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Tahsin Oner

Ege University: Ege Universitesi

Tugce Katican

Ege Universitesi

Salviya Svanidze

Ege Universitesi

Akbar Rezaei (✉ [rezaei@pnu.ac.ir](mailto:rezaei@pnu.ac.ir))

Payame Noor University <https://orcid.org/0000-0002-6003-3993>

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## Research Article

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# Neutrosophic $\mathcal{N}$ –structures on Sheffer stroke BE-algebras

Tahsin Oner · Tugce Katican · Salviya  
Svanidze · Akbar Rezaei

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**Abstract** In this study, a neutrosophic  $\mathcal{N}$ –subalgebra, a (implicative) neutrosophic  $\mathcal{N}$ –filter, level sets of these neutrosophic  $\mathcal{N}$ –structures and their properties are introduced on a Sheffer stroke BE-algebras (briefly, SBE-algebras). It is proved that the level set of neutrosophic  $\mathcal{N}$ –subalgebras ((implicative) neutrosophic  $\mathcal{N}$ –filter) of this algebra is the SBE-subalgebra ((implicative) SBE-filter) and vice versa. Then it is proved that the family of all neutrosophic  $\mathcal{N}$ –subalgebras of a SBE-algebra forms a complete distributive modular lattice. We present relationships between upper sets and neutrosophic  $\mathcal{N}$ –filters of this algebra. Also, it is given that every neutrosophic  $\mathcal{N}$ –filter of a SBE-algebra is its neutrosophic  $\mathcal{N}$ –subalgebra but the inverse is generally not true. It is demonstrated that a neutrosophic  $\mathcal{N}$ –structure on a SBE-algebra defined

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T. Oner  
Department of Mathematics,  
Ege University,  
İzmir, Turkey  
E-mail: tahsin.oner@ege.edu.tr

T. Katican  
Department of Mathematics,  
Ege University,  
İzmir, Turkey  
E-mail: tugcektcn@gmail.com

S. Svanidze  
Department of Mathematics,  
Ege University,  
İzmir, Turkey  
E-mail: selviye.svanidze@gmail.com

A. Rezaei  
Department of Mathematics,  
Payame Noor University,  
P.O. Box. 19395-3697, Tehran, Iran.  
E-mail: rezaei@pnu.ac.ir

by a (implicative) neutrosophic  $\mathcal{N}$ -filter of another SBE-algebra and a surjective SBE-homomorphism is a (implicative) neutrosophic  $\mathcal{N}$ -filter. We present relationships between a neutrosophic  $\mathcal{N}$ -filter and an implicative neutrosophic  $\mathcal{N}$ -filter of a SBE-algebra in detail. Finally, certain subsets of a SBE-algebra are determined by means of  $\mathcal{N}$ -functions and some properties are examined.

**Keywords** SBE-algebra · (implicative) SBE-filter · neutrosophic  $\mathcal{N}$ -subalgebra · (implicative) neutrosophic  $\mathcal{N}$ -filter.

## 1 Introduction

Sheffer stroke which is one of the two operators that can be used by itself, without any other logical operators, was originally introduced by H. M. Sheffer to build a logical formal system [22]. Since it provides new, basic and easily applicable axiom systems for many algebraic structures owing to its commutative property, this operation has many applications in algebraic structures such as orthoimplication algebras [1], ortholattices [4], Boolean algebras [11], strong Sheffer stroke non-associative MV-algebras [5], filters [14] and neutrosophic  $N$ -structures [17], Sheffer Stroke Hilbert Algebras [12], fuzzy filters [13] and neutrosophic  $N$ -structures [18], (fuzzy) filters of Sheffer stroke BL-algebras [15] and Sheffer stroke UP-algebras [16]. On the other hand, H S. Kim and Y. H. Kim introduced BE-algebras as a generalization of a dual BCK-algebra and defined filters and upper sets on this algebra [10]. Also, some types of filters in BEalgebras [3] and some results in BE-algebras [19]. Recently, Katican et al. introduced BE-algebras with Sheffer stroke and investigated upper sets, SBE-filters and SBE-homomorphisms [8].

The fuzzy set theory which has the truth (t) (membership) function and positive meaning of information was introduced by L. Zadeh [26]. Therefore, researchers deal with negative meaning of information. For this propose, Atanassov introduced the intuitionistic fuzzy set theory [2] which is a type of fuzzy sets and has truth (t) (membership) and the falsehood (f) (nonmembership) functions. Then, Smarandache introduced the neutrosophic set theory which is the intuitionistic fuzzy set theory and has the indeterminacy/neutrality (i) function with membership and nonmembership functions [23, ?]. Hence, neutrosophic sets are defined on three components  $(t, i, f)$  [27]. In recent times, neutrosophic sets are applied to the algebraic structures such as BCK/BCI-algebras and BE-algebras ([6], [7], [9], [20], [25]).

In the second section, basic definitions and notions on Sheffer stroke BE-algebras, neutrosophic  $\mathcal{N}$ -functions and neutrosophic  $\mathcal{N}$ -structures are presented (briefly, SBE-algebra). In third section, an implicative SBE-filter, a neutrosophic  $\mathcal{N}$ -subalgebra, a (implicative) neutrosophic  $\mathcal{N}$ -filter and a level set on neutrosophic  $\mathcal{N}$ -structures are defined on SBE-algebras. Then it is shown that the level set of a neutrosophic  $\mathcal{N}$ -subalgebra on a SBE-algebra is its SBE-subalgebra and vice versa, and that the family of all neutrosophic  $\mathcal{N}$ -subalgebras of the algebraic structure forms a complete distributive modular lattice. By defining a (implicative) neutrosophic  $\mathcal{N}$ -filter of a SBE-algebra,

some properties are investigated. Also, a neutrosophic  $\mathcal{N}$ -filter of a SBE-algebra is restated by means of upper sets. We demonstrate that every implicative neutrosophic  $\mathcal{N}$ -filter of a SBE-algebra is the neutrosophic  $\mathcal{N}$ -filter but the inverse is generally not true. It is propounded that level set of a (implicative) neutrosophic  $\mathcal{N}$ -filter of a SBE-algebra is its (implicative) SBE-filter and the inverse always holds. Indeed, it is proved that a neutrosophic  $\mathcal{N}$ -structure on a SBE-algebra defined by means of a (implicative) neutrosophic  $\mathcal{N}$ -filter of another SBE-algebra and a surjective SBE-homomorphism is a (implicative) neutrosophic  $\mathcal{N}$ -filter. We illustrate that every neutrosophic  $\mathcal{N}$ -filter of a SBE-algebra is the neutrosophic  $\mathcal{N}$ -subalgebra but the inverse is mostly not valid. Besides, the cases which a neutrosophic  $\mathcal{N}$ -filter of a SBE-algebra is an implicative neutrosophic  $\mathcal{N}$ -filter are analyzed. Special subsets of a SBE-algebra are described by  $\mathcal{N}$ -functions and it is shown that these subsets are (implicative) SBE-filters of SBE-algebra for its (implicative)  $\mathcal{N}$ -filter. Finally, we determine new subsets by means of the  $\mathcal{N}$ -functions and some elements of a SBE-algebra and prove that these subsets are (implicative) SBE-filters of a SBE-algebra for a (implicative) neutrosophic  $\mathcal{N}$ -filter of this algebraic structure but the inverse does not hold in general.

## 2 Preliminaries

In this section, basic definitions and notions about Sheffer stroke BE-algebras (for short, SBE-algebras) and neutrosophic  $\mathcal{N}$ -structures.

**Definition 1** [4] Let  $\mathcal{S} = \langle S, \circ \rangle$  be a groupoid. The operation  $\circ$  on  $S$  is said to be a *Sheffer operation* (or *Sheffer stroke*) if it satisfies the following conditions for all  $x, y, z \in S$ :

- (S1)  $x \circ y = y \circ x$ ,
- (S2)  $(x \circ x) \circ (x \circ y) = x$ ,
- (S3)  $x \circ ((y \circ z) \circ (y \circ z)) = ((x \circ y) \circ (x \circ y)) \circ z$ ,
- (S4)  $(x \circ ((x \circ x) \circ (y \circ y))) \circ (x \circ ((x \circ x) \circ (y \circ y))) = x$ .

**Definition 2** [8] A Sheffer stroke BE-algebra (shortly, SBE-algebra) is a structure  $\langle S; \circ, 1 \rangle$  of type  $(2, 0)$  such that 1 is the constant in  $S$ ,  $\circ$  is a Sheffer operation on  $S$  and the following axioms are satisfied for all  $x, y, z \in S$ :

- (SBE-1)  $x \circ (x \circ x) = 1$ ,
- (SBE-2)  $x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))) = y \circ ((x \circ (z \circ z)) \circ (x \circ (z \circ z)))$ .

**Lemma 1** [8] Let  $\langle S; \circ, 1 \rangle$  be a SBE-algebra. Then the following hold for all  $x, y \in S$ :

- (i)  $x \circ (1 \circ 1) = 1$ ,
- (ii)  $1 \circ (x \circ x) = x$ ,
- (iii)  $x \circ ((y \circ (x \circ x)) \circ (y \circ (x \circ x))) = 1$ .
- (iv)  $x \circ (((x \circ (y \circ y)) \circ (y \circ y)) \circ ((x \circ (y \circ y)) \circ (y \circ y))) = 1$ ,
- (v)  $(x \circ 1) \circ (x \circ 1) = x$ ,
- (vi)  $((x \circ y) \circ (x \circ y)) \circ (x \circ x) = 1$  and  $((x \circ y) \circ (x \circ y)) \circ (y \circ y) = 1$ ,
- (vii)  $x \circ ((x \circ y) \circ (x \circ y)) = x \circ y = ((x \circ y) \circ (x \circ y)) \circ y$ .

**Definition 3** [8] A SBE-algebra  $\langle S; \circ, 1 \rangle$  is called self-distributive if

$$x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))) = (x \circ (y \circ y)) \circ ((x \circ (z \circ z)) \circ (x \circ (z \circ z))),$$

for any  $x, y, z \in S$ .

**Definition 4** [8] Let  $\langle S; \circ, 1 \rangle$  be a SBE-algebra. Define a relation  $\preceq$  on  $S$  by

$$x \preceq y \text{ if and only if } x \circ (y \circ y) = 1,$$

for all  $x, y \in S$ . The relation is not a partial order on  $S$ , since it is only reflexive by  $(SBE - 1)$ .

**Lemma 2** [8] Let  $\langle S; \circ, 1 \rangle$  be a SBE-algebra. Then

1. If  $x \preceq y$ , then  $y \circ y \preceq x \circ x$ ,
2.  $x \preceq y \circ (x \circ x)$ ,
3.  $y \preceq (y \circ (x \circ x)) \circ (x \circ x)$ ,
4. If  $S$  is self-distributive, then  $x \preceq y$  implies  $y \circ z \preceq x \circ z$ ,
5. If  $S$  is self-distributive, then  $y \circ (z \circ z) \preceq (z \circ (x \circ x)) \circ ((y \circ (x \circ x)) \circ (y \circ (x \circ x)))$ .

**Definition 5** [8] Let  $\langle S; \circ, 1 \rangle$  be a SBE-algebra. Then a nonempty subset  $F \subseteq S$  is called a SBE-filter of  $S$  if it satisfies the following properties:

$$(SBEf - 1) \quad 1 \in F,$$

$$(SBEf - 2) \quad \text{For all } x, y \in S, x \circ (y \circ y) \in F \text{ and } x \in F \text{ imply } y \in F.$$

**Lemma 3** [8] Let  $\langle S; \circ, 1 \rangle$  be a SBE-algebra. Then a nonempty subset  $F \subseteq S$  is a SBE-filter of  $S$  if and only if for all  $x, y \in S$

- (i)  $x \in F$  and  $y \in F$  imply  $(x \circ y) \circ (x \circ y) \in F$ ,
- (ii)  $x \in F$  and  $x \preceq y$  imply  $y \in F$ .

**Definition 6** [8] Let  $\langle S; \circ, 1 \rangle$  be a SBE-algebra,  $x, y \in S$  and define  $U(x, y) = \{z \in S : x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))) = 1\}$ . Then  $U(x, y)$  is called an upper set of  $x$  and  $y$ . For  $x, y \in S$ ,  $U(x, y)$  is not a SBE-filter of  $S$  in general.

**Definition 7** [8] A subset  $T$  of a SBE-algebra  $\langle S; \circ, 1 \rangle$  is called a SBE-subalgebra of  $S$  if  $x \circ (y \circ y) \in T$ , for  $x, y \in T$ . Clearly,  $S$  itself and  $\{1\}$  are SBE-subalgebras of  $S$ .

**Definition 8** [8] Let  $\langle S; \circ_S, 1_S \rangle$  and  $\langle P; \circ_P, 1_P \rangle$  be SBE-algebras. A mapping  $f : S \rightarrow P$  is called a SBE-homomorphism if  $f(x \circ_S y) = f(x) \circ_P f(y)$ , for all  $x, y \in S$  and  $f(1_S) = 1_P$ .

**Definition 9** [21] A modular lattice is any lattice which satisfies  $x \leq y \rightarrow x \vee (y \wedge z) = y \wedge (x \vee z)$ .

**Theorem 1** [21] Every distributive lattice is a modular lattice.

**Definition 10** [6]  $\mathcal{F}(X, [-1, 0])$  denotes the collection of functions from a set  $X$  to  $[-1, 0]$  and a element of  $\mathcal{F}(X, [-1, 0])$  is called a negative-valued function from  $X$  to  $[-1, 0]$  (briefly,  $\mathcal{N}$ -function on  $X$ ). An  $\mathcal{N}$ -structure refers to an ordered pair  $(X, f)$  of  $X$  and  $\mathcal{N}$ -function  $f$  on  $X$ .

**Definition 11** [9] A neutrosophic  $\mathcal{N}$ -structure over a nonempty universe  $X$  is defined by  $X_N := \frac{X}{(T_N, I_N, F_N)} = \{ \frac{x}{(T_N(x), I_N(x), F_N(x))} : x \in X \}$  where  $T_N, I_N$  and  $F_N$  are  $\mathcal{N}$ -function on  $X$ , called the negative truth membership function, the negative indeterminacy membership function and the negative falsity membership function, respectively.

Every neutrosophic  $\mathcal{N}$ -structure  $X_N$  over  $X$  satisfies the condition

$$(\forall x \in X)(-3 \leq T_N(x) + I_N(x) + F_N(x) \leq 0).$$

**Definition 12** [7] Let  $X_N$  be a neutrosophic  $\mathcal{N}$ -structure on a set  $X$  and  $\alpha, \beta, \gamma$  be any elements of  $[-1, 0]$  such that  $-3 \leq \alpha + \beta + \gamma \leq 0$ . Consider the following sets:

$$T_N^\alpha := \{x \in X : T_N(x) \leq \alpha\},$$

$$I_N^\beta := \{x \in X : I_N(x) \geq \beta\}$$

and

$$F_N^\gamma := \{x \in X : F_N(x) \leq \gamma\}.$$

The set

$$X_N(\alpha, \beta, \gamma) := \{x \in X : T_N(x) \leq \alpha, I_N(x) \geq \beta \text{ and } F_N(x) \leq \gamma\}$$

is called the  $(\alpha, \beta, \gamma)$ -level set of  $X_N$ . Moreover,  $X_N(\alpha, \beta, \gamma) = T_N^\alpha \cap I_N^\beta \cap F_N^\gamma$ .

Consider sets

$$X_N^{w_t} := \{x \in X : T_N(x) \leq T_N(w_t),$$

$$X_N^{w_i} := \{x \in X : I_N(x) \geq I_N(w_i)$$

and

$$X_N^{w_f} := \{x \in X : F_N(x) \leq F_N(w_f),$$

for any  $w_t, w_i, w_f \in X$ . Obviously,  $w_t \in X_N^{w_t}, w_i \in X_N^{w_i}$  and  $w_f \in X_N^{w_f}$  [7].

### 3 Neutrosophic $\mathcal{N}$ -structures

In this section, we present implicative SBE-filters, neutrosophic  $\mathcal{N}$ -subalgebras and (implicative) neutrosophic  $\mathcal{N}$ -filters of SBE-algebras. Unless indicated otherwise,  $S$  states a SBE-algebra.

**Definition 13** A nonempty subset  $F$  of a SBE-algebra  $S$  is called a implicative SBE-filter of  $S$  if it satisfies

$$(SIF - 1) \ 1 \in F,$$

$$(SIF - 2) \ x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))) \in F \text{ and } x \circ (y \circ y) \in F \text{ imply } x \circ (z \circ z) \in F, \text{ for all } x, y, z \in S.$$

*Example 1* Consider a SBE-algebra  $S$  in which the set  $S = \{0, u, v, w, t, 1\}$  and the Sheffer operation  $\circ$  on  $S$  has the following Cayley table [8]:

**Table 1** Cayley table of Sheffer operation  $\circ$  on  $S$ 

$\circ$	0	$u$	$v$	$w$	$t$	1
0	1	1	1	1	1	1
$u$	1	$t$	$w$	1	1	$t$
$v$	1	$w$	$w$	1	1	$w$
$w$	1	1	1	$v$	$u$	$v$
$t$	1	1	1	$u$	$u$	$u$
1	1	$t$	$w$	$v$	$u$	0

Then  $\{w, t, 1\}$  is an implicative SBE-filter of  $S$  while  $\{v, 1\}$  is not since  $w \circ (u \circ u) = u \notin \{v, 1\}$  when  $w \circ ((v \circ (u \circ u)) \circ (v \circ (u \circ u))) = 1 \in \{v, 1\}$  and  $w \circ (v \circ v) = v \in \{v, 1\}$ .

**Lemma 4** Every implicative SBE-filter of a SBE-algebra  $S$  is a SBE-filter of  $S$ .

*Proof* Let  $F$  be an implicative SBE-filter of  $S$ . It is obvious from (SIF-1) that  $1 \in F$ . Assume that  $x, x \circ (y \circ y) \in F$ . Since  $1 \circ ((x \circ (y \circ y)) \circ (x \circ (y \circ y))) = x \circ (y \circ y) \in F$  and  $1 \circ (x \circ x) = x \in F$  from Lemma 1 (ii), it follows from Lemma 1 (ii) and (SIF-2) that  $y = 1 \circ (y \circ y) \in F$ . Thus,  $F$  is a SBE-filter of  $S$ .

The inverse of Lemma 4 is not true in general.

*Example 2* [8] Consider a SBE-algebra  $S$  where the set  $S = \{0, u, v, w, t, 1\}$  and the Sheffer operation  $\circ$  on  $S$  has the following Cayley table [8]:

**Table 2** Cayley table of Sheffer operation  $\circ$  on  $S$ 

$\circ$	0	$u$	$v$	$w$	$t$	1
0	1	1	1	1	1	1
$u$	1	$v$	1	1	1	$v$
$v$	1	1	$u$	1	1	$u$
$w$	1	1	1	$t$	1	$t$
$t$	1	1	1	1	$w$	$w$
1	1	$v$	$u$	$t$	$w$	0

Then  $\{1\}$  is a SBE-filter of  $S$  but it is not implicative since  $u \circ (v \circ v) = v \notin \{1\}$  when  $u \circ ((w \circ (v \circ v)) \circ (w \circ (v \circ v))) = 1 \in \{1\}$  and  $u \circ (w \circ w) = 1 \in \{1\}$ .

**Definition 14** A neutrosophic  $\mathcal{N}$ -subalgebra  $S_N$  of a SBE-algebra  $S$  is a neutrosophic  $\mathcal{N}$ -structure on  $S$  satisfying the condition

$$\begin{aligned} \min\{T_N(x), T_N(y)\} &\leq T_N(x \circ (y \circ y)), \\ I_N(x \circ (y \circ y)) &\leq \max\{I_N(x), I_N(y)\} \\ &\text{and} \\ F_N(x \circ (y \circ y)) &\leq \max\{F_N(x), F_N(y)\}, \end{aligned} \quad (1)$$

for all  $x, y \in S$ .

*Example 3* Consider the SBE-algebra  $S$  in Example 2. Then a neutrosophic  $\mathcal{N}$ -structure

$$S_N = \left\{ \frac{x}{(-0.87, -0.1, -0.2)} : x \in S - \{1\} \right\} \cup \left\{ \frac{1}{(-0.03, -1, -0.78)} \right\}$$

on  $S$  is a neutrosophic  $\mathcal{N}$ -subalgebra of  $S$ .

**Definition 15** Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -structure on a SBE-algebra  $S$  and  $\alpha, \beta, \gamma$  be any elements of  $[-1, 0]$  such that  $-3 \leq \alpha + \beta + \gamma \leq 0$ . For the sets

$$T_N^\alpha := \{x \in S : \alpha \leq T_N(x)\},$$

$$I_N^\beta := \{x \in S : I_N(x) \leq \beta\}$$

and

$$F_N^\gamma := \{x \in S : F_N(x) \leq \gamma\},$$

the set

$$S_N(\alpha, \beta, \gamma) := \{x \in S : \alpha \leq T_N(x), I_N(x) \leq \beta \text{ and } F_N(x) \leq \gamma\}$$

is called the  $(\alpha, \beta, \gamma)$ -level set of  $S_N$ . Also,  $S_N(\alpha, \beta, \gamma) = T_N^\alpha \cap I_N^\beta \cap F_N^\gamma$ .

**Theorem 2** Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -structure on a SBE-algebra  $S$  and  $\alpha, \beta, \gamma$  be any elements of  $[-1, 0]$  with  $-3 \leq \alpha + \beta + \gamma \leq 0$ . If  $S_N$  is a neutrosophic  $\mathcal{N}$ -subalgebra of  $S$ , then the nonempty level set  $S_N(\alpha, \beta, \gamma)$  of  $S_N$  is a SBE-subalgebra of  $S$ .

*Proof* Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -subalgebra of  $S$  and  $x, y$  be any elements of  $S_N(\alpha, \beta, \gamma)$ , for  $\alpha, \beta, \gamma \in [-1, 0]$  with  $-3 \leq \alpha + \beta + \gamma \leq 0$ . Then  $\alpha \leq T_N(x), T_N(y); I_N(x), I_N(y) \leq \beta$  and  $F_N(x), F_N(y) \leq \gamma$ . Since

$$\alpha \leq \min\{T_N(x), T_N(y)\} \leq T_N(x \circ (y \circ y)),$$

$$I_N(x \circ (y \circ y)) \leq \max\{I_N(x), I_N(y)\} \leq \beta$$

and

$$F_N(x \circ (y \circ y)) \leq \max\{F_N(x), F_N(y)\} \leq \gamma,$$

for all  $x, y \in S$ , it is obtained that  $x \circ (y \circ y) \in T_N^\alpha, I_N^\beta, F_N^\gamma$ , and so,  $x \circ (y \circ y) \in T_N^\alpha \cap I_N^\beta \cap F_N^\gamma = S_N(\alpha, \beta, \gamma)$ . Hence,  $S_N(\alpha, \beta, \gamma)$  is a SBE-subalgebra of  $S$ .

**Theorem 3** Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -structure on a SBE-algebra  $S$  and  $T_N^\alpha, I_N^\beta$  and  $F_N^\gamma$  be SBE-subalgebras of  $S$ , for all  $\alpha, \beta, \gamma \in [-1, 0]$  with  $-3 \leq \alpha + \beta + \gamma \leq 0$ . Then  $S_N$  is a neutrosophic  $\mathcal{N}$ -subalgebra of  $S$ .



*Proof* Let  $T_N^\alpha, I_N^\beta$  and  $F_N^\gamma$  be SBE-subalgebras of  $S$ , for all  $\alpha, \beta, \gamma \in [-1, 0]$  with  $-3 \leq \alpha + \beta + \gamma \leq 0$ . Suppose that  $\alpha_1 = T_N(x \circ (y \circ y)) < \min\{T_N(x), T_N(y)\} = \alpha_2$ . If  $\alpha = \frac{1}{2}(\alpha_1 + \alpha_2) \in [-1, 0)$ , then  $\alpha_1 < \alpha < \alpha_2$ . So,  $x, y \in T_N^\alpha$  but  $x \circ (y \circ y) \notin T_N^\alpha$  which is a contradiction. Thus,  $\min\{T_N(x), T_N(y)\} \leq T_N(x \circ (y \circ y))$ , for all  $x, y \in S$ . Assume that  $\beta_1 = \max\{I_N(x), I_N(y)\} < I_N(x \circ (y \circ y)) = \beta_2$ . If  $\beta = \frac{1}{2}(\beta_1 + \beta_2) \in [-1, 0)$ , then  $\beta_1 < \beta < \beta_2$ . Hence,  $x, y \in I_N^\beta$  but  $x \circ (y \circ y) \notin I_N^\beta$  which is a contradiction. Thus,  $I_N(x \circ (y \circ y)) \leq \max\{I_N(x), I_N(y)\}$ , for all  $x, y \in S$ . Moreover, suppose that  $\gamma_1 = \max\{F_N(x), F_N(y)\} < F_N(x \circ (y \circ y)) = \gamma_2$ . If  $\gamma = \frac{1}{2}(\gamma_1 + \gamma_2) \in [-1, 0)$ , then  $\gamma_1 < \gamma < \gamma_2$ . Thus,  $x, y \in F_N^\gamma$  whereas  $x \circ (y \circ y) \notin F_N^\gamma$  which is a contradiction. Thereby,  $F_N(x \circ (y \circ y)) \leq \max\{F_N(x), F_N(y)\}$ , for all  $x, y \in S$ . Therefore,  $S_N$  is a neutrosophic  $\mathcal{N}$ -subalgebra of  $S$ .

**Theorem 4** Let  $\{S_{N_i} : i \in \mathbb{N}\}$  be a family of all neutrosophic  $\mathcal{N}$ -subalgebras of a SBE-algebra  $S$ . Then  $\{S_{N_i} : i \in \mathbb{N}\}$  forms a complete distributive modular lattice.

*Proof* Let  $T$  be a nonempty subset of  $\{S_{N_i} : i \in \mathbb{N}\}$ . Since  $S_{N_i}$  is a neutrosophic  $\mathcal{N}$ -subalgebra of  $S$ , for all  $i \in \mathbb{N}$ , it satisfies the condition (1) for all  $x, y \in S$ . Then  $\bigcap T$  satisfies the condition (1). Thus,  $\bigcap T$  is a neutrosophic  $\mathcal{N}$ -subalgebra of  $S$ . Let  $P$  be a family of all neutrosophic  $\mathcal{N}$ -subalgebras of  $S$  containing  $\bigcup\{S_{N_i} : i \in \mathbb{N}\}$ . So,  $\bigcap P$  is a neutrosophic  $\mathcal{N}$ -subalgebra of  $S$ . If  $\bigwedge_{i \in \mathbb{N}} S_{N_i} = \bigcap_{i \in \mathbb{N}} S_{N_i}$  and  $\bigvee_{i \in \mathbb{N}} S_{N_i} = \bigcap P$ , then  $(\{S_{N_i} : i \in \mathbb{N}\}, \bigvee, \bigwedge)$  forms a complete lattice. Moreover, it is distributive by the definitions of  $\bigvee$  and  $\bigwedge$ . By Theorem 1, the lattice is modular.

**Lemma 5** Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -subalgebra of a SBE-algebra  $S$ . Then

$$T_N(x) \leq T_N(1), I_N(1) \leq I_N(x) \text{ and } F_N(1) \leq F_N(x), \quad (2)$$

for all  $x \in S$ .

*Proof* Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -subalgebra of  $S$ . Then it follows from (SBE-1) that

$$T_N(x) = \min\{T_N(x), T_N(x)\} \leq T_N(x \circ (x \circ x)) = T_N(1),$$

$$I_N(1) = I_N(x \circ (x \circ x)) \leq \max\{I_N(x), I_N(x)\} = I_N(x)$$

and

$$F_N(1) = F_N(x \circ (x \circ x)) \leq \max\{F_N(x), F_N(x)\} = F_N(x),$$

for all  $x \in S$ .

The inverse of Lemma 5 does not usually hold.

*Example 4* Consider the SBE-algebra  $A$  in Example 2. Then a neutrosophic  $\mathcal{N}$ -structure

$$S_N = \left\{ \frac{v}{(-0.91, -0.4, -0.5)} \right\} \cup \left\{ \frac{x}{(0, -0.7, -0.8)} : x \in S - \{v\} \right\}$$

on  $S$  satisfies the condition (2) but it is not a neutrosophic  $\mathcal{N}$ -subalgebra of  $S$  since  $\max\{F_N(u), F_N(0)\} = -0.8 < -0.5 = F_N(v) = F_N(u \circ (0 \circ 0))$ .

**Lemma 6** *A neutrosophic  $\mathcal{N}$ -subalgebra  $S_N$  of a SBE-algebra  $S$  satisfies  $T_N(x) \leq T_N(x \circ (y \circ y))$ ,  $I_N(x \circ (y \circ y)) \leq I_N(x)$  and  $F_N(x \circ (y \circ y)) \leq F_N(x)$ , for all  $x, y \in S$  if and only if  $T_N, I_N$  and  $F_N$  are constant.*

*Proof* Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -subalgebra of a SBE-algebra  $S$  satisfying  $T_N(x) \leq T_N(x \circ (y \circ y))$ ,  $I_N(x \circ (y \circ y)) \leq I_N(x)$  and  $F_N(x \circ (y \circ y)) \leq F_N(x)$ , for all  $x, y \in S$ . Since  $T_N(1) \leq T_N(1 \circ (x \circ x)) = T_N(x)$ ,  $I_N(x) = I_N(1 \circ (x \circ x)) \leq I_N(1)$  and  $F_N(x) = F_N(1 \circ (x \circ x)) \leq F_N(1)$  from Lemma 1 (ii), it follows from Lemma 5 that  $T_N(x) = T_N(1)$ ,  $I_N(x) = I_N(1)$  and  $F_N(x) = F_N(1)$ , for all  $x \in A$ . Hence,  $T_N, I_N$  and  $F_N$  are constant.

Conversely, it is obvious by the fact that  $T_N, I_N$  and  $F_N$  are constant.

**Definition 16** A neutrosophic  $\mathcal{N}$ -structure  $S_N$  on a SBE-algebra  $S$  is called a neutrosophic  $\mathcal{N}$ -filter of  $S$  if

$$\begin{aligned} \min\{T_N(x \circ (y \circ y)), T_N(x)\} &\leq T_N(y) \leq T_N(1), \\ I_N(1) &\leq I_N(y) \leq \max\{I_N(x \circ (y \circ y)), I_N(x)\} \\ &\text{and} \\ F_N(1) &\leq F_N(y) \leq \max\{F_N(x \circ (y \circ y)), F_N(x)\}, \end{aligned} \quad (3)$$

for all  $x, y \in S$ .

*Example 5* Consider the SBE-algebra  $S$  in Example 1. Then a neutrosophic  $\mathcal{N}$ -structure

$$S_N = \left\{ \frac{x}{(-1, -0.02, 0)} : x = 0, w, t \right\} \cup \left\{ \frac{x}{(0, -0.2, -0.5)} : x = u, v, 1 \right\}$$

on  $S$  is a neutrosophic  $\mathcal{N}$ -filter of  $S$ .

**Lemma 7** *Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -structure on a SBE-algebra  $S$ . Then  $S_N$  is a neutrosophic  $\mathcal{N}$ -filter of  $S$  if and only if*

1.  $x \preceq y$  implies  $T_N(x) \leq T_N(y)$ ,  $I_N(y) \leq I_N(x)$  and  $F_N(y) \leq F_N(x)$ ,
2.  $\min\{T_N(x), T_N(y)\} \leq T_N((x \circ y) \circ (x \circ y))$ ,  $I_N((x \circ y) \circ (x \circ y)) \leq \max\{I_N(x), I_N(y)\}$  and  $F_N((x \circ y) \circ (x \circ y)) \leq \max\{F_N(x), F_N(y)\}$ ,

for all  $x, y \in S$ .

*Proof* Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -filter of  $S$ .

1. Assume that  $x \preceq y$ . Then  $x \circ (y \circ y) = 1$ . Thus,

$$T_N(x) = \min\{T_N(1), T_N(x)\} = \min\{T_N(x \circ (y \circ y)), T_N(x)\} \leq T_N(y),$$

$$I_N(y) \leq \max\{I_N(x \circ (y \circ y)), I_N(x)\} = \max\{I_N(1), I_N(x)\} \leq I_N(x)$$

and

$$F_N(y) \leq \max\{F_N(x \circ (y \circ y)), F_N(x)\} = \max\{F_N(1), F_N(x)\} \leq F_N(x),$$

for all  $x, y \in S$ .

2. Since  $y \preceq y \circ ((x \circ x) \circ (x \circ x)) \circ ((x \circ x) \circ (x \circ x)) = x \circ (x \circ y)$  from Lemma 2 (3), (S1) and (S2), it is obtained from (1) that  $\min\{T_N(x), T_N(y)\} \leq \min\{T_N(x), T_N(x \circ (x \circ y))\} = \min\{T_N(x), T_N(x \circ ((x \circ y) \circ (x \circ y)) \circ ((x \circ y) \circ (x \circ y)))\} \leq T_N((x \circ y) \circ (x \circ y)), I_N((x \circ y) \circ (x \circ y)) \leq \max\{I_N(x), I_N(x \circ ((x \circ y) \circ (x \circ y)) \circ ((x \circ y) \circ (x \circ y)))\} = \max\{I_N(x), I_N(x \circ (x \circ y))\} \leq \max\{I_N(x), I_N(y)\}$  and  $F_N((x \circ y) \circ (x \circ y)) \leq \max\{F_N(x), F_N(x \circ ((x \circ y) \circ (x \circ y)) \circ ((x \circ y) \circ (x \circ y)))\} = \max\{F_N(x), F_N(x \circ (x \circ y))\} \leq \max\{F_N(x), F_N(y)\}$ , for all  $x, y \in S$ .

Conversely, let  $S_N$  be a neutrosophic  $\mathcal{N}$ -structure on  $S$  satisfying (1) and (2). Since  $x \preceq 1$  from Lemma 1 (i), we have from (1) that  $T_N(x) \leq T_N(1)$ ,  $I_N(1) \leq I_N(x)$  and  $F_N(1) \leq F_N(x)$ , for all  $x \in S$ . Also,  $((x \circ (x \circ (y \circ y))) \circ (x \circ (x \circ (y \circ y)))) \circ (y \circ y) = (x \circ (y \circ y)) \circ ((x \circ (y \circ y)) \circ (x \circ (y \circ y))) = 1$  from (S1), (SBE-1) and (SBE-2), and so, it follows that  $(x \circ (x \circ (y \circ y))) \circ (x \circ (x \circ (y \circ y))) \preceq y$ . Then it follows from (1) and (2) that

$$\min\{T_N(x \circ (y \circ y)), T_N(x)\} \leq T_N((x \circ (x \circ (y \circ y))) \circ (x \circ (x \circ (y \circ y)))) \leq T_N(y),$$

$$I_N(y) \leq I_N((x \circ (x \circ (y \circ y))) \circ (x \circ (x \circ (y \circ y)))) \leq \max\{I_N(x \circ (y \circ y)), I_N(x)\}$$

and

$$F_N(y) \leq F_N((x \circ (x \circ (y \circ y))) \circ (x \circ (x \circ (y \circ y)))) \leq \max\{F_N(x \circ (y \circ y)), F_N(x)\},$$

for all  $x, y \in S$ . Therefore,  $S_N$  is a neutrosophic  $\mathcal{N}$ -filter of  $S$ .

**Lemma 8** Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -filter of a SBE-algebra  $S$ . Then

1.  $T_N(x) \leq T_N(y \circ (x \circ x))$ ,  $I_N(y \circ (x \circ x)) \leq I_N(x)$  and  $F_N(y \circ (x \circ x)) \leq F_N(x)$ ,
2.  $\min\{T_N(x), T_N(y)\} \leq T_N(x \circ (y \circ y))$ ,  $I_N(x \circ (y \circ y)) \leq \max\{I_N(x), I_N(y)\}$  and  $F_N(x \circ (y \circ y)) \leq \max\{F_N(x), F_N(y)\}$ ,
3.  $T_N(x) \leq T_N((x \circ (y \circ y)) \circ (y \circ y))$ ,  $I_N((x \circ (y \circ y)) \circ (y \circ y)) \leq I_N(x)$  and  $F_N((x \circ (y \circ y)) \circ (y \circ y)) \leq F_N(x)$ ,
4.  $\min\{T_N(x), T_N(y)\} \leq T_N((x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z)))) \circ (z \circ z))$ ,  $I_N((x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z)))) \circ (z \circ z)) \leq \max\{I_N(x), I_N(y)\}$  and  $F_N((x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z)))) \circ (z \circ z)) \leq \max\{F_N(x), F_N(y)\}$ ,

for all  $x, y, z \in S$ .

*Proof* Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -filter of a SBE-algebra  $S$ . Then

1. It is proved from Lemma 2 (2) and Lemma 7 (1).
2. It follows from (1) that  $\min\{T_N(x), T_N(y)\} \leq T_N(y) \leq T_N(x \circ (y \circ y))$ ,  $I_N(x \circ (y \circ y)) \leq I_N(y) \leq \max\{I_N(x), I_N(y)\}$  and  $F_N(x \circ (y \circ y)) \leq F_N(y) \leq \max\{F_N(x), F_N(y)\}$ , for all  $x, y \in S$ .
3. We get from Lemma 2 (3) and Lemma 7 (1).
4. It is obtained from (3) and (SBE-2) that

$$\begin{aligned} \min\{T_N(x), T_N(y)\} &\leq \min\{T_N(x), T_N((y \circ ((x \circ (z \circ z)) \circ (x \circ \\ &\quad (z \circ z)))) \circ ((x \circ (z \circ z)) \circ (x \circ (z \circ z))))\} \\ &\leq \min\{T_N(x), T_N(x \circ (((x \circ ((y \circ (z \circ z)) \\ &\quad \circ (y \circ (z \circ z)))) \circ (z \circ z)) \circ ((x \circ ((y \circ \\ &\quad (z \circ z)) \circ (y \circ (z \circ z)))) \circ (z \circ z))))\} \\ &\leq T_N((x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z)))) \circ (z \circ z)), \end{aligned}$$

$$\begin{aligned} &I_N((x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z)))) \circ (z \circ z)) \\ &\leq \max\{I_N(x), I_N(x \circ (((x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z)))) \circ \\ &\quad (z \circ z)) \circ ((x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z)))) \circ (z \circ z))))\} \\ &= \max\{I_N(x), I_N((y \circ ((x \circ (z \circ z)) \circ (x \circ (z \circ z)))) \circ ((x \circ (z \circ z)) \circ (x \circ (z \circ z))))\} \\ &\leq \max\{I_N(x), I_N(y)\} \\ &\text{and} \\ &F_N((x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z)))) \circ (z \circ z)) \\ &\leq \max\{F_N(x), F_N(x \circ (((x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z)))) \circ \\ &\quad (z \circ z)) \circ ((x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z)))) \circ (z \circ z))))\} \\ &= \max\{F_N(x), F_N((y \circ ((x \circ (z \circ z)) \circ (x \circ (z \circ z)))) \circ ((x \circ (z \circ z)) \circ (x \circ (z \circ z))))\} \\ &\leq \max\{F_N(x), F_N(y)\}, \\ &\text{for all } x, y, z \in S. \end{aligned}$$

**Theorem 5** Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -structure on a SBE-algebra  $S$ . Then  $S_N$  is a neutrosophic  $\mathcal{N}$ -filter of  $S$  if and only if

$$\begin{aligned} z \in U(x, y) \text{ implies } \min\{T_N(x), T_N(y)\} &\leq T_N(z), I_N(z) \leq \max\{I_N(x), I_N(y)\} \\ \text{and } F_N(z) &\leq \max\{F_N(x), F_N(y)\}, \end{aligned} \quad (4)$$

for all  $x, y, z \in S$ .

*Proof* Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -filter of  $S$  and  $z \in U(x, y)$ . Since  $x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))) = 1$ , it is obtained that  $x \preceq y \circ (z \circ z)$ . Then it follows from Lemma 7 (1) that  $\min\{T_N(x), T_N(y)\} \leq \min\{T_N(y \circ (z \circ z)), T_N(y)\} \leq T_N(z)$ ,  $I_N(z) \leq \max\{I_N(y \circ (z \circ z)), I_N(y)\} \leq \max\{I_N(x), I_N(y)\}$  and  $F_N(z) \leq \max\{F_N(y \circ (z \circ z)), F_N(y)\} \leq \max\{F_N(x), F_N(y)\}$ , for all  $x, y, z \in S$ .

Conversely, let  $S_N$  be a neutrosophic  $\mathcal{N}$ -structure on  $S$  satisfying the condition (4). Since  $x \circ ((x \circ (1 \circ 1)) \circ (x \circ (1 \circ 1))) = 1$  from Lemma 1 (i), we have that  $1 \in U(x, x)$ , for all  $x \in S$ . Then  $T_N(x) = \min\{T_N(x), T_N(x)\} \leq T_N(1)$ ,  $I_N(1) \leq \max\{I_N(x), I_N(x)\} = I_N(x)$  and  $F_N(1) \leq \max\{F_N(x), F_N(x)\} = F_N(x)$ , for all  $x \in S$ . Since  $x \circ (((x \circ (y \circ y)) \circ (y \circ y)) \circ ((x \circ (y \circ y)) \circ (y \circ y))) = 1$

Lemma 1 (iv), we obtain that  $y \in U(x, x \circ (y \circ y))$ . Thus, it follows from the condition (4) that  $\min\{T_N(x \circ (y \circ y)), T_N(x)\} \leq T_N(y)$ ,  $I_N(y) \leq \max\{I_N(x \circ (y \circ y)), I_N(x)\}$  and  $F_N(y) \leq \max\{F_N(x \circ (y \circ y)), F_N(x)\}$ , for all  $x, y \in S$ . Hence,  $S_N$  is a neutrosophic  $\mathcal{N}$ -filter of  $S$ .

**Definition 17** A neutrosophic  $\mathcal{N}$ -structure  $S_N$  on a SBE-algebra  $S$  is called an implicative neutrosophic  $\mathcal{N}$ -filter of  $S$  if it satisfies

(inf-1)  $T_N(x) \leq T_N(1)$ ,  $I_N(1) \leq I_N(x)$  and  $F_N(1) \leq F_N(x)$ ,  
 (inf-2)  $\min\{T_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))))), T_N(x \circ (y \circ y))\} \leq T_N(x \circ (z \circ z))$ ,  
 $I_N(x \circ (z \circ z)) \leq \max\{I_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))))), I_N(x \circ (y \circ y))\}$  and  
 $F_N(x \circ (z \circ z)) \leq \max\{F_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))))), F_N(x \circ (y \circ y))\}$ ,  
 for all  $x, y, z \in S$ .

*Example 6* Consider the SBE-algebra  $S$  in Example 1. Then a neutrosophic  $\mathcal{N}$ -structure

$$S_N = \left\{ \frac{x}{(-0.87, 0, -0.97)} : x = 0, u, v \right\} \cup \left\{ \frac{x}{(-0.13, -1, -0.99)} : x = w, t, 1 \right\}$$

on  $S$  is an implicative neutrosophic  $\mathcal{N}$ -filter of  $S$ .

**Lemma 9** Every implicative neutrosophic  $\mathcal{N}$ -filter of a SBE-algebra  $S$  is a neutrosophic  $\mathcal{N}$ -filter of  $S$ .

*Proof* Let  $S_N$  be an implicative neutrosophic  $\mathcal{N}$ -filter of  $S$ . Then it is clear from (inf-1) that  $T_N(x) \leq T_N(1)$ ,  $I_N(1) \leq I_N(x)$  and  $F_N(1) \leq F_N(x)$ , for all  $x \in S$ . Since it follows from Lemma 1 (ii) and (inf-2) that  $\min\{T_N(x \circ (y \circ y)), T_N(x)\} = \min\{T_N(1 \circ ((x \circ (y \circ y)) \circ (x \circ (y \circ y))))), T_N(1 \circ (x \circ x))\} \leq T_N(1 \circ (y \circ y)) = T_N(y)$ ,  $I_N(y) = I_N(1 \circ (y \circ y)) \leq \max\{I_N(1 \circ ((x \circ (y \circ y)) \circ (x \circ (y \circ y))))), I_N(1 \circ (x \circ x))\}$  and  $F_N(y) = F_N(1 \circ (y \circ y)) \leq \max\{F_N(1 \circ ((x \circ (y \circ y)) \circ (x \circ (y \circ y))))), F_N(1 \circ (x \circ x))\}$ , for all  $x, y, z \in S$ ,  $S_N$  is a neutrosophic  $\mathcal{N}$ -filter of  $S$ .

The inverse of Lemma 9 is mostly not true.

*Example 7* Consider the SBE-algebra  $A$  in Example 2. Then a neutrosophic  $\mathcal{N}$ -structure

$$S_N = \left\{ \frac{1}{(0, -1, -1)} \right\} \cup \left\{ \frac{x}{(-1, 0, 0)} : x \in S - \{1\} \right\}$$

on  $S$  is a neutrosophic  $\mathcal{N}$ -subalgebra of  $S$  but it is not implicative since  $T_N(u \circ (v \circ v)) = T_N(v) = -1 < 0 = T_N(1) \min\{T_N(u \circ ((w \circ (v \circ v)) \circ (w \circ (v \circ v))))), T_N(u \circ (w \circ yw))\}$ .

**Theorem 6** Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -structure on a SBE-algebra  $S$  and  $\alpha, \beta, \gamma$  be any elements of  $[-1, 0]$  with  $-3 \leq \alpha + \beta + \gamma \leq 0$ . If  $S_N$  is a (implicative) neutrosophic  $\mathcal{N}$ -filter of  $S$ , then the nonempty  $(\alpha, \beta, \gamma)$ -level set  $S_N(\alpha, \beta, \gamma)$  of  $S_N$  is a (implicative) SBE-filter of  $S$ .

*Proof* Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -filter of  $S$  and  $S_N(\alpha, \beta, \gamma) \neq \emptyset$ , for  $\alpha, \beta, \gamma \in [-1, 0]$  with  $-3 \leq \alpha + \beta + \gamma \leq 0$ . Since  $\alpha \leq T_N(x) \leq T_N(1)$ ,  $I_N(1) \leq I_N(x) \leq \beta$  and  $F_N(1) \leq F_N(x) \leq \gamma$ , for all  $x \in S$ , we obtain that  $1 \in S_N(\alpha, \beta, \gamma)$ . Let  $x \circ (y \circ y), x \in S_N(\alpha, \beta, \gamma)$ . Since  $\alpha \leq T_N(x), T_N(x \circ (y \circ y)), I_N(x), I_N(x \circ (y \circ y)) \leq \beta$  and  $F_N(x), F_N(x \circ (y \circ y)) \leq \gamma$ , it follows that

$$\alpha \leq \min\{T_N(x \circ (y \circ y)), T_N(x)\} \leq T_N(y),$$

$$I_N(y) \leq \max\{I_N(x \circ (y \circ y)), I_N(x)\} \leq \beta$$

and

$$F_N(y) \leq \max\{F_N(x \circ (y \circ y)), F_N(x)\} \leq \gamma,$$

for all  $x, y \in S$ , which imply that  $y \in S_N(\alpha, \beta, \gamma)$ . So,  $S_N(\alpha, \beta, \gamma)$  is a SBE-filter of  $S$ .

Let  $S_N$  be an implicative neutrosophic  $\mathcal{N}$ -filter of  $S$  and  $x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))), x \circ (y \circ y) \in S_N(\alpha, \beta, \gamma)$ . Since  $\alpha \leq T_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))), T_N(x \circ (y \circ y)), I_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))), I_N(x \circ (y \circ y)) \leq \beta$  and  $F_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))), F_N(x \circ (y \circ y)) \leq \gamma$ , it is obtained that  $\alpha \leq \min\{T_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))), T_N(x \circ (y \circ y))\} \leq T_N(x \circ (z \circ z))$ ,  $I_N(x \circ (z \circ z)) \leq \max\{I_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))), I_N(x \circ (y \circ y))\} \leq \beta$  and  $F_N(x \circ (z \circ z)) \leq \max\{F_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))), F_N(x \circ (y \circ y))\} \leq \gamma$ . Thus,  $x \circ (z \circ z) \in T_N^\alpha, I_N^\beta, F_N^\gamma$ , and so,  $x \circ (z \circ z) \in T_N^\alpha \cap I_N^\beta \cap F_N^\gamma = S_N(\alpha, \beta, \gamma)$ . Hence,  $S_N(\alpha, \beta, \gamma)$  is an implicative SBE-filter of  $S$ .

**Theorem 7** Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -structure on a SBE-algebra  $S$  and  $T_N^\alpha, I_N^\beta, F_N^\gamma$  be (implicative) SBE-filters of  $S$ , for all  $\alpha, \beta, \gamma \in [-1, 0]$  with  $-3 \leq \alpha + \beta + \gamma \leq 0$ . Then  $S_N$  is a (implicative) neutrosophic  $\mathcal{N}$ -filter of  $S$ .

*Proof* Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -structure on a SBE-algebra  $S$  and  $T_N^\alpha, I_N^\beta, F_N^\gamma$  be SBE-filters of  $S$ , for all  $\alpha, \beta, \gamma \in [-1, 0]$  with  $-3 \leq \alpha + \beta + \gamma \leq 0$ . Suppose that  $T_N(1) < T_N(x)$ ,  $I_N(x) < I_N(1)$  and  $F_N(x) < F_N(1)$ , for some  $x \in S$ . If  $\alpha = \frac{1}{2}(T_N(1) + T_N(x))$ ,  $\beta = \frac{1}{2}(I_N(1) + I_N(x))$  and  $\gamma = \frac{1}{2}(F_N(1) + F_N(x))$  in  $[-1, 0)$ , then  $T_N(1) < \alpha < T_N(x)$ ,  $I_N(x) < \beta < I_N(1)$  and  $F_N(x) < \gamma < F_N(1)$ , which imply that  $1 \notin T_N^\alpha, I_N^\beta, F_N^\gamma$ . This contradicts with (SBEf-1). So,  $T_N(x) \leq T_N(1)$ ,  $I_N(1) \leq I_N(x)$  and  $F_N(1) \leq F_N(x)$ , for all  $x \in S$ . Assume that

$$\alpha_1 = T_N(y) < \min\{T_N(x \circ (y \circ y)), T_N(x)\} = \alpha_2,$$

$$\beta_1 = \max\{I_N(x \circ (y \circ y)), I_N(x)\} < I_N(y) = \beta_2,$$

and

$$\gamma_1 = \max\{F_N(x \circ (y \circ y)), F_N(x)\} < F_N(y) = \gamma_2.$$

If  $\alpha' = \frac{1}{2}(\alpha_1 + \alpha_2)$ ,  $\beta' = \frac{1}{2}(\beta_1 + \beta_2)$  and  $\gamma' = \frac{1}{2}(\gamma_1 + \gamma_2)$  in  $[-1, 0)$ , then  $\alpha_1 < \alpha' < \alpha_2$ ,  $\beta_1 < \beta' < \beta_2$  and  $\gamma_1 < \gamma' < \gamma_2$ . Thus,  $x \circ (y \circ y), x \in T_N^{\alpha'}, I_N^{\beta'}, F_N^{\gamma'}$  but  $y \notin T_N^{\alpha'}, I_N^{\beta'}, F_N^{\gamma'}$ , which contradicts with (SBEf-2). Hence,

$$\min\{T_N(x \circ (y \circ y)), T_N(x)\} \leq T_N(y),$$

$$I_N(y) \leq \max\{I_N(x \circ (y \circ y)), I_N(x)\}$$

and

$$F_N(y) \leq \max\{F_N(x \circ (y \circ y)), F_N(x)\},$$

for all  $x, y \in S$ . Therefore,  $S_N$  is a neutrosophic  $\mathcal{N}$ -filter of  $S$ .

Let  $T_N^\alpha, I_N^\beta, F_N^\gamma$  be implicative SBE-filters of  $S$ , for all  $\alpha, \beta, \gamma \in [-1, 0]$  with  $-3 \leq \alpha + \beta + \gamma \leq 0$ . Suppose that  $a_1 = T_N(x \circ (z \circ z)) < \min\{T_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))))), T_N(x \circ (y \circ y))\} = a_2$ ,  $b_1 = \max\{I_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))))), I_N(x \circ (y \circ y))\} < I_N(x \circ (z \circ z)) = b_2$  and  $c_1 = \max\{F_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))))), F_N(x \circ (y \circ y))\} < F_N(x \circ (z \circ z)) = c_2$ , for some  $x, y, z \in S$ . If  $\alpha_0 = \frac{1}{2}(a_1 + a_2)$ ,  $\beta_0 = \frac{1}{2}(b_1 + b_2)$  and  $\gamma_0 = \frac{1}{2}(c_1 + c_2)$  in  $[-1, 0]$ , then  $a_1 < \alpha_0 < a_2$ ,  $b_1 < \beta_0 < b_2$  and  $c_1 < \gamma_0 < c_2$ . So,  $x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))), x \circ (y \circ y) \in T_N^{\alpha_0}, I_N^{\beta_0}, F_N^{\gamma_0}$  but  $x \circ (z \circ z) \notin T_N^{\alpha_0}, I_N^{\beta_0}, F_N^{\gamma_0}$ , which is a contradiction. Thus,  $\min\{T_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))))), T_N(x \circ (y \circ y))\} \leq T_N(x \circ (z \circ z))$ ,  $I_N(x \circ (z \circ z)) \leq \max\{I_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))))), I_N(x \circ (y \circ y))\}$  and  $F_N(x \circ (z \circ z)) \leq \max\{F_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))))), F_N(x \circ (y \circ y))\}$ , for all  $x, y, z \in S$ .

**Theorem 8** Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -structure on a SBE-algebra  $S$ . Then  $S_N$  is a neutrosophic  $\mathcal{N}$ -filter of  $S$  if and only if

$$x, y \in S_N(\alpha, \beta, \gamma) \Leftrightarrow U(x, y) \subseteq S_N(\alpha, \beta, \gamma), \quad (5)$$

for all  $x, y \in S$  and  $\alpha, \beta, \gamma \in [-1, 0]$  with  $-3 \leq \alpha + \beta + \gamma \leq 0$ .

*Proof* Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -filter of  $S$ . Assume that  $x, y \in S_N(\alpha, \beta, \gamma)$ , for any  $x, y \in S$  and  $\alpha, \beta, \gamma \in [-1, 0]$  with  $-3 \leq \alpha + \beta + \gamma \leq 0$ , and  $z \in U(x, y)$ . Since  $\alpha \leq T_N(x), T_N(y), I_N(x), I_N(y) \leq \beta$  and  $F_N(x), F_N(y) \leq \gamma$ , it follows from Theorem 5 that  $\alpha \leq \min\{T_N(x), T_N(y)\} \leq T_N(z)$ ,  $I_N(z) \leq \max\{I_N(x), I_N(y)\} \leq \beta$  and  $F_N(z) \leq \max\{F_N(x), F_N(y)\} \leq \gamma$ . Thus,  $z \in T_N^\alpha, I_N^\beta, F_N^\gamma$ , and so,  $z \in T_N^\alpha \cap I_N^\beta \cap F_N^\gamma = S_N(\alpha, \beta, \gamma)$ . Hence,  $U(x, y) \subseteq S_N(\alpha, \beta, \gamma)$ . Suppose that  $U(x, y) \subseteq S_N(\alpha, \beta, \gamma)$ . Since  $x \circ ((y \circ (x \circ x)) \circ (y \circ (x \circ x))) = 1$  and  $x \circ ((y \circ (y \circ y)) \circ (y \circ (y \circ y))) = x \circ (1 \circ 1) = 1$  from (SBE-1), Lemma 1 (i) and (iii), we have that  $x, y \in U(x, y) \subseteq S_N(\alpha, \beta, \gamma)$ .

Conversely, Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -structure on  $S$  satisfying the condition (5). Then it is obtained from Lemma 1 (i) and the condition (5) that  $1 \in U(x, y) \subseteq S_N(\alpha, \beta, \gamma)$ . Assume that  $x, x \circ (y \circ y) \in S_N(\alpha, \beta, \gamma)$ . Thus,  $U(x, x \circ (y \circ y)) \subseteq S_N(\alpha, \beta, \gamma)$ . Since  $x \circ (((x \circ (y \circ y)) \circ (y \circ y)) \circ ((x \circ (y \circ y)) \circ (y \circ y))) = 1$  from Lemma 1 (iv), it follows that  $y \in U(x, x \circ (y \circ y)) \subseteq S_N(\alpha, \beta, \gamma)$ . Thereby,  $S_N(\alpha, \beta, \gamma) = T_N^\alpha \cap I_N^\beta \cap F_N^\gamma$  is a SBE-filter of  $S$ , and so,  $S_N$  be a neutrosophic  $\mathcal{N}$ -filter of  $S$  by Theorem 7.

**Corollary 1** Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -structure on a SBE-algebra  $S$ . Then  $S_N$  is a neutrosophic  $\mathcal{N}$ -filter of  $S$  if and only if

$$\emptyset \neq S_N(\alpha, \beta, \gamma) = \bigcup_{x, y \in S_N(\alpha, \beta, \gamma)} U(x, y), \quad (6)$$

for all  $x, y \in S$  and  $\alpha, \beta, \gamma \in [-1, 0]$  with  $-3 \leq \alpha + \beta + \gamma \leq 0$ .

*Proof* Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -filter of  $S$ . By Theorem 8, it is obvious that it is obvious that  $\bigcup_{x,y \in S_N(\alpha,\beta,\gamma)} U(x,y) \subseteq S_N(\alpha,\beta,\gamma)$ , for all  $x, y \in S$  and  $\alpha, \beta, \gamma \in [-1, 0]$  with  $-3 \leq \alpha + \beta + \gamma \leq 0$ . Then it is sufficient to show that  $S_N(\alpha, \beta, \gamma) \subseteq \bigcup_{x,y \in S_N(\alpha,\beta,\gamma)} U(x,y)$ . Since  $S_N$  is a neutrosophic  $\mathcal{N}$ -filter of  $S$ , it follows from Theorem 6 that  $S_N(\alpha, \beta, \gamma)$  is a SBE-filter of  $S$ . Assume that  $x \in S_N(\alpha, \beta, \gamma)$ . Since  $x \circ ((1 \circ (x \circ x)) \circ (1 \circ (x \circ x))) = x \circ (x \circ x) = 1$  from Lemma 1 (ii) and (SBE-1), we have that  $x \in U(x, 1)$ . Thus,

$$S_N(\alpha, \beta, \gamma) \subseteq U(x, 1) \subseteq \bigcup_{x \in S_N(\alpha,\beta,\gamma)} U(x, 1) \subseteq \bigcup_{x,y \in S_N(\alpha,\beta,\gamma)} U(x, y).$$

Conversely, let  $S_N$  be a neutrosophic  $\mathcal{N}$ -structure on  $S$  satisfying the condition (6). Since  $1 \in U(x, y)$  from Lemma 1 (i), we get that

$$1 \in \bigcup_{x,y \in S_N(\alpha,\beta,\gamma)} U(x, y) = S_N(\alpha, \beta, \gamma).$$

Assume that  $x, x \circ (y \circ y) \in S_N(\alpha, \beta, \gamma)$ . Since  $\bigcup_{x,x \circ (y \circ y) \in S_N(\alpha,\beta,\gamma)} U(x, x \circ (y \circ y)) = S_N(\alpha, \beta, \gamma)$  and  $x \circ (((x \circ (y \circ y)) \circ (y \circ y)) \circ ((x \circ (y \circ y)) \circ (y \circ y))) = 1$  from the condition (6) and Lemma 1 (iv), it is obtained that  $y \in \bigcup_{x,x \circ (y \circ y) \in S_N(\alpha,\beta,\gamma)} U(x, x \circ (y \circ y)) = S_N(\alpha, \beta, \gamma)$ . Hence,  $S_N(\alpha, \beta, \gamma)$  is a SBE-filter of  $S$ , and so,  $S_N$  is a neutrosophic  $\mathcal{N}$ -filter of  $S$  from Theorem 7.

**Theorem 9** Let  $\langle S; \circ_S, 1_S \rangle$  and  $\langle P; \circ_P, 1_P \rangle$  be SBE-algebras,  $f : S \rightarrow P$  be a surjective SBE-homomorphism and  $P_N = \frac{P}{(T_N, I_N, F_N)}$  be a neutrosophic  $\mathcal{N}$ -structure on  $P$ . Then  $P_N$  is a (implicative) neutrosophic  $\mathcal{N}$ -filter of  $P$  if and only if  $P_N^f = \frac{S}{(T_N^f, I_N^f, F_N^f)}$  is a (implicative) neutrosophic  $\mathcal{N}$ -filter of  $S$  where the  $\mathcal{N}$ -functions  $T_N^f, I_N^f, F_N^f : S \rightarrow [-1, 0]$  on  $S$  are defined by  $T_N^f(x) = T_N(f(x))$ ,  $I_N^f(x) = I_N(f(x))$  and  $F_N^f(x) = F_N(f(x))$ , for all  $x \in S$ , respectively.

*Proof* Let  $\langle S; \circ_S, 1_S \rangle$  and  $\langle P; \circ_P, 1_P \rangle$  be SBE-algebras,  $f : S \rightarrow P$  be a surjective SBE-homomorphism and  $P_N = \frac{P}{(T_N, I_N, F_N)}$  be a neutrosophic  $\mathcal{N}$ -filter of  $P$ . Then  $T_N^f(x) = T_N(f(x)) \leq T_N(1_P) = T_N(f(1_S)) = T_N^f(1_S)$ ,  $I_N^f(1_S) = I_N(f(1_S)) = I_N(1_P) \leq I_N(f(x)) = I_N^f(x)$  and  $F_N^f(1_S) = F_N(f(1_S)) = F_N(1_P) \leq F_N(f(x)) = F_N^f(x)$ , for all  $x \in S$ . Also,

$$\begin{aligned} \min\{T_N^f(x), T_N^f(x \circ_S (y \circ_S y))\} &= \min\{T_N(f(x)), T_N(f(x \circ_S (y \circ_S y)))\} \\ &= \min\{T_N(f(x)), T_N(f(x) \circ_P (f(y) \circ_P f(y)))\} \\ &\leq T_N(f(y)) \\ &= T_N^f(y), \end{aligned}$$

$$\begin{aligned} I_N^f(y) &= I_N(f(y)) \\ &\leq \max\{I_N(f(x)), I_N(f(x) \circ_P (f(y) \circ_P f(y)))\} \\ &= \max\{I_N(f(x)), I_N(f(x \circ_S (y \circ_S y)))\} \\ &= \max\{I_N^f(x), I_N^f(x \circ_S (y \circ_S y))\} \end{aligned}$$



and

$$\begin{aligned} F_N^f(y) &= F_N(f(y)) \\ &\leq \max\{F_N(f(x)), F_N(f(x) \circ_P (f(y) \circ_P f(y)))\} \\ &= \max\{F_N(f(x)), F_N(f(x \circ_S (y \circ_S y)))\} \\ &= \max\{F_N^f(x), F_N^f(x \circ_S (y \circ_S y))\}, \end{aligned}$$

for all  $x, y \in S$ . Thus,  $P_N^f = \frac{S}{(T_N^f, I_N^f, F_N^f)}$  is a neutrosophic  $\mathcal{N}$ -filter of  $S$ .

Suppose that  $P_N$  is an implicative neutrosophic  $\mathcal{N}$ -filter of  $P$ . Since

$$\begin{aligned} &\min\{T_N^f(x \circ_S ((y \circ_S (z \circ_S z)) \circ_S (y \circ_S (z \circ_S z))))), T_N^f(x \circ_S (y \circ_S y))\} \\ &= \min\{T_N(f(x \circ_S ((y \circ_S (z \circ_S z)) \circ_S (y \circ_S (z \circ_S z))))), T_N(f(x \circ_S (y \circ_S y)))\} \\ &= \min\{T_N(f(x) \circ_P ((f(y) \circ_P (f(z) \circ_P f(z))) \circ_P (f(y) \\ &\quad \circ_P (f(z) \circ_P f(z))))), T_N(f(x) \circ_P (f(y) \circ_P f(y)))\} \\ &\leq T_N(f(x) \circ_P (f(z) \circ_P f(z))) \\ &= T_N(f(x \circ_S (z \circ_S z))) \\ &= T_N^f(x \circ_S (z \circ_S z)), \end{aligned}$$

$$\begin{aligned} &I_N^f(x \circ_S (z \circ_S z)) \\ &= I_N(f(x \circ_S (z \circ_S z))) \\ &= I_N(f(x) \circ_P (f(z) \circ_P f(z))) \\ &\leq \max\{I_N(f(x) \circ_P ((f(y) \circ_P (f(z) \circ_P f(z))) \circ_P (f(y) \\ &\quad \circ_P (f(z) \circ_P f(z))))), I_N(f(x) \circ_P (f(y) \circ_P f(y)))\} \\ &= \max\{I_N(f(x \circ_S ((y \circ_S (z \circ_S z)) \circ_S (y \circ_S (z \circ_S z))))), I_N(f(x \circ_S (y \circ_S y)))\} \\ &= \max\{I_N^f(x \circ_S ((y \circ_S (z \circ_S z)) \circ_S (y \circ_S (z \circ_S z))))), I_N^f(x \circ_S (y \circ_S y))\} \end{aligned}$$

and

$$\begin{aligned} &F_N^f(x \circ_S (z \circ_S z)) \\ &= F_N(f(x \circ_S (z \circ_S z))) \\ &= F_N(f(x) \circ_P (f(z) \circ_P f(z))) \\ &\leq \max\{F_N(f(x) \circ_P ((f(y) \circ_P (f(z) \circ_P f(z))) \circ_P (f(y) \\ &\quad \circ_P (f(z) \circ_P f(z))))), F_N(f(x) \circ_P (f(y) \circ_P f(y)))\} \\ &= \max\{F_N(f(x \circ_S ((y \circ_S (z \circ_S z)) \circ_S (y \circ_S (z \circ_S z))))), F_N(f(x \circ_S (y \circ_S y)))\} \\ &= \max\{F_N^f(x \circ_S ((y \circ_S (z \circ_S z)) \circ_S (y \circ_S (z \circ_S z))))), F_N^f(x \circ_S (y \circ_S y))\} \end{aligned}$$

for all  $x, y, z \in S$ . Hence,  $P_N^f$  is an implicative neutrosophic  $\mathcal{N}$ -filter of  $S$ .

Conversely, let  $P_N^f$  be a neutrosophic  $\mathcal{N}$ -filter of  $S$ . So,  $T_N(y) = T_N(f(x)) = T_N^f(x) \leq T_N^f(1_S) = T_N(f(1_S)) = T_N(1_P)$ ,  $I_N(1_P) = I_N(f(1_S)) = I_N^f(1_S) \leq I_N^f(x) = I_N(f(x)) = I_N(y)$  and  $F_N(1_P) = F_N(f(1_S)) = F_N^f(1_S) \leq F_N^f(x) = F_N(f(x)) = F_N(y)$ , for all  $y \in P$ . Besides,

$$\begin{aligned} \min\{T_N(y_1), T_N(y_1 \circ_P (y_2 \circ_P y_2))\} &= \min\{T_N(f(x_1)), T_N(f(x_1) \\ &\quad \circ_P (f(x_2) \circ_P f(x_2)))\} \\ &= \min\{T_N(f(x_1)), T_N(f(x_1 \circ_S (x_2 \circ_S x_2)))\} \\ &= \min\{T_N^f(x_1), T_N^f(x_1 \circ_S (x_2 \circ_S x_2))\} \\ &\leq T_N^f(x_2) \\ &= T_N(f(x_2)) \\ &= T_N(y_2), \end{aligned}$$

$$\begin{aligned}
I_N(y_2) &= I_N(f(x_2)) \\
&= I_N^f(x_2) \\
&\leq \max\{I_N^f(x_1), I_N^f(x_1 \circ_S (x_2 \circ_S x_2))\} \\
&= \max\{I_N(f(x_1)), I_N(f(x_1 \circ_S (x_2 \circ_S x_2)))\} \\
&= \max\{I_N(f(x_1)), I_N(f(x_1) \circ_P (f(x_2) \circ_P f(x_2)))\} \\
&= \max\{I_N(y_1), I_N(y_1 \circ_P (y_2 \circ_P y_2))\}
\end{aligned}$$

and

$$\begin{aligned}
F_N(y_2) &= F_N(f(x_2)) \\
&= F_N^f(x_2) \\
&\leq \max\{F_N^f(x_1), F_N^f(x_1 \circ_S (x_2 \circ_S x_2))\} \\
&= \max\{F_N(f(x_1)), F_N(f(x_1 \circ_S (x_2 \circ_S x_2)))\} \\
&= \max\{F_N(f(x_1)), F_N(f(x_1) \circ_P (f(x_2) \circ_P f(x_2)))\} \\
&= \max\{F_N(y_1), F_N(y_1 \circ_P (y_2 \circ_P y_2))\},
\end{aligned}$$

for all  $y_1, y_2 \in P$ . Thereby,  $P_N = \frac{S}{(T_N, I_N, F_N)}$  is a neutrosophic  $\mathcal{N}$ -filter of

$P$ . Assume that  $P_N^f$  is an implicative neutrosophic  $\mathcal{N}$ -filter of  $S$ . Since

$$\begin{aligned}
&\min\{T_N(y_1 \circ_P ((y_2 \circ_P (y_3 \circ_P y_3)) \circ_P (y_2 \\
&\circ_P (y_3 \circ_P y_3))))), T_N(y_1 \circ_P (y_2 \circ_P y_2))\} \\
&= \min\{T_N(f(x_1) \circ_P ((f(x_2) \circ_P (f(x_3) \circ_P f(x_3))) \circ_P (f(x_2) \\
&\circ_P (f(x_3) \circ_P f(x_3))))), T_N(f(x_1) \circ_P (f(x_2) \circ_P f(x_2)))\} \\
&= \min\{T_N(f(x_1 \circ_S ((x_2 \circ_S (x_3 \circ_S x_3)) \circ_S (x_2 \\
&\circ_S (x_3 \circ_S x_3))))), T_N(f(x_1 \circ_S (x_2 \circ_S x_2)))\} \\
&= \min\{T_N^f(x_1 \circ_S ((x_2 \circ_S (x_3 \circ_S x_3)) \circ_S (x_2 \\
&\circ_S (x_3 \circ_S x_3))))), T_N^f(x_1 \circ_S (x_2 \circ_S x_2))\} \\
&\leq T_N^f(x_1 \circ_S (x_3 \circ_S x_3)) \\
&= T_N(f(x_1 \circ_S (x_3 \circ_S x_3))) \\
&= T_N(f(x_1) \circ_P (f(x_3) \circ_P f(x_3))) \\
&= T_N(y_1 \circ_P (y_3 \circ_P y_3)),
\end{aligned}$$

$$\begin{aligned}
&I_N(y_1 \circ_P (y_3 \circ_P y_3)) \\
&= I_N(f(x_1) \circ_P (f(x_3) \circ_P f(x_3))) \\
&= I_N(f(x_1 \circ_S (x_3 \circ_S x_3))) \\
&= I_N^f(x_1 \circ_S (x_3 \circ_S x_3)) \\
&\leq \max\{I_N^f(x_1 \circ_S ((x_2 \circ_S (x_3 \circ_S x_3)) \circ_S (x_2 \\
&\circ_S (x_3 \circ_S x_3))))), I_N^f(x_1 \circ_S (x_2 \circ_S x_2))\} \\
&= \max\{I_N(f(x_1 \circ_S ((x_2 \circ_S (x_3 \circ_S x_3)) \circ_S (x_2 \\
&\circ_S (x_3 \circ_S x_3))))), I_N(f(x_1 \circ_S (x_2 \circ_S x_2)))\} \\
&= \max\{I_N(f(x_1) \circ_P ((f(x_2) \circ_P (f(x_3) \circ_P f(x_3))) \circ_P (f(x_2) \\
&\circ_P (f(x_3) \circ_P f(x_3))))), I_N(f(x_1) \circ_P (f(x_2) \circ_P f(x_2)))\} \\
&= \max\{I_N(y_1 \circ_P ((y_2 \circ_P (y_3 \circ_P y_3)) \circ_P (y_2 \\
&\circ_P (y_3 \circ_P y_3))))), I_N(y_1 \circ_P (y_2 \circ_P y_2))\}
\end{aligned}$$

and

$$\begin{aligned}
& F_N(y_1 \circ_P (y_3 \circ_P y_3)) \\
&= F_N(f(x_1) \circ_P (f(x_3) \circ_P f(x_3))) \\
&= F_N(f(x_1 \circ_S (x_3 \circ_S x_3))) \\
&= F_N^f(x_1 \circ_S (x_3 \circ_S x_3)) \\
&\leq \max\{F_N^f(x_1 \circ_S ((x_2 \circ_S (x_3 \circ_S x_3)) \circ_S (x_2 \\
&\quad \circ_S (x_3 \circ_S x_3))))), F_N^f(x_1 \circ_S (x_2 \circ_S x_2))\} \\
&= \max\{F_N(f(x_1 \circ_S ((x_2 \circ_S (x_3 \circ_S x_3)) \circ_S (x_2 \\
&\quad \circ_S (x_3 \circ_S x_3))))), F_N(f(x_1 \circ_S (x_2 \circ_S x_2)))\} \\
&= \max\{F_N(f(x_1) \circ_P ((f(x_2) \circ_P (f(x_3) \circ_P f(x_3))) \circ_P (f(x_2) \\
&\quad \circ_P (f(x_3) \circ_P f(x_3)))))\}, F_N(f(x_1) \circ_P (f(x_2) \circ_P f(x_2)))\} \\
&= \max\{F_N(y_1 \circ_P ((y_2 \circ_P (y_3 \circ_P y_3)) \circ_P (y_2 \\
&\quad \circ_P (y_3 \circ_P y_3))))), F_N(y_1 \circ_P (y_2 \circ_P y_2))\},
\end{aligned}$$

for all  $y_1, y_2, y_3 \in P$ . Therefore,  $P_N$  is an implicative neutrosophic  $\mathcal{N}$ -filter of  $P$ .

**Theorem 10** Every neutrosophic  $\mathcal{N}$ -filter of a SBE-algebra  $S$  is a neutrosophic  $\mathcal{N}$ -subalgebra of  $S$ .

*Proof* Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -filter of  $S$ . Since  $((x \circ y) \circ (x \circ y)) \circ (y \circ y) = 1$  from Lemma 1 (vi), we get that  $(x \circ y) \circ (x \circ y) \leq y$ , for all  $x, y \in S$ . Then it follows from Lemma 7 and Lemma 2 (2) that  $\min\{T_N(x), T_N(y)\} \leq T_N((x \circ y) \circ (x \circ y)) \leq T_N(y) \leq T_N(x \circ (y \circ y))$ ,  $I_N(x \circ (y \circ y)) \leq I_N(y) \leq I_N((x \circ y) \circ (x \circ y)) \leq \max\{I_N(x), I_N(y)\}$  and  $F_N(x \circ (y \circ y)) \leq F_N(y) \leq F_N((x \circ y) \circ (x \circ y)) \leq \max\{F_N(x), F_N(y)\}$ , for all  $x, y \in S$ . Thereby,  $S_N$  is a neutrosophic  $\mathcal{N}$ -subalgebra of  $S$ .

The inverse of Theorem 10 does not generally hold.

*Example 8* Consider the SBE-algebra  $S$  in Example 1. Then a neutrosophic  $\mathcal{N}$ -structure

$$S_N = \left\{ \frac{0}{(-1, -0.88, 0)}, \frac{1}{(0, -1, -0.73)} \right\} \cup \left\{ \frac{x}{(-0.55, -0.91, -0.43)} : x \in S - \{0, 1\} \right\}$$

on  $S$  is a neutrosophic  $\mathcal{N}$ -subalgebra of  $S$  but it is not a neutrosophic  $\mathcal{N}$ -filter of  $S$  since  $T_N(0) - 1 < -0.55 = \min\{T_N(u \circ (0 \circ 0)), T_N(u)\}$ .

**Lemma 10** Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -subalgebra of a SBE-algebra  $S$  satisfying

$$\begin{aligned}
& \min\{T_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))))), T_N(x \circ (y \circ y))\} \leq T_N(x \circ (z \circ z)) \\
& I_N(x \circ (z \circ z)) \leq \max\{I_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))))), I_N(x \circ (y \circ y))\} \\
& \text{and} \\
& F_N(x \circ (z \circ z)) \leq \max\{F_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))))), F_N(x \circ (y \circ y))\},
\end{aligned} \tag{7}$$

for all  $x, y, z \in S$ . Then  $S_N$  is a neutrosophic  $\mathcal{N}$ -filter of  $S$ .

*Proof* Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -subalgebra of  $S$  satisfying the condition (7). By Lemma 5, it is obvious that  $T_N(x) \leq T_N(1)$ ,  $I_N(1) \leq I_N(x)$  and  $F_N(1) \leq F_N(x)$ , for all  $x \in S$ . Then it is obtained from Lemma 1 (ii) that  $\min\{T_N(x \circ (y \circ y)), T_N(x)\} = \min\{T_N(1 \circ ((x \circ (y \circ y)) \circ (x \circ (y \circ y))))\}$ ,  $T_N(1 \circ (x \circ x)) \leq T_N(1 \circ (y \circ y)) = T_N(y)$ ,  $I_N(y) = I_N(1 \circ (y \circ y)) \leq \max\{I_N(1 \circ ((x \circ (y \circ y)) \circ (x \circ (y \circ y))))\}$ ,  $I_N(1 \circ (x \circ x)) \leq \max\{I_N(x \circ (y \circ y)), I_N(x)\}$  and  $F_N(y) = F_N(1 \circ (y \circ y)) \leq \max\{F_N(1 \circ ((x \circ (y \circ y)) \circ (x \circ (y \circ y))))\}$ ,  $F_N(1 \circ (x \circ x)) \leq \max\{F_N(x \circ (y \circ y)), F_N(x)\}$ , for all  $x, y \in S$ . Hence,  $S_N$  is a neutrosophic  $\mathcal{N}$ -filter of  $S$ .

**Theorem 11** *Let  $S$  be a self-distributive SBE-algebra. Then every neutrosophic  $\mathcal{N}$ -filter of  $S$  is an implicative neutrosophic  $\mathcal{N}$ -filter of  $S$ .*

*Proof* Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -filter of a self-distributive SBE-algebra  $S$ . Since  $S_N$  be a neutrosophic  $\mathcal{N}$ -filter of  $S$ , it is clear that  $T_N(x) \leq T_N(1)$ ,  $I_N(1) \leq I_N(x)$  and  $F_N(1) \leq F_N(x)$ , for all  $x \in S$ . Then it follows from Definition 3 that  $\min\{T_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))))\}$ ,  $T_N(x \circ (y \circ y)) \leq \min\{T_N((x \circ (y \circ y)) \circ ((x \circ (z \circ z)) \circ (x \circ (z \circ z))))\}$ ,  $T_N(x \circ (y \circ y)) \leq T_N(x \circ (z \circ z))$ ,  $I_N(x \circ (z \circ z)) \leq \max\{I_N((x \circ (y \circ y)) \circ ((x \circ (z \circ z)) \circ (x \circ (z \circ z))))\}$ ,  $I_N(x \circ (y \circ y)) \leq \max\{I_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))))\}$ ,  $I_N(x \circ (y \circ y)) \leq \max\{I_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))))\}$  and  $F_N(x \circ (z \circ z)) \leq \max\{F_N((x \circ (y \circ y)) \circ ((x \circ (z \circ z)) \circ (x \circ (z \circ z))))\}$ ,  $F_N(x \circ (y \circ y)) \leq \max\{F_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))))\}$ ,  $F_N(x \circ (y \circ y)) \leq \max\{F_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))))\}$ , for all  $x, y, z \in S$ . Thus,  $S_N$  is an implicative neutrosophic  $\mathcal{N}$ -filter of  $S$ .

**Lemma 11** *Let  $S_N$  be a (implicative) neutrosophic  $\mathcal{N}$ -filter of a SBE-algebra  $S$ . Then the subsets  $S_{T_N} = \{x \in S : T_N(x) = T_N(1)\}$ ,  $S_{I_N} = \{x \in S : I_N(x) = I_N(1)\}$  and  $S_{F_N} = \{x \in S : F_N(x) = F_N(1)\}$  of  $S$  are (implicative) SBE-filters of  $S$ .*

*Proof* Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -filter of  $S$ . Then it is obvious that  $1 \in S_{T_N}, S_{I_N}, S_{F_N}$ . Assume that  $x, x \circ (y \circ y) \in S_{T_N}, S_{I_N}, S_{F_N}$ . Since  $T_N(x) = T_N(1) = T_N(x \circ (y \circ y))$ ,  $I_N(x) = I_N(1) = I_N(x \circ (y \circ y))$  and  $F_N(x) = F_N(1) = F_N(x \circ (y \circ y))$ , it is obtained that  $T_N(1) = \min\{T_N(1), T_N(1)\} = \min\{T_N(x \circ (y \circ y)), T_N(x)\} \leq T_N(y)$ ,  $I_N(y) \leq \max\{I_N(x \circ (y \circ y)), I_N(x)\} = \max\{I_N(1), I_N(1)\} = I_N(1)$  and  $F_N(y) \leq \max\{F_N(x \circ (y \circ y)), F_N(x)\} = \max\{F_N(1), F_N(1)\} = F_N(1)$ , which imply that  $T_N(y) = T_N(1)$ ,  $I_N(y) = I_N(1)$  and  $F_N(y) = F_N(1)$ . Then  $y \in S_{T_N}, S_{I_N}, S_{F_N}$ . Hence,  $S_{T_N}, S_{I_N}$  and  $S_{F_N}$  are SBE-filters of  $S$ .

Let  $S_N$  be an implicative neutrosophic  $\mathcal{N}$ -filter of  $S$ . Suppose that  $x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))), x \circ (y \circ y) \in S_{T_N}, S_{I_N}, S_{F_N}$ . Since  $T_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z)))) = T_N(1) = T_N(x \circ (y \circ y))$ ,  $I_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z)))) = I_N(1) = I_N(x \circ (y \circ y))$  and  $F_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z)))) = F_N(1) = F_N(x \circ (y \circ y))$ , it follows that  $T_N(1) = \min\{T_N(1), T_N(1)\} = \min\{T_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))))\}$ ,  $T_N(x \circ (y \circ y)) \leq T_N(x \circ (z \circ z))$ ,  $I_N(x \circ (z \circ z)) \leq \max\{I_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))))\}$ ,  $I_N(x \circ (y \circ y)) \leq \max\{I_N(1), I_N(1)\} = I_N(1)$  and  $F_N(x \circ (z \circ z)) \leq \max\{F_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))))\}$ ,  $F_N(x \circ (y \circ y)) \leq \max\{F_N(1), F_N(1)\} = F_N(1)$ , which imply that  $T_N(x \circ (z \circ z)) = T_N(1)$ ,  $I_N(x \circ (z \circ z)) = I_N(1)$  and

$F_N(x \circ (z \circ z)) = F_N(1)$ . Thus,  $x \circ (z \circ z) \in S_{T_N}, S_{I_N}, S_{F_N}$ . Therefore,  $S_{T_N}, S_{I_N}$  and  $S_{F_N}$  are implicative SBE-filters of  $S$ .

**Definition 18** Let  $S$  be a SBE-algebra. Define the subsets

$$S_N^{s_t} := \{x \in S : T_N(s_t) \leq T_N(x)\},$$

$$S_N^{s_i} := \{x \in S : I_N(x) \leq I_N(s_i)\}$$

and

$$S_N^{s_f} := \{x \in S : F_N(x) \leq F_N(s_f)\}$$

of  $S$ , for all  $s_t, s_i, s_f \in S$ . Also, it is obvious that  $s_t \in S_N^{s_t}, s_i \in S_N^{s_i}$  and  $s_f \in S_N^{s_f}$ .

*Example 9* Consider the SBE-algebra  $S$  in Example 2. Let

$$T_N(x) = \begin{cases} -0.99, & \text{if } x = u, w \\ -0.72, & \text{if } x = 1 \\ 0, & \text{otherwise,} \end{cases} \quad I_N(x) = \begin{cases} 0, & \text{if } x = t, 1 \\ -1, & \text{otherwise,} \end{cases}$$

$$F_N(x) = \begin{cases} -0.011, & \text{if } x = 0, u, v \\ -0.1, & \text{otherwise,} \end{cases} \quad s_t = v, s_i = u \text{ and } s_f = w.$$

Then

$$S_N^{s_t} = \{x \in S : T_N(v) \leq T_N(x)\} = \{0, v, t\},$$

$$S_N^{s_i} = \{x \in S : I_N(x) \leq I_N(u)\} = \{0, u, v, w\}$$

and

$$S_N^{s_f} = \{x \in S : F_N(x) \leq F_N(w)\} = \{w, t, 1\}.$$

**Theorem 12** Let  $s_t, s_i$  and  $s_f$  be any elements of a SBE-algebra  $S$ . If  $S_N$  is a (implicative) neutrosophic  $\mathcal{N}$ -filter of  $S$ , then  $S_N^{s_t}, S_N^{s_i}$  and  $S_N^{s_f}$  are (implicative) SBE-filters of  $S$ .

*Proof* Let  $s_t, s_i$  and  $s_f$  be any elements of  $S$  and  $S_N$  be a neutrosophic  $\mathcal{N}$ -filter of  $S$ . Since  $T_N(s_t) \leq T_N(1)$ ,  $I_N(1) \leq I_N(s_i)$  and  $F_N(1) \leq F_N(s_f)$ , for any  $s_t, s_i, s_f \in S$ , it follows that  $1 \in S_N^{s_t}, S_N^{s_i}, S_N^{s_f}$ . Assume that  $x \circ (y \circ y), x \in S_N^{s_t}, S_N^{s_i}, S_N^{s_f}$ . Since  $T_N(s_t) \leq T_N(x \circ (y \circ y)), T_N(x), I_N(x \circ (y \circ y)), I_N(x) \leq I_N(s_i)$  and  $F_N(x \circ (y \circ y)), F_N(x) \leq F_N(s_f)$ , it is obtained that  $T_N(s_t) \leq \min\{T_N(x \circ (y \circ y)), T_N(x)\} \leq T_N(y)$ ,  $I_N(y) \leq \max\{I_N(x \circ (y \circ y)), I_N(x)\} \leq I_N(s_i)$  and  $F_N(y) \leq \max\{F_N(x \circ (y \circ y)), F_N(x)\} \leq F_N(s_f)$ , which imply that  $y \in S_N^{s_t}, S_N^{s_i}, S_N^{s_f}$ . Thus,  $S_N^{s_t}, S_N^{s_i}$  and  $S_N^{s_f}$  are SBE-filters of  $S$ .

Let  $S_N$  be an implicative neutrosophic  $\mathcal{N}$ -filter of  $S$ . Suppose that  $x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))), x \circ (y \circ y) \in S_N^{s_t}, S_N^{s_i}, S_N^{s_f}$ . Since  $T_N(s_t) \leq T_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))))$ ,  $T_N(x \circ (y \circ y)), I_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))))$ ,  $I_N(x \circ (y \circ y)) \leq I_N(s_i)$  and  $F_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))))$ ,  $F_N(x \circ (y \circ y)) \leq F_N(s_f)$ , we get that  $T_N(s_t) \leq \min\{T_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))))$ ,  $T_N(x \circ (y \circ y))\} \leq T_N(x \circ (z \circ z))$ ,  $I_N(x \circ (z \circ z)) \leq \max\{I_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))))$ ,  $I_N(x \circ (y \circ y))\} \leq I_N(s_i)$  and  $F_N(x \circ (z \circ z)) \leq \max\{F_N(x \circ ((y \circ (z \circ z)) \circ (y \circ (z \circ z))))$ ,  $F_N(x \circ (y \circ y))\} \leq F_N(s_f)$ , which means that  $x \circ (z \circ z) \in S_N^{s_t}, S_N^{s_i}, S_N^{s_f}$ . Hence,  $S_N^{s_t}, S_N^{s_i}$  and  $S_N^{s_f}$  are implicative SBE-filters of  $S$ .

*Example 10* Consider the SBE-algebra  $S$  in Example 1. For a (implicative) neutrosophic  $\mathcal{N}$ -filter

$$S_N = \left\{ \frac{x}{(-0.86, -0.75, -0.64)} : x = 0, u, v \right\} \cup \left\{ \frac{x}{(-0.52, -0.8, -0.7)} : x = w, t, 1 \right\}$$

of  $S$  and  $s_t = 0, s_i = u, s_f = w \in S$ , the subsets

$$S_N^{s_t} = \{x \in S : T_N(0) \leq T_N(x)\} = S,$$

$$S_N^{s_i} = \{x \in S : I_N(x) \leq I_N(u)\} = S$$

and

$$S_N^{s_f} = \{x \in S : F_N(x) \leq F_N(w)\} = \{w, t, 1\}$$

of  $A$  are (implicative) SBE-filters of  $S$ .

**Theorem 13** Let  $s_t, s_i$  and  $s_f$  be any elements of a SBE-algebra  $S$  and  $S_N$  be a neutrosophic  $\mathcal{N}$ -structure on  $S$ .

1. If  $S_N^{s_t}, S_N^{s_i}$  and  $S_N^{s_f}$  are SBE-filters of  $S$ , then

$$\begin{aligned} T_N(x) \leq \min\{T_N(y \circ (z \circ z)), T_N(y)\} &\Rightarrow T_N(x) \leq T_N(z), \\ \max\{I_N(y \circ (z \circ z)), I_N(y)\} &\leq I_N(x) \Rightarrow I_N(z) \leq I_N(x) \quad \text{and} \\ \max\{F_N(y \circ (z \circ z)), F_N(y)\} &\leq F_N(x) \Rightarrow F_N(z) \leq F_N(x), \end{aligned} \quad (8)$$

for all  $x, y, z \in S$ .

2. If  $S_N$  satisfies the condition (8) and

$$T_N(x) \leq T_N(1), \quad I_N(1) \leq I_N(x) \quad \text{and} \quad F_N(1) \leq F_N(x), \quad (9)$$

for all  $x \in S$ , then  $S_N^{s_t}, S_N^{s_i}$  and  $S_N^{s_f}$  are SBE-filters of  $S$ , for all  $s_t \in T_N^{-1}$ ,  $s_i \in I_N^{-1}$  and  $s_f \in F_N^{-1}$ .

*Proof* Let  $s_t, s_i$  and  $s_f$  be any elements of  $S$  and  $S_N$  be a neutrosophic  $\mathcal{N}$ -structure on  $S$ .

1. Assume that  $S_N^{s_t}, S_N^{s_i}$  and  $S_N^{s_f}$  be SBE-filters of  $S$  and  $T_N(x) \leq \min\{T_N(y \circ (z \circ z)), T_N(y)\}$ ,  $\max\{I_N(y \circ (z \circ z)), I_N(y)\} \leq I_N(x)$  and  $\max\{F_N(y \circ (z \circ z)), F_N(y)\} \leq F_N(x)$ . Since  $y \circ (z \circ z), y \in S_N^{s_t}, S_N^{s_i}, S_N^{s_f}$  where  $s_t = s_i = s_f = x$ , it follows that  $z \in S_N^{s_t}, S_N^{s_i}, S_N^{s_f}$  where  $s_t = s_i = s_f = x$ . Therefore,  $T_N(x) \leq T_N(z)$ ,  $I_N(z) \leq I_N(x)$  and  $F_N(z) \leq F_N(x)$ , for all  $x, y, z \in S$ .
2. Let  $S_N$  be a neutrosophic  $\mathcal{N}$ -structure on  $S$  satisfying the conditions (8) and (9), for  $s_t \in T_N^{-1}$ ,  $s_i \in I_N^{-1}$  and  $s_f \in F_N^{-1}$ . It is obvious from from the condition (9) that  $1 \in S_N^{s_t}, S_N^{s_i}, S_N^{s_f}$ . Suppose that  $x \circ (y \circ y), x \in S_N^{s_t}, S_N^{s_i}, S_N^{s_f}$ . Then  $T_N(s_t) \leq T_N(x \circ (y \circ y)), T_N(x), I_N(x \circ (y \circ y)), I_N(x) \leq I_N(s_i)$  and  $F_N(x \circ (y \circ y)), F_N(x) \leq F_N(s_f)$ . Since  $T_N(s_t) \leq \min\{T_N(x \circ (y \circ y)), T_N(x)\}$ ,  $\max\{I_N(x \circ (y \circ y)), I_N(x)\} \leq I_N(s_i)$  and  $\max\{F_N(x \circ (y \circ y)), F_N(x)\} \leq F_N(s_f)$ , it is obtained from the condition (8) that  $T_N(s_t) \leq T_N(y)$ ,  $I_N(y) \leq I_N(s_i)$  and  $F_N(y) \leq F_N(s_f)$ , which imply that  $y \in S_N^{s_t}, S_N^{s_i}, S_N^{s_f}$ . Thereby,  $S_N^{s_t}, S_N^{s_i}$  and  $S_N^{s_f}$  are SBE-filters of  $S$ .

*Example 11* Consider the SBE-algebra  $S$  in Example 1. Let

$$T_N(x) = \begin{cases} -0.83, & \text{if } x = 0, w, t \\ 0, & \text{otherwise,} \end{cases} \quad I_N(x) = \begin{cases} -0.79, & \text{if } x = u, v, 1 \\ -0.3, & \text{otherwise,} \end{cases}$$

$$F_N(x) = \begin{cases} -0.67, & \text{if } x = 1 \\ -0.17, & \text{otherwise,} \end{cases} \quad \text{and } s_t = t, s_i = u, s_f = v \in S.$$

Then the SBE-filters

$$S_N^{s_t} = S, S_N^{s_i} = \{u, v, 1\} \text{ and } S_N^{s_f} = S$$

of  $S$  satisfy the condition (8).

Also, let

$$S_N = \left\{ \frac{x}{(0, -1, -1)} : x = w, t, 1 \right\} \cup \left\{ \frac{x}{(-0.09, 0, -0.9)} : x = 0, u, v \right\}$$

be a neutrosophic  $\mathcal{N}$ -structure on  $S$  satisfying the conditions (8) and (9). Then the subsets  $S_N^{s_t} = S$ ,  $S_N^{s_i} = \{w, t, 1\}$  and  $S_N^{s_f} = \{w, t, 1\}$  of  $S$  are SBE-filters of  $S$ , where  $s_t = u$ ,  $s_i = w$  and  $s_f = 1$ .

#### 4 Conclusion

In the study, an implicative SBE-filter, a neutrosophic  $\mathcal{N}$ -subalgebra, a (implicative) neutrosophic  $\mathcal{N}$ -filter and a level set on neutrosophic  $\mathcal{N}$ -structures are introduced on SBE-algebras. Then we prove that the level set of a neutrosophic  $\mathcal{N}$ -subalgebra (a (implicative) neutrosophic  $\mathcal{N}$ -filter) of a SBE-algebra is its SBE-subalgebra (a (implicative) SBE-filter) and vice versa, and that the family of all neutrosophic  $\mathcal{N}$ -subalgebras of the algebraic structure forms a complete distributive modular lattice. We present the situations which  $\mathcal{N}$ -functions are constant. Additionally, the new statement equivalent to the definition of a neutrosophic  $\mathcal{N}$ -filter of a SBE-algebra is given. We restate a neutrosophic  $\mathcal{N}$ -filter of a SBE-algebra by means of upper sets on this algebra. It is illustrated that every implicative neutrosophic  $\mathcal{N}$ -filter of a SBE-algebra is the neutrosophic  $\mathcal{N}$ -filter but the inverse does not mostly hold, and that level set of a (implicative) neutrosophic  $\mathcal{N}$ -filter of a SBE-algebra is its (implicative) SBE-filter and vice versa. Infact, we reveal relationships between (implicative) neutrosophic  $\mathcal{N}$ -filters of two SBE-algebras by the hepl of an onto SBE-homomorphism. It is demonstrated that every neutrosophic  $\mathcal{N}$ -filter of a SBE-algebra is the neutrosophic  $\mathcal{N}$ -subalgebra but the inverse is not valid in general. Also, it is shown that a neutrosophic  $\mathcal{N}$ -filter of a self-distributive SBE-algebra is its implicative neutrosophic  $\mathcal{N}$ -filter. Besides, the subsets  $S_{T_N}$ ,  $S_{I_N}$  and  $S_{F_N}$  of a SBE-algebra are its (implicative) SBE-filters for the (implicative)  $\mathcal{N}$ -filter. At the end, it is proved that the subsets  $S_N^{s_t}$ ,  $S_N^{s_i}$  and  $S_N^{s_f}$  of a SBE-algebra defined by any elements  $s_t, s_i, s_f$  of the algebraic structure and  $\mathcal{N}$ -functions are its (implicative) SBE-filters, if a neutrosophic  $\mathcal{N}$ -structure on this algebraic structure is the (implicative) neutrosophic  $\mathcal{N}$ -filter.

In future works, we plan to study on plithogenic structures and relationships between neutrosophic  $\mathcal{N}$ -structures on some algebraic structures.

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### Conflict of interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

### Ethical approval

This article does not contain any studies with animals performed by any of the authors.

### Informed consent

Informed consent was obtained from all individual participants included in the study.

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