts to get a unique optimal option based on IFRWA, IFRWG, IFROWA, IFROWG, IFRHA and IFRHG operators and by applying EDAS method to illustrate the DM application. Later on, Yhya [59] developed the IFR frank aggregation operators and discussed it basic properties. From the above analysis and discussion, it is clear that Dombi operations has natural resilience and flexibility to demonstrate the datum and questionable real life issues more effectively. Furthermore, the behavior of general operational parameter β in Dombi operations has more importance to express the decision maker's attitude. Therefore, motivated from the existing literature, in the current work we introduced the combine study of IFR averaging and geometric aggregation operators based on Dombi operations to get the novel concepts of IFRDWA and IFRDWG aggregation operators. Moreover, we developed IFRDOWA, IFRDHA, IFRDOWG and IFRDHG operators and investigated their desirable properties with details. The remaining portion of the manuscript is managed as.

In Section 2, of the manuscript some basic concepts are given which will be helpful for onward sections. Section 3, consists of Dombi operations and proposed some new operational laws based on Dombi operations to aggregate averaging operators and geometric operators. In Section 4, we investigated the notion of IFRDWA, IFRDOWA and IFRDHA operators. Moreover, in Section 5, we developed the concept of IFRDWG, IFRDOWG and IFRDHA operators. The fundamental related characteristics of the developed operators are presented in detailed. Section 6, we developed a step algorithm of TOPSIS method for MCGDM based on for IF rough Dombi averaging and geometric operators. In Section 7, an illustrative example is proposed to solve MCGDM problem to diagnose the most severe patient of COVID-19 by applying TOPSIS technique. Finally, a comparative analysis of the developed model is presented with different previous models in literature which presents that the investigated concepts are more resilience and flexible than the developed models.

2. Preliminaries

This section, includes the review of some elementary definitions, operations and their score values, which associate the existing literature with the developed concepts.

Definition 1 [6]. Consider K be a universal set, and IFS $\mathfrak{G}$ on the set K is given as;

$$\mathfrak{G} = \{\langle \mathfrak{g}, \mathscr{k}_{\mathfrak{G}}(\mathfrak{g}), \delta_{\mathfrak{G}}(\mathfrak{g})\rangle | \mathfrak{g} \in K\},$$

where $\mathscr{k}_{\mathfrak{G}}, \delta_{\mathfrak{G}}: K \to [0,1]$, represent the MeD and NonMeD of an object $\mathfrak{g} \in K$, to the set $\mathfrak{G}$ with $0 \le \mathscr{k}_{\mathfrak{G}}(\mathfrak{g}) + \delta_{\mathfrak{G}}(\mathfrak{g}) \le 1$. Moreover, $\pi_{\mathfrak{G}}(\mathfrak{g}) = 1 - (\mathscr{k}_{\mathfrak{G}}(\mathfrak{g}) + \delta_{\mathfrak{G}}(\mathfrak{g}))$ denotes the hesitancy degree of an alternative $\mathfrak{g} \in K$. For simplicity $\mathfrak{G} = \langle \mathfrak{g}, \mathscr{k}_{\mathfrak{G}}(\mathfrak{g}), \delta_{\mathfrak{G}}(\mathfrak{g})\rangle$ is denoted by $\mathfrak{G} = (\mathscr{k}_{\mathfrak{G}}, \delta_{\mathfrak{G}})$ and is known is IF number (IFN) for $\mathfrak{g} \in K$.

Assume to IFNs $\mathfrak{G} = (\mathscr{k}_{\mathfrak{G}}, \delta_{\mathfrak{G}})$ and $\mathfrak{G}_1 = (\mathscr{k}_{\mathfrak{G}_1}, \delta_{\mathfrak{G}_1})$, then some fundamental operations on them are defined as:

(i) $\mathfrak{G} \cup \mathfrak{G}_1 = (\max(\mathscr{k}_{\mathfrak{G}}(\mathfrak{g}), \mathscr{k}_{\mathfrak{G}_1}(\mathfrak{g})), min((\delta_{\mathfrak{G}}(\mathfrak{g}), \delta_{\mathfrak{G}_1}(\mathfrak{g}))))$;
(ii) $\mathfrak{G} \cap \mathfrak{G}_1 = (\min(\mathscr{k}_{\mathfrak{G}}(\mathfrak{g}), \mathscr{k}_{\mathfrak{G}_1}(\mathfrak{g})), max(\delta_{\mathfrak{G}}(\mathfrak{g}), \delta_{\mathfrak{G}_1}(\mathfrak{g})))$;
(iii) $\mathfrak{G} \oplus \mathfrak{G}_1 = (\mathscr{k}_{\mathfrak{G}} + \mathscr{k}_{\mathfrak{G}_1} - \mathscr{k}_{\mathfrak{G}}\mathscr{k}_{\mathfrak{G}_1}, \delta_{\mathfrak{G}}\delta_{\mathfrak{G}_1})$;
(iv) $\mathfrak{G} \otimes \mathfrak{G}_1 = (\mathscr{k}_{\mathfrak{G}}\mathscr{k}_{\mathfrak{G}_1}, \delta_{\mathfrak{G}} + \delta_{\mathfrak{G}_1} - \delta_{\mathfrak{G}}\delta_{\mathfrak{G}_1})$;
(v) $\mathfrak{G} \le \mathfrak{G}_1$ if $\mathscr{k}_{\mathfrak{G}}(\mathfrak{g}) \le \mathscr{k}_{\mathfrak{G}_1}(\mathfrak{g}), \delta_{\mathfrak{G}}(\mathfrak{g}) \ge \delta_{\mathfrak{G}_1}(\mathfrak{g})$ for all $\mathfrak{g} \in K$;
(vi) $\mathfrak{G}^c = (\delta_{\mathfrak{G}}, \mathscr{k}_{\mathfrak{G}})$ where $\mathfrak{G}^c$ represents the complement of $\mathfrak{G}$;
(vii) $\alpha\mathfrak{G} = (1 - (1 - \mathscr{k}_{\mathfrak{G}})^{\alpha}, \delta_{\mathfrak{G}}^{\alpha})$ for $\alpha \ge 1$;
(viii) $\mathfrak{G}^{\alpha} = (\mathscr{k}_{\mathfrak{G}}^{\alpha}, 1 - (1 - \delta_{\mathfrak{G}})^{\alpha})$ for $\alpha \ge 1$.

Definition 2 [60]**.** Consider the score function for IFN $\mathfrak{G} = (\mathscr{k}_{\mathfrak{G}}, \delta_{\mathfrak{G}})$, is denoted and defined as;

$$\overline{\overline{S}}(\mathfrak{G}) = \mathscr{k}_{\mathfrak{G}} - \delta_{\mathfrak{G}} \quad for\, \overline{\overline{S}}(\mathfrak{G}) \in [-1,1].$$

Greater the score batter the IFN is.

Definition 3 [61]. The accuracy function for IFN $\mathfrak{G} = (\mathscr{k}_{\mathfrak{G}}, \delta_{\mathfrak{G}})$, is denoted and defined as;

$$\overline{\overline{A}}(\mathfrak{G}) = \mathscr{k}_{\mathfrak{G}} + \delta_{\mathfrak{G}} \quad for\, \overline{\overline{A}}(\mathfrak{G}) \in [0,1].$$

Definition 4 [52]. Assume a fixed set K and crisp IF relation $\psi \in IFS(K \times K)$. Then

(i) For all $\mathfrak{g} \in K$, the relation ψ is reflexive, if $\hbar_{\psi}(\mathfrak{g},\mathfrak{g}) = 1$ and $\delta_{\psi}(\mathfrak{g},\mathfrak{g}) = 0$.
(ii) For all $(\mathfrak{g},c) \in K \times K$, the relation ψ is symmetric, if $\hbar_{\psi}(\mathfrak{g},c) = \hbar_{\psi}(\mathfrak{g},c)$ and $\delta_{\psi}(\mathfrak{g},c) = \delta_{\psi}(\mathfrak{g},c)$.
(iii) For all $(\mathfrak{g},d) \in K \times K$, the relation ψ is transitive, if $\hbar_{\psi}(\mathfrak{g},d) \geq \bigvee_{c \in K}\{\hbar_{\psi}(\mathfrak{g},c) \vee \hbar_{\psi}(\mathfrak{g},c)\}$ and $\delta_{\psi}(\mathfrak{g},d) \geq \bigwedge_{c \in K}\{\delta_{\psi}(\mathfrak{g},c) \wedge \delta_{\psi}(c,d)\}$.

Definition 5 [58]. Consider K as a universal of discourse such that ψ be IF relation over K i.e. $\psi \in IFS(K \times K)$. Then the order pair (K,ψ) is known to be IF approximation space. Now any normal decision object $\mathfrak{B} \subseteq IFS(K)$, the lower and upper approximation of $\mathfrak{B}$ w.r.t IF approximation space (K,ψ) are represented by $\underline{\psi}(\mathfrak{B})$ and $\overline{\psi}(\mathfrak{B})$ which is defined as:

$$\underline{\psi}(\mathfrak{B}) = \{\langle \mathfrak{g}, \hbar_{\underline{\psi}(\mathfrak{B})}(\mathfrak{g}), \delta_{\underline{\psi}(\mathfrak{B})}(\mathfrak{g})\rangle | \mathfrak{g} \in K\}$$
$$\overline{\psi}(\mathfrak{B}) = \{\langle \mathfrak{g}, \hbar_{\overline{\psi}(\mathfrak{B})}(\mathfrak{g}), \delta_{\overline{\psi}(\mathfrak{B})}(\mathfrak{g})\rangle | \mathfrak{g} \in K\}$$

where

$$\hbar_{\underline{\psi}(\mathfrak{B})}(\mathfrak{g}) = \bigwedge_{c \in K}\{\hbar_{\psi}(\mathfrak{g},c) \wedge \hbar_{\mathfrak{B}}(c)\},\ \delta_{\underline{\psi}(\mathfrak{B})}(\mathfrak{g}) = \bigvee_{c \in K}\{\delta_{\psi}(\mathfrak{g},c) \vee \delta_{\mathfrak{B}}(c)\}$$

$$\hbar_{\overline{\psi}(\mathfrak{B})}(\mathfrak{g}) = \bigvee_{c \in K}\{\hbar_{\psi}(\mathfrak{g},c) \vee \hbar_{\mathfrak{B}}(c)\},\ \delta_{\overline{\psi}(\mathfrak{B})}(\mathfrak{g}) = \bigwedge_{c \in K}\{\delta_{\psi}(\mathfrak{g},c) \wedge \delta_{\mathfrak{B}}(c)\}$$

with $0 \leq \hbar_{\underline{\psi}(\mathfrak{B})}(\mathfrak{g}) + \delta_{\underline{\psi}(\mathfrak{B})}(\mathfrak{g}) \leq 1$ and $0 \leq \hbar_{\overline{\psi}(\mathfrak{B})}(\mathfrak{g}) + \delta_{\overline{\psi}(\mathfrak{B})}(\mathfrak{g}) \leq 1$. As $\underline{\psi}(\mathfrak{B})$ and $\overline{\psi}(\mathfrak{B})$ are IFS, so $\underline{\psi}(\mathfrak{B}),\overline{\psi}(\mathfrak{B})$ $:IFS(K) \to IFS(K)$ are lower and upper approximation operators. Therefore, the pair $\psi(\mathfrak{B}) = (\underline{\psi}(\mathfrak{B}),\overline{\psi}(\mathfrak{B})) = \{(\mathfrak{g}, \langle \hbar_{\underline{\psi}(\mathfrak{B})}(\mathfrak{g}), \delta_{\underline{\psi}(\mathfrak{B})}(\mathfrak{g})\rangle, \langle \hbar_{\overline{\psi}(\mathfrak{B})}(\mathfrak{g}), \delta_{\overline{\psi}(\mathfrak{B})}(\mathfrak{g})\rangle) | \mathfrak{g} \in K\}$ is called IF rough set (IFRS). For simplicity $\psi(\mathfrak{B}) = (\underline{\psi}(\mathfrak{B}),\overline{\psi}(\mathfrak{B})) = (\langle \underline{\hbar}, \underline{\delta}\rangle, \langle \overline{\hbar}, \overline{\delta}\rangle)$ denotes the IF rough number (IFRN).

Definition 6 [58]. Consider the score function for IFRN $\psi(\mathfrak{B}) = (\underline{\psi}(\mathfrak{B}),\overline{\psi}(\mathfrak{B})) = (\langle \underline{\hbar}, \underline{\delta}\rangle, \langle \overline{\hbar}, \overline{\delta}\rangle)$, is denoted and defined as;

$$\overline{\overline{S}}(\mathfrak{B}) = \frac{1}{4}(2 + \underline{\hbar} + \overline{\hbar} - \underline{\delta} - \overline{\delta}) \quad for\ \overline{\overline{S}}(\mathfrak{B}) \in [0,1].$$

Greater the score batter the IFRN is.

Definition 7 [59]. The accuracy function for IFRN $\psi(\mathfrak{B}) = (\underline{\psi}(\mathfrak{B}),\overline{\psi}(\mathfrak{B})) = (\langle \underline{\hbar}, \underline{\delta}\rangle, \langle \overline{\hbar}, \overline{\delta}\rangle)$, is denoted and defined as;

$$\overline{\overline{A}}(\mathfrak{B}) = \frac{1}{4}(\underline{\hbar} + \overline{\hbar} + \underline{\delta} + \overline{\delta}) \quad for\ \overline{\overline{A}}(\mathfrak{B}) \in [0,1].$$

3. Dombi Operations

Dombi presented the pioneer concept Dombi operations known as Dombi product and Dombi sum, which are the special form of t-norms and t-conorms given in the following definition.

Definition 8 [43]. Consider that ξ and υ belong to real numbers with $\beta \geq 1$. Then Dombi operations are elaborated as:

$$T_D(\xi,\upsilon) = \frac{1}{1 + \left\{\left(\frac{1-\xi}{\xi}\right)^{\beta} + \left(\frac{1-\upsilon}{\upsilon}\right)^{\beta}\right\}^{\frac{1}{\beta}}}$$

$$T_{D'}(\xi,\upsilon) = 1 - \frac{1}{1 + \left\{ \left(\frac{\xi}{1-\xi} \right)^{\beta} + \left(\frac{\upsilon}{1-\upsilon} \right)^{\beta} \right\}^{\frac{1}{\beta}}}$$

Definition 9. Let $\psi(\mathfrak{B}_1) = \left(\underline{\psi}(\mathfrak{B}_1),\overline{\psi}(\mathfrak{B}_1)\right) = \left(\langle \underline{\mathscr{k}_1},\underline{\delta_1}\rangle,\langle \overline{\mathscr{k}_1},\overline{\delta_1}\rangle\right)$ and $\psi(\mathfrak{B}_2) = \left(\underline{\psi}(\mathfrak{B}_2),\overline{\psi}(\mathfrak{B}_2)\right) = \left(\langle \underline{\mathscr{k}_2},\underline{\delta_2}\rangle,\langle \overline{\mathscr{k}_2},\overline{\delta_2}\rangle\right)$ be two IFRNs and $\alpha > 0$. Then some basic operation based on Dombi t-norms and t-conorms operations are give as:

(i) $\psi(\mathfrak{B}_1) \oplus \psi(\mathfrak{B}_2) =$

$$\left\{ \left(1 - \frac{1}{1 + \left\{ \left(\frac{\underline{\mathscr{k}_1}}{1-\underline{\mathscr{k}_1}} \right)^{\beta} + \left(\frac{\underline{\mathscr{k}_2}}{1-\underline{\mathscr{k}_2}} \right)^{\beta} \right\}^{\frac{1}{\beta}}}, \frac{1}{1 + \left\{ \left(\frac{1-\underline{\delta_1}}{\underline{\delta_1}} \right)^{\beta} + \left(\frac{1-\underline{\delta_2}}{\underline{\delta_2}} \right)^{\beta} \right\}^{\frac{1}{\beta}}} \right), \left(1 - \frac{1}{1 + \left\{ \left(\frac{\overline{\mathscr{k}_1}}{1-\overline{\mathscr{k}_1}} \right)^{\beta} + \left(\frac{\overline{\mathscr{k}_2}}{1-\overline{\mathscr{k}_2}} \right)^{\beta} \right\}^{\frac{1}{\beta}}}, \frac{1}{1 + \left\{ \left(\frac{1-\overline{\delta_1}}{\overline{\delta_1}} \right)^{\beta} + \left(\frac{1-\overline{\delta_2}}{\overline{\delta_2}} \right)^{\beta} \right\}^{\frac{1}{\beta}}} \right) \right\},$$

(ii) $\psi(\mathfrak{B}_1) \otimes \psi(\mathfrak{B}_2) =$

$$\left\{ \left(\frac{1}{1 + \left\{ \left(\frac{1-\underline{\mathscr{k}_1}}{\underline{\mathscr{k}_1}} \right)^{\beta} + \left(\frac{1-\underline{\mathscr{k}_2}}{\underline{\mathscr{k}_2}} \right)^{\beta} \right\}^{\frac{1}{\beta}}}, 1 - \frac{1}{1 + \left\{ \left(\frac{\underline{\delta_1}}{1-\underline{\delta_1}} \right)^{\beta} + \left(\frac{\underline{\delta_2}}{1-\underline{\delta_2}} \right)^{\beta} \right\}^{\frac{1}{\beta}}} \right), \left(\frac{1}{1 + \left\{ \left(\frac{1-\overline{\mathscr{k}_1}}{\overline{\mathscr{k}_1}} \right)^{\beta} + \left(\frac{1-\overline{\mathscr{k}_2}}{\overline{\mathscr{k}_2}} \right)^{\beta} \right\}^{\frac{1}{\beta}}}, 1 - \frac{1}{1 + \left\{ \left(\frac{\overline{\delta_1}}{1-\overline{\delta_1}} \right)^{\beta} + \left(\frac{\overline{\delta_2}}{1-\overline{\delta_2}} \right)^{\beta} \right\}^{\frac{1}{\beta}}} \right) \right\},$$

(iii) $$\alpha\psi(\mathfrak{B}_1) = \left\{ \left(1 - \frac{1}{1 + \left\{ \alpha \left(\frac{\underline{\mathscr{k}_1}}{1-\underline{\mathscr{k}_1}} \right)^{\beta} \right\}^{\frac{1}{\beta}}}, \frac{1}{1 + \left\{ \alpha \left(\frac{1-\underline{\delta_1}}{\underline{\delta_1}} \right)^{\beta} \right\}^{\frac{1}{\beta}}} \right), \left(1 - \frac{1}{1 + \left\{ \alpha \left(\frac{\overline{\mathscr{k}_1}}{1-\overline{\mathscr{k}_1}} \right)^{\beta} \right\}^{\frac{1}{\beta}}}, \frac{1}{1 + \left\{ \alpha \left(\frac{1-\overline{\delta_1}}{\overline{\delta_1}} \right)^{\beta} \right\}^{\frac{1}{\beta}}} \right) \right\},$$

(iv) $$\left(\psi(\mathfrak{B}_1)\right)^{\alpha} = \left\{ \left(\frac{1}{1 + \left\{ \alpha \left(\frac{1-\underline{\mathscr{k}_1}}{\underline{\mathscr{k}_1}} \right)^{\beta} \right\}^{\frac{1}{\beta}}}, 1 - \frac{1}{1 + \left\{ \alpha \left(\frac{\underline{\delta_1}}{1-\underline{\delta_1}} \right)^{\beta} \right\}^{\frac{1}{\beta}}} \right), \left(\frac{1}{1 + \left\{ \alpha \left(\frac{1-\overline{\mathscr{k}_1}}{\overline{\mathscr{k}_1}} \right)^{\beta} \right\}^{\frac{1}{\beta}}}, 1 - \frac{1}{1 + \left\{ \alpha \left(\frac{\overline{\delta_1}}{1-\overline{\delta_1}} \right)^{\beta} \right\}^{\frac{1}{\beta}}} \right) \right\}.$$

Theorem 1. Let $\psi(\mathfrak{B}_1) = \left(\underline{\psi}(\mathfrak{B}_1),\overline{\psi}(\mathfrak{B}_1)\right)$ and $\psi(\mathfrak{B}_2) = \left(\underline{\psi}(\mathfrak{B}_2),\overline{\psi}(\mathfrak{B}_2)\right)$ be two IFRNs and $\alpha_1,\alpha_2 > 0$. Then the following results are hold:

(i) $\psi(\mathfrak{B}_1) \oplus \psi(\mathfrak{B}_2) = \psi(\mathfrak{B}_2) \oplus \psi(\mathfrak{B}_1)$,
(ii) $\psi(\mathfrak{B}_1) \otimes \psi(\mathfrak{B}_2) = \psi(\mathfrak{B}_2) \otimes \psi(\mathfrak{B}_1)$,
(iii) $\alpha_1\left(\psi(\mathfrak{B}_1) \oplus \psi(\mathfrak{B}_2)\right) = \alpha_1\psi(\mathfrak{B}_1) \oplus \alpha_1\psi(\mathfrak{B}_2)$,
(iv) $(\alpha_1 + \alpha_2)\psi(\mathfrak{B}_1) = \alpha_1\psi(\mathfrak{B}_1) \oplus \alpha_2\psi(\mathfrak{B}_1)$,
(v) $\left(\psi(\mathfrak{B}_1) \otimes \psi(\mathfrak{B}_2)\right)^{\alpha_1} = \left(\psi(\mathfrak{B}_1)\right)^{\alpha_1} \otimes \left(\psi(\mathfrak{B}_2)\right)^{\alpha_1}$,
(vi) $\left(\psi(\mathfrak{B}_1)\right)^{\alpha_1} \otimes \left(\psi(\mathfrak{B}_1)\right)^{\alpha_2} = \left(\psi(\mathfrak{B}_1)\right)^{(\alpha_1 + \alpha_2)}$.

4. Average Aggregation Operators

The process of aggregation plays a key role to in DM to aggregate the multiple input information of different specialists into a single value. Here we will address the concept of IFRDWA, IFRDOWA and IFRDHA aggregation operators and presented the important properties of these operators.

4.1. IF rough Dombi weighted averaging operators

Definition 10. Assume that $\psi(\mathfrak{B}_i) = \left(\underline{\psi}(\mathfrak{B}_i),\overline{\psi}(\mathfrak{B}_i)\right)$ be the family of IFRNs. Let $\varepsilon = (\varepsilon_1,\varepsilon_2,\varepsilon_3,\dots,\varepsilon_n)^T$ be the weight vector (WV) such that $\sum_{i=1}^{n}\varepsilon_i = 1$ and $\varepsilon_i \in [0,1]$. Then the IFRDWA operator is a mapping $(\psi(\mathfrak{B}))^n \to \psi(\mathfrak{B})$, which is given as:

$$IFRDWA(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\psi(\mathfrak{B}_3),\ldots,\psi(\mathfrak{B}_n)) = \left(\oplus_{i=1}^{n}\varepsilon_i\underline{\psi}(\mathfrak{B}_i),\oplus_{i=1}^{n}\varepsilon_i\overline{\psi}(\mathfrak{B}_i)\right).$$

Theorem 2. Let $\psi(\mathfrak{B}_i) = (\underline{\psi}(\mathfrak{B}_i),\overline{\psi}(\mathfrak{B}_i))$ be the family of IFRNs and $\varepsilon = (\varepsilon_1,\varepsilon_2,\varepsilon_3,\ldots,\varepsilon_n)^T$ be the WV such that $\sum_{i=1}^{n}\varepsilon_i = 1$ and $\varepsilon_i \in [0,1]$. Then by using IFRDWA operator, the aggregated result is described as:

$$IFRDWA(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\psi(\mathfrak{B}_3),\ldots,\psi(\mathfrak{B}_n)) = \left(\oplus_{i=1}^{n}\varepsilon_i\underline{\psi}(\mathfrak{B}_i),\oplus_{i=1}^{n}\varepsilon_i\overline{\psi}(\mathfrak{B}_i)\right)$$

$$= \left\{\left(1-\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{\underline{k}_i}{1-\underline{k}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\underline{\delta}_i}{\underline{\delta}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right),\left(1-\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{\overline{k}_i}{1-\overline{k}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\overline{\delta}_i}{\overline{\delta}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\right\}.$$

Proof. By applying induction method to prove the required result.

Let $n = 2$, and now using the Dombi operational laws, we get

$$IFRDWA(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2)) = \left(\oplus_{i=1}^{2}\varepsilon_i\underline{\psi}(\mathfrak{B}_i),\oplus_{i=1}^{2}\varepsilon_i\overline{\psi}(\mathfrak{B}_i)\right)$$

$$= \left\{\left(1-\frac{1}{1+\left\{\varepsilon_1\left(\frac{\underline{k}_1}{1-\underline{k}_1}\right)^{\beta}+\varepsilon_2\left(\frac{\underline{k}_2}{1-\underline{k}_2}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\varepsilon_1\left(\frac{1-\underline{\delta}_1}{\underline{\delta}_1}\right)^{\beta}+\varepsilon_2\left(\frac{1-\underline{\delta}_2}{\underline{\delta}_2}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right),\right.$$

$$\left.\left(1-\frac{1}{1+\left\{\varepsilon_1\left(\frac{\overline{k}_1}{1-\overline{k}_1}\right)^{\beta}+\varepsilon_2\left(\frac{\overline{k}_2}{1-\overline{k}_2}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\varepsilon_1\left(\frac{1-\overline{\delta}_1}{\overline{\delta}_1}\right)^{\beta}+\varepsilon_2\left(\frac{1-\overline{\delta}_2}{\overline{\delta}_2}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\right\}$$

$$= \left\{\left(1-\frac{1}{1+\left\{\sum_{i=1}^{2}\varepsilon_i\left(\frac{\underline{k}_i}{1-\underline{k}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\sum_{i=1}^{2}\varepsilon_i\left(\frac{1-\underline{\delta}_i}{\underline{\delta}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right),\left(1-\frac{1}{1+\left\{\sum_{i=1}^{2}\varepsilon_i\left(\frac{\overline{k}_i}{1-\overline{k}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\sum_{i=1}^{2}\varepsilon_i\left(\frac{1-\overline{\delta}_i}{\overline{\delta}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\right\}.$$

The result is true for $n = 2$.

Assume that the required result holds for $n = k$, so we have

$$IFRDWA(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\ldots,\psi(\mathfrak{B}_k)) = \left(\oplus_{i=1}^{k}\varepsilon_i\underline{\psi}(\mathfrak{B}_i),\oplus_{i=1}^{k}\varepsilon_i\overline{\psi}(\mathfrak{B}_i)\right)$$

$$= \left\{\left(1-\frac{1}{1+\left\{\sum_{i=1}^{k}\varepsilon_i\left(\frac{\underline{k}_i}{1-\underline{k}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\sum_{i=1}^{k}\varepsilon_i\left(\frac{1-\underline{\delta}_i}{\underline{\delta}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right),\left(1-\frac{1}{1+\left\{\sum_{i=1}^{k}\varepsilon_i\left(\frac{\overline{k}_i}{1-\overline{k}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\sum_{i=1}^{k}\varepsilon_i\left(\frac{1-\overline{\delta}_i}{\overline{\delta}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\right\}.$$

Further, to prove for $n = k + 1$, so we have

$$\begin{aligned} &IFRDWA\{(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\ldots,\psi(\mathfrak{B}_k)),\psi(\mathfrak{B}_{k+1})\} \\ &= \left(\oplus_{i=1}^{k}\varepsilon_i\underline{\psi}(\mathfrak{B}_i),\oplus_{i=1}^{k}\varepsilon_i\overline{\psi}(\mathfrak{B}_i)\right)\oplus\left(\varepsilon_{k+1}\underline{\psi}(\mathfrak{B}_{k+1}),\varepsilon_{k+1}\overline{\psi}(\mathfrak{B}_{k+1})\right) \end{aligned}$$

$$=\left\{\left(1-\frac{1}{1+\left\{\sum_{i=1}^{k}\varepsilon_i\left(\frac{\underline{\Bbbk}_i}{1-\underline{\Bbbk}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\sum_{i=1}^{k}\varepsilon_i\left(\frac{1-\underline{\delta}_i}{\underline{\delta}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right),\left(1-\frac{1}{1+\left\{\sum_{i=1}^{k}\varepsilon_i\left(\frac{\overline{\Bbbk}_i}{1-\overline{\Bbbk}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\sum_{i=1}^{k}\varepsilon_i\left(\frac{1-\overline{\delta}_i}{\overline{\delta}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\right\}\oplus$$

$$\left\{\left(1-\frac{1}{1+\left\{\varepsilon_{k+1}\left(\frac{\underline{\Bbbk}_{k+1}}{1-\underline{\Bbbk}_{k+1}}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\varepsilon_{k+1}\left(\frac{1-\underline{\delta}_{k+1}}{\underline{\delta}_{k+1}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right),\left(1-\frac{1}{1+\left\{\varepsilon_{k+1}\left(\frac{\overline{\Bbbk}_{k+1}}{1-\overline{\Bbbk}_{k+1}}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\varepsilon_{k+1}\left(\frac{1-\overline{\delta}_{\varepsilon_{k+1}}}{\overline{\delta}_{k+1}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\right\}$$

$$=\left\{\left(1-\frac{1}{1+\left\{\sum_{i=1}^{k+1}\varepsilon_i\left(\frac{\underline{\Bbbk}_i}{1-\underline{\Bbbk}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\sum_{i=1}^{k+1}\varepsilon_i\left(\frac{1-\underline{\delta}_i}{\underline{\delta}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right),\left(1-\frac{1}{1+\left\{\sum_{i=1}^{k+1}\varepsilon_i\left(\frac{\overline{\Bbbk}_i}{1-\overline{\Bbbk}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\sum_{i=1}^{k+1}\varepsilon_i\left(\frac{1-\overline{\delta}_i}{\overline{\delta}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\right\}.$$

Hence the condition is true for $n \geq k+1$. Therefore, by induction principle the result holds $\forall\, n \geq 1$.

As $\underline{\psi}(\mathfrak{B})$ and $\overline{\psi}(\mathfrak{B})$ are IFRNs, this implies $\oplus_{i=1}^{n}\varepsilon_i\underline{\psi}(\mathfrak{B}_i)$ and $\oplus_{i=1}^{n}\varepsilon_i\overline{\psi}(\mathfrak{B}_i)$ is also IFRNs. Therefore, from the above analysis $IFRDWA(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),...,\psi(\mathfrak{B}_n))$ also represents an IFRN based on IFR approximation space (K,ψ).

Theorem 3. Let $\psi(\mathfrak{B}_i) = (\underline{\psi}(\mathfrak{B}_i),\overline{\psi}(\mathfrak{B}_i))$ be the family of IFRNs and $\varepsilon = (\varepsilon_1,\varepsilon_2,\varepsilon_3,...,\varepsilon_n)^T$ be the WV such that $\sum_{i=1}^{n}\varepsilon_i = 1$ and $\varepsilon_i \in [0,1]$. Then some elementary properties are satisfied for IFRDWA operator.

(i) **Idempotency**. Let $\psi(\mathfrak{B}_i) = E(\mathfrak{B})\quad \forall\, i = 1,2,...,n$ with $E(\mathfrak{B}) = (\underline{E}(\mathfrak{B}),\overline{E}(\mathfrak{B})) = (\langle\underline{\Bbbk},\underline{\delta}\rangle,\langle\overline{\Bbbk},\overline{\delta}\rangle)$. Then

$$IFRDWA(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),...,\psi(\mathfrak{B}_n)) = E(\mathfrak{B}).$$

(ii) **Boundedness**.
Let $(\psi(\mathfrak{B}_i))^- = \left(\min_i \underline{\psi}(\mathfrak{B}_i),\min_i \overline{\psi}(\mathfrak{B}_i)\right)$ and $(\psi(\mathfrak{B}_i))^+ = \left(\max_i \underline{\psi}(\mathfrak{B}_i),\max_i \overline{\psi}(\mathfrak{B}_i)\right)$. Then

$$(\psi(\mathfrak{B}_i))^- \leq IFRDWA(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),...,\psi(\mathfrak{B}_n)) \leq (\psi(\mathfrak{B}_i))^+.$$

(iii) **Monotonicity**. Consider the another family $\psi(\mathfrak{B}_i^{'}) = (\underline{\psi}(\mathfrak{B}_i^{'}),\overline{\psi}(\mathfrak{B}_i^{'}))$ of IFRNs, such that $\underline{\psi}(\mathfrak{B}_i^{'}) \leq \underline{\psi}(\mathfrak{B}_i)$ and $\overline{\psi}(\mathfrak{B}_i^{'}) \leq \overline{\psi}(\mathfrak{B}_i)$. Then

$$IFRDWA(\psi(\mathfrak{B}_1^{'}),\psi(\mathfrak{B}_2^{'}),...,\psi(\mathfrak{B}_n^{'})) \leq IFRDWA(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),...,\psi(\mathfrak{B}_n)).$$

(iv) **Shift Invariance**. Assume that $E(\mathfrak{B}^{'}) = (\underline{E}(\mathfrak{B}^{'}),\overline{E}(\mathfrak{B}^{'})) = (\langle\underline{\Bbbk}^{'},\underline{\delta}^{'}\rangle,\langle\overline{\Bbbk}^{'},\overline{\delta}^{'}\rangle)$ be another IFRN. Then

$$IFRDWA(\psi(\mathfrak{B}_1)\oplus E(\mathfrak{B}^{'}),\psi(\mathfrak{B}_2)\oplus E(\mathfrak{B}^{'}),...,\psi(\mathfrak{B}_n)\oplus E(\mathfrak{B}^{'})) = IFRDWA(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),...,\psi(\mathfrak{B}_n))\oplus E(\mathfrak{B}^{'}).$$

(v) **Homogeneity**. For a real number $\alpha > 0$,

$$IFRDWA(\alpha\psi(\mathfrak{B}_1),\alpha\psi(\mathfrak{B}_2),...,\alpha\psi(\mathfrak{B}_n)) = \alpha\, IFRDWA(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),...,\psi(\mathfrak{B}_n)).$$

Proof. ***(i) Idempotency***. Since $\psi(\mathfrak{B}_i) = E(\mathfrak{B})\ \forall\, i = 1,2,...,n$ where $E(\mathfrak{B}) = (\underline{E}(\mathfrak{B}),\overline{E}(\mathfrak{B})) = (\langle\underline{\Bbbk},\underline{\delta}\rangle,\langle\overline{\Bbbk},\overline{\delta}\rangle)$. Then by applying Theorem 2, we have

$$IFRDWA(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\psi(\mathfrak{B}_3),...,\psi(\mathfrak{B}_n)) = (\oplus_{i=1}^{n}\varepsilon_i\underline{\psi}(\mathfrak{B}_i),\oplus_{i=1}^{n}\varepsilon_i\overline{\psi}(\mathfrak{B}_i))$$

$$
=\left\{\left(1-\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{\underline{\mathscr{k}}_i}{1-\underline{\mathscr{k}}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\underline{\delta}_i}{\underline{\delta}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right),\right.
$$

$$
\left.\left(1-\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{\overline{\mathscr{k}}_i}{1-\overline{\mathscr{k}}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\overline{\delta}_i}{\overline{\delta}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\right\}
$$

$$
=\left\{\left(1-\frac{1}{1+\left\{\left(\frac{\underline{\mathscr{k}}}{1-\underline{\mathscr{k}}}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\left(\frac{1-\underline{\delta}}{\underline{\delta}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right),\left(1-\frac{1}{1+\left\{\left(\frac{\overline{\mathscr{k}}}{1-\overline{\mathscr{k}}}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\left(\frac{1-\overline{\delta}}{\overline{\delta}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\right\}
$$

$$
=(\underline{E}(\mathfrak{B}),\overline{E}(\mathfrak{B}))
$$

$$
=E(\mathfrak{B})
$$

(ii) ***Boundedness***. As $(\psi(\mathfrak{B}_i))^{-}=\left((\underline{\psi}(\mathfrak{B}_i))^{-},(\overline{\psi}(\mathfrak{B}_i))^{-}\right)=\left[\left(\min_i\{\underline{\mathscr{k}_i}\},\ \max_i\{\underline{\delta_i}\}\right),\left(\min_i\{\overline{\mathscr{k}_i}\},\max_i\{\overline{\delta_i}\}\right)\right]$

and $(\psi(\mathfrak{B}_i))^{+}=\left((\underline{\psi}(\mathfrak{B}_i))^{+},(\overline{\psi}(\mathfrak{B}_i))^{+}\right)=\left[\left(\max_i\{\underline{\mathscr{k}_i}\},\ \min_i\{\underline{\delta_i}\}\right),\left(\max_i\{\overline{\mathscr{k}_i}\},\min_i\{\overline{\delta_i}\}\right)\right]$

and $\psi(\mathfrak{B}_i)=[(\underline{\mathscr{k}_i},\underline{\delta_i}),(\overline{\mathscr{k}_i},\overline{\delta_i})]$. To verify that

$$
(\psi(\mathfrak{B}_i))^{-}\le IFRDWA(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\ldots,\psi(\mathfrak{B}_n))\le(\psi(\mathfrak{B}_i))^{+}
$$

Since for each $i=1,2,\ldots,n$, we have

$$
\min_i\underline{\mathscr{k}_i}\le\underline{\mathscr{k}_i}\le\max_i\underline{\mathscr{k}_i}\Rightarrow\frac{\min_i\underline{\mathscr{k}_i}}{1-\min_i\underline{\mathscr{k}_i}}\le\frac{\underline{\mathscr{k}_i}}{1-\underline{\mathscr{k}_i}}\le\frac{\max_i\underline{\mathscr{k}_i}}{1-\max_i\underline{\mathscr{k}_i}}
$$

$$
\Rightarrow1+\frac{\min_i\underline{\mathscr{k}_i}}{1-\min_i\underline{\mathscr{k}_i}}\le1+\frac{\underline{\mathscr{k}_i}}{1-\underline{\mathscr{k}_i}}\le1+\frac{\max_i\underline{\mathscr{k}_i}}{1-\max_i\underline{\mathscr{k}_i}}\Rightarrow\frac{1}{1+\frac{\max_i\underline{\mathscr{k}_i}}{1-\max_i\underline{\mathscr{k}_i}}}\le\frac{1}{1+\frac{\underline{\mathscr{k}_i}}{1-\underline{\mathscr{k}_i}}}\le\frac{1}{1+\frac{\min_i\underline{\mathscr{k}_i}}{1-\min_i\underline{\mathscr{k}_i}}}
$$

$$
\Rightarrow1-\frac{1}{1+\left(\sum_{i=1}^{n}\varepsilon_i\left(\frac{\min_i\underline{\mathscr{k}_i}}{1-\min_i\underline{\mathscr{k}_i}}\right)^{\beta}\right)^{\frac{1}{\beta}}}\le1-\frac{1}{1+\left(\sum_{i=1}^{n}\varepsilon_i\left(\frac{\underline{\mathscr{k}_i}}{1-\underline{\mathscr{k}_i}}\right)^{\beta}\right)^{\frac{1}{\beta}}}\le1-\frac{1}{1+\left(\sum_{i=1}^{n}\varepsilon_i\left(\frac{\max_i\underline{\mathscr{k}_i}}{1-\max_i\underline{\mathscr{k}_i}}\right)^{\beta}\right)^{\frac{1}{\beta}}}
$$

$$\Rightarrow 1-\frac{1}{1+\frac{\min\limits_{i}\underline{\mathscr{k}_i}}{1-\min\limits_{i}\underline{\mathscr{k}_i}}}\le 1-\frac{1}{1+\left(\sum_{i=1}^{n}\varepsilon_i\left(\frac{\underline{\mathscr{k}_i}}{1-\underline{\mathscr{k}_i}}\right)^{\beta}\right)^{\frac{1}{\beta}}}\le 1-\frac{1}{1+\frac{\max\limits_{i}\underline{\mathscr{k}_i}}{1-\max\limits_{i}\underline{\mathscr{k}_i}}}$$

$$\Rightarrow \min_{i}\underline{\mathscr{k}_i}\le 1-\frac{1}{1+\left(\sum_{i=1}^{n}\varepsilon_i\left(\frac{\underline{\mathscr{k}_i}}{1-\underline{\mathscr{k}_i}}\right)^{\beta}\right)^{\frac{1}{\beta}}}\le \max_{i}\underline{\mathscr{k}_i}$$

Next consider for every $i=1,2,\dots,n$, consider that

$$\max_{i}\{\underline{\delta_i}\}\ge\underline{\delta_i}\ge\min_{i}\{\underline{\delta_i}\}\Rightarrow 1-\min_{i}\{\underline{\delta_i}\}\ge 1-\underline{\delta_i}\ge 1-\max_{i}\{\underline{\delta_i}\}$$

$$\Rightarrow 1+\frac{1-\min\limits_{i}\{\underline{\delta_i}\}}{\min\limits_{i}\{\underline{\delta_i}\}}\ge 1+\frac{1-\underline{\delta_i}}{\underline{\delta_i}}\ge 1+\frac{1-\max\limits_{i}\{\underline{\delta_i}\}}{\max\limits_{i}\{\underline{\delta_i}\}}$$

$$\Rightarrow 1+\left(\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\min\limits_{i}\{\underline{\delta_i}\}}{\min\limits_{i}\{\underline{\delta_i}\}}\right)^{\beta}\right)^{\frac{1}{\beta}}\ge 1+\left(\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\underline{\delta_i}}{\underline{\delta_i}}\right)^{\beta}\right)^{\frac{1}{\beta}}\ge 1+\left(\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\max\limits_{i}\{\underline{\delta_i}\}}{\max\limits_{i}\{\underline{\delta_i}\}}\right)^{\beta}\right)^{\frac{1}{\beta}}$$

$$\Rightarrow\frac{1}{1+\left(\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\max\limits_{i}\{\underline{\delta_i}\}}{\max\limits_{i}\{\underline{\delta_i}\}}\right)^{\beta}\right)^{\frac{1}{\beta}}}\ge\frac{1}{1+\left(\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\underline{\delta_i}}{\underline{\delta_i}}\right)^{\beta}\right)^{\frac{1}{\beta}}}\ge\frac{1}{1+\left(\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\min\limits_{i}\{\underline{\delta_i}\}}{\min\limits_{i}\{\underline{\delta_i}\}}\right)^{\beta}\right)^{\frac{1}{\beta}}}$$

$$\Rightarrow\frac{1}{1+\frac{1-\max\limits_{i}\{\underline{\delta_i}\}}{\max\limits_{i}\{\underline{\delta_i}\}}}\ge\frac{1}{1+\left(\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\underline{\delta_i}}{\underline{\delta_i}}\right)^{\beta}\right)^{\frac{1}{\beta}}}\ge\frac{1}{1+\frac{1-\min\limits_{i}\{\underline{\delta_i}\}}{\min\limits_{i}\{\underline{\delta_i}\}}}$$

$$\Rightarrow\max_{i}\{\underline{\delta_i}\}\ge\frac{1}{1+\left(\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\underline{\delta_i}}{\underline{\delta_i}}\right)^{\beta}\right)^{\frac{1}{\beta}}}\ge\min_{i}\{\underline{\delta_i}\}$$

In the same way, we can prove that

$$\Rightarrow \min_{i}\overline{\mathscr{k}_i}\le 1-\frac{1}{1+\left(\sum_{i=1}^{n}\varepsilon_i\left(\frac{\overline{\mathscr{k}_i}}{1-\overline{\mathscr{k}_i}}\right)^{\beta}\right)^{\frac{1}{\beta}}}\le \max_{i}\overline{\mathscr{k}_i}$$

and

$$\max_i \{ \overline{\delta_i} \} \geq \frac{1}{1 + \left(\sum_{i=1}^{n} \varepsilon_i \left(\frac{1 - \overline{\delta_i}}{\overline{\delta_i}} \right)^{\beta} \right)^{\frac{1}{\beta}}} \geq \min_i \{ \overline{\delta_i} \}$$

Thus from the above analysis, we have

$$(\psi(\mathfrak{B}_i))^- \leq IFRDWA(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\ldots,\psi(\mathfrak{B}_n)) \leq (\psi(\mathfrak{B}_i))^+$$

The proofs of (iii), (iv) and (v) can be follows from (i) and (ii).

4.2. IF rough Dombi ordered weighted averaging operators

Definition 11. Assume that $\psi(\mathfrak{B}_i) = (\underline{\psi}(\mathfrak{B}_i),\overline{\psi}(\mathfrak{B}_i))$ be the family of IFRNs. Let $\varepsilon = (\varepsilon_1,\varepsilon_2,\varepsilon_3,\ldots,\varepsilon_n)^T$ be the WV such that $\sum_{i=1}^{n} \varepsilon_i = 1$ and $\varepsilon_i \in [0,1]$. Then the aggregated result for IFRDOWA operator is a mapping $(\psi(\mathfrak{B}))^n \rightarrow \psi(\mathfrak{B})$, which is given as:

$$IFRDOWA(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\psi(\mathfrak{B}_3),\ldots,\psi(\mathfrak{B}_n)) = (\oplus_{i=1}^{n} \varepsilon_i \underline{\psi}(\mathfrak{B}_{\sigma i}),\oplus_{i=1}^{n} \varepsilon_i \overline{\psi}(\mathfrak{B}_{\sigma i})).$$

Theorem 4. Let $\psi(\mathfrak{B}_i) = (\underline{\psi}(\mathfrak{B}_i),\overline{\psi}(\mathfrak{B}_i))$ be the family of IFRNs and $\varepsilon = (\varepsilon_1,\varepsilon_2,\varepsilon_3,\ldots,\varepsilon_n)^T$ be the WV such that $\sum_{i=1}^{n} \varepsilon_i = 1$ and $\varepsilon_i \in [0,1]$. Then by using IFRDOWA operator, the aggregated result is described as:

$$IFRDOWA(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\psi(\mathfrak{B}_3),\ldots,\psi(\mathfrak{B}_n)) = (\oplus_{i=1}^{n} \varepsilon_i \underline{\psi}(\mathfrak{B}_{\sigma i}),\oplus_{i=1}^{n} \varepsilon_i \overline{\psi}(\mathfrak{B}_{\sigma i}))$$

$$= \left\{ \left(1 - \frac{1}{1 + \left\{ \sum_{i=1}^{n} \varepsilon_i \left(\frac{\underline{\mathscr{k}_{\sigma i}}}{1 - \underline{\mathscr{k}_{\sigma i}}} \right)^{\beta} \right\}^{\frac{1}{\beta}}}, \frac{1}{1 + \left\{ \sum_{i=1}^{n} \varepsilon_i \left(\frac{1 - \underline{\delta_{\sigma i}}}{\underline{\delta_{\sigma i}}} \right)^{\beta} \right\}^{\frac{1}{\beta}}} \right), \left(1 - \frac{1}{1 + \left\{ \sum_{i=1}^{n} \varepsilon_i \left(\frac{\overline{\mathscr{k}_{\sigma i}}}{1 - \overline{\mathscr{k}_{\sigma i}}} \right)^{\beta} \right\}^{\frac{1}{\beta}}}, \frac{1}{1 + \left\{ \sum_{i=1}^{n} \varepsilon_i \left(\frac{1 - \overline{\delta_{\sigma i}}}{\overline{\delta_{\sigma i}}} \right)^{\beta} \right\}^{\frac{1}{\beta}}} \right) \right\},$$

where the IFRN $\psi(\mathfrak{B}_{\sigma i}) = (\underline{\psi}(\mathfrak{B}_{\sigma i}),\overline{\psi}(\mathfrak{B}_{\sigma i}))$ represent the largest permutation of the collection $\psi(\mathfrak{B}_i)$.

Theorem 5. Let $\psi(\mathfrak{B}_i) = (\underline{\psi}(\mathfrak{B}_i),\overline{\psi}(\mathfrak{B}_i))$ be the family of IFRNs and $\varepsilon = (\varepsilon_1,\varepsilon_2,\varepsilon_3,\ldots,\varepsilon_n)^T$ be the WV such that $\sum_{i=1}^{n} \varepsilon_i = 1$ and $\varepsilon_i \in [0,1]$. Then some rudimentary axioms are discussed for IFRDOWA operator.

(i) **Idempotency**. Let $\psi(\mathfrak{B}_i) = E(\mathfrak{B})\ \forall\ i = 1,2,\ldots,n$ such that $E(\mathfrak{B}) = (\underline{E}(\mathfrak{B}),\overline{E}(\mathfrak{B})) = (\langle \underline{\mathscr{k}},\underline{\delta} \rangle,\langle \overline{\mathscr{k}},\overline{\delta} \rangle)$. Then
$$IFRDOWA(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\ldots,\psi(\mathfrak{B}_n)) = E(\mathfrak{B}).$$

(ii) **Boundedness**.
Let $(\psi(\mathfrak{B}_i))^- = \left(\min_i \underline{\psi}(\mathfrak{B}_i),\min_i \overline{\psi}(\mathfrak{B}_i) \right)$ and $(\psi(\mathfrak{B}_i))^+ = \left(\max_i \underline{\psi}(\mathfrak{B}_i),\max_i \overline{\psi}(\mathfrak{B}_i) \right)$. Then
$$(\psi(\mathfrak{B}_i))^- \leq IFRDOWA(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\ldots,\psi(\mathfrak{B}_n)) \leq (\psi(\mathfrak{B}_i))^+.$$

(iii) **Monotonicity**. Consider the another family $\psi(\mathfrak{B}_i^{'}) = (\underline{\psi}(\mathfrak{B}_i^{'}),\overline{\psi}(\mathfrak{B}_i^{'}))$ of IFRNs, such that $\underline{\psi}(\mathfrak{B}_i^{'}) \leq \underline{\psi}(\mathfrak{B}_i)$ and $\overline{\psi}(\mathfrak{B}_i^{'}) \leq \overline{\psi}(\mathfrak{B}_i)$. Then
$$IFRDOWA(\psi(\mathfrak{B}_1^{'}),\psi(\mathfrak{B}_2^{'}),\ldots,\psi(\mathfrak{B}_n^{'})) \leq IFRDOWA(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\ldots,\psi(\mathfrak{B}_n)).$$

(iv) **Shift Invariance**. Assume that $E(\mathfrak{B}^{'}) = (\underline{E}(\mathfrak{B}^{'}),\overline{E}(\mathfrak{B}^{'})) = (\langle \underline{\mathscr{k}^{'}},\underline{\delta^{'}} \rangle,\langle \overline{\mathscr{k}^{'}},\overline{\delta^{'}} \rangle)$ be another IFRN. Then
$$IFRDOWA(\psi(\mathfrak{B}_1) \oplus E(\mathfrak{B}^{'}),\psi(\mathfrak{B}_2) \oplus E(\mathfrak{B}^{'}),\ldots,\psi(\mathfrak{B}_n) \oplus E(\mathfrak{B}^{'})) = IFRDOWA(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\ldots,\psi(\mathfrak{B}_n)) \oplus E(\mathfrak{B}^{'}).$$

(v) **Homogeneity**. For a real number $\alpha > 0$,
$$IFRDOWA(\alpha\psi(\mathfrak{B}_1),\alpha\psi(\mathfrak{B}_2),\ldots,\alpha\psi(\mathfrak{B}_n)) = \alpha\ IFRDOWA(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\ldots,\psi(\mathfrak{B}_n)).$$

4.3. IF rough Dombi hybrid averaging operators

In this portion of the manuscript, to examine relation of the hybrid aggregation operators with IFRDWA and IFRDOWA operators which weight both the ordered position and the arguments value itself, that is IFRDHA generalized both the operations. This subsection consists of the study of IFRDHA operator and discuss its rudimentary properties.

Definition 12. Assume that $\psi(\mathfrak{B}_i)=\left(\underline{\psi}(\mathfrak{B}_i),\overline{\psi}(\mathfrak{B}_i)\right)$ be the family of IFRNs such that $\rho=(\rho_1,\rho_2,\ldots,\rho_n)^T$ be the WV such that $\sum_{i=1}^{n}\rho_i=1$ and $\rho_i\in[0,1]$. Let $\varepsilon=\ (\varepsilon_1,\varepsilon_2,\varepsilon_3,\ldots,\varepsilon_n)^T$ be the associated WV such that $\sum_{i=1}^{n}\varepsilon_i=1$ and $\varepsilon_i\in[0,1]$. Then the aggregated result for IFRDHA operator is a mapping $(\psi(\mathfrak{B}))^n\rightarrow\psi(\mathfrak{B})$, which is given as:

$$IFRDHA\left(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\psi(\mathfrak{B}_3),\ldots,\psi(\mathfrak{B}_n)\right)=\left(\oplus_{i=1}^{n}\varepsilon_i\underline{\psi}(\mathfrak{B}_{\tilde{\sigma}i}),\oplus_{i=1}^{n}\varepsilon_i\overline{\psi}(\mathfrak{B}_{\tilde{\sigma}i})\right).$$

Theorem 6. Let $\psi(\mathfrak{B}_i)=\left(\underline{\psi}(\mathfrak{B}_i),\overline{\psi}(\mathfrak{B}_i)\right)$ be the family of IFRNs such that $\rho=(\rho_1,\rho_2,\ldots,\rho_n)^T$ be the WV such that $\sum_{i=1}^{n}\rho_i=1$ and $\in[0,1]$. Let $\varepsilon=\ (\varepsilon_1,\varepsilon_2,\varepsilon_3,\ldots,\varepsilon_n)^T$ be the associated WV such that $\sum_{i=1}^{n}\varepsilon_i=1$ and $\varepsilon_i\in[0,1]$. Then by using IFRDHA operator, the aggregated result is described as:

$$IFRDHA\left(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\psi(\mathfrak{B}_3),\ldots,\psi(\mathfrak{B}_n)\right)=\left(\oplus_{i=1}^{n}\varepsilon_i\underline{\psi}(\mathfrak{B}_{\tilde{\sigma}i}),\oplus_{i=1}^{n}\varepsilon_i\overline{\psi}(\mathfrak{B}_{\tilde{\sigma}i})\right)$$

$$=\left\{\left(1-\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{\underline{\Bbbk}_{\tilde{\sigma}i}}{1-\underline{\Bbbk}_{\tilde{\sigma}i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\underline{\delta}_{\tilde{\sigma}i}}{\underline{\delta}_{\tilde{\sigma}i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right),\left(1-\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{\overline{\Bbbk}_{\tilde{\sigma}i}}{1-\overline{\Bbbk}_{\tilde{\sigma}i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}},\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\overline{\delta}_{\tilde{\sigma}i}}{\overline{\delta}_{\tilde{\sigma}i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\right\},$$

where the IFRN $\psi(\mathfrak{B}_{\tilde{\sigma}i})=n\rho_i\psi(\mathfrak{B}_i)=n\rho_i\left(\underline{\psi}(\mathfrak{B}_i),\overline{\psi}(\mathfrak{B}_i)\right)$ represent the largest permutation of the collection $\psi(\mathfrak{B}_i)$.

Theorem 7. Let $\psi(\mathfrak{B}_i)=\left(\underline{\psi}(\mathfrak{B}_i),\overline{\psi}(\mathfrak{B}_i)\right)$ be the family of IFRNs and $\varepsilon=\ (\varepsilon_1,\varepsilon_2,\varepsilon_3,\ldots,\varepsilon_n)^T$ be the WV such that $\sum_{i=1}^{n}\varepsilon_i=1$ and $\varepsilon_i\in[0,1]$. Then some rudimentary characteristics are discussed for IFRDHA operator.

(i) **Idempotency**. Let $\psi(\mathfrak{B}_i)=E(\mathfrak{B})\ \ \forall\ i=1,2,\ldots,n$ such that $E(\mathfrak{B})=(\underline{E}(\mathfrak{B}),\overline{E}(\mathfrak{B}))=(\langle\underline{\Bbbk},\underline{\delta}\rangle,\langle\overline{\Bbbk},\overline{\delta}\rangle)$. Then
$$IFRDHA\left(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\ldots,\psi(\mathfrak{B}_n)\right)=E(\mathfrak{B}).$$

(ii) **Boundedness**.
Let $(\psi(\mathfrak{B}_i))^-=\left(\min_i\underline{\psi}(\mathfrak{B}_i),\min_i\overline{\psi}(\mathfrak{B}_i)\right)$ and $(\psi(\mathfrak{B}_i))^+=\left(\max_i\underline{\psi}(\mathfrak{B}_i),\max_i\overline{\psi}(\mathfrak{B}_i)\right)$. Then
$$(\psi(\mathfrak{B}_i))^-\leq IFRDHA\left(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\ldots,\psi(\mathfrak{B}_n)\right)\leq(\psi(\mathfrak{B}_i))^+.$$

(iii) **Monotonicity**. Consider the another family $\psi(\mathfrak{B}_i^{'})=\left(\underline{\psi}(\mathfrak{B}_i^{'}),\overline{\psi}(\mathfrak{B}_i^{'})\right)$ of IFRNs, such that $\underline{\psi}(\mathfrak{B}_i^{'})\leq\underline{\psi}(\mathfrak{B}_i)$ and $\overline{\psi}(\mathfrak{B}_i^{'})\leq\overline{\psi}(\mathfrak{B}_i)$. Then
$$IFRDHA\left(\psi(\mathfrak{B}_1^{'}),\psi(\mathfrak{B}_2^{'}),\ldots,\psi(\mathfrak{B}_n^{'})\right)\leq IFRDHA\left(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\ldots,\psi(\mathfrak{B}_n)\right).$$

(iv) **Shift Invariance**. Assume that $E(\mathfrak{B}^{'})=\left(\underline{E}(\mathfrak{B}^{'}),\overline{E}(\mathfrak{B}^{'})\right)=\left(\langle\underline{\Bbbk}^{'},\underline{\delta}^{'}\rangle,\langle\overline{\Bbbk}^{'},\overline{\delta}^{'}\rangle\right)$ be another IFRN. Then
$$IFRDHA\left(\psi(\mathfrak{B}_1)\oplus E(\mathfrak{B}^{'}),\psi(\mathfrak{B}_2)\oplus E(\mathfrak{B}^{'}),\ldots,\psi(\mathfrak{B}_n)\oplus E(\mathfrak{B}^{'})\right)=IFRHWA\left(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\ldots,\psi(\mathfrak{B}_n)\right)\oplus E(\mathfrak{B}^{'}).$$

(v) **Homogeneity**. For a real number $\alpha>0$,
$$IFRDHA\left(\alpha\psi(\mathfrak{B}_1),\alpha\psi(\mathfrak{B}_2),\ldots,\alpha\psi(\mathfrak{B}_n)\right)=\alpha\ IFRDHA\left(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\ldots,\psi(\mathfrak{B}_n)\right).$$

5. Geometric Aggregation Operators

Here we will originate the novel notion of IFRDWG, IFRDOWG and IFRDHG aggregation operators and presented the important properties of these operators.

5.1. IF rough Dombi weighted geometric operators

Definition 13. Assume that $\psi(\mathfrak{B}_i)=\left(\underline{\psi}(\mathfrak{B}_i),\overline{\psi}(\mathfrak{B}_i)\right)$ be the family of IFRNs. Let $\varepsilon=\ (\varepsilon_1,\varepsilon_2,\varepsilon_3,\ldots,\varepsilon_n)^T$ be the WV such that $\sum_{i=1}^{n}\varepsilon_i=1$ and $\varepsilon_i\in[0,1]$. Then the aggregated result for IFRDWG operator is a mapping $(\psi(\mathfrak{B}))^n\rightarrow\psi(\mathfrak{B})$, which is given as:

$$IFRDWG\left(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\psi(\mathfrak{B}_3),\ldots,\psi(\mathfrak{B}_n)\right)=\left(\otimes_{i=1}^{n}\left(\underline{\psi}(\mathfrak{B}_i)\right)^{\varepsilon_i},\otimes_{i=1}^{n}\left(\overline{\psi}(\mathfrak{B}_i)\right)^{\varepsilon_i}\right).$$

Theorem 8. Let $\psi(\mathfrak{B}_i)=(\underline{\psi}(\mathfrak{B}_i),\overline{\psi}(\mathfrak{B}_i))$ be the family of IFRNs and $\varepsilon=\ (\varepsilon_1,\varepsilon_2,\varepsilon_3,\ldots,\varepsilon_n)^T$ be the WV such that $\sum_{i=1}^{n}\varepsilon_i=1$ and $\varepsilon_i\in[0,1]$. Then by using IFRDWG operator, the aggregated result is described as:

$$IFRDWG(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\psi(\mathfrak{B}_3),\ldots,\psi(\mathfrak{B}_n))=\left(\otimes_{i=1}^{n}(\underline{\psi}(\mathfrak{B}_i))^{\varepsilon_i},\otimes_{i=1}^{n}(\overline{\psi}(\mathfrak{B}_i))^{\varepsilon_i}\right)$$

$$=\left\{\left(\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\underline{\hbar}_i}{\underline{\hbar}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}},1-\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{\underline{\delta}_i}{1-\underline{\delta}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right),\left(\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\overline{\hbar}_i}{\overline{\hbar}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}},1-\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{\overline{\delta}_i}{1-\overline{\delta}_i}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\right\}.$$

Proof. Proof followed from Theorem 2.

Theorem 9. Let $\psi(\mathfrak{B}_i)=(\underline{\psi}(\mathfrak{B}_i),\overline{\psi}(\mathfrak{B}_i))$ be the family of IFRNs and $\varepsilon=\ (\varepsilon_1,\varepsilon_2,\varepsilon_3,\ldots,\varepsilon_n)^T$ be the WV such that $\sum_{i=1}^{n}\varepsilon_i=1$ and $\varepsilon_i\in[0,1]$. Then the elementary result for IFRDWG operator are given as.

(i) **Idempotency**. Let $\psi(\mathfrak{B}_i)=E(\mathfrak{B})\ \forall\ i=1,2,\ldots,n$ such that $E(\mathfrak{B})=(\underline{E}(\mathfrak{B}),\overline{E}(\mathfrak{B}))=(\langle\underline{\hbar},\underline{\delta}\rangle,\langle\overline{\hbar},\overline{\delta}\rangle)$. Then
$$IFRDWG(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\ldots,\psi(\mathfrak{B}_n))=E(\mathfrak{B}).$$

(ii) **Boundedness**.
Let $(\psi(\mathfrak{B}_i))^-=\left(\min_i\underline{\psi}(\mathfrak{B}_i),\min_i\overline{\psi}(\mathfrak{B}_i)\right)$ and $(\psi(\mathfrak{B}_i))^+=\left(\max_i\underline{\psi}(\mathfrak{B}_i),\max_i\overline{\psi}(\mathfrak{B}_i)\right)$. Then
$$(\psi(\mathfrak{B}_i))^-\leq IFRDWG(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\ldots,\psi(\mathfrak{B}_n))\leq(\psi(\mathfrak{B}_i))^+.$$

(iii) **Monotonicity**. Consider the another family $\psi(\mathfrak{B}_i^{'})=(\underline{\psi}(\mathfrak{B}_i^{'}),\overline{\psi}(\mathfrak{B}_i^{'}))$ of IFRNs, such that $\underline{\psi}(\mathfrak{B}_i^{'})\leq\underline{\psi}(\mathfrak{B}_i)$ and $\overline{\psi}(\mathfrak{B}_i^{'})\leq\overline{\psi}(\mathfrak{B}_i)$. Then
$$IFRDWG(\psi(\mathfrak{B}_1^{'}),\psi(\mathfrak{B}_2^{'}),\ldots,\psi(\mathfrak{B}_n^{'}))\leq IFRDWG(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\ldots,\psi(\mathfrak{B}_n)).$$

(iv) **Shift Invariance**. Assume that $E(\mathfrak{B}^{'})=(\underline{E}(\mathfrak{B}^{'}),\overline{E}(\mathfrak{B}^{'}))=(\langle\underline{\hbar}^{'},\underline{\delta}^{'}\rangle,\langle\overline{\hbar}^{'},\overline{\delta}^{'}\rangle)$ be another IFRN. Then
$$IFRDWG(\psi(\mathfrak{B}_1)\oplus E(\mathfrak{B}^{'}),\psi(\mathfrak{B}_2)\oplus E(\mathfrak{B}^{'}),\ldots,\psi(\mathfrak{B}_n)\oplus E(\mathfrak{B}^{'}))=IFRDWG(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\ldots,\psi(\mathfrak{B}_n))\oplus E(\mathfrak{B}^{'}).$$

(v) **Homogeneity**. For a real number $\alpha>0$,
$$IFRDWG(\alpha\psi(\mathfrak{B}_1),\alpha\psi(\mathfrak{B}_2),\ldots,\alpha\psi(\mathfrak{B}_n))=\alpha\ IFRDWG(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\ldots,\psi(\mathfrak{B}_n)).$$

Proof. Proofs are e easy and straightforward.

5.2. IF rough Dombi ordered weighted geometric operators

Definition 14. Assume that $\psi(\mathfrak{B}_i)=(\underline{\psi}(\mathfrak{B}_i),\overline{\psi}(\mathfrak{B}_i))$ be the family of IFRNs. Let $\varepsilon=\ (\varepsilon_1,\varepsilon_2,\varepsilon_3,\ldots,\varepsilon_n)^T$ be the WV such that $\sum_{i=1}^{n}\varepsilon_i=1$ and $\varepsilon_i\in[0,1]$. Then the aggregated result for IFRDOWG operator is a mapping $(\psi(\mathfrak{B}))^n\to\psi(\mathfrak{B})$, which is given as:

$$IFRDOWG(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\psi(\mathfrak{B}_3),\ldots,\psi(\mathfrak{B}_n))=\left(\otimes_{i=1}^{n}(\underline{\psi}(\mathfrak{B}_{\sigma i}))^{\varepsilon_i},\otimes_{i=1}^{n}(\overline{\psi}(\mathfrak{B}_{\sigma i}))^{\varepsilon_i}\right).$$

Theorem 10. Let $\psi(\mathfrak{B}_i)=(\underline{\psi}(\mathfrak{B}_i),\overline{\psi}(\mathfrak{B}_i))$ be the family of IFRNs and $\varepsilon=\ (\varepsilon_1,\varepsilon_2,\varepsilon_3,\ldots,\varepsilon_n)^T$ be the WV such that $\sum_{i=1}^{n}\varepsilon_i=1$ and $\varepsilon_i\in[0,1]$. Then by using IFRDOWG operator, the aggregated result is described as:

$$IFRDOWG(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\ldots,\psi(\mathfrak{B}_n))=\left(\otimes_{i=1}^{n}(\underline{\psi}(\mathfrak{B}_{\sigma i}))^{\varepsilon_i},\otimes_{i=1}^{n}(\overline{\psi}(\mathfrak{B}_{\sigma i}))^{\varepsilon_i}\right)$$

$$=\left\{\left(\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\underline{\hbar}_{\sigma i}}{\underline{\hbar}_{\sigma i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}},1-\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{\underline{\delta}_{\sigma i}}{1-\underline{\delta}_{\sigma i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right),\left(\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\overline{\hbar}_{\sigma i}}{\overline{\hbar}_{\sigma i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}},1-\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{\overline{\delta}_{\sigma i}}{1-\overline{\delta}_{\sigma i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\right\},$$

where the IFRN $\psi(\mathfrak{B}_{\sigma i}) = (\underline{\psi}(\mathfrak{B}_{\sigma i}),\overline{\psi}(\mathfrak{B}_{\sigma i}))$ represent the largest permutation of the collection $\psi(\mathfrak{B}_i)$.

Theorem 11. Let $\psi(\mathfrak{B}_i) = (\underline{\psi}(\mathfrak{B}_i),\overline{\psi}(\mathfrak{B}_i))$ be the family of IFRNs and $\varepsilon = (\varepsilon_1,\varepsilon_2,\varepsilon_3,...,\varepsilon_n)^T$ be the WV such that $\sum_{i=1}^{n}\varepsilon_i = 1$ and $\varepsilon_i \in [0,1]$. Then some basic results are satisfied for the collection $\psi(\mathfrak{B}_i)$ by applying IFRDOWA operator.

(i) **Idempotency**. Let $\psi(\mathfrak{B}_i) = E(\mathfrak{B})\ \forall\ i = 1,2,...,n$ such that $E(\mathfrak{B}) = (\underline{E}(\mathfrak{B}),\overline{E}(\mathfrak{B})) = (\langle \underline{\hbar},\underline{\delta}\rangle,\langle \overline{\hbar},\overline{\delta}\rangle)$. Then

$$IFRDOWG(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),...,\psi(\mathfrak{B}_n)) = E(\mathfrak{B}).$$

(ii) **Boundedness**.
Let $(\psi(\mathfrak{B}_i))^- = \left(\min_i \underline{\psi}(\mathfrak{B}_i),\min_i \overline{\psi}(\mathfrak{B}_i)\right)$ and $(\psi(\mathfrak{B}_i))^+ = \left(\max_i \underline{\psi}(\mathfrak{B}_i),\max_i \overline{\psi}(\mathfrak{B}_i)\right)$. Then

$$(\psi(\mathfrak{B}_i))^- \leq IFRDOWG(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),...,\psi(\mathfrak{B}_n)) \leq (\psi(\mathfrak{B}_i))^+.$$

(iii) **Monotonicity**. Consider the another family $\psi(\mathfrak{B}_i^{'}) = (\underline{\psi}(\mathfrak{B}_i^{'}),\overline{\psi}(\mathfrak{B}_i^{'}))$ of IFRNs, such that $\underline{\psi}(\mathfrak{B}_i^{'}) \leq \underline{\psi}(\mathfrak{B}_i)$ and $\overline{\psi}(\mathfrak{B}_i^{'}) \leq \overline{\psi}(\mathfrak{B}_i)$. Then

$$IFRDOWG(\psi(\mathfrak{B}_1^{'}),\psi(\mathfrak{B}_2^{'}),...,\psi(\mathfrak{B}_n^{'})) \leq IFRDOWG(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),...,\psi(\mathfrak{B}_n)).$$

(iv) **Shift Invariance**. Assume that $E(\mathfrak{B}^{'}) = (\underline{E}(\mathfrak{B}^{'}),\overline{E}(\mathfrak{B}^{'})) = (\langle \underline{\hbar}^{'},\underline{\delta}^{'}\rangle,\langle \overline{\hbar}^{'},\overline{\delta}^{'}\rangle)$ be another IFRN. Then

$$IFRDOWG(\psi(\mathfrak{B}_1)\oplus E(\mathfrak{B}^{'}),\psi(\mathfrak{B}_2)\oplus E(\mathfrak{B}^{'}),...,\psi(\mathfrak{B}_n)\oplus E(\mathfrak{B}^{'})) = IFRDOWG(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),...,\psi(\mathfrak{B}_n))\oplus E(\mathfrak{B}^{'}).$$

(v) **Homogeneity**. For a real number $\alpha > 0$,

$$IFRDOWG(\alpha\psi(\mathfrak{B}_1),\alpha\psi(\mathfrak{B}_2),...,\alpha\psi(\mathfrak{B}_n)) = \alpha\ IFRDOWG(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),...,\psi(\mathfrak{B}_n)).$$

5.3. IF rough Dombi hybrid geometric operators

In this portion of the manuscript, to examine relation of the hybrid geometric operators with IFRDWG and IFRDOWG operators which weight both the ordered position and the arguments value itself, that is IFRDHG generalized both the operations. This subsection consists of the study of IFRDHG operator and discuss its rudimentary properties.

Definition 15. Assume that $\psi(\mathfrak{B}_i) = (\underline{\psi}(\mathfrak{B}_i),\overline{\psi}(\mathfrak{B}_i))$ be the family of IFRNs such that $\rho = (\rho_1,\rho_2,...,\rho_n)^T$ be the WV such that $\sum_{i=1}^{n}\rho_i = 1$ and $\rho_i \in [0,1]$. Let $\varepsilon = (\varepsilon_1,\varepsilon_2,\varepsilon_3,...,\varepsilon_n)^T$ be the associated WV such that $\sum_{i=1}^{n}\varepsilon_i = 1$ and $\varepsilon_i \in [0,1]$. Then the IFRDHG operator is a mapping $(\psi(\mathfrak{B}))^n \rightarrow \psi(\mathfrak{B})$, which is given as:

$$IFRDHG(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\psi(\mathfrak{B}_3),...,\psi(\mathfrak{B}_n)) = \left(\otimes_{i=1}^{n}(\underline{\psi}(\mathfrak{B}_{\tilde{\sigma}i}))^{\varepsilon_i},\otimes_{i=1}^{n}(\overline{\psi}(\mathfrak{B}_{\tilde{\sigma}i}))^{\varepsilon_i}\right).$$

Theorem 12. Let $\psi(\mathfrak{B}_i) = (\underline{\psi}(\mathfrak{B}_i),\overline{\psi}(\mathfrak{B}_i))$ be the family of IFRNs such that $\rho = (\rho_1,\rho_2,...,\rho_n)^T$ be the WV such that $\sum_{i=1}^{n}\rho_i = 1$ and $\in [0,1]$. Let $\varepsilon = (\varepsilon_1,\varepsilon_2,\varepsilon_3,...,\varepsilon_n)^T$ be the associated WV such that $\sum_{i=1}^{n}\varepsilon_i = 1$ and $\varepsilon_i \in [0,1]$. Then by using IFRDHG operator, the aggregated result is described as:

$$IFRDHG(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),\psi(\mathfrak{B}_3),...,\psi(\mathfrak{B}_n)) = \left(\otimes_{i=1}^{n}(\underline{\psi}(\mathfrak{B}_{\tilde{\sigma}i}))^{\varepsilon_i},\otimes_{i=1}^{n}(\overline{\psi}(\mathfrak{B}_{\tilde{\sigma}i}))^{\varepsilon_i}\right)$$

$$= \left\{\left(\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\underline{\hbar}_{\tilde{\sigma}i}}{\underline{\hbar}_{\tilde{\sigma}i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}},1-\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{\underline{\delta}_{\tilde{\sigma}i}}{1-\underline{\delta}_{\tilde{\sigma}i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right),\left(\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{1-\overline{\hbar}_{\tilde{\sigma}i}}{\overline{\hbar}_{\tilde{\sigma}i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}},1-\frac{1}{1+\left\{\sum_{i=1}^{n}\varepsilon_i\left(\frac{\overline{\delta}_{\tilde{\sigma}i}}{1-\overline{\delta}_{\tilde{\sigma}i}}\right)^{\beta}\right\}^{\frac{1}{\beta}}}\right)\right\},$$

where the IFRN $\psi(\mathfrak{B}_{\tilde{\sigma}i}) = n\rho_i\psi(\mathfrak{B}_i) = n\rho_i(\underline{\psi}(\mathfrak{B}_i),\overline{\psi}(\mathfrak{B}_i))$ represent the largest permutation of the collection $\psi(\mathfrak{B}_i)$.

Theorem 13. Let $\psi(\mathfrak{B}_i) = (\underline{\psi}(\mathfrak{B}_i),\overline{\psi}(\mathfrak{B}_i))$ be the family of IFRNs and $\varepsilon = (\varepsilon_1,\varepsilon_2,\varepsilon_3,...,\varepsilon_n)^T$ be the WV such that $\sum_{i=1}^{n}\varepsilon_i = 1$ and $\varepsilon_i \in [0,1]$. Then some basic results are satisfied for the collection $\psi(\mathfrak{B}_i)$ by applying IFRDOWG operator.

(i) **Idempotency**. Let $\psi(\mathfrak{B}_i) = E(\mathfrak{B})\ \forall\ i = 1,2,...,n$ such that $E(\mathfrak{B}) = (\underline{E}(\mathfrak{B}),\overline{E}(\mathfrak{B})) = (\langle \underline{\hbar},\underline{\delta}\rangle,\langle \overline{\hbar},\overline{\delta}\rangle)$. Then

$$IFRDHG(\psi(\mathfrak{B}_1),\psi(\mathfrak{B}_2),...,\psi(\mathfrak{B}_n)) = E(\mathfrak{B}).$$

(ii) **Boundedness**.

Let $(\psi(\mathfrak{B}_i))^- = \left(\min_i \underline{\psi}(\mathfrak{B}_i), \min_i \overline{\psi}(\mathfrak{B}_i)\right)$ and $(\psi(\mathfrak{B}_i))^+ = \left(\max_i \underline{\psi}(\mathfrak{B}_i), \max_i \overline{\psi}(\mathfrak{B}_i)\right)$. Then

$$(\psi(\mathfrak{B}_i))^- \le IFRDHG(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n)) \le (\psi(\mathfrak{B}_i))^+.$$

(iii) **Monotonicity**. Consider the another family $\psi(\mathfrak{B}_i^{'}) = \left(\underline{\psi}(\mathfrak{B}_i^{'}), \overline{\psi}(\mathfrak{B}_i^{'})\right)$ of IFRNs, such that $\underline{\psi}(\mathfrak{B}_i^{'}) \le \underline{\psi}(\mathfrak{B}_i)$ and $\overline{\psi}(\mathfrak{B}_i^{'}) \le \overline{\psi}(\mathfrak{B}_i)$. Then

$$IFRDHG\left(\psi(\mathfrak{B}_1^{'}), \psi(\mathfrak{B}_2^{'}), \ldots, \psi(\mathfrak{B}_n^{'})\right) \le IFRDHG(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n)).$$

(iv) **Shift Invariance**. Assume that $E(\mathfrak{B}^{'}) = \left(\underline{E}(\mathfrak{B}^{'}), \overline{E}(\mathfrak{B}^{'})\right) = \left(\langle \underline{k}^{'}, \underline{\delta}^{'} \rangle, \langle \overline{k}^{'}, \overline{\delta}^{'} \rangle\right)$ be another IFRN. Then

$$IFRDHG\left(\psi(\mathfrak{B}_1) \oplus E(\mathfrak{B}^{'}), \psi(\mathfrak{B}_2) \oplus E(\mathfrak{B}^{'}), \ldots, \psi(\mathfrak{B}_n) \oplus E(\mathfrak{B}^{'})\right) = IFRHWG(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n)) \oplus E(\mathfrak{B}^{'}).$$

(v) **Homogeneity**. For a real number $\alpha > 0$,

$$IFRDHG(\alpha\psi(\mathfrak{B}_1), \alpha\psi(\mathfrak{B}_2), \ldots, \alpha\psi(\mathfrak{B}_n)) = \alpha\, IFRDHG(\psi(\mathfrak{B}_1), \psi(\mathfrak{B}_2), \ldots, \psi(\mathfrak{B}_n)).$$

6. TOPSIS approach to MCGDM based of IFR Dombi aggregation operators

In this portion of the manuscript, we will present the general structure of the TOPSIS and step wise algorithm for TOPSIS technique based of MCGDM.

In real life group DM is one of the most significant process, in which the professional experts of different genre present their input evaluations for every alternative against all criteria to get the most desirable solution. Assume that the set $K = \{\mathfrak{g}_1, \mathfrak{g}_2, \ldots, \mathfrak{g}_n\}$ of n objects and let $\tilde{C} = \{\tilde{C}_1, \tilde{C}_2, \ldots, \tilde{C}_m\}$ be the set corresponding criteria with WV $\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_m)^T$ such that $\sum_{j=1}^{m} \varepsilon_j$ with $\varepsilon_j \in [0,1]$. Let $G = \{G_1, G_2, \ldots, G_t\}$ be a set of professional specialist who assign their personal views for each alternatives with respect to corresponding criteria with WV $v = (v_1, v_2, \ldots, v_t)^T$ such that $\sum_{l=1}^{t} v_l$ with $v_l \in [0,1]$. The decision experts present their evaluation in the form of IFRNs and collectively represented in the form of decision matrix $\mathcal{M} = [\psi(\mathfrak{B}_{ij})]_{m \times n}$. Then defined the accumulated geometric operator to transform the IFRNs into IFNs which is defined by:

Definition 16. Assume an IFRN of the form $\psi(\mathfrak{B}) = (\underline{\psi}(\mathfrak{B}), \overline{\psi}(\mathfrak{B})) = (\langle \underline{k}, \underline{\delta} \rangle, \langle \overline{k}, \overline{\delta} \rangle)$. Then transform the IFRN into IFN by applying accumulated geometric operator (AGO), which is defined as:

$$\mathfrak{G} = (k_{\mathfrak{G}}, \delta_{\mathfrak{G}}) = (\underline{\psi}(\mathfrak{B}), \overline{\psi}(\mathfrak{B}))^{0.5} = \left((\underline{k}\overline{k})^{0.5}, (\underline{\delta}\overline{\delta})^{0.5}\right)$$

The combined opinions of decision experts are expressed in the form of IF rough decision matrix $\mathcal{M} = [\psi(\mathfrak{B}_{ij})]_{m \times n}$ is transformed into an IF decision matrix $\mathcal{M} = \left[\mathfrak{G}\left(k_{\mathfrak{G}_{ij}}, \delta_{\mathfrak{G}_{ij}}\right)\right]_{m \times n}$ by applying AGO.

Furthermore, by applying the technique of TOPSIS method to calculate the IF-PIS $\mathcal{P}^+$ and IF-NIS $\mathcal{P}^-$ of the transformed decision matrix via the score function which is defined as:

$$\mathcal{P}^+ = \left\{\left\langle \tilde{C}_j, max\left\{\overline{\overline{S}}\left(\tilde{C}_j(\mathfrak{g}_i)\right)\right\}\right\rangle \middle| i = 1, \ldots, n, \ \ j = 1, \ldots, m\right\}$$

$$= \left\{\left\langle \tilde{C}_1, \left(k_1^+, \delta_1^+\right)\right\rangle, \left\langle \tilde{C}_2, \left(k_2^+, \delta_2^+\right)\right\rangle, \ldots, \left\langle \tilde{C}_m, \left(k_m^+, \delta_m^+\right)\right\rangle\right\}$$

$$\mathcal{P}^- = \left\{\left\langle \tilde{C}_j, min\left\{\overline{\overline{S}}\left(\tilde{C}_j(\mathfrak{g}_i)\right)\right\}\right\rangle \middle| i = 1, \ldots, n, \ \ j = 1, \ldots, m\right\}$$

$$= \left\{\left\langle \tilde{C}_1, \left(k_1^-, \delta_1^-\right)\right\rangle, \left\langle \tilde{C}_2, \left(k_2^-, \delta_2^-\right)\right\rangle, \ldots, \left\langle \tilde{C}_m, \left(k_m^-, \delta_m^-\right)\right\rangle\right\}$$

Calculate the shortest distance D^+ and farthest distance D^- between the each object $\mathfrak{g}_i$ and the IF-PIS and IF-NIS

$$D^+(\mathfrak{g}_i, \mathcal{P}^+) = \sum_{j=1}^{n} \varepsilon_j d\left(\tilde{C}_j(\mathfrak{g}_i), \tilde{C}_j(\mathcal{P}^+)\right)$$

$$= \frac{1}{2} \sum_{j=1}^{n} \varepsilon_j \left(\left|k_{ij} - k^+{}_j\right| + \left|\delta_{ij} - \delta^+{}_j\right| + \left|\pi_{ij} - \pi^+{}_j\right|\right) \quad for\ p > 1$$

$$D^-(\mathfrak{g}_i,\mathcal{P}^-)=\sum_{j=1}^{n}\varepsilon_j d\left(\tilde{C}_j(\mathfrak{g}_i),\tilde{C}_j(\mathcal{P}^-)\right)$$

$$=\frac{1}{2}\sum_{j=1}^{n}\varepsilon_j\left(\left|\Bbbk_{ij}-\Bbbk^-{}_j\right|+\left|\delta_{ij}-\delta^-{}_j\right|+\left|\pi_{ij}-\pi^-{}_j\right|\right)\quad for\ p>1$$

Generally the objects having smaller the value of shortest distance $D^+(\mathfrak{g}_i,\mathcal{P}^+)$ is better the one and bigger the value of farthest distance $D^-(\mathfrak{g}_i,\mathcal{P}^-)$ better that alternative is.

$$D^+_{min}(\mathfrak{g}_i,\mathcal{P}^+)=\min_{1\le i\le n}D^+(\mathfrak{g}_i,\mathcal{P}^+),\ D^-_{max}(\mathfrak{g}_i,\mathcal{P}^-)=\max_{1\le i\le n}D^-(\mathfrak{g}_i,\mathcal{P}^-)$$

Finally, from the above analysis calculate the ranking of all alternatives according to the corresponding criteria and arranged them in a specific ordered to get the optimum value.

$$\xi(\mathfrak{g}_i)=\frac{D^-(\mathfrak{g}_i,\mathcal{P}^-)}{D^-_{max}(\mathfrak{g}_i,\mathcal{P}^-)}-\frac{D^+(\mathfrak{g}_i,\mathcal{P}^+)}{D^+_{min}(\mathfrak{g}_i,\mathcal{P}^+)}.$$

Table 1, IFR evaluation information D_1

	$\tilde{C}_1$	$\tilde{C}_2$	$\tilde{C}_3$	$\tilde{C}_4$	$\tilde{C}_5$
$\mathfrak{g}_1$	((0.7,0.1),(0.4,0.2))	((0.7,0.2),(0.6,0.4))	((0.5,0.2),(0.8,0.2))	((0.8,0.1),(0.7,0.2))	((0.4,0.2),(0.5,0.3))
$\mathfrak{g}_2$	((0.6,0.2),(0.9,0.1))	((0.5,0.3),(0.9,0.1))	((0.9,0.1),(0.6,0.3))	((0.4,0.1),(0.5,0.1))	((0.8,0.1),(0.6,0.2))
$\mathfrak{g}_3$	((0.8,0.2),(0.7,0.2))	((0.4,0.1),(0.3,0.2))	((0.4,0.2),(0.7,0.1))	((0.9,0.1),(0.6,0.2))	((0.6,0.3),(0.7,0.2))
$\mathfrak{g}_4$	((0.4,0.1),(0.6,0.3))	((0.8,0.1),(0.7,0.3))	((0.9,0.1),(0.6,0.2))	((0.5,0.2),(0.8,0.1))	((0.7,0.1),(0.9,0.1))

Table 2, IFR evaluation information D_2

	$\tilde{C}_1$	$\tilde{C}_2$	$\tilde{C}_3$	$\tilde{C}_4$	$\tilde{C}_5$
$\mathfrak{g}_1$	((0.4,0.1),(0.3,0.3))	((0.9,0.1),(0.5,0.2))	((0.8,0.1),(0.7,0.2))	((0.9,0.1),(0.4,0.1))	((0.6,0.1),(0.8,0.2))
$\mathfrak{g}_2$	((0.5,0.3),(0.8,0.2))	((0.7,0.1),(0.9,0.1))	((0.7,0.2),(0.5,0.2))	((0.4,0.3),(0.3,0.2))	((0.5,0.2),(0.9,0.1))
$\mathfrak{g}_3$	((0.7,0.3),(0.6,0.1))	((0.6,0.2),(0.7,0.3))	((0.8,0.1),(0.6,0.3))	((0.8,0.2),(0.9,0.1))	((0.4,0.1),(0.3,0.2))
$\mathfrak{g}_4$	((0.2,0.1),(0.5,0.2))	((0.9,0.1),(0.6,0.2))	((0.3,0.1),(0.4,0.3))	((0.5,0.4),(0.7,0.3))	((0.6,0.2),(0.8,0.1))

6.1. Algorithm

From the above analysis, the step wise decision algorithm for the developed approach consists of the following steps:

Step 1. The decision experts present their evaluation in the form of IFRNs and collectively expressed in the form of IFR decision matrix given by:

$$\mathcal{M}=[\psi(\mathfrak{B}_{ij})]_{m\times n}$$

Step 2. Aggregate the expressed combine decision assessment of the professional experts by applying the developed approach to get a single decision matric in the form of IFR decision matrix.

Step 3. The collective aggregated evaluation information of decision experts in the form of IF rough decision matrix $\mathcal{M} = [\psi(\mathfrak{B}_{ij})]_{m \times n}$ is transformed into an IF decision matrix $\mathcal{M} = \left[\mathfrak{G}\left(k_{\mathfrak{G}_{ij}}, \delta_{\mathfrak{G}_{ij}}\right)\right]_{m \times n}$ by applying AGO.

Step 4. Calculate the IF-PIS $\mathcal{P}^{+}$ and IF-NIS $\mathcal{P}^{-}$ of the transformed decision matrix via the score function.

Step 5. Calculate the shortest distance D^{+} and farthest distance D^{-} between the alternative $\mathfrak{g}_i$ and the IF-PIS and IF-NIS.

Step 6. Finally, by applying the ranking function $\xi(\mathfrak{g}_i)$ and arrange the ranking information in a specific ordered get the optimum object.

7. Illustrative Example

The massive outbreak of the pandemic COVID-19 promoted the challenging scenario for the world organizations including scientists, laboratories and researchers to conduct special clinical treatment strategies to prevent the people from COVID-19 pandemic. Globally, COVID-19 pandemic affected the human race and hit hard on them in term of health and economy. The most severe symptoms, which need medical attention are, low level of oxygen in the body, pneumonia, sometime failure of vital organs such as kidneys, heart, and liver. Studies also reported loss of taste and smell. The common symptoms reported by CDC is mentioned somewhere in this article, but we here studied the symptoms with sever disease that are associated to most distinctive comorbidities SARS-CoV-2 infection. The severe-ness of disease with symptoms we linked via illustrative analysis.

Assume that a team of experts doctors including D_1,D_2 and D_3 are called to diagnose the most severe illness of COVID-19 patient with WV $\varepsilon = (0.326, 0.352, 0.322)^T$ such that $\sum_{j=1}^{m} \varepsilon_j$ with $\varepsilon_j \in [0,1]$. The experts examined four patients $\mathfrak{g}_1$,$\mathfrak{g}_2$,$\mathfrak{g}_3$ and $\mathfrak{g}_4$. According to the recent study by the collaboration of different organizations a majority exhibited clinical criteria such as $\tilde{C}_1 =$ fever, $\tilde{C}_2 =$ dry cough, $\tilde{C}_3 =$ fatigue, $\tilde{C}_4 =$ diarrhea and $\tilde{C}_5 =$ shortness of breath, with WV $\vartheta = (0.215, 0.218, 0.212,\ 0.231, 0.124)^T$. Further the decision maker presented their evaluation report in the form of IFRNs for each patient $\mathfrak{g}_i$ with respect to their corresponding criteria. Now by applying the step wise algorithm for the developed approach to diagnose the most severe ill patient by taking the operational parameter $\beta = 2$.

Table 3, IFR evaluation information D_3

	$\tilde{C}_1$	$\tilde{C}_2$	$\tilde{C}_3$	$\tilde{C}_4$	$\tilde{C}_5$
$\mathfrak{g}_1$	((0.6,0.3),(0.8,0.1))	((0.6,0.2),(0.9,0.1))	((0.7,0.3),(0.9,0.1))	((0.5,0.3),(0.9,0.1))	((0.8,0.1),(0.4,0.3))
$\mathfrak{g}_2$	((0.7,0.1),(0.4,0.2))	((0.8,0.1),(0.7,0.2))	((0.4,0.2),(0.7,0.2))	((0.3,0.2),(0.8,0.2))	((0.7,0.3),(0.5,0.1))
$\mathfrak{g}_3$	((0.5,0.3),(0.8,0.2))	((0.3,0.2),(0.5,0.3))	((0.8,0.1),(0.4,0.3))	((0.7,0.2),(0.5,0.3))	((0.9,0.1),(0.2,0.3))
$\mathfrak{g}_4$	((0.7,0.2),(0.6,0.4))	((0.2,0.1),(0.4,0.1))	((0.6,0.4),(0.5,0.2))	((0.6,0.1),(0.9,0.1))	((0.7,0.2),(0.8,0.1))

Table 4, aggregated result by applying IFRWA operator

	$\tilde{C}_1$	$\tilde{C}_2$	$\tilde{C}_3$
$\mathfrak{g}_1$	((0.6197,0.1173),(0.6984,0.1479))	((0.8478,0.1382),(0.8390,0.1493))	((0.7352,0.1438),(0.8521,0.1413))

$\mathfrak{g}_2$	((0.6275,0.1479),(0.8501,0.1409))	((0.7311,0.1175),(0.8827,0.1143))	((0.8422,0.1409),(0.6275,0.2201))
$\mathfrak{g}_3$	((0.7319,0.2512),(0.7353,0.1382))	((0.4995,0.1409),(0.6026,0.2512))	((0.7678,0.1145),(0.6221,0.1542))
$\mathfrak{g}_4$	((0.5808,0.1143),(0.5736,0.2596))	((0.8531,0.1),(0.6221,0.1473))	((0.8391,0.1182),(0.5240,0.2220))

	$\tilde{C}_4$	$\tilde{C}_5$
$\mathfrak{g}_1$	((0.8537,0.1173),(0.8411,0.1145))	((0.7116,0.1145),(0.7118,0.2484))
$\mathfrak{g}_2$	((0.3752,0.1474),(0.7019,0.1409))	((0.7302,0.1468),(0.8447,0.1145))
$\mathfrak{g}_3$	((0.8532,0.1409),(0.8447,0.1438))	((0.8385,0.1175),(0.5769,0.2198))
$\mathfrak{g}_4$	((0.5822,0.1501),(0.8521,0.1194))	((0.6752,0.1409),(0.8591,0.1))

For IFRDWA/ IFRDWG operator

Step 1. The decision experts expressed their judgement in the form of IFRNs and collectively represented in the form of IFR decision matrix given in Tables 1-3.

Step 2. Aggregate the collective decision information of the professional experts given in Tables 1-3, by applying the IFRDWA/IFRDWG operator to get a single decision matric in the form of IFR decision matrix which is given in Table 4.

Step 3. The collective aggregated evaluation information of decision experts in the form of IF rough decision matrix $\mathcal{M} = [\psi(\mathfrak{B}_{ij})]_{m \times n}$ is transformed into an IF decision matrix $\mathcal{M} = \left[\mathfrak{G}\left(\mathscr{k}_{\mathfrak{G}_{ij}}, \delta_{\mathfrak{G}_{ij}}\right)\right]_{m \times n}$ by applying AGO, which is defined as:

$$\mathfrak{G} = (\mathscr{k}_{\mathfrak{G}}, \delta_{\mathfrak{G}}) = (\underline{\psi}(\mathfrak{B}), \overline{\psi}(\mathfrak{B}))^{0.5} = ((\underline{\mathscr{k}}\overline{\mathscr{k}})^{0.5}, (\underline{\delta}\overline{\delta})^{0.5}).$$

The collective evaluation information of decision experts in the form of IF decision matrix by applying AGO is given in Table 5.

Step 4. Determine the IF-PIS $\mathcal{P}^+$ and IF-NIS $\mathcal{P}^-$ of the transformed decision matrix given in Table 5, by applying the score function given in Definition 6.

$$\mathcal{P}^+ = \{(0.7304,0.1443),(0.8434,0.1437),(0.7915,0.1425),(0.8474,0.1159),(0.7853,0.1296)\}$$

$$\mathcal{P}^- = \{(0.5772,0.1723),(0.5486,0.1881),(0.6631,0.1620),(0.5131,0.1441),(0.6955,0.1607)\}$$

Step 5. Calculate the shortest distance $D^+(\mathfrak{g}_i, \mathcal{P}^+)$ and farthest distance $D^-(\mathfrak{g}_i, \mathcal{P}^-)$ between the alternative $\mathfrak{g}_i$ and the IF-PIS and IF-NIS, which is given in Table 6.

Step 6. Finally, by applying the ranking function $\xi(\mathfrak{g}_i)$ and arrange the ranking information in a specific ordered to get the optimum object, which is illustrated in Table 7.

Table 5, IF decision matrix after the use of AGO

	$\tilde{C}_1$	$\tilde{C}_2$	$\tilde{C}_3$	$\tilde{C}_4$	$\tilde{C}_5$
$\mathfrak{g}_1$	(0.6579,0.1317)	(0.8434,0.1437)	(0.7915,0.1425)	(0.8474,0.1159)	(0.7117,0.1686)
$\mathfrak{g}_2$	(0.7304,0.1443)	(0.8034,0.1159)	(0.7270,0.1761)	(0.5131,0.1441)	(0.7853,0.1296)

$\mathfrak{g}_3$	(0.7336,0.1863)	(0.5486,0.1881)	(0.6911,0.1329)	(0.8489,0.1423)	(0.6955,0.1607)
$\mathfrak{g}_4$	(0.5772,0.1723)	(0.7285,0.1214)	(0.6631,0.1620)	(0.6797,0.1338)	(0.7616,0.1187)

Table 7, Ranking ordered of function $\xi(\mathfrak{g}_i)$ for optimum object,

Proposed method	*Score Values* $\xi(\mathfrak{g}_i)$				Ranking
	$\mathfrak{g}_1$	$\mathfrak{g}_2$	$\mathfrak{g}_3$	$\mathfrak{g}_4$	
IFRDWA proposed	0	− 3.2327	− 3.5469	− 4.3899	$\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$
IFRDWG proposed	0.0000,	− 0.3131,	− 0.4616,	− 0.8582	$\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_4 \geq \mathfrak{g}_3$

Table 6, result obtained for IFRDWA operator by applying IFR TOPSIS method

	$D^+(\mathfrak{g}_i,\mathcal{P}^+)$	$D^-(\mathfrak{g}_i,\mathcal{P}^-)$	$\xi(\mathfrak{g}_i)$	Ranking
$\mathfrak{g}_1$	0.0275	0.1891	0	1
$\mathfrak{g}_2$	0.1057	0.1162	− 3.2327	2
$\mathfrak{g}_3$	0.1149	0.1204	− 3.5469	3
$\mathfrak{g}_4$	0.1331	0.0859	− 4.3899	4

Table 7, result obtained for IFRDWG operator by applying IFR TOPSIS method

	$D^+(\mathfrak{g}_i,\mathcal{P}^+)$	$D^-(\mathfrak{g}_i,\mathcal{P}^-)$	$\xi(\mathfrak{g}_i)$	Ranking
$\mathfrak{g}_1$	0.0785	0.1760	0	1
$\mathfrak{g}_2$	0.1351	0.1514	− 0.8600	2
$\mathfrak{g}_3$	0.1858	0.1397	− 1.5732	3
$\mathfrak{g}_4$	0.1496	0.1317	− 1.1570	4

Table 8, Comparative study of the proposed model with some existing approaches

Methods	$\xi(\mathfrak{g}_i)$	Ranking
IFWA [11]	0.6523, 0.5781, 0.5646, 0.5361	$\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$
IF TOPSIS [62]	0.8041, 0.4434, 0.5694, 0.4631	$\mathfrak{g}_1 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4 \geq \mathfrak{g}_2$
IFRFWA based on EDAS [59]	0.8966, 0.6654, 0.247, 0.3567	$\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_4 \geq \mathfrak{g}_3$
IFRWA based on EDAS [58]	0.8584, 0.5703, 0.2734, 0.2234	$\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$

IFDWA [44]	0.6747, 0.6081, 0.6113, 0.5530	$\mathfrak{g}_1 \geq \mathfrak{g}_3 \geq \mathfrak{g}_2 \geq \mathfrak{g}_4$
IFWG [12]	0.6356, 0.5507, 0.5362, 0.5253	$\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$
IFDWG [44]	0.6136, 0.5064, 0.5022, 0.5107	$\mathfrak{g}_1 \geq \mathfrak{g}_4 \geq \mathfrak{g}_3 \geq \mathfrak{g}_2$
IFRWG based on EDAS [58]	0.7789, 0.6357, 0.3677, 0.2043	$\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$
IFRDWA proposed	0.0000, $-$ 3.2327, $-$ 3.5469, $-$ 4.3899	$\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$
IFRDWG proposed	0.0000, $-$ 0.1313, $-$ 0.4616, $-$ 0.8582	$\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$

Table 9, Ranking result based on different parameter β, for IFRWA and IFRWG operators

	The IFRDWA Operator		The IFRDWG Operator	
β	Score value for $\xi(\mathfrak{g}_i)$	Ranking Result	Score value for $\xi(\mathfrak{g}_i)$	Ranking Result
$\beta = 2$	$0, -3.2327, -3.5469, -4.3899$	$\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$	$0, -0.3131, -0.4616, -0.8582$	$\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$
$\beta = 3$	$0, -4.4821, -4.6355, -5.7402$	$\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$	$0, -0.3943, -0.4598, -0.8727$	$\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$
$\beta = 5$	$0, -4.7495, -4.6601, -6.2513$	$\mathfrak{g}_1 \geq \mathfrak{g}_3 \geq \mathfrak{g}_2 \geq \mathfrak{g}_4$	$0, -0.3291, -0.4541, -0.7768$	$\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$
$\beta = 8$	$0, -5.1842, -4.9268, -6.7728$	$\mathfrak{g}_1 \geq \mathfrak{g}_3 \geq \mathfrak{g}_2 \geq \mathfrak{g}_4$	$0, -0.3397, -0.4569, -0.7674$	$\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$
$\beta = 10$	$0, -5.3057, -4.9896, -6.9064$	$\mathfrak{g}_1 \geq \mathfrak{g}_3 \geq \mathfrak{g}_2 \geq \mathfrak{g}_4$	$0, -0.3446, -0.4592, -0.7674$	$\mathfrak{g}_1 \geq \mathfrak{g}_2 \geq \mathfrak{g}_3 \geq \mathfrak{g}_4$

7.1. Comparative study for the effectiveness of the proposed approaches

The TOPSIS method is one of the most significant technique to cope MCDM problems, in which the target is to get the optimal object having highest score vale known as PIS and the object with the least score value is known as NIF. To present the ability and resilience of proposed approach by applying IF rough aggregation operators based on Dombi t-norms and t-conorms hybrid with TOPSIS method, we made a comparison of the investigated concept with several previous models in literature such as IFWA operator by Xu [10], IFWG operator by Xu and Yager [11], IF TOPSIS method by Yinghui and Wenlu [62], IFRWA operator by Yahya et al. [59], IFRWA and IFRWG operators based on EDAS method by chinram et al. [58], IFDWA and IFDWG operators by Seikh and Mandal [44]. If we consider the Tables $1-4$, then the aggregation operators presented in [10, 11, 44, 62] are not capable to aggregate the illustrative example presented in Section 7. However, the aggregation operators investigated by chinram et al. [58] work but these operators are the special cases of the investigated operators. Furthermore, the influence of operational parameter β provides additional space to the decision makers to use their skill and expertise. Dombi operators has general capability and provides additional space in evaluation process to the decision makers. Some of the existing models such as [10, 11, 44, 58, 62] have lake of this operational parameter. The collectively aggregated ranking result of the existing and developed approaches are given in Table 8. The influence of operational parameter β plays significant role in DM. Different values are used for the operational parameter β to judge the ranking result of proposed approaches IFRDWA and IFRDWG operators. The raking result based on different values of operational parameter β in the range of $2 \leq \beta \leq 10$, for both IFRWA and IFRWG operators are shown in Table 9. From Table 9, it is clear that the ranking results and best optimal object is same that is g_1. From the analysis of existing models and proposed approaches it is clear that the investigated approach provides extra flexibility and capability than the previous methods.

7.2. Conclusion

The MCGDM is one of the prominent methodology, in which a team of professional experts evaluate alternatives for the selection of best optimal object based on multiple criteria. Group DM has the ability and capability to improve the assessment process by evaluating multiple conflicting criteria based on the performance of each objects from independent aspect. In DM it's hard to avoid the uncertainty due to the imprecise judgement by the decision makers. For this shortcoming, Atanassov presented the dominant notion of intuitionistic fuzzy sets (IFS) which brought revolution in different field of science scene their inception. The aim of this manuscript is proposed IF rough TOPSIS method based on Dombi operations. For this, first we proposed some new operational laws based on Dombi operations to aggregate averaging and geometric aggregation operators. On the proposed concept, we presented IFRDWA, IFRDOWA and IFRDHA operators. Moreover, on the developed concept we presented IFRDWG, IFRDOWG and IFRDHG operators. The basic related properties of the developed operators are presented in detailed. Then the algorithm for MCGDM based on TOPSIS method for IF rough Dombi averaging and geometric operators is presented. By applying accumulated geometric operator, the IF rough numbers are converted into the IF numbers. The massive outbreak of the pandemic COVID-19 promoted the challenging scenario for the world organizations including scientists, laboratories and researchers to conduct special clinical treatment strategies to prevent the people from COVID-19 pandemic. In addition, an illustrative example is proposed to solve MCGDM problem to diagnose the most severe patient of COVID-19 by applying TOPSIS. Finally, a comparative analysis of the developed model is presented with some existing approaches which shows the applicability and preeminence of the investigated model.

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