A Lecture On Some Complex Neutrosophic Functions

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Abstract:

In This lecture, we try to give the reader a new insight into some new functions by using neutrosophic complex field.

Neutrosophic Gamma function.

Definition

Let z be any Neutrosophic complex number, We define a neutrosophic Gamma function as follows:

$$\ldots\ldots (2.1)\Gamma(z+I) = \int_0^\infty t^{(z+I)-1}\, e^{-t}\, dt \,; Re(z+I) > 0$$

Properites

1). $\Gamma(1+I) = 1$.

Proof:

$$\Gamma(1+I) = \int_0^\infty t^0\, e^{-t}\, dt = [-e^{-t}]_0^\infty = 1$$

2). $\Gamma(z+I) = \frac{1}{z+I}\Gamma\big((z+I)+1\big)$.

By (1) we have:

$$, dv = t^{(z+I)-1}\, dt \Rightarrow v = \frac{t^{(z+I)}}{(z+I)}, \text{then:} u = e^{-t} \Rightarrow du = -e^{-t}$$

$$\Gamma(z+I) = \left[-\frac{t^{(z+I)}}{(z+I)}e^{-t}\right]_0^\infty + \int_0^\infty \frac{t^{(z+I)}}{(z+I)}e^{-t}\ dt = 0 + \frac{1}{z+I}\int_0^\infty t^{(z+I)}e^{-t}\ dt =$$
$$.\frac{1}{z+I}\Gamma\big((z+I)+1\big)$$

Then for $Re(z+I) > 0$ we have:

$$\ldots\ldots (2.2)\Gamma\big((z+I)+1\big) = (z+I)\Gamma(z+I)$$

$$\ldots\ldots (2.3)\Gamma(z+I) = \frac{\Gamma\big((z+I)+1\big)}{z+I}$$

Remark

Let $z = n\, ; n \in N$, then.

$$\Gamma(n+I) = \int_0^\infty t^{(n+I)-1} e^{-t}\ dt = \big((n+I)-1\big)!$$

Remark

Since $\Gamma\big((z+I)+1\big)$is convergent for $Re(z+I) > -1$ results from (2.2) and (2.3) that (2.3) represents the analytic continuation of the function $\Gamma(z+I)$ to the entire neutrosophic complex plane except for points

, this can be obtained gradually and conclude that these $(z+I) = 0, -1, -2, -3, \dots \dots$ points represent simple poles of the function $\Gamma(z+I)$. According $(2.s3)$ we have:

$$\Gamma(z+I) = \frac{1}{z+I}\Gamma\big((z+I)+1\big);\ Re(z+I) > -1$$

$$\Gamma(z+I) = \frac{1}{z+I}\frac{1}{(z+I)+1}\Gamma\big((z+I)+2\big);\ Re(z+I) > -2$$

$$\Gamma(z+I) = \frac{1}{z+I}\frac{1}{(z+I)+1}\frac{1}{(z+I)+2}\Gamma\big((z+I)+3\big);\ Re(z+I) > -3$$

$$\Gamma(z+I) = \ \dots \dots \dots \dots \dots \dots.$$

Theorem

The function $\Gamma(z+I)$in the neutrosophiccpmplex plane has simple poles in points $z + I = -n$,where

and the value of the residue in it is:$(z+I) = 0, -1, -2, -3, \dots \dots$

$$Res[\Gamma(z+I) - n] = \frac{(-1)^n}{n!}\ ; n = 0,1,2, \dots \dots$$

Proof:

Let function $\Gamma(z+I)$ by form:

$$\Gamma(z+I) = \int_0^\infty t^{(z+I)-1}\, e^{-t}\, dt = \int_0^1 t^{(z+I)-1}\, e^{-t}\, dt + \int_1^\infty t^{(z+I)-1}\, e^{-t}\, dt$$
$$= \Gamma_1(z+I) + \Gamma_2(z+I)$$

As $t^{(z+I)-1} = e^{[(z+I)-1]\ln(t)}$ and $\ln(t)$ is a real number where $t > 0$, then $\Gamma_2(z+I)$ is analytical function in the neutrosophic cpmplex plane, hence.

$$\Gamma_1(z+I)=\int_0^1 t^{(z+I)-1}\,e^{-t}\,dt;\ Re(z+I)>0$$

As $e^{-t}=\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}t^n$, this series is convergenuniformaly then sum and integral can be exchanged, then.

$$\Gamma_1(z+I)=\int_0^1 t^{(z+I)-1}\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}t^n\,dt=\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\int_0^1 t^{(z+I)-1+n}\,dt$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\frac{1}{(z+I)+n};Re(z+I)>0$$

Now we have.

$$\Gamma(z+I)=\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\frac{1}{(z+I)+n}+\Gamma_2(z+I)$$

The series$\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\frac{1}{(z+I)+n}$ converges absolutely and uniformaly in every finite arena of neutrosophic complex plane when some of its first limits are neglected, as there are simple poles at points $(z+I)=-n$, and thus the required is fulfilled.

Remark

For $(z+I)\neq 0,\mp 1,\mp 2,\dots$ ...be.

$$\Gamma(z+I)\Gamma\big(1-(z+I)\big)=\frac{\pi}{sin\pi(z+I)}\ \dots\dots\ (2.4)$$

Remark

By (4) we have.

, $\Gamma\left(-\frac{1}{2}\right)=-2\sqrt{\pi}$, $\Gamma\left(\frac{3}{2}\right)=\Gamma\left(\frac{1}{2}+1\right)=\frac{1}{2}\sqrt{\pi}$, $\Gamma\left(\frac{5}{2}\right)=\Gamma\left(\frac{3}{2}+1\right)=\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$ $\frac{3}{2}.\frac{1}{2}\sqrt{\pi}$, then for $n=1,2,\dots$ we have.

$$\Gamma\left((n+I)+\frac{1}{2}\right)=\Gamma\left(\frac{3}{2}+n\right)=\frac{(2n+1).(2n-1)\dots\dots.3.1}{2^{n+1}}\sqrt{\pi}$$

Remark

$$\Gamma(z+I)\Gamma\big(-(z+I)\big)=\frac{-\pi}{(z+I).sin\pi(z+I)};(z+I\neq 0,\mp 1,\mp 2,\dots\dots)\ \dots\dots\ (2.5)$$

$$\Gamma\left(\frac{1}{2}+(z+I)\right)\Gamma\left(\frac{1}{2}-(z+I)\right)$$
$$=\frac{\pi}{(z+I).\cos\pi(z+I)};\left(z+I\neq 0,\mp\frac{1}{2},\mp\frac{3}{2},\dots\dots\right)\ \dots\dots\ (2.6)$$

$$\Gamma\left(\frac{1-(z+I)}{2}\right)\Gamma\left(\frac{1+(z+I)}{2}\right)=\frac{-\pi}{\cos\frac{\pi(z+I)}{2}};(z+I\neq\mp1,\mp2,\dots\dots)\ \dots\dots\ (2.7)$$

$$\boldsymbol{\Gamma\big(n(z+I)\big)=(2\pi)^{\frac{(1-n)}{2}}n^{n(z+I)-\frac{1}{2}}\Gamma(z+I).\Gamma\left(\frac{1}{2}+(z+I)\right).\Gamma\left(\frac{3}{2}\right.}$$
$$\boldsymbol{\left.+(z+I)\right)\dots.\Gamma\left(\frac{(n-1)}{n}+(z+I)\right)\dots\dots\ (2.8)}$$

Neutrosophic Beta function.

Definition

Let p, q be any two Neutrosophic complex numbers, We define a neutrosophic Beta function as follows:

$$\dots\dots\ \mathrm{B}(p+I,q+I)=\int_0^1 t^{(p+I)-1}(1-t)^{(q+I)-1}\,dt\ ;Re(p+I)>0,Re(q+I)>0$$

Properites

1. $\mathbf{B}(\boldsymbol{p}+\boldsymbol{I},\boldsymbol{q}+\boldsymbol{I})=\mathbf{B}(\boldsymbol{q}+\boldsymbol{I},\boldsymbol{p}+\boldsymbol{I})$.
2. $\mathbf{B}(\boldsymbol{p}+\boldsymbol{I},\boldsymbol{q}+\boldsymbol{I})=\mathbf{B}\big((\boldsymbol{p}+\boldsymbol{I})+\mathbf{1},\boldsymbol{q}+\boldsymbol{I}\big)$.
3. $(\boldsymbol{p}+\boldsymbol{I}).\mathbf{B}(\boldsymbol{p}+\boldsymbol{I},(\boldsymbol{q}+\boldsymbol{I})+\mathbf{1})=(\boldsymbol{q}+\boldsymbol{I}).\mathbf{B}\big((\boldsymbol{p}+\boldsymbol{I})+\mathbf{1},\boldsymbol{q}+\boldsymbol{I}\big)$

Theorem

A neutrosophic Beta function can be written as follows**:**

$$\dots\dots\ \mathrm{B}(p+I,q+I)=\int_0^\infty\frac{t^{(p+I)-1}}{(1-t)^{(p+I)+(q+I)}}\,dt\ ;Re(p+I)>0,Re(q+I)>0$$

Proof.

let $\boldsymbol{t}=\frac{\boldsymbol{\tau}}{\mathbf{1}+\boldsymbol{\tau}}$ then.

$$\mathbf{B}(\boldsymbol{p}+\boldsymbol{I},\boldsymbol{q}+\boldsymbol{I})=\int_{\mathbf{0}}^{\mathbf{1}}\boldsymbol{t}^{(\boldsymbol{p}+\boldsymbol{I})-\mathbf{1}}(\mathbf{1}-\boldsymbol{t})^{(\boldsymbol{q}+\boldsymbol{I})-\mathbf{1}}\,\boldsymbol{dt}$$
$$=\int_{\mathbf{0}}^{\infty}\left(\frac{\boldsymbol{\tau}}{\mathbf{1}+\boldsymbol{\tau}}\right)^{(\boldsymbol{p}+\boldsymbol{I})-\mathbf{1}}\left(\mathbf{1}-\frac{\boldsymbol{\tau}}{\mathbf{1}+\boldsymbol{\tau}}\right)^{(\boldsymbol{q}+\boldsymbol{I})-\mathbf{1}}\frac{\boldsymbol{d\tau}}{(\mathbf{1}+\boldsymbol{\tau})^{\mathbf{2}}}$$

$$\mathbf{B}(\boldsymbol{p}+\boldsymbol{I},\boldsymbol{q}+\boldsymbol{I}) = \int_{\mathbf{0}}^{\infty} \frac{\boldsymbol{\tau}^{(\boldsymbol{p}+\boldsymbol{I})-\mathbf{1}}}{(\mathbf{1}+\boldsymbol{\tau})^{(\boldsymbol{p}+\boldsymbol{I})+(\boldsymbol{q}+\boldsymbol{I})}} \boldsymbol{d\tau}$$

Remark 3.4:

For $(\boldsymbol{q}+\boldsymbol{I}) = \mathbf{1} - (\boldsymbol{p}+\boldsymbol{I})$, we have.

$$\mathrm{B}(p+I,q+I) = \int_0^\infty \frac{t^{(p+I)-1}}{1+t}\, dt = \frac{\pi}{\sin \pi(p+I)} \;; 0 < Re(p+I) < 1$$

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