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# Regularity of Every Element to Function in the Type of Domination in Neutrosophic Graphs

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## Abstract

New setting is introduced to study k-number-dominating number and neutrosophic k-number-dominating number arising from k-number-dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. Minimum number of k-number-dominated vertices, is a number which is representative based on those vertices. Minimum neutrosophic number of k-number-dominated vertices corresponded to k-number-dominating set is called neutrosophic k-number-dominating number. Forming sets from k-number-dominated vertices to figure out different types of number of vertices in the sets from k-number-dominated sets in the terms of minimum number of vertices to get minimum number to assign to neutrosophic graphs is key type of approach to have these notions namely k-number-dominating number and neutrosophic k-number-dominating number arising from k-number-dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. Two numbers and one set are assigned to a neutrosophic graph, are obtained but now both settings lead to approach is on demand which is to compute and to find representatives of sets having smallest number of k-number-dominated vertices from different types of sets in the terms of minimum number and minimum neutrosophic number forming it to get minimum number to assign to a neutrosophic graph. Let  $NTG : (V, E, \sigma, \mu)$  be a neutrosophic graph. Then for given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum cardinality between all k-number-dominating sets is called k-number-dominating number and it's denoted by  $\mathcal{N}^k(NTG)$ ; for given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called neutrosophic k-number-dominating set. The minimum neutrosophic cardinality between all k-number-dominating sets is called neutrosophic k-number-dominating number and it's denoted by  $\mathcal{N}_n^k(NTG)$ . As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs,

complete-bipartite-neutrosophic graphs, complete-t-partite-neutrosophic graphs, and wheel-neutrosophic graphs. The clarifications are also presented in both sections “Setting of k-number-dominating number,” and “Setting of neutrosophic k-number-dominating number,” for introduced results and used classes. This approach facilitates identifying sets which form k-number-dominating number and neutrosophic k-number-dominating number arising from k-number-dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. In both settings, some classes of well-known neutrosophic graphs are studied. Some clarifications for each result and each definition are provided. The cardinality of set of k-number-dominated vertices and neutrosophic cardinality of set of k-number-dominated vertices corresponded to k-number-dominating set have eligibility to define k-number-dominating number and neutrosophic k-number-dominating number but different types of set of k-number-dominated vertices to define k-number-dominating sets. Some results get more frameworks and more perspectives about these definitions. The way in that, different types of set of k-number-dominated vertices in the terms of minimum number to assign to neutrosophic graphs, opens the way to do some approaches. These notions are applied into neutrosophic graphs as individuals but not family of them as drawbacks for these notions. Finding special neutrosophic graphs which are well-known, is an open way to pursue this study. Neutrosophic k-number-dominating notion is applied to different settings and classes of neutrosophic graphs. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.

**Keywords:** k-number-dominating Number, Neutrosophic k-number-dominating Number, Classes of Neutrosophic Graphs

**AMS Subject Classification:** 05C17, 05C22, 05E45

## 1 Background

Fuzzy set in **Ref. [22]** by Zadeh (1965), intuitionistic fuzzy sets in **Ref. [3]** by Atanassov (1986), a first step to a theory of the intuitionistic fuzzy graphs in **Ref. [17]** by Shannon and Atanassov (1994), a unifying field in logics neutrosophy: neutrosophic probability, set and logic, rehoboth in **Ref. [18]** by Smarandache (1998), single-valued neutrosophic sets in **Ref. [20]** by Wang et al. (2010), single-valued neutrosophic graphs in **Ref. [6]** by Broumi et al. (2016), operations on single-valued neutrosophic graphs in **Ref. [1]** by Akram and Shahzadi (2017), neutrosophic soft graphs in **Ref. [16]** by Shah and Hussain (2016), bounds on the average and minimum attendance in preference-based activity scheduling in **Ref. [2]** by Aronshtam and Ilani (2022), investigating the recoverable robust single machine scheduling problem under interval uncertainty in **Ref. [5]** by Bold and Goerigk (2022),  $k$ -domination and total  $k$ -domination numbers in catacondensed hexagonal systems in **Ref. [4]** by S. Bermudo et al. (2022), the minus total  $k$ -domination numbers in graphs in **Ref. [7]** by J. Dayap et al. (2022), weighted top- $k$  dominating queries on highly incomplete data in **Ref. [8]** by H.M.A. Fattah et al. (2022), a note on the  $k$ -tuple domination number of graphs in **Ref. [13]** by A.C. Martinez (2022), improved bounds on the  $k$ -tuple (Roman) domination number of a graph in **Ref. [14]** by A.A. Noor et al. (2022), a restart local search algorithm with relaxed configuration checking strategy for the minimum  $k$ -dominating set problem in **Ref. [15]** by L. Ruizhi et al. (2022), Zeroth-order general Randić index of trees with given distance  $k$ -domination number in **Ref. [19]** by T. Vetric et al. (2022), top- $k$  dominating queries on incomplete large dataset in graphs in **Ref. [21]** by J.M.T. Wu et al. (2012), dimension and coloring alongside domination in neutrosophic hypergraphs in **Ref. [10]** by Henry Garrett (2022), three types of

neutrosophic alliances based on connectedness and (strong) edges in **Ref. [12]** by Henry Garrett (2022), properties of SuperHyperGraph and neutrosophic SuperHyperGraph in **Ref. [11]** by Henry Garrett (2022), are studied. Also, some studies and researches about neutrosophic graphs, are proposed as a book in **Ref. [9]** by Henry Garrett (2022).

In this section, I use two subsections to illustrate a perspective about the background of this study.

## 1.1 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

**Question 1.1.** *Is it possible to use mixed versions of ideas concerning “k-number-dominating number”, “neutrosophic k-number-dominating number” and “Neutrosophic Graph” to define some notions which are applied to neutrosophic graphs?*

It's motivation to find notions to use in any classes of neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Having connection amid two vertices have key roles to assign k-number-dominating number and neutrosophic k-number-dominating number arising from k-number-dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. Thus they're used to define new ideas which conclude to the structure of k-number-dominating number and neutrosophic k-number-dominating number arising from k-number-dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. The concept of having smallest number of k-number-dominated vertices in the terms of crisp setting and in the terms of neutrosophic setting inspires us to study the behavior of all k-number-dominated vertices in the way that, some types of numbers, k-number-dominating number and neutrosophic k-number-dominating number arising from k-number-dominated vertices in neutrosophic graphs assigned to neutrosophic graphs, are the cases of study in the setting of individuals. In both settings, corresponded numbers conclude the discussion. Also, there are some avenues to extend these notions.

The framework of this study is as follows. In the beginning, I introduce basic definitions to clarify about preliminaries. In subsection “Preliminaries”, new notions of k-number-dominating number and neutrosophic k-number-dominating number arising from k-number-dominated vertices in neutrosophic graphs assigned to neutrosophic graphs, are highlighted, are introduced and are clarified as individuals. In section “Preliminaries”, minimum number of k-number-dominated vertices, is a number which is representative based on those vertices, have the key role in this way. General results are obtained and also, the results about the basic notions of k-number-dominating number and neutrosophic k-number-dominating number arising from k-number-dominated vertices in neutrosophic graphs assigned to neutrosophic graphs, are elicited. Some classes of neutrosophic graphs are studied in the terms of k-number-dominating number and neutrosophic k-number-dominating number arising from k-number-dominated vertices in neutrosophic graphs assigned to neutrosophic graphs, in section “Setting of k-number-dominating number,” as individuals. In section “Setting of k-number-dominating number,” k-number-dominating number is applied into individuals. As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs, complete-t-partite-neutrosophic graphs, and wheel-neutrosophic graphs. The clarifications are also presented in both sections “Setting of k-number-dominating number,” and “Setting of neutrosophic k-number-dominating number,” for introduced results and used classes. In section “Applications in Time Table and Scheduling”, two applications are posed for

quasi-complete and complete notions, namely complete-neutrosophic graphs and complete-t-partite-neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects and the mentioned models are considered as individual. In section “Open Problems”, some problems and questions for further studies are proposed. In section “Conclusion and Closing Remarks”, gentle discussion about results and applications is featured. In section “Conclusion and Closing Remarks”, a brief overview concerning advantages and limitations of this study alongside conclusions is formed.

## 1.2 Preliminaries

In this subsection, basic material which is used in this article, is presented. Also, new ideas and their clarifications are elicited.

Basic idea is about the model which is used. First definition introduces basic model.

**Definition 1.2.** (Graph).

$G = (V, E)$  is called a **graph** if  $V$  is a set of objects and  $E$  is a subset of  $V \times V$  ( $E$  is a set of 2-subsets of  $V$ ) where  $V$  is called **vertex set** and  $E$  is called **edge set**. Every two vertices have been corresponded to at most one edge.

Neutrosophic graph is the foundation of results in this paper which is defined as follows. Also, some related notions are demonstrated.

**Definition 1.3.** (Neutrosophic Graph And Its Special Case).

$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$  is called a **neutrosophic graph** if it's graph,  $\sigma_i : V \rightarrow [0, 1]$ , and  $\mu_i : E \rightarrow [0, 1]$ . We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every  $v_i v_j \in E$ ,

$$\mu(v_i v_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

(i) :  $\sigma$  is called **neutrosophic vertex set**.

(ii) :  $\mu$  is called **neutrosophic edge set**.

(iii) :  $|V|$  is called **order** of NTG and it's denoted by  $\mathcal{O}(NTG)$ .

(iv) :  $\sum_{v \in V} \sum_{i=1}^3 \sigma_i(v)$  is called **neutrosophic order** of NTG and it's denoted by  $\mathcal{O}_n(NTG)$ .

(v) :  $|E|$  is called **size** of NTG and it's denoted by  $\mathcal{S}(NTG)$ .

(vi) :  $\sum_{e \in E} \sum_{i=1}^3 \mu_i(e)$  is called **neutrosophic size** of NTG and it's denoted by  $\mathcal{S}_n(NTG)$ .

Some classes of well-known neutrosophic graphs are defined. These classes of neutrosophic graphs are used to form this study and the most results are about them.

**Definition 1.4.** Let  $NTG : (V, E, \sigma, \mu)$  be a neutrosophic graph. Then

(i) : a sequence of consecutive vertices  $P : x_0, x_1, \dots, x_{\mathcal{O}(NTG)}$  is called **path** where  $x_i x_{i+1} \in E$ ,  $i = 0, 1, \dots, \mathcal{O}(NTG) - 1$ ;

(ii) : **strength** of path  $P : x_0, x_1, \dots, x_{\mathcal{O}(NTG)}$  is  $\bigwedge_{i=0, \dots, \mathcal{O}(NTG)-1} \mu(x_i x_{i+1})$ ;

(iii) : **connectedness** amid vertices  $x_0$  and  $x_t$  is

$$\mu^\infty(x_0, x_t) = \bigvee_{P: x_0, x_1, \dots, x_t} \bigwedge_{i=0, \dots, t-1} \mu(x_i x_{i+1});$$

- (iv) : a sequence of consecutive vertices  $P : x_0, x_1, \dots, x_{\mathcal{O}(NTG)}, x_0$  is called **cycle** where  $x_i x_{i+1} \in E$ ,  $i = 0, 1, \dots, \mathcal{O}(NTG) - 1$ ,  $x_{\mathcal{O}(NTG)} x_0 \in E$  and there are two edges  $xy$  and  $uv$  such that  $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\dots,n-1} \mu(v_i v_{i+1})$ ;
- (v) : it's **t-partite** where  $V$  is partitioned to  $t$  parts,  $V_1^{s_1}, V_2^{s_2}, \dots, V_t^{s_t}$  and the edge  $xy$  implies  $x \in V_i^{s_i}$  and  $y \in V_j^{s_j}$  where  $i \neq j$ . If it's complete, then it's denoted by  $K_{\sigma_1, \sigma_2, \dots, \sigma_t}$  where  $\sigma_i$  is  $\sigma$  on  $V_i^{s_i}$  instead  $V$  which mean  $x \notin V_i$  induces  $\sigma_i(x) = 0$ . Also,  $|V_j^{s_j}| = s_j$ ;
- (vi) : t-partite is **complete bipartite** if  $t = 2$ , and it's denoted by  $K_{\sigma_1, \sigma_2}$ ;
- (vii) : complete bipartite is **star** if  $|V_1| = 1$ , and it's denoted by  $S_{1, \sigma_2}$ ;
- (viii) : a vertex in  $V$  is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by  $W_{1, \sigma_2}$ ;
- (ix) : it's **complete** where  $\forall uv \in V$ ,  $\mu(uv) = \sigma(u) \wedge \sigma(v)$ ;
- (x) : it's **strong** where  $\forall uv \in E$ ,  $\mu(uv) = \sigma(u) \wedge \sigma(v)$ .

To make them concrete, I bring preliminaries of this article in two upcoming definitions in other ways.

**Definition 1.5.** (Neutrosophic Graph And Its Special Case).

$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$  is called a **neutrosophic graph** if it's graph,  $\sigma_i : V \rightarrow [0, 1]$ , and  $\mu_i : E \rightarrow [0, 1]$ . We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every  $v_i v_j \in E$ ,

$$\mu(v_i v_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

$|V|$  is called **order** of NTG and it's denoted by  $\mathcal{O}(NTG)$ .  $\sum_{v \in V} \sigma(v)$  is called **neutrosophic order** of NTG and it's denoted by  $\mathcal{O}_n(NTG)$ .

**Definition 1.6.** Let  $NTG : (V, E, \sigma, \mu)$  be a neutrosophic graph. Then it's **complete** and denoted by  $CMT_\sigma$  if  $\forall x, y \in V, xy \in E$  and  $\mu(xy) = \sigma(x) \wedge \sigma(y)$ ; a sequence of consecutive vertices  $P : x_0, x_1, \dots, x_{\mathcal{O}(NTG)}$  is called **path** and it's denoted by  $PTH$  where  $x_i x_{i+1} \in E$ ,  $i = 0, 1, \dots, n - 1$ ; a sequence of consecutive vertices  $P : x_0, x_1, \dots, x_{\mathcal{O}(NTG)}, x_0$  is called **cycle** and denoted by  $CYC$  where  $x_i x_{i+1} \in E$ ,  $i = 0, 1, \dots, n - 1$ ,  $x_{\mathcal{O}(NTG)} x_0 \in E$  and there are two edges  $xy$  and  $uv$  such that  $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\dots,n-1} \mu(v_i v_{i+1})$ ; it's **t-partite** where  $V$  is partitioned to  $t$  parts,  $V_1^{s_1}, V_2^{s_2}, \dots, V_t^{s_t}$  and the edge  $xy$  implies  $x \in V_i^{s_i}$  and  $y \in V_j^{s_j}$  where  $i \neq j$ . If it's **complete**, then it's denoted by  $CMT_{\sigma_1, \sigma_2, \dots, \sigma_t}$  where  $\sigma_i$  is  $\sigma$  on  $V_i^{s_i}$  instead  $V$  which mean  $x \notin V_i$  induces  $\sigma_i(x) = 0$ . Also,  $|V_j^{s_j}| = s_j$ ; t-partite is **complete bipartite** if  $t = 2$ , and it's denoted by  $CMT_{\sigma_1, \sigma_2}$ ; complete bipartite is **star** if  $|V_1| = 1$ , and it's denoted by  $STR_{1, \sigma_2}$ ; a vertex in  $V$  is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by  $WHL_{1, \sigma_2}$ .

*Remark 1.7.* Using notations which is mixed with literatures, are reviewed.

1.  $NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ ,  $\mathcal{O}(NTG)$ , and  $\mathcal{O}_n(NTG)$ ;
2.  $CMT_\sigma, PTH, CYC, STR_{1, \sigma_2}, CMT_{\sigma_1, \sigma_2}, CMT_{\sigma_1, \sigma_2, \dots, \sigma_t}$ , and  $WHL_{1, \sigma_2}$ .

**Definition 1.8.** (k-number-dominating numbers).

Let  $NTG : (V, E, \sigma, \mu)$  be a neutrosophic graph. Then

- (i) for given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called **k-number-dominating set**. The minimum cardinality between all k-number-dominating sets is called **k-number-dominating number** and it's denoted by  $\mathcal{N}^k(NTG)$ ;
- (ii) for given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called **neutrosophic k-number-dominating set**. The minimum neutrosophic cardinality between all k-number-dominating sets is called **neutrosophic k-number-dominating number** and it's denoted by  $\mathcal{N}_n^k(NTG)$ .

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually. In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

**Example 1.9.** In Figure (1), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given neutrosophic vertex,  $s$ , there's an edge with other vertices;
- (ii) in the setting of complete, a vertex of dominating set corresponded to dominating number dominates as if it doesn't k-number-dominate so as dominating is different from k-number-dominating. Dominating number and k-number-dominating number are the same if  $k = 1$ ;
- (iii) all k-number-dominating sets corresponded to k-number-dominating number are

$$\begin{aligned} &\{n_1\}^1, \{n_2\}^1, \{n_3\}^1, \\ &\{n_4\}^1, \{n_1, n_2\}^2, \{n_1, n_3\}^2, \\ &\{n_1, n_4\}^2, \{n_2, n_3\}^2, \{n_2, n_4\}^2, \\ &\{n_3, n_4\}^2, \{n_1, n_2, n_3\}^3, \{n_1, n_2, n_4\}^3, \\ &\{n_2, n_3, n_4\}^3, \{n_1, n_2, n_3, n_4\}^4. \end{aligned}$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum cardinality between all k-number-dominating sets is called k-number-dominating number and it's denoted by  $\mathcal{N}^k(NTG) = k$ ,  $k = 1, 2, \dots, \mathcal{O}(NTG)$ ; and corresponded to

k-number-dominating sets are

$$\begin{aligned} &\{n_1\}^1, \{n_2\}^1, \{n_3\}^1, \\ &\{n_4\}^1, \{n_1, n_2\}^2, \{n_1, n_3\}^2, \\ &\{n_1, n_4\}^2, \{n_2, n_3\}^2, \{n_2, n_4\}^2, \\ &\{n_3, n_4\}^2, \{n_1, n_2, n_3\}^3, \{n_1, n_2, n_4\}^3, \\ &\{n_2, n_3, n_4\}^3, \{n_1, n_2, n_3, n_4\}^4; \end{aligned}$$

(iv) there are some k-number-dominating sets

$$\begin{aligned} &\{n_1\}^{1,2,3,4}, \{n_2\}^{1,2,3,4}, \{n_3\}^{1,2,3,4}, \\ &\{n_4\}^{1,2,3,4}, \{n_1, n_2\}^{2,3,4}, \{n_1, n_3\}^{2,3,4}, \\ &\{n_1, n_4\}^{2,3,4}, \{n_2, n_3\}^{2,3,4}, \{n_2, n_4\}^{2,3,4}, \\ &\{n_3, n_4\}^{2,3,4}, \{n_1, n_2, n_3\}^{3,4}, \{n_1, n_2, n_4\}^{3,4}, \\ &\{n_2, n_3, n_4\}^{3,4}, \{n_1, n_2, n_3, n_4\}^4. \end{aligned}$$

so as it's possible to have one of them as a set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is characteristic;

(v) there are some k-number-dominating sets

$$\begin{aligned} &\{n_1\}^1, \{n_2\}^1, \{n_3\}^1, \\ &\{n_4\}^1, \{n_1, n_2\}^2, \{n_1, n_3\}^2, \\ &\{n_1, n_4\}^2, \{n_2, n_3\}^2, \{n_2, n_4\}^2, \\ &\{n_3, n_4\}^2, \{n_1, n_2, n_3\}^3, \{n_1, n_2, n_4\}^3, \\ &\{n_2, n_3, n_4\}^3, \{n_1, n_2, n_3, n_4\}^4, \end{aligned}$$

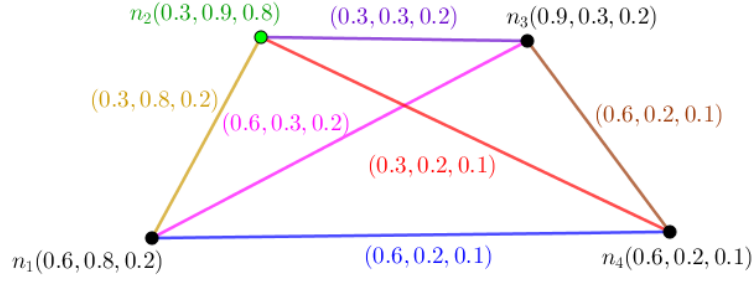
corresponded to k-number-dominating number as if there's one k-number-dominating set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is the determiner;

(vi) all k-number-dominating sets corresponded to k-number-dominating number are

$$\begin{aligned} &\{n_1\}^1, \{n_2\}^1, \{n_3\}^1, \\ &\{n_4\}^1, \{n_1, n_2\}^2, \{n_1, n_3\}^2, \\ &\{n_1, n_4\}^2, \{n_2, n_3\}^2, \{n_2, n_4\}^2, \\ &\{n_3, n_4\}^2, \{n_1, n_2, n_3\}^3, \{n_1, n_2, n_4\}^3, \\ &\{n_2, n_3, n_4\}^3, \{n_1, n_2, n_3, n_4\}^4. \end{aligned}$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum neutrosophic cardinality between all k-number-dominating sets is called neutrosophic k-number-dominating number and it's denoted by  $\mathcal{N}_n^k(NTG) = 0.9^1, 2.3^2, 3.9^3, 5.9^4$ ; and corresponded to k-number-dominating sets are

$$\begin{aligned} &\{n_4\}^1, \{n_4, n_3\}^2, \{n_4, n_3, n_1\}^3, \\ &\{n_1, n_2, n_3, n_4\}^4. \end{aligned}$$



**Figure 1.** A Neutrosophic Graph in the Viewpoint of its k-number-dominating number and its neutrosophic k-number-dominating number.

## 2 Setting of k-number-dominating number

In this section, I provide some results in the setting of k-number-dominating number. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, star-neutrosophic graph, bipartite-neutrosophic graph, t-partite-neutrosophic graph, and wheel-neutrosophic graph, are both of cases of study and classes which the results are about them.

**Proposition 2.1.** Let  $NTG : (V, E, \sigma, \mu)$  be a complete-neutrosophic graph. Then

$$\mathcal{N}^k(CMT_\sigma) = k, \quad k = 1, 2, 3, \dots, \mathcal{O}(CMT_\sigma).$$

Thus,

$$\mathcal{N}^1(CMT_\sigma) = 1, \mathcal{N}^2(CMT_\sigma) = 2, \dots, \mathcal{N}^{\mathcal{O}(CMT_\sigma)}(CMT_\sigma) = \mathcal{O}(CMT_\sigma).$$

*Proof.* Suppose  $CMT_\sigma : (V, E, \sigma, \mu)$  is a complete-neutrosophic graph. By  $CMT_\sigma : (V, E, \sigma, \mu)$  is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. In the setting of complete, a vertex of dominating set corresponded to dominating number dominates as if it doesn't k-number-dominate so as dominating is different from k-number-dominating. Dominating number and k-number-dominating number are the same if  $k = 1$ . All k-number-dominating sets corresponded to k-number-dominating number are

$$\begin{aligned} & \{n_1\}^1, \{n_2\}^1, \{n_3\}^1, \dots, \{n_{\mathcal{O}(CMT_\sigma)-2}\}^1, \{n_{\mathcal{O}(CMT_\sigma)-1}\}^1, \{n_{\mathcal{O}(CMT_\sigma)}\}^1, \\ & \{n_1, n_2\}^2, \{n_1, n_3\}^2, \{n_1, n_4\}^2, \dots, \{n_1, n_{\mathcal{O}(CMT_\sigma)-1}\}^2, \{n_1, n_{\mathcal{O}(CMT_\sigma)}\}^2, \dots, \\ & \{n_1, n_2, n_3\}^3, \{n_1, n_2, n_4\}^3, \dots, \{n_1, n_2, n_{\mathcal{O}(CMT_\sigma)-1}\}^3, \{n_1, n_2, n_{\mathcal{O}(CMT_\sigma)}\}^3, \dots, \\ & \dots \\ & \{n_1, n_2, \dots, n_{\mathcal{O}(CMT_\sigma)}\}^{\mathcal{O}(CMT_\sigma)}. \end{aligned}$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum cardinality between all k-number-dominating sets is called k-number-dominating number and it's denoted by

$$\mathcal{N}^k(CMT_\sigma) = k, \quad k = 1, 2, 3, \dots, \mathcal{O}(CMT_\sigma).$$

Thus,

$$\mathcal{N}^1(CMT_\sigma) = 1, \mathcal{N}^2(CMT_\sigma) = 2, \dots, \mathcal{N}^{\mathcal{O}(CMT_\sigma)}(CMT_\sigma) = \mathcal{O}(CMT_\sigma);$$

and corresponded to k-number-dominating sets are

$$\begin{aligned} & \{n_1\}^1, \{n_2\}^1, \{n_3\}^1, \dots, \{n_{\mathcal{O}(CMT_\sigma)-2}\}^1, \{n_{\mathcal{O}(CMT_\sigma)-1}\}^1, \{n_{\mathcal{O}(CMT_\sigma)}\}^1, \\ & \{n_1, n_2\}^2, \{n_1, n_3\}^2, \{n_1, n_4\}^2, \dots, \{n_1, n_{\mathcal{O}(CMT_\sigma)-1}\}^2, \{n_1, n_{\mathcal{O}(CMT_\sigma)}\}^2, \dots, \\ & \{n_1, n_2, n_3\}^3, \{n_1, n_2, n_4\}^3, \dots, \{n_1, n_2, n_{\mathcal{O}(CMT_\sigma)-1}\}^3, \{n_1, n_2, n_{\mathcal{O}(CMT_\sigma)}\}^3, \dots, \\ & \dots \\ & \{n_1, n_2, \dots, n_{\mathcal{O}(CMT_\sigma)}\}^{\mathcal{O}(CMT_\sigma)}. \end{aligned}$$

Thus

$$\mathcal{N}^k(CMT_\sigma) = k, \quad k = 1, 2, 3, \dots, \mathcal{O}(CMT_\sigma).$$

Thus,

$$\mathcal{N}^1(CMT_\sigma) = 1, \mathcal{N}^2(CMT_\sigma) = 2, \dots, \mathcal{N}^{\mathcal{O}(CMT_\sigma)}(CMT_\sigma) = \mathcal{O}(CMT_\sigma).$$

□ 214

**Proposition 2.2.** Let  $NTG : (V, E, \sigma, \mu)$  be a complete-neutrosophic graph. Then  $k$ -number-dominating number isn't equal to dominating number where  $k > 1$ .

**Proposition 2.3.** Let  $NTG : (V, E, \sigma, \mu)$  be a complete-neutrosophic graph. Then the number of  $k$ -number-dominating sets corresponded to  $k$ -number-dominating number is  $\mathcal{O}(CMT_\sigma)$  choose  $k$ .

**Proposition 2.4.** Let  $NTG : (V, E, \sigma, \mu)$  be a complete-neutrosophic graph. Then the number of  $k$ -number-dominating sets is  $\mathcal{O}(CMT_\sigma)$  choose  $k$  plus  $\mathcal{O}(CMT_\sigma)$  choose  $k - 1$  plus  $\mathcal{O}(CMT_\sigma)$  choose  $k - 2$  plus ... plus  $\mathcal{O}(CMT_\sigma)$  choose 1.

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

**Example 2.5.** In Figure (2), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given neutrosophic vertex,  $s$ , there's an edge with other vertices;
- (ii) in the setting of complete, a vertex of dominating set corresponded to dominating number dominates as if it doesn't  $k$ -number-dominate so as dominating is different from  $k$ -number-dominating. Dominating number and  $k$ -number-dominating number are the same if  $k = 1$ ;
- (iii) all  $k$ -number-dominating sets corresponded to  $k$ -number-dominating number are

$$\begin{aligned} & \{n_1\}^1, \{n_2\}^1, \{n_3\}^1, \\ & \{n_4\}^1, \{n_1, n_2\}^2, \{n_1, n_3\}^2, \\ & \{n_1, n_4\}^2, \{n_2, n_3\}^2, \{n_2, n_4\}^2, \\ & \{n_3, n_4\}^2, \{n_1, n_2, n_3\}^3, \{n_1, n_2, n_4\}^3, \\ & \{n_2, n_3, n_4\}^3, \{n_1, n_2, n_3, n_4\}^4. \end{aligned}$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$   $k$ -number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside

triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum cardinality between all k-number-dominating sets is called k-number-dominating number and it's denoted by  $\mathcal{N}^k(CMT_\sigma) = k$ ,  $k = 1, 2, \dots, \mathcal{O}(CMT_\sigma)$ ; and corresponded to k-number-dominating sets are

$$\begin{aligned} &\{n_1\}^1, \{n_2\}^1, \{n_3\}^1, \\ &\{n_4\}^1, \{n_1, n_2\}^2, \{n_1, n_3\}^2, \\ &\{n_1, n_4\}^2, \{n_2, n_3\}^2, \{n_2, n_4\}^2, \\ &\{n_3, n_4\}^2, \{n_1, n_2, n_3\}^3, \{n_1, n_2, n_4\}^3, \\ &\{n_2, n_3, n_4\}^3, \{n_1, n_2, n_3, n_4\}^4; \end{aligned}$$

(iv) there are some k-number-dominating sets

$$\begin{aligned} &\{n_1\}^{1,2,3,4}, \{n_2\}^{1,2,3,4}, \{n_3\}^{1,2,3,4}, \\ &\{n_4\}^{1,2,3,4}, \{n_1, n_2\}^{2,3,4}, \{n_1, n_3\}^{2,3,4}, \\ &\{n_1, n_4\}^{2,3,4}, \{n_2, n_3\}^{2,3,4}, \{n_2, n_4\}^{2,3,4}, \\ &\{n_3, n_4\}^{2,3,4}, \{n_1, n_2, n_3\}^{3,4}, \{n_1, n_2, n_4\}^{3,4}, \\ &\{n_2, n_3, n_4\}^{3,4}, \{n_1, n_2, n_3, n_4\}^4. \end{aligned}$$

so as it's possible to have one of them as a set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is characteristic;

(v) there are some k-number-dominating sets

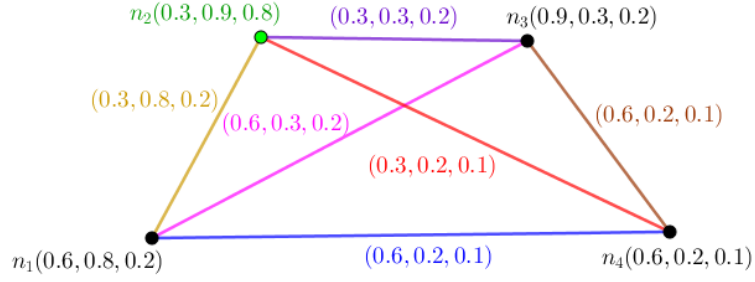
$$\begin{aligned} &\{n_1\}^1, \{n_2\}^1, \{n_3\}^1, \\ &\{n_4\}^1, \{n_1, n_2\}^2, \{n_1, n_3\}^2, \\ &\{n_1, n_4\}^2, \{n_2, n_3\}^2, \{n_2, n_4\}^2, \\ &\{n_3, n_4\}^2, \{n_1, n_2, n_3\}^3, \{n_1, n_2, n_4\}^3, \\ &\{n_2, n_3, n_4\}^3, \{n_1, n_2, n_3, n_4\}^4, \end{aligned}$$

corresponded to k-number-dominating number as if there's one k-number-dominating set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is the determiner;

(vi) all k-number-dominating sets corresponded to k-number-dominating number are

$$\begin{aligned} &\{n_1\}^1, \{n_2\}^1, \{n_3\}^1, \\ &\{n_4\}^1, \{n_1, n_2\}^2, \{n_1, n_3\}^2, \\ &\{n_1, n_4\}^2, \{n_2, n_3\}^2, \{n_2, n_4\}^2, \\ &\{n_3, n_4\}^2, \{n_1, n_2, n_3\}^3, \{n_1, n_2, n_4\}^3, \\ &\{n_2, n_3, n_4\}^3, \{n_1, n_2, n_3, n_4\}^4. \end{aligned}$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum neutrosophic cardinality



**Figure 2.** A Neutrosophic Graph in the Viewpoint of its k-number-dominating number and its neutrosophic k-number-dominating number.

between all k-number-dominating sets is called neutrosophic k-number-dominating number and it's denoted by  $\mathcal{N}_n^k(CMT_\sigma) = 0.9^1, 2.3^2, 3.9^3, 5.9^4$ ; and corresponded to k-number-dominating sets are

$$\{n_4\}^1, \{n_4, n_3\}^2, \{n_4, n_3, n_1\}^3, \\ \{n_1, n_2, n_3, n_4\}^4.$$

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

**Proposition 2.6.** Let  $NTG : (V, E, \sigma, \mu)$  be a path-neutrosophic graph. Then

$$\mathcal{N}^1(PTH) = \lfloor \frac{\mathcal{O}(PTH)}{3} \rfloor.$$

$$\mathcal{N}^2(PTH) = \lfloor \frac{\mathcal{O}(PTH)}{2} \rfloor.$$

$$k \leq 2.$$

*Proof.* Suppose  $PTH : (V, E, \sigma, \mu)$  is a path-neutrosophic graph. Let  $n_1, n_2, \dots, n_{\mathcal{O}(PTH)}$  be a path-neutrosophic graph. For given two vertices,  $x$  and  $y$ , there's one path from  $x$  to  $y$ . In the setting of path, a vertex of dominating set corresponded to dominating number dominates as if it doesn't k-number-dominate in the setting of dominating. All minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\{n_1, n_4, \dots\}_{\lfloor \frac{\mathcal{O}(PTH)}{3} \rfloor}^1, \{n_2, n_5, \dots\}_{\lfloor \frac{\mathcal{O}(PTH)}{3} \rfloor}^1, \{n_2, n_4, \dots\}_{\lfloor \frac{\mathcal{O}(PTH)}{3} \rfloor}^1, \dots \\ \{n_1, n_{\mathcal{O}(PTH)}, n_3, \dots\}_{\lfloor \frac{\mathcal{O}(PTH)}{2} \rfloor}^{1,2}, \{n_1, n_{\mathcal{O}(PTH)}, n_2, n_4, \dots\}_{\lfloor \frac{\mathcal{O}(PTH)}{2} \rfloor}^{1,2}, \dots$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum cardinality between all k-number-dominating sets is called k-number-dominating number and it's denoted by

$$\mathcal{N}^1(PTH) = \lfloor \frac{\mathcal{O}(PTH)}{3} \rfloor;$$

$$\mathcal{N}^2(PTH) = \lfloor \frac{\mathcal{O}(PTH)}{2} \rfloor;$$

$$k \leq 2;$$

and corresponded to k-number-dominating sets are

$$\{n_1, n_4, \dots\}_{\lfloor \frac{\mathcal{O}(PTH)}{3} \rfloor}^1, \{n_2, n_5, \dots\}_{\lfloor \frac{\mathcal{O}(PTH)}{3} \rfloor}^1, \{n_2, n_4, \dots\}_{\lfloor \frac{\mathcal{O}(PTH)}{3} \rfloor}^1, \dots \\ \{n_1, n_{\mathcal{O}(PTH)}, n_3, \dots\}_{\lfloor \frac{\mathcal{O}(PTH)}{2} \rfloor}^{1,2}, \{n_1, n_{\mathcal{O}(PTH)}, n_2, n_4, \dots\}_{\lfloor \frac{\mathcal{O}(PTH)}{2} \rfloor}^{1,2}, \dots$$

Thus

$$\mathcal{N}^1(PTH) = \lfloor \frac{\mathcal{O}(PTH)}{3} \rfloor.$$

$$\mathcal{N}^2(PTH) = \lfloor \frac{\mathcal{O}(PTH)}{2} \rfloor.$$

$$k \leq 2.$$

□ 271

**Proposition 2.7.** Let  $NTG : (V, E, \sigma, \mu)$  be a path-neutrosophic graph. If  $k$  isn't equal to one, then all leaves belong k-number-dominating sets corresponded to k-number-dominating number. 272  
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**Proposition 2.8.** Let  $NTG : (V, E, \sigma, \mu)$  be a path-neutrosophic graph. If at least one leaf doesn't belong k-number-dominating sets corresponded to k-number-dominating number, then  $k$  is equal to one. 275  
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**Example 2.9.** There are two sections for clarifications. 278

- (a) In Figure (3), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. 279  
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- (i) For given neutrosophic vertex,  $s$ , there's only one path with other vertices; 281
  - (ii) in the setting of path, a vertex of dominating set corresponded to dominating number dominates as if it doesn't k-number-dominate in the setting of dominating; 282  
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  - (iii) all minimal k-number-dominating sets corresponded to k-number-dominating number are 285  
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$$\{n_1, n_4\}^1, \{n_2, n_5\}^1, \{n_2, n_4\}^1, \\ \{n_1, n_5, n_3\}^{1,2}.$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum cardinality between all k-number-dominating sets is called k-number-dominating number and it's denoted by  $\mathcal{N}^k(PTH) = k + 1$ ,  $k = 1, 2$ ; and corresponded to k-number-dominating sets are 287  
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$$\{n_1, n_4\}^1, \{n_2, n_5\}^1, \{n_2, n_4\}^1, \\ \{n_1, n_5, n_3\}^{1,2};$$

(iv) there are thirteen k-number-dominating sets

$$\begin{aligned} &\{n_1, n_4\}^1, \{n_2, n_5\}^1, \{n_2, n_4\}^1, \\ &\{n_1, n_4, n_2\}^1, \{n_1, n_4, n_3\}^1, \{n_1, n_4, n_5\}^1, \\ &\{n_2, n_4, n_3\}^1, \{n_2, n_4, n_5\}^1, \{n_2, n_5, n_1\}^1, \\ &\{n_2, n_5, n_3\}^1, \{n_1, n_5, n_3, n_2\}^{1,2}, \{n_1, n_5, n_3, n_4\}^{1,2}, \\ &\{n_1, n_5, n_3, n_4, n_2\}^{1,2}, \end{aligned}$$

so as it's possible to have one of them as a set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is characteristic;

(v) there are four k-number-dominating sets

$$\begin{aligned} &\{n_1, n_4\}^1, \{n_2, n_5\}^1, \{n_2, n_4\}^1, \\ &\{n_1, n_5, n_3\}^{1,2}, \end{aligned}$$

corresponded to k-number-dominating number as if there's one k-number-dominating set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is the determiner;

(vi) all minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\begin{aligned} &\{n_1, n_4\}^1, \{n_2, n_5\}^1, \{n_2, n_4\}^1, \\ &\{n_1, n_5, n_3\}^{1,2}, \{n_1, n_5, n_4\}^{1,2}, \{n_1, n_5, n_2\}^{1,2}. \end{aligned}$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum neutrosophic cardinality between all k-number-dominating sets is called neutrosophic k-number-dominating number and it's denoted by  $\mathcal{N}_n^1(PTH) = 2.6$ ,  $\mathcal{N}_n^2(PTH) = 3.3$ ; and corresponded to k-number-dominating sets are

$$\{n_1, n_4\}^1, \{n_1, n_5, n_3\}^{1,2}.$$

(b) In Figure (4), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given neutrosophic vertex,  $s$ , there's only one path with other vertices;
- (ii) in the setting of path, a vertex of dominating set corresponded to dominating number dominates as if it doesn't k-number-dominate in the setting of dominating;
- (iii) all minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\{n_2, n_5\}^1, \{n_1, n_6, n_3, n_5\}^{1,2}, \{n_1, n_6, n_3, n_4\}^{1,2}, \{n_1, n_6, n_2, n_4\}^{1,2}.$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every

neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum cardinality between all k-number-dominating sets is called k-number-dominating number and it's denoted by  $\mathcal{N}^1(PTH) = 2$ ,  $\mathcal{N}^2(PTH) = 4$ ; and corresponded to k-number-dominating sets are

$$\{n_2, n_5\}^1, \{n_1, n_6, n_3, n_5\}^{1,2}, \{n_1, n_6, n_3, n_4\}^{1,2}, \{n_1, n_6, n_2, n_4\}^{1,2};$$

(iv) there are twenty k-number-dominating sets

$$\begin{aligned} &\{n_2, n_5\}^1, \{n_2, n_5, n_1\}^1, \{n_2, n_5, n_3\}^1, \\ &\{n_2, n_5, n_4\}^1, \{n_2, n_5, n_6\}^1, \{n_2, n_5, n_1, n_3\}^1, \\ &\{n_2, n_5, n_1, n_4\}^1, \{n_2, n_5, n_1, n_6\}^{1,2}, \{n_2, n_5, n_3, n_4\}^1, \\ &\{n_2, n_5, n_3, n_6\}^1, \{n_2, n_5, n_4, n_6\}^1, \{n_2, n_5, n_1, n_3, n_4\}^1, \\ &\{n_2, n_5, n_1, n_3, n_6\}^{1,2}, \{n_2, n_5, n_1, n_3, n_4\}^1, \{n_2, n_5, n_3, n_4, n_6\}^1, \\ &\{n_2, n_5, n_4, n_6, n_3, n_1\}^{1,2}, \{n_1, n_6, n_3, n_5, n_4\}^{1,2}, \{n_1, n_6, n_3, n_5\}^{1,2}, \\ &\{n_1, n_6, n_3, n_4\}^{1,2}, \{n_1, n_6, n_2, n_4\}^{1,2}, \end{aligned}$$

so as it's possible to have one of them as a set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is characteristic;

(v) there are four k-number-dominating sets

$$\{n_2, n_5\}^1, \{n_1, n_6, n_3, n_5\}^{1,2}, \{n_1, n_6, n_3, n_4\}^{1,2}, \{n_1, n_6, n_2, n_4\}^{1,2},$$

corresponded to k-number-dominating number as if there's one k-number-dominating set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is the determiner;

(vi) all minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\{n_2, n_5\}^1, \{n_1, n_6, n_3, n_5\}^{1,2}, \{n_1, n_6, n_3, n_4\}^{1,2}, \{n_1, n_6, n_2, n_4\}^{1,2}.$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum neutrosophic cardinality between all k-number-dominating sets is called neutrosophic k-number-dominating number and it's denoted by  $\mathcal{N}_n^1(PTH) = 3.8, \mathcal{N}_n^2(PTH) = 2.2$ ; and corresponded to k-number-dominating sets are

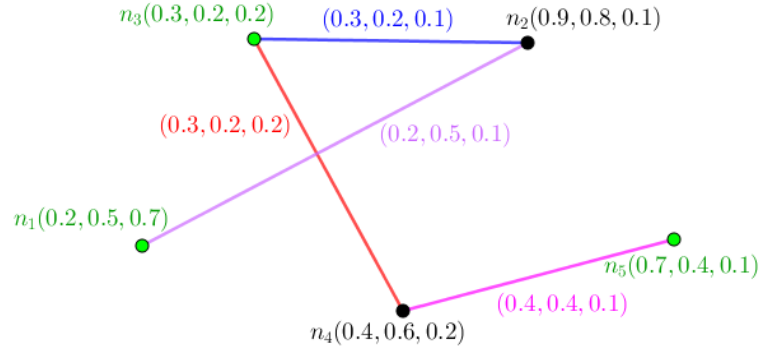
$$\{n_2, n_5\}^1, \{n_1, n_6, n_3, n_4\}^{1,2}.$$

**Proposition 2.10.** Let  $NTG : (V, E, \sigma, \mu)$  be a cycle-neutrosophic graph where  $\mathcal{O}(CYC) \geq 3$ . Then

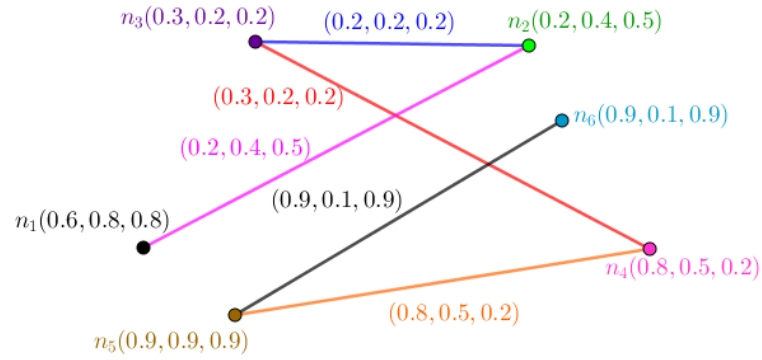
$$\mathcal{N}^1(CYC) = \lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor.$$

$$\mathcal{N}^2(CYC) = \lfloor \frac{\mathcal{O}(CYC)}{2} \rfloor.$$

$$k \leq 2.$$



**Figure 3.** A Neutrosophic Graph in the Viewpoint of its k-number-dominating number and its neutrosophic k-number-dominating number.



**Figure 4.** A Neutrosophic Graph in the Viewpoint of its k-number-dominating number and its neutrosophic k-number-dominating number.

*Proof.* Suppose  $CYC : (V, E, \sigma, \mu)$  is a cycle-neutrosophic graph. For given two vertices,  $x$  and  $y$ , there are only two paths with distinct edges from  $x$  to  $y$ . Let

$$n_1, n_2, \dots, n_{\mathcal{O}(CYC)-1}, n_{\mathcal{O}(CYC)}, n_1$$

be a cycle-neutrosophic graph  $CYC : (V, E, \sigma, \mu)$ . In the setting of cycle, a vertex of dominating set corresponded to dominating number dominates as if it doesn't k-number-dominate in the setting of dominating. All minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\begin{aligned} & \{n_1, n_4, \dots\}_{\lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor}^1, \{n_2, n_5, \dots\}_{\lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor}^1, \{n_2, n_4, \dots\}_{\lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor}^1, \dots \\ & \{n_1, n_{\mathcal{O}(CYC)}, n_3, \dots\}_{\lfloor \frac{\mathcal{O}(CYC)}{2} \rfloor}^{1,2}, \{n_1, n_{\mathcal{O}(CYC)}, n_2, n_4, \dots\}_{\lfloor \frac{\mathcal{O}(CYC)}{2} \rfloor}^{1,2}, \dots \end{aligned}$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum cardinality between all k-number-dominating sets is called k-number-dominating number and it's denoted by

$$\mathcal{N}^1(CYC) = \lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor;$$

$$\mathcal{N}^2(CYC) = \lfloor \frac{\mathcal{O}(CYC)}{2} \rfloor;$$

$$k \leq 2;$$

and corresponded to k-number-dominating sets are

$$\begin{aligned} & \{n_1, n_4, \dots\}_{\lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor}^1, \{n_2, n_5, \dots\}_{\lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor}^1, \{n_2, n_4, \dots\}_{\lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor}^1, \dots \\ & \{n_1, n_{\mathcal{O}(CYC)}, n_3, \dots\}_{\lfloor \frac{\mathcal{O}(CYC)}{2} \rfloor}^{1,2}, \{n_1, n_{\mathcal{O}(CYC)}, n_2, n_4, \dots\}_{\lfloor \frac{\mathcal{O}(CYC)}{2} \rfloor}^{1,2}, \dots \end{aligned}$$

Thus

$$\mathcal{N}^1(CYC) = \lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor.$$

$$\mathcal{N}^2(CYC) = \lfloor \frac{\mathcal{O}(CYC)}{2} \rfloor.$$

$$k \leq 2.$$

□ 358

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

**Example 2.11.** There are two sections for clarifications.

(a) In Figure (5), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given neutrosophic vertex,  $s$ , there's only one path with other vertices;

- (ii) in the setting of cycle, a vertex of dominating set corresponded to dominating number dominates as if it doesn't k-number-dominate in the setting of dominating;
- (iii) all minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\{n_2, n_5\}^1, \{n_1, n_4\}^1, \{n_3, n_6\}^1, \\ \{n_1, n_3, n_5\}^{1,2}, \{n_2, n_4, n_6\}^{1,2}.$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum cardinality between all k-number-dominating sets is called k-number-dominating number and it's denoted by  $\mathcal{N}^k(CYC) = k + 1$ ,  $k = 1, 2$ ; and corresponded to k-number-dominating sets are

$$\{n_2, n_5\}^1, \{n_1, n_4\}^1, \{n_3, n_6\}^1, \\ \{n_1, n_3, n_5\}^{1,2}, \{n_2, n_4, n_6\}^{1,2};$$

- (iv) there are some k-number-dominating sets

$$\{n_2, n_5\}^1, \{n_2, n_5, n_1\}^1, \{n_2, n_5, n_3\}^1, \\ \{n_2, n_5, n_4\}^1, \{n_2, n_5, n_6\}^1, \{n_2, n_5, n_1, n_3\}^{1,2}, \\ \{n_2, n_5, n_1, n_4\}^{1,2}, \{n_2, n_5, n_1, n_6\}^1, \{n_2, n_5, n_3, n_4\}^1, \\ \{n_2, n_5, n_3, n_6\}^{1,2}, \{n_2, n_5, n_4, n_6\}^{1,2}, \{n_2, n_5, n_1, n_3, n_4\}^{1,2}, \\ \{n_2, n_5, n_1, n_3, n_6\}^{1,2}, \{n_2, n_5, n_1, n_3, n_4\}^{1,2}, \{n_2, n_5, n_3, n_4, n_6\}^{1,2}, \\ \{n_2, n_5, n_4, n_6, n_3, n_1\}^{1,2}, \dots,$$

so as it's possible to have one of them as a set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is characteristic;

- (v) there are five k-number-dominating sets

$$\{n_2, n_5\}^1, \{n_1, n_4\}^1, \{n_3, n_6\}^1, \\ \{n_1, n_3, n_5\}^{1,2}, \{n_2, n_4, n_6\}^{1,2},$$

corresponded to k-number-dominating number as if there's one k-number-dominating set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is the determiner;

- (vi) all minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\{n_2, n_5\}^1, \{n_1, n_4\}^1, \{n_3, n_6\}^1, \\ \{n_1, n_3, n_5\}^{1,2}, \{n_2, n_4, n_6\}^{1,2}.$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices

$s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum neutrosophic cardinality between all k-number-dominating sets is called neutrosophic k-number-dominating number and it's denoted by  $\mathcal{N}_n^1(CYC) = 2.2$ ,  $\mathcal{N}_n^2(CYC) = 3.2$ ; and corresponded to k-number-dominating sets are

$$\{n_1, n_4\}^1, \{n_1, n_3, n_5\}^{1,2}.$$

(b) In Figure (6), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given neutrosophic vertex,  $s$ , there's only one path with other vertices;
- (ii) in the setting of cycle, a vertex of dominating set corresponded to dominating number dominates as if it doesn't k-number-dominate in the setting of dominating;
- (iii) all minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\begin{aligned} &\{n_1, n_4\}^1, \{n_1, n_3\}^1, \{n_2, n_4\}^1, \\ &\{n_2, n_5\}^1, \{n_3, n_5\}^1, \{n_1, n_3, n_5\}^{1,2}, \\ &\{n_1, n_2, n_4\}^{1,2}, \{n_2, n_3, n_5\}^{1,2}, \{n_3, n_4, n_1\}^{1,2}, \\ &\{n_2, n_4, n_5\}^{1,2}. \end{aligned}$$

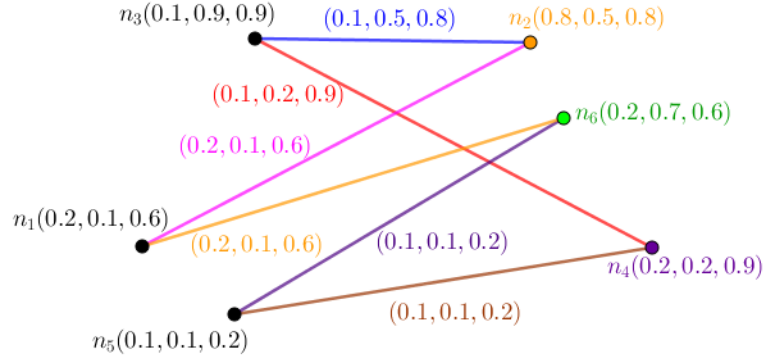
For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum cardinality between all k-number-dominating sets is called k-number-dominating number and it's denoted by  $\mathcal{N}^k(CYC) = k + 1$ ,  $k = 1, 2$ ; and corresponded to k-number-dominating sets are

$$\begin{aligned} &\{n_1, n_4\}^1, \{n_1, n_3\}^1, \{n_2, n_4\}^1, \\ &\{n_2, n_5\}^1, \{n_3, n_5\}^1, \{n_1, n_3, n_5\}^{1,2}, \\ &\{n_1, n_2, n_4\}^{1,2}, \{n_2, n_3, n_5\}^{1,2}, \{n_3, n_4, n_1\}^{1,2}, \\ &\{n_2, n_4, n_5\}^{1,2}; \end{aligned}$$

(iv) there are thirteen k-number-dominating sets

$$\begin{aligned} &\{n_1, n_4\}^1, \{n_2, n_5\}^1, \{n_2, n_4\}^1, \\ &\{n_1, n_4, n_2\}^1, \{n_1, n_4, n_3\}^1, \{n_1, n_4, n_5\}^1, \\ &\{n_2, n_4, n_3\}^1, \{n_2, n_4, n_5\}^1, \{n_2, n_5, n_1\}^1, \\ &\{n_2, n_5, n_3\}^1, \{n_1, n_5, n_3, n_2\}^{1,2}, \{n_1, n_5, n_3, n_4\}^{1,2}, \\ &\{n_1, n_5, n_3, n_4, n_2\}^{1,2}, \end{aligned}$$

so as it's possible to have one of them as a set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is characteristic;



**Figure 5.** A Neutrosophic Graph in the Viewpoint of its k-number-dominating number and its neutrosophic k-number-dominating number.

(v) there are twenty-three k-number-dominating sets

$$\begin{aligned} &\{n_1, n_4\}^1, \{n_2, n_5\}^1, \{n_2, n_4\}^1, \\ &\{n_1, n_4, n_2\}^1, \{n_1, n_4, n_3\}^1, \{n_1, n_4, n_5\}^1, \\ &\{n_2, n_4, n_3\}^1, \{n_2, n_4, n_5\}^1, \{n_2, n_5, n_1\}^1, \\ &\{n_2, n_5, n_3\}^1, \{n_1, n_5, n_3, n_2\}^{1,2}, \{n_1, n_5, n_3, n_4\}^{1,2}, \\ &\{n_1, n_5, n_3, n_4, n_2\}^{1,2}, \{n_1, n_2, n_3\}^1, \{n_2, n_3, n_4\}^1, \\ &\{n_3, n_4, n_5\}^1, \{n_5, n_1, n_2\}^1, \{n_1, n_2, n_3, n_4\}^{1,2}, \\ &\{n_2, n_3, n_4, n_5\}^{1,2}, \{n_3, n_4, n_5, n_1\}^{1,2}, \{n_4, n_5, n_1, n_2\}^{1,2}, \\ &\{n_5, n_1, n_2, n_3\}^{1,2}, \{n_1, n_2, n_3, n_4, n_5\}^{1,2}, \end{aligned}$$

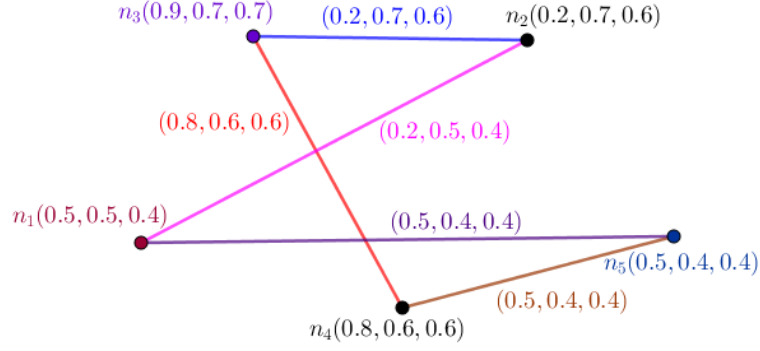
corresponded to k-number-dominating number as if there's one  
k-number-dominating set corresponded to neutrosophic k-number-dominating  
number so as neutrosophic cardinality is the determiner;

(vi) all minimal k-number-dominating sets corresponded to k-number-dominating  
number are

$$\begin{aligned} &\{n_1, n_4\}^1, \{n_1, n_3\}^1, \{n_2, n_4\}^1, \\ &\{n_2, n_5\}^1, \{n_3, n_5\}^1, \{n_1, n_3, n_5\}^{1,2}, \\ &\{n_1, n_2, n_4\}^{1,2}, \{n_2, n_3, n_5\}^{1,2}, \{n_3, n_4, n_1\}^{1,2}, \\ &\{n_2, n_4, n_5\}^{1,2}. \end{aligned}$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$   
k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex  
alongside triple pair of its values is called neutrosophic vertex.]. If for every  
neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  
 $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set  
of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum  
neutrosophic cardinality between all k-number-dominating sets is called  
neutrosophic k-number-dominating number and it's denoted by  
 $\mathcal{N}_n^1(CYC) = 2.8$ ,  $\mathcal{N}_n^2(CYC) = 4.8$ ; and corresponded to  
k-number-dominating sets are

$$\{n_2, n_5\}^1, \{n_2, n_4, n_5\}^{1,2}.$$



**Figure 6.** A Neutrosophic Graph in the Viewpoint of its k-number-dominating number and its neutrosophic k-number-dominating number.

**Proposition 2.12.** Let  $NTG : (V, E, \sigma, \mu)$  be a star-neutrosophic graph with center  $c$ . Then

$$\mathcal{N}^1(STR_{1,\sigma_2}) = 1.$$

$$\mathcal{N}^{\mathcal{O}(STR_{1,\sigma_2})-1}(STR_{1,\sigma_2}) = \mathcal{O}(STR_{1,\sigma_2}) - 1.$$

$$k = 1, \mathcal{O}(STR_{1,\sigma_2}) - 1.$$

*Proof.* Suppose  $STR_{1,\sigma_2} : (V, E, \sigma, \mu)$  is a star-neutrosophic graph. An edge always has center,  $c$ , as one of its endpoints where  $n_{\mathcal{O}(STR_{1,\sigma_2})} = c$ . All paths have one as their lengths, forever. In the setting of star, a vertex of dominating set corresponded to dominating number dominates as if it doesn't k-number-dominate in the setting of dominating. All minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\{n_{\mathcal{O}(STR_{1,\sigma_2})}\}^1, \{n_1, n_2, n_3, \dots, n_{\mathcal{O}(STR_{1,\sigma_2})-1}\}^{1, \mathcal{O}(STR_{1,\sigma_2})-1}.$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum cardinality between all k-number-dominating sets is called k-number-dominating number and it's denoted by

$$\mathcal{N}^1(STR_{1,\sigma_2}) = 1;$$

$$\mathcal{N}^{\mathcal{O}(STR_{1,\sigma_2})-1}(STR_{1,\sigma_2}) = \mathcal{O}(STR_{1,\sigma_2}) - 1;$$

$$k = 1, \mathcal{O}(STR_{1,\sigma_2}) - 1;$$

and corresponded to k-number-dominating sets are

$$\{n_{\mathcal{O}(STR_{1,\sigma_2})}\}^1, \{n_1, n_2, n_3, \dots, n_{\mathcal{O}(STR_{1,\sigma_2})-1}\}^{1, \mathcal{O}(STR_{1,\sigma_2})-1}.$$

Thus

$$\mathcal{N}^1(STR_{1,\sigma_2}) = 1.$$

$$\mathcal{N}^{\mathcal{O}(STR_{1,\sigma_2})-1}(STR_{1,\sigma_2}) = \mathcal{O}(STR_{1,\sigma_2}) - 1.$$

$$k = 1, \mathcal{O}(STR_{1,\sigma_2}) - 1.$$

□ 447

**Proposition 2.13.** Let  $NTG : (V, E, \sigma, \mu)$  be a star-neutrosophic graph. Then  $k$ -number-dominating number isn't equal to dominating number where  $k \neq 1$ .

**Proposition 2.14.** Let  $NTG : (V, E, \sigma, \mu)$  be a star-neutrosophic graph with center  $c$ . Then

- (i) the number of  $k$ -number-dominating sets is  $2^{\mathcal{O}(STR_{1,\sigma_2})-1}$  where  $k = 1$ ;
- (ii) the number of  $k$ -number-dominating sets is one where  $k \neq 1$ .

**Proposition 2.15.** Let  $NTG : (V, E, \sigma, \mu)$  be a star-neutrosophic graph with center  $c$ . Then

- (i) the number of  $k$ -number-dominating sets corresponded to  $k$ -number-dominating number is one where  $k = 1$ ;
- (ii) the number of  $k$ -number-dominating sets corresponded to  $k$ -number-dominating number is one where  $k \neq 1$ .

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

**Example 2.16.** There is one section for clarifications. In Figure (7), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given two neutrosophic vertices,  $s$  and  $n_1$ , there's only one path, precisely one edge between them and there's no path despite them;
- (ii) in the setting of star, a vertex of dominating set corresponded to dominating number dominates as if it doesn't  $k$ -number-dominate in the setting of dominating;
- (iii) all minimal  $k$ -number-dominating sets corresponded to  $k$ -number-dominating number are

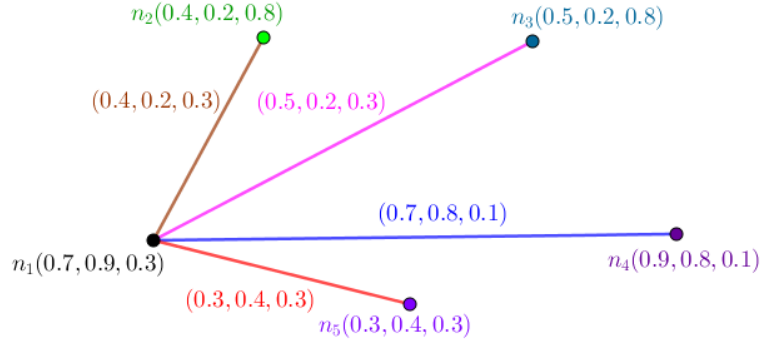
$$\{n_1\}^1, \{n_2, n_3, n_4, n_5\}^{1,4}.$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$   $k$ -number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$   $k$ -number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called  $k$ -number-dominating set. The minimum cardinality between all  $k$ -number-dominating sets is called  $k$ -number-dominating number and it's denoted by  $\mathcal{N}^1(STR_{1,\sigma_2}) = 1$ ,  $\mathcal{N}^4(STR_{1,\sigma_2}) = 4$ ; and corresponded to  $k$ -number-dominating sets are

$$\{n_1\}^1, \{n_2, n_3, n_4, n_5\}^{1,4};$$

- (iv) there are seventeen  $k$ -number-dominating sets

$$\begin{aligned} &\{n_1\}^1, \{n_1, n_2\}^1, \{n_1, n_3\}^1, \\ &\{n_1, n_4\}^1, \{n_1, n_5\}^1, \{n_1, n_2, n_3\}^1, \\ &\{n_1, n_2, n_4\}^1, \{n_1, n_2, n_5\}^1, \{n_2, n_3, n_4\}^1, \\ &\{n_2, n_3, n_5\}^1, \{n_3, n_4, n_5\}^1, \{n_2, n_3, n_4, n_5\}^1, \\ &\{n_1, n_3, n_4, n_5\}^1, \{n_1, n_2, n_4, n_5\}^1, \{n_1, n_2, n_3, n_5\}^1, \\ &\{n_1, n_2, n_3, n_4\}^1, \{n_2, n_3, n_4, n_5\}^{1,4}, \end{aligned}$$



**Figure 7.** A Neutrosophic Graph in the Viewpoint of its k-number-dominating number and its neutrosophic k-number-dominating number.

so as it's possible to have one of them as a set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is characteristic;

(v) there are two k-number-dominating sets

$$\{n_1\}^1, \{n_2, n_3, n_4, n_5\}^{1,4},$$

corresponded to k-number-dominating number as if there's one k-number-dominating set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is the determiner;

(vi) all minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\{n_1\}^1, \{n_2, n_3, n_4, n_5\}^{1,4}.$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum neutrosophic cardinality between all k-number-dominating sets is called neutrosophic k-number-dominating number and it's denoted by  $\mathcal{N}_n^1(STR_{1,\sigma_2}) = 1.9$ ,  $\mathcal{N}_n^4(STR_{1,\sigma_2}) = 5.7$ ; and corresponded to k-number-dominating sets are

$$\{n_1\}^1, \{n_2, n_3, n_4, n_5\}^{1,4}.$$

**Proposition 2.17.** Let  $NTG : (V, E, \sigma, \mu)$  be a complete-bipartite-neutrosophic graph which isn't star-neutrosophic graph which means  $|V_1|, |V_2| \geq 2$ . Then

$$\mathcal{N}^k(CMC_{\sigma_1, \sigma_2}) = 2k$$

where  $k = 1, 2, \dots, \min\{|V_1|, |V_2|\}$ .

*Proof.* Suppose  $CMC_{\sigma_1, \sigma_2} : (V, E, \sigma, \mu)$  is a complete-bipartite-neutrosophic graph. Every vertex in a part and another vertex in opposite part k-number-dominates any given vertex. Assume same parity for same partition of vertex set which means  $V_1$  has odd indexes and  $V_2$  has even indexes. In the setting of complete-bipartite, a vertex of dominating set corresponded to dominating number dominates as if it doesn't

k-number-dominate so as dominating is different from k-number-dominating. All minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\begin{aligned}
& \{n_1, n_2\}^1, \{n_1, n_4\}^1, \{n_1, n_6\}^1, \dots, \{n_1, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})}(n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1})\}^1, \\
& \{n_2, n_3\}^1, \{n_2, n_5\}^1, \{n_2, n_7\}^1, \dots, \{n_2, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1}(n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})})\}^1, \\
& \{n_3, n_4\}^1, \{n_3, n_6\}^1, \{n_3, n_8\}^1, \dots, \{n_3, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})}(n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1})\}^1, \\
& \dots \\
& \{n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})}\}^1, \\
& \dots, \\
& \{n_1, n_3, n_2, n_4\}^{1,2}, \dots, \{n_1, n_3, n_2, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})}(n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1})\}^{1,2}, \\
& \{n_1, n_3, n_6, n_8\}^{1,2}, \dots, \{n_1, n_3, n_6, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})}(n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1})\}^{1,2}, \\
& \{n_1, n_3, n_8, n_{10}\}^{1,2}, \dots, \{n_1, n_3, n_8, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})}(n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1})\}^{1,2}, \\
& \dots, \\
& \{n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-3}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-2}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})}\}^{1,2}, \\
& \dots, \\
& \{n_{i+1}, n_{i+2}, n_{i+3}, \dots, n_{i+2 \min\{|V_1|, |V_2|\}}\}^{1,2, \dots, \min\{|V_1|, |V_2|\}}.
\end{aligned}$$

For given vertex  $n$ , if  $s_1 n, s_2 n, \dots, s_k n \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum cardinality between all k-number-dominating sets is called k-number-dominating number and it's denoted by

$$\mathcal{N}^k(CMC_{\sigma_1, \sigma_2}) = 2k$$

where  $k = 1, 2, \dots, \min\{|V_1|, |V_2|\}$ ; and corresponded to k-number-dominating sets are

$$\begin{aligned}
& \{n_1, n_2\}^1, \{n_1, n_4\}^1, \{n_1, n_6\}^1, \dots, \{n_1, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})}(n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1})\}^1, \\
& \{n_2, n_3\}^1, \{n_2, n_5\}^1, \{n_2, n_7\}^1, \dots, \{n_2, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1}(n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})})\}^1, \\
& \{n_3, n_4\}^1, \{n_3, n_6\}^1, \{n_3, n_8\}^1, \dots, \{n_3, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})}(n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1})\}^1, \\
& \dots \\
& \{n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})}\}^1, \\
& \dots, \\
& \{n_1, n_3, n_2, n_4\}^{1,2}, \dots, \{n_1, n_3, n_2, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})}(n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1})\}^{1,2}, \\
& \{n_1, n_3, n_6, n_8\}^{1,2}, \dots, \{n_1, n_3, n_6, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})}(n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1})\}^{1,2}, \\
& \{n_1, n_3, n_8, n_{10}\}^{1,2}, \dots, \{n_1, n_3, n_8, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})}(n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1})\}^{1,2}, \\
& \dots, \\
& \{n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-3}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-2}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})}\}^{1,2}, \\
& \dots, \\
& \{n_{i+1}, n_{i+2}, n_{i+3}, \dots, n_{i+2 \min\{|V_1|, |V_2|\}}\}^{1,2, \dots, \min\{|V_1|, |V_2|\}}.
\end{aligned}$$

Thus

$$\mathcal{N}^k(CMC_{\sigma_1, \sigma_2}) = 2k$$

where  $k = 1, 2, \dots, \min\{|V_1|, |V_2|\}$ .

□ 510

**Proposition 2.18.** Let  $NTG : (V, E, \sigma, \mu)$  be a complete-bipartite-neutrosophic graph. Then  $k$ -number-dominating number isn't equal to dominating number where  $k \neq 1$ .

**Proposition 2.19.** Let  $NTG : (V, E, \sigma, \mu)$  be a complete-bipartite-neutrosophic graph. Then the number of  $k$ -number-dominating sets is multiplying  $2^{|V_1|+|V_2|-2k}$  by multiplying  $|V_1|$  choose  $k$  by  $|V_2|$  choose  $k$ .

**Proposition 2.20.** Let  $NTG : (V, E, \sigma, \mu)$  be a complete-bipartite-neutrosophic graph. Then the number of  $k$ -number-dominating sets corresponded to  $k$ -number-dominating number is multiplying  $|V_1|$  choose  $k$  by  $|V_2|$  choose  $k$ .

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

**Example 2.21.** There is one section for clarifications. In Figure (8), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given two neutrosophic vertices,  $n$  and  $n'$ , there is either one path with length one or one path with length two between them;
- (ii) in the setting of complete-bipartite, a vertex of dominating set corresponded to dominating number dominates as if it doesn't  $k$ -number-dominate so as dominating is different from  $k$ -number-dominating;
- (iii) all minimal  $k$ -number-dominating sets corresponded to  $k$ -number-dominating number are

$$\{n_1, n_2\}^1, \{n_1, n_3\}^1, \{n_4, n_2\}^1, \\ \{n_4, n_3\}^1, \{n_1, n_2, n_3, n_4\}^{1,2}.$$

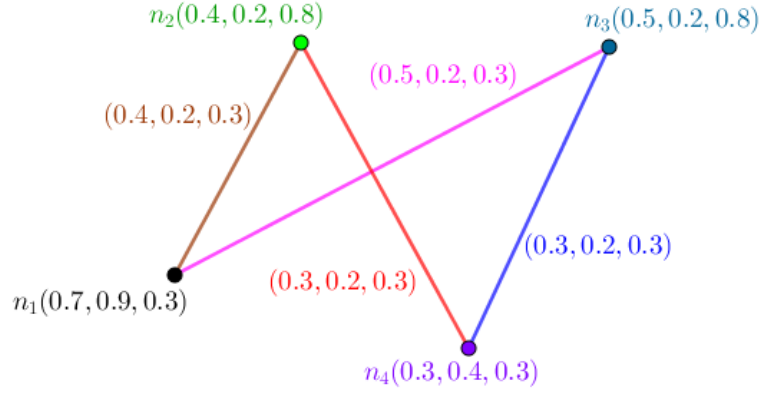
For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$   $k$ -number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$   $k$ -number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called  $k$ -number-dominating set. The minimum cardinality between all  $k$ -number-dominating sets is called  $k$ -number-dominating number and it's denoted by  $\mathcal{N}^k(CMC_{\sigma_1, \sigma_2}) = 2k$ ,  $k = 1, 2$ ; and corresponded to  $k$ -number-dominating sets are

$$\{n_1, n_2\}^1, \{n_1, n_3\}^1, \{n_4, n_2\}^1, \\ \{n_4, n_3\}^1, \{n_1, n_2, n_3, n_4\}^{1,2};$$

- (iv) there are nine  $k$ -number-dominating sets

$$\{n_1, n_2\}^1, \{n_1, n_3\}^1, \{n_4, n_2\}^1, \\ \{n_4, n_3\}^1, \{n_1, n_2, n_3, n_4\}^{1,2}, \{n_1, n_2, n_3\}^1, \\ \{n_1, n_2, n_4\}^1, \{n_1, n_3, n_4\}^1, \{n_4, n_2, n_3\}^1,$$

so as it's possible to have one of them as a set corresponded to neutrosophic  $k$ -number-dominating number so as neutrosophic cardinality is characteristic;



**Figure 8.** A Neutrosophic Graph in the Viewpoint of its k-number-dominating number and its neutrosophic k-number-dominating number.

(v) there are five k-number-dominating sets

$$\{n_1, n_2\}^1, \{n_1, n_3\}^1, \{n_4, n_2\}^1, \\ \{n_4, n_3\}^1, \{n_1, n_2, n_3, n_4\}^{1,2},$$

corresponded to k-number-dominating number as if there's one  
k-number-dominating set corresponded to neutrosophic k-number-dominating  
number so as neutrosophic cardinality is the determiner;

(vi) all minimal k-number-dominating sets corresponded to k-number-dominating  
number are

$$\{n_1, n_2\}^1, \{n_1, n_3\}^1, \{n_4, n_2\}^1, \\ \{n_4, n_3\}^1, \{n_1, n_2, n_3, n_4\}^{1,2}.$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$   
k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside  
triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic  
vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such  
that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$   
is called k-number-dominating set. The minimum neutrosophic cardinality  
between all k-number-dominating sets is called neutrosophic k-number-dominating  
number and it's denoted by  $\mathcal{N}_n^1(CMC_{\sigma_1, \sigma_2}) = 2.4$ ,  $\mathcal{N}_n^2(CMC_{\sigma_1, \sigma_2}) = 5.8$ ; and  
corresponded to k-number-dominating sets are

$$\{n_4, n_2\}^1, \{n_1, n_2, n_3, n_4\}^{1,2}.$$

**Proposition 2.22.** Let  $NTG : (V, E, \sigma, \mu)$  be a complete- $t$ -partite-neutrosophic graph  
where  $t \geq 3$ . Then

$$\mathcal{N}^k(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t}) = 2k$$

where  $k = 1, 2, \dots, \min\{|V_1|, |V_2|, \dots, |V_t|\}$ .

*Proof.* Suppose  $CMC_{\sigma_1, \sigma_2, \dots, \sigma_t} : (V, E, \sigma, \mu)$  is a complete- $t$ -partite-neutrosophic graph.  
Every vertex in a part is k-number-dominated by another vertex in another part.  
Assume same parity for same partition of vertex set which means  $V_i$  has odd indexes  
and  $V_j$  has even indexes. In the setting of complete- $t$ -partite, a vertex of dominating set

corresponded to dominating number dominates as if it doesn't k-number-dominate so as dominating is different from k-number-dominating. All minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\begin{aligned}
& \{n_1, n_2\}^1, \{n_1, n_4\}^1, \{n_1, n_6\}^1, \dots, \{n_1, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})} (n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1})\}^1, \\
& \{n_2, n_3\}^1, \{n_2, n_5\}^1, \{n_2, n_7\}^1, \dots, \{n_2, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1} (n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})})\}^1, \\
& \{n_3, n_4\}^1, \{n_3, n_6\}^1, \{n_3, n_8\}^1, \dots, \{n_3, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})} (n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1})\}^1, \\
& \dots \\
& \{n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})}\}^1, \\
& \dots, \\
& \{n_1, n_3, n_2, n_4\}^{1,2}, \dots, \{n_1, n_3, n_2, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})} (n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1})\}^{1,2}, \\
& \{n_1, n_3, n_6, n_8\}^{1,2}, \dots, \{n_1, n_3, n_6, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})} (n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1})\}^{1,2}, \\
& \{n_1, n_3, n_8, n_{10}\}^{1,2}, \dots, \{n_1, n_3, n_8, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})} (n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1})\}^{1,2}, \\
& \dots, \\
& \{n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-3}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-2}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})}\}^{1,2}, \\
& \dots, \\
& \{n_{i+1}, n_{i+2}, n_{i+3}, \dots, n_{i+2 \min\{|V_1|, |V_2|, \dots, |V_t|\}}\}^{1,2, \dots, \min\{|V_1|, |V_2|, \dots, |V_t|\}}.
\end{aligned}$$

For given vertex  $n$ , if  $s_1 n, s_2 n, \dots, s_k n \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum cardinality between all k-number-dominating sets is called k-number-dominating number and it's denoted by

$$\mathcal{N}^k(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t}) = 2k$$

where  $k = 1, 2, \dots, \min\{|V_1|, |V_2|, \dots, |V_t|\}$ ; and corresponded to k-number-dominating sets are

$$\begin{aligned}
& \{n_1, n_2\}^1, \{n_1, n_4\}^1, \{n_1, n_6\}^1, \dots, \{n_1, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})} (n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1})\}^1, \\
& \{n_2, n_3\}^1, \{n_2, n_5\}^1, \{n_2, n_7\}^1, \dots, \{n_2, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1} (n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})})\}^1, \\
& \{n_3, n_4\}^1, \{n_3, n_6\}^1, \{n_3, n_8\}^1, \dots, \{n_3, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})} (n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1})\}^1, \\
& \dots \\
& \{n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})}\}^1, \\
& \dots, \\
& \{n_1, n_3, n_2, n_4\}^{1,2}, \dots, \{n_1, n_3, n_2, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})} (n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1})\}^{1,2}, \\
& \{n_1, n_3, n_6, n_8\}^{1,2}, \dots, \{n_1, n_3, n_6, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})} (n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1})\}^{1,2}, \\
& \{n_1, n_3, n_8, n_{10}\}^{1,2}, \dots, \{n_1, n_3, n_8, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})} (n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1})\}^{1,2}, \\
& \dots, \\
& \{n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-3}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-2}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})}\}^{1,2}, \\
& \dots, \\
& \{n_{i+1}, n_{i+2}, n_{i+3}, \dots, n_{i+2 \min\{|V_1|, |V_2|, \dots, |V_t|\}}\}^{1,2, \dots, \min\{|V_1|, |V_2|, \dots, |V_t|\}}.
\end{aligned}$$

Thus

$$\mathcal{N}^k(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t}) = 2k$$

where  $k = 1, 2, \dots, \min\{|V_1|, |V_2|, \dots, |V_t|\}$ . □

**Proposition 2.23.** Let  $NTG : (V, E, \sigma, \mu)$  be a complete-t-partite-neutrosophic graph. Then  $k$ -number-dominating number isn't equal to dominating number where  $k \neq 1$ .

**Proposition 2.24.** Let  $NTG : (V, E, \sigma, \mu)$  be a complete-t-partite-neutrosophic graph. Then the number of  $k$ -number-dominating sets is multiplying  $2^{|V_i|+|V_j|-2k}$  by the summation of multiplying  $|V_i|$  choose  $k$  by  $|V_j|$  choose  $k$  on  $i$  and  $j$ .

**Proposition 2.25.** Let  $NTG : (V, E, \sigma, \mu)$  be a complete-t-partite-neutrosophic graph. Then the number of  $k$ -number-dominating sets corresponded to  $k$ -number-dominating number is the summation of multiplying  $|V_i|$  choose  $k$  by  $|V_j|$  choose  $k$  on  $i$  and  $j$ .

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

**Example 2.26.** There is one section for clarifications. In Figure (9), a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given two neutrosophic vertices,  $n$  and  $n'$ , there is either one path with length one or one path with length two between them;
- (ii) in the setting of complete-t-partite, a vertex of dominating set corresponded to dominating number dominates as if it doesn't  $k$ -number-dominate so as dominating is different from  $k$ -number-dominating;
- (iii) all minimal  $k$ -number-dominating sets corresponded to  $k$ -number-dominating number are

$$\begin{aligned} &\{n_1, n_2\}^1, \{n_1, n_3\}^1, \{n_1, n_5\}^1, \\ &\{n_4, n_2\}^1, \{n_4, n_3\}^1, \{n_4, n_5\}^1, \\ &\{n_1, n_4, n_2, n_3\}^{1,2}, \{n_1, n_4, n_2, n_5\}^{1,2}, \{n_1, n_4, n_3, n_5\}^{1,2}. \end{aligned}$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$   $k$ -number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$   $k$ -number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called  $k$ -number-dominating set. The minimum cardinality between all  $k$ -number-dominating sets is called  $k$ -number-dominating number and it's denoted by  $\mathcal{N}^k(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t}) = 2k$ ,  $k = 1, 2$ ; and corresponded to  $k$ -number-dominating sets are

$$\begin{aligned} &\{n_1, n_2\}^1, \{n_1, n_3\}^1, \{n_1, n_5\}^1, \\ &\{n_4, n_2\}^1, \{n_4, n_3\}^1, \{n_4, n_5\}^1, \\ &\{n_1, n_4, n_2, n_3\}^{1,2}, \{n_1, n_4, n_2, n_5\}^{1,2}, \{n_1, n_4, n_3, n_5\}^{1,2}; \end{aligned}$$

(iv) there are eighteen k-number-dominating sets

$$\begin{aligned} &\{n_1, n_2\}^1, \{n_1, n_3\}^1, \{n_1, n_5\}^1, \\ &\{n_4, n_2\}^1, \{n_4, n_3\}^1, \{n_4, n_5\}^1, \\ &\{n_1, n_2, n_3, n_4\}^{1,2}, \{n_1, n_2, n_3, n_5\}^{1,2}, \{n_1, n_2, n_4, n_5\}^{1,2}, \\ &\{n_1, n_2, n_3\}^1, \{n_1, n_2, n_4\}^1, \{n_1, n_2, n_5\}^1, \\ &\{n_1, n_3, n_4\}^1, \{n_1, n_3, n_5\}^1, \{n_4, n_2, n_3\}^1, \\ &\{n_4, n_2, n_5\}^1, \{n_4, n_3, n_5\}^1, \{n_1, n_2, n_3, n_4, n_5\}^{1,2}, \end{aligned}$$

so as it's possible to have one of them as a set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is characteristic;

(v) there are nine k-number-dominating sets

$$\begin{aligned} &\{n_1, n_2\}^1, \{n_1, n_3\}^1, \{n_1, n_5\}^1, \\ &\{n_4, n_2\}^1, \{n_4, n_3\}^1, \{n_4, n_5\}^1, \\ &\{n_1, n_4, n_2, n_3\}^{1,2}, \{n_1, n_4, n_2, n_5\}^{1,2}, \{n_1, n_4, n_3, n_5\}^{1,2}, \end{aligned}$$

corresponded to k-number-dominating number as if there's one k-number-dominating set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is the determiner;

(vi) all minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\begin{aligned} &\{n_1, n_2\}^1, \{n_1, n_3\}^1, \{n_1, n_5\}^1, \\ &\{n_4, n_2\}^1, \{n_4, n_3\}^1, \{n_4, n_5\}^1, \\ &\{n_1, n_4, n_2, n_3\}^{1,2}, \{n_1, n_4, n_2, n_5\}^{1,2}, \{n_1, n_4, n_3, n_5\}^{1,2}. \end{aligned}$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum neutrosophic cardinality between all k-number-dominating sets is called neutrosophic k-number-dominating number and it's denoted by  $\mathcal{N}_n^1(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t}) = 2.4$ ,  $\mathcal{N}_n^2(CMC_{\sigma_1, \sigma_2}) = 5.7$ ; and corresponded to k-number-dominating sets are

$$\{n_4, n_2\}^1, \{n_4, n_5\}^1, \{n_1, n_4, n_2, n_5\}^{1,2}.$$

**Proposition 2.27.** Let  $NTG : (V, E, \sigma, \mu)$  be a wheel-neutrosophic graph. Then

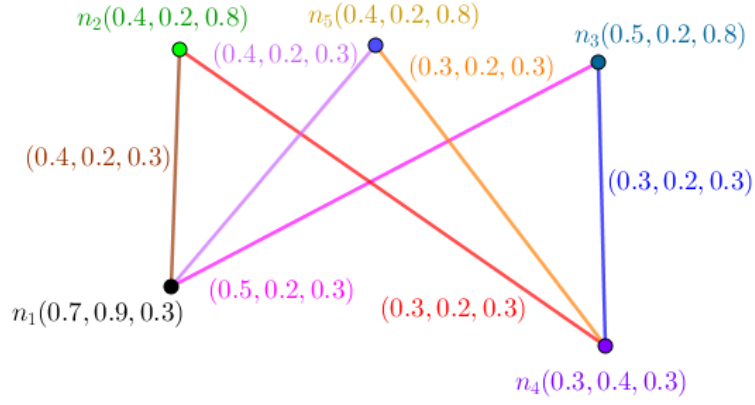
$$\mathcal{N}^1(WHL_{1, \sigma_2}) = 1.$$

$$\mathcal{N}^2(WHL_{1, \sigma_2}) = \lfloor \frac{\mathcal{O}(WHL_{1, \sigma_2})}{2} \rfloor.$$

$$\mathcal{N}^3(WHL_{1, \sigma_2}) = \lfloor \frac{\mathcal{O}(WHL_{1, \sigma_2})}{2} \rfloor + 1.$$

$$\mathcal{N}^{\mathcal{O}(WHL_{1, \sigma_2})-1}(WHL_{1, \sigma_2}) = \mathcal{O}(WHL_{1, \sigma_2}) - 1.$$

$$k = 1, 2, 3, \mathcal{O}(WHL_{1, \sigma_2}) - 1.$$



**Figure 9.** A Neutrosophic Graph in the Viewpoint of its k-number-dominating number and its neutrosophic k-number-dominating number.

*Proof.* Suppose  $WHL_{1,\sigma_2} : (V, E, \sigma, \mu)$  is a wheel-neutrosophic graph. The argument is elementary. All vertices of a cycle

$$n_1, n_2, n_3, \dots, n_{\mathcal{O}(WHL_{1,\sigma_2})-3}, n_{\mathcal{O}(WHL_{1,\sigma_2})-2}, n_{\mathcal{O}(WHL_{1,\sigma_2})-1}, n_1$$

join to one vertex,  $c = n_{\mathcal{O}(WHL_{1,\sigma_2})}$ . For every vertices, the minimum number of edges amid them is either one or two because of center and the notion of neighbors. In the setting of wheel, a vertex of dominating set corresponded to dominating number dominates as if it doesn't k-number-dominate so as dominating is different from k-number-dominating. All minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\begin{aligned} & \{n_{\mathcal{O}(WHL_{1,\sigma_2})}\}^1, \{n_2, n_4, \dots, n_{\mathcal{O}(WHL_{1,\sigma_2})-1}(n_{\mathcal{O}(WHL_{1,\sigma_2})-2})\}^{1,2}, \\ & \{n_3, n_5, \dots, n_{\mathcal{O}(WHL_{1,\sigma_2})-2}(n_{\mathcal{O}(WHL_{1,\sigma_2})-1})\}^{1,2}, \\ & \{n_1, n_2, n_4, \dots, n_{\mathcal{O}(WHL_{1,\sigma_2})-1}(n_{\mathcal{O}(WHL_{1,\sigma_2})-2})\}^{1,2,3}, \\ & \{n_1, n_3, n_5, \dots, n_{\mathcal{O}(WHL_{1,\sigma_2})-2}(n_{\mathcal{O}(WHL_{1,\sigma_2})-1})\}^{1,2,3}, \\ & \{n_1, n_2, n_3, \dots, n_{\mathcal{O}(WHL_{1,\sigma_2})-3}, n_{\mathcal{O}(WHL_{1,\sigma_2})-2}, n_{\mathcal{O}(WHL_{1,\sigma_2})-1}\}^{\mathcal{O}(WHL_{1,\sigma_2})-1}. \end{aligned}$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum cardinality between all k-number-dominating sets is called k-number-dominating number and it's denoted by

$$\mathcal{N}^1(WHL_{1,\sigma_2}) = 1.$$

$$\mathcal{N}^2(WHL_{1,\sigma_2}) = \lfloor \frac{\mathcal{O}(WHL_{1,\sigma_2})}{2} \rfloor.$$

$$\mathcal{N}^3(WHL_{1,\sigma_2}) = \lfloor \frac{\mathcal{O}(WHL_{1,\sigma_2})}{2} \rfloor + 1.$$

$$\mathcal{N}^{\mathcal{O}(WHL_{1,\sigma_2})-1}(WHL_{1,\sigma_2}) = \mathcal{O}(WHL_{1,\sigma_2}) - 1.$$

$$k = 1, 2, 3, \mathcal{O}(WHL_{1,\sigma_2}) - 1;$$

and corresponded to k-number-dominating sets are

$$\begin{aligned} & \{n_{\mathcal{O}(WHL_{1,\sigma_2})}\}^1, \{n_2, n_4, \dots, n_{\mathcal{O}(WHL_{1,\sigma_2})-1}(n_{\mathcal{O}(WHL_{1,\sigma_2})-2})\}^{1,2}, \\ & \{n_3, n_5, \dots, n_{\mathcal{O}(WHL_{1,\sigma_2})-2}(n_{\mathcal{O}(WHL_{1,\sigma_2})-1})\}^{1,2}, \\ & \{n_1, n_2, n_4, \dots, n_{\mathcal{O}(WHL_{1,\sigma_2})-1}(n_{\mathcal{O}(WHL_{1,\sigma_2})-2})\}^{1,2,3}, \\ & \{n_1, n_3, n_5, \dots, n_{\mathcal{O}(WHL_{1,\sigma_2})-2}(n_{\mathcal{O}(WHL_{1,\sigma_2})-1})\}^{1,2,3}, \\ & \{n_1, n_2, n_3, \dots, n_{\mathcal{O}(WHL_{1,\sigma_2})-3}, n_{\mathcal{O}(WHL_{1,\sigma_2})-2}, n_{\mathcal{O}(WHL_{1,\sigma_2})-1}\}^{\mathcal{O}(WHL_{1,\sigma_2})-1}. \end{aligned}$$

Thus

$$\begin{aligned} \mathcal{N}^1(WHL_{1,\sigma_2}) &= 1. \\ \mathcal{N}^2(WHL_{1,\sigma_2}) &= \lfloor \frac{\mathcal{O}(WHL_{1,\sigma_2})}{2} \rfloor. \\ \mathcal{N}^3(WHL_{1,\sigma_2}) &= \lfloor \frac{\mathcal{O}(WHL_{1,\sigma_2})}{2} \rfloor + 1. \\ \mathcal{N}^{\mathcal{O}(WHL_{1,\sigma_2})-1}(WHL_{1,\sigma_2}) &= \mathcal{O}(WHL_{1,\sigma_2}) - 1. \\ k &= 1, 2, 3, \mathcal{O}(WHL_{1,\sigma_2}) - 1. \end{aligned}$$

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**Proposition 2.28.** Let  $NTG : (V, E, \sigma, \mu)$  be a wheel-neutrosophic graph. Then  $k$ -number-dominating number isn't equal to dominating number where  $k > 1$ .

**Proposition 2.29.** Let  $NTG : (V, E, \sigma, \mu)$  be a wheel-partite-neutrosophic graph. Then the number of  $k$ -number-dominating sets corresponded to  $k$ -number-dominating number is one where  $k = 1$ .

**Proposition 2.30.** Let  $NTG : (V, E, \sigma, \mu)$  be a wheel-partite-neutrosophic graph. Then the number of  $k$ -number-dominating sets corresponded to  $k$ -number-dominating number is one where  $k = \mathcal{O}(WHL_{1,\sigma_2}) - 1$ .

The clarifications about results are in progress as follows. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

**Example 2.31.** There is one section for clarifications. In Figure (10), a wheel-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given two neutrosophic vertices,  $s$  and  $n_1$ , there's only one edge between them;
- (ii) in the setting of wheel, a vertex of dominating set corresponded to dominating number dominates as if it doesn't  $k$ -number-dominate so as dominating is different from  $k$ -number-dominating;
- (iii) all minimal  $k$ -number-dominating sets corresponded to  $k$ -number-dominating number are

$$\begin{aligned} & \{n_1\}^1, \{n_2, n_4\}^{1,2}, \{n_3, n_5\}^{1,2}, \\ & \{n_2, n_4, n_1\}^{1,2,3}, \{n_3, n_5, n_1\}^{1,2,3}, \{n_2, n_3, n_4, n_5\}^{1,2,3,4}. \end{aligned}$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$   $k$ -number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside

triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum cardinality between all k-number-dominating sets is called k-number-dominating number and it's denoted by  $\mathcal{N}^k(WHL_{1,\sigma_2}) = k$ ,  $k = 1, 2, 3, 4$ ; and corresponded to k-number-dominating sets are

$$\{n_1\}^1, \{n_2, n_4\}^{1,2}, \{n_3, n_5\}^{1,2}, \\ \{n_2, n_4, n_1\}^{1,2,3}, \{n_3, n_5, n_1\}^{1,2,3}, \{n_2, n_3, n_4, n_5\}^{1,2,3,4};$$

(iv) there are twenty k-number-dominating sets

$$\{n_1\}^1, \{n_1, n_2\}^1, \{n_1, n_3\}^1, \\ \{n_1, n_4\}^1, \{n_1, n_5\}^1, \{n_2, n_3\}^1, \\ \{n_2, n_4\}^{1,2}, \{n_2, n_5\}^1, \{n_3, n_4\}^1, \\ \{n_3, n_5\}^{1,2}, \{n_4, n_5\}^1, \{n_1, n_2, n_3\}^{1,2}, \\ \{n_1, n_2, n_4\}^{1,2,3}, \{n_1, n_2, n_5\}^{1,2}, \{n_2, n_3, n_4\}^{1,2}, \\ \{n_2, n_3, n_5\}^{1,2}, \{n_3, n_4, n_5\}^{1,2}, \{n_1, n_2, n_3, n_4\}^{1,2,3}, \\ \{n_1, n_2, n_3, n_5\}^{1,2,3}, \{n_2, n_3, n_4, n_5\}^{1,2,3,4};$$

so as it's possible to have one of them as a set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is characteristic;

(v) there are six k-number-dominating sets

$$\{n_1\}^1, \{n_2, n_4\}^{1,2}, \{n_3, n_5\}^{1,2}, \\ \{n_2, n_4, n_1\}^{1,2,3}, \{n_3, n_5, n_1\}^{1,2,3}, \{n_2, n_3, n_4, n_5\}^{1,2,3,4};$$

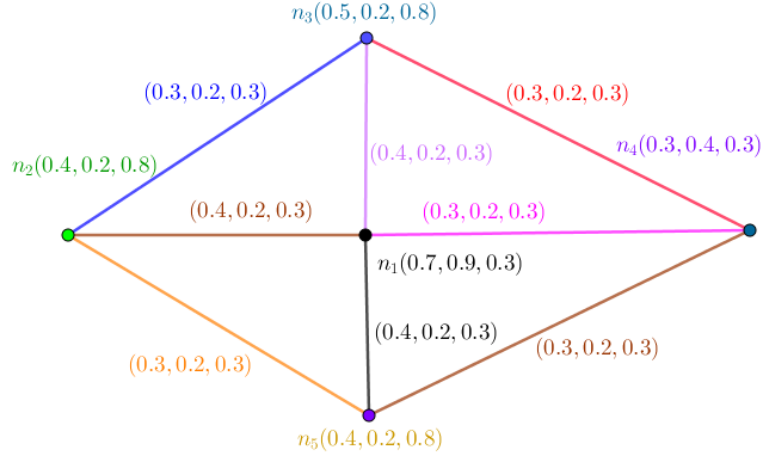
corresponded to k-number-dominating number as if there's one k-number-dominating set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is the determiner;

(vi) all minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\{n_1\}^1, \{n_2, n_4\}^{1,2}, \{n_3, n_5\}^{1,2}, \\ \{n_2, n_4, n_1\}^{1,2,3}, \{n_3, n_5, n_1\}^{1,2,3}, \{n_2, n_3, n_4, n_5\}^{1,2,3,4}.$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum neutrosophic cardinality between all k-number-dominating sets is called neutrosophic k-number-dominating number and it's denoted by  $\mathcal{N}_n^1(WHL_{1,\sigma_2}) = 1.9, \mathcal{N}_n^2(WHL_{1,\sigma_2}) = 2.4, \mathcal{N}_n^3(WHL_{1,\sigma_2}) = 4.3, \mathcal{N}_n^4(WHL_{1,\sigma_2}) = 5.3$ ; and corresponded to k-number-dominating sets are

$$\{n_1\}^1, \{n_2, n_4\}^{1,2}, \{n_2, n_4, n_1\}^{1,2,3}, \\ \{n_2, n_3, n_4, n_5\}^{1,2,3,4}.$$



**Figure 10.** A Neutrosophic Graph in the Viewpoint of its k-number-dominating number and its neutrosophic k-number-dominating number.

### 3 Setting of neutrosophic k-number-dominating number

In this section, I provide some results in the setting of neutrosophic k-number-dominating number. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, star-neutrosophic graph, bipartite-neutrosophic graph, t-partite-neutrosophic graph, and wheel-neutrosophic graph, are both of cases of study and classes which the results are about them.

**Proposition 3.1.** Let  $NTG : (V, E, \sigma, \mu)$  be a complete-neutrosophic graph. Then

$$\mathcal{N}_n^k(CMT_\sigma) = \min_{x_j \in \{x_1, x_2, \dots, x_k\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^k \sigma_i(x_j), \quad k = 1, 2, 3, \dots, \mathcal{O}(CMT_\sigma).$$

Thus,

$$\mathcal{N}_n^1(CMT_\sigma) = \min_{x \in \{x\} \subseteq V} \sum_{i=1}^3 \sigma_i(x), \quad \mathcal{N}_n^2(CMT_\sigma) = \min_{x_j \in \{x_1, x_2\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^2 \sigma_i(x_j), \dots,$$

$$\mathcal{N}_n^{\mathcal{O}(CMT_\sigma)}(CMT_\sigma) = \min_{x_j \in \{x_1, x_2, \dots, x_{\mathcal{O}(CMT_\sigma)}\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^{\mathcal{O}(CMT_\sigma)} \sigma_i(x_j).$$

*Proof.* Suppose  $CMT_\sigma : (V, E, \sigma, \mu)$  is a complete-neutrosophic graph. By  $CMT_\sigma : (V, E, \sigma, \mu)$  is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. In the setting of complete, a vertex of dominating set corresponded to dominating number dominates as if it doesn't k-number-dominate so as dominating is different from k-number-dominating. Dominating number and k-number-dominating number are the same if  $k = 1$ . All

k-number-dominating sets corresponded to k-number-dominating number are

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$$\begin{aligned} & \{n_1\}^1, \{n_2\}^1, \{n_3\}^1, \dots, \{n_{\mathcal{O}(CMT_\sigma)-2}\}^1, \{n_{\mathcal{O}(CMT_\sigma)-1}\}^1, \{n_{\mathcal{O}(CMT_\sigma)}\}^1, \\ & \{n_1, n_2\}^2, \{n_1, n_3\}^2, \{n_1, n_4\}^2, \dots, \{n_1, n_{\mathcal{O}(CMT_\sigma)-1}\}^2, \{n_1, n_{\mathcal{O}(CMT_\sigma)}\}^2, \dots, \\ & \{n_1, n_2, n_3\}^3, \{n_1, n_2, n_4\}^3, \dots, \{n_1, n_2, n_{\mathcal{O}(CMT_\sigma)-1}\}^3, \{n_1, n_2, n_{\mathcal{O}(CMT_\sigma)}\}^3, \dots, \\ & \dots \\ & \{n_1, n_2, \dots, n_{\mathcal{O}(CMT_\sigma)}\}^{\mathcal{O}(CMT_\sigma)}. \end{aligned}$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum neutrosophic cardinality between all k-number-dominating sets is called neutrosophic k-number-dominating number and it's denoted by

$$\mathcal{N}_n^k(CMT_\sigma) = \min_{x_j \in \{x_1, x_2, \dots, x_k\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^k \sigma_i(x_j), \quad k = 1, 2, 3, \dots, \mathcal{O}(CMT_\sigma).$$

Thus,

$$\mathcal{N}_n^1(CMT_\sigma) = \min_{x \in \{x\} \subseteq V} \sum_{i=1}^3 \sigma_i(x), \mathcal{N}_n^2(CMT_\sigma) = \min_{x_j \in \{x_1, x_2\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^2 \sigma_i(x_j), \dots,$$

$$\mathcal{N}_n^{\mathcal{O}(CMT_\sigma)}(CMT_\sigma) = \min_{x_j \in \{x_1, x_2, \dots, x_{\mathcal{O}(CMT_\sigma)}\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^{\mathcal{O}(CMT_\sigma)} \sigma_i(x_j);$$

and corresponded to k-number-dominating sets are

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$$\begin{aligned} & \{n_1\}^1, \{n_2\}^1, \{n_3\}^1, \dots, \{n_{\mathcal{O}(CMT_\sigma)-2}\}^1, \{n_{\mathcal{O}(CMT_\sigma)-1}\}^1, \{n_{\mathcal{O}(CMT_\sigma)}\}^1, \\ & \{n_1, n_2\}^2, \{n_1, n_3\}^2, \{n_1, n_4\}^2, \dots, \{n_1, n_{\mathcal{O}(CMT_\sigma)-1}\}^2, \{n_1, n_{\mathcal{O}(CMT_\sigma)}\}^2, \dots, \\ & \{n_1, n_2, n_3\}^3, \{n_1, n_2, n_4\}^3, \dots, \{n_1, n_2, n_{\mathcal{O}(CMT_\sigma)-1}\}^3, \{n_1, n_2, n_{\mathcal{O}(CMT_\sigma)}\}^3, \dots, \\ & \dots \\ & \{n_1, n_2, \dots, n_{\mathcal{O}(CMT_\sigma)}\}^{\mathcal{O}(CMT_\sigma)}. \end{aligned}$$

Thus

$$\mathcal{N}_n^k(CMT_\sigma) = \min_{x_j \in \{x_1, x_2, \dots, x_k\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^k \sigma_i(x_j), \quad k = 1, 2, 3, \dots, \mathcal{O}(CMT_\sigma).$$

Thus,

$$\mathcal{N}_n^1(CMT_\sigma) = \min_{x \in \{x\} \subseteq V} \sum_{i=1}^3 \sigma_i(x), \mathcal{N}_n^2(CMT_\sigma) = \min_{x_j \in \{x_1, x_2\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^2 \sigma_i(x_j), \dots,$$

$$\mathcal{N}_n^{\mathcal{O}(CMT_\sigma)}(CMT_\sigma) = \min_{x_j \in \{x_1, x_2, \dots, x_{\mathcal{O}(CMT_\sigma)}\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^{\mathcal{O}(CMT_\sigma)} \sigma_i(x_j).$$

□ 698

**Proposition 3.2.** Let  $NTG : (V, E, \sigma, \mu)$  be a complete-neutrosophic graph. Then  $k$ -number-dominating number isn't equal to dominating number where  $k > 1$ .

**Proposition 3.3.** Let  $NTG : (V, E, \sigma, \mu)$  be a complete-neutrosophic graph. Then the number of  $k$ -number-dominating sets corresponded to  $k$ -number-dominating number is  $\mathcal{O}(CMT_\sigma)$  choose  $k$ .

**Proposition 3.4.** Let  $NTG : (V, E, \sigma, \mu)$  be a complete-neutrosophic graph. Then the number of  $k$ -number-dominating sets is  $\mathcal{O}(CMT_\sigma)$  choose  $k$  plus  $\mathcal{O}(CMT_\sigma)$  choose  $k - 1$  plus  $\mathcal{O}(CMT_\sigma)$  choose  $k - 2$  plus ... plus  $\mathcal{O}(CMT_\sigma)$  choose 1.

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

**Example 3.5.** In Figure (11), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given neutrosophic vertex,  $s$ , there's an edge with other vertices;
- (ii) in the setting of complete, a vertex of dominating set corresponded to dominating number dominates as if it doesn't  $k$ -number-dominate so as dominating is different from  $k$ -number-dominating. Dominating number and  $k$ -number-dominating number are the same if  $k = 1$ ;
- (iii) all  $k$ -number-dominating sets corresponded to  $k$ -number-dominating number are

$$\begin{aligned} &\{n_1\}^1, \{n_2\}^1, \{n_3\}^1, \\ &\{n_4\}^1, \{n_1, n_2\}^2, \{n_1, n_3\}^2, \\ &\{n_1, n_4\}^2, \{n_2, n_3\}^2, \{n_2, n_4\}^2, \\ &\{n_3, n_4\}^2, \{n_1, n_2, n_3\}^3, \{n_1, n_2, n_4\}^3, \\ &\{n_2, n_3, n_4\}^3, \{n_1, n_2, n_3, n_4\}^4. \end{aligned}$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$   $k$ -number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$   $k$ -number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called  $k$ -number-dominating set. The minimum cardinality between all  $k$ -number-dominating sets is called  $k$ -number-dominating number and it's denoted by  $\mathcal{N}^k(CMT_\sigma) = k$ ,  $k = 1, 2, \dots, \mathcal{O}(CMT_\sigma)$ ; and corresponded to  $k$ -number-dominating sets are

$$\begin{aligned} &\{n_1\}^1, \{n_2\}^1, \{n_3\}^1, \\ &\{n_4\}^1, \{n_1, n_2\}^2, \{n_1, n_3\}^2, \\ &\{n_1, n_4\}^2, \{n_2, n_3\}^2, \{n_2, n_4\}^2, \\ &\{n_3, n_4\}^2, \{n_1, n_2, n_3\}^3, \{n_1, n_2, n_4\}^3, \\ &\{n_2, n_3, n_4\}^3, \{n_1, n_2, n_3, n_4\}^4; \end{aligned}$$

(iv) there are some k-number-dominating sets

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$$\begin{aligned} &\{n_1\}^{1,2,3,4}, \{n_2\}^{1,2,3,4}, \{n_3\}^{1,2,3,4}, \\ &\{n_4\}^{1,2,3,4}, \{n_1, n_2\}^{2,3,4}, \{n_1, n_3\}^{2,3,4}, \\ &\{n_1, n_4\}^{2,3,4}, \{n_2, n_3\}^{2,3,4}, \{n_2, n_4\}^{2,3,4}, \\ &\{n_3, n_4\}^{2,3,4}, \{n_1, n_2, n_3\}^{3,4}, \{n_1, n_2, n_4\}^{3,4}, \\ &\{n_2, n_3, n_4\}^{3,4}, \{n_1, n_2, n_3, n_4\}^4. \end{aligned}$$

so as it's possible to have one of them as a set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is characteristic;

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(v) there are some k-number-dominating sets

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$$\begin{aligned} &\{n_1\}^1, \{n_2\}^1, \{n_3\}^1, \\ &\{n_4\}^1, \{n_1, n_2\}^2, \{n_1, n_3\}^2, \\ &\{n_1, n_4\}^2, \{n_2, n_3\}^2, \{n_2, n_4\}^2, \\ &\{n_3, n_4\}^2, \{n_1, n_2, n_3\}^3, \{n_1, n_2, n_4\}^3, \\ &\{n_2, n_3, n_4\}^3, \{n_1, n_2, n_3, n_4\}^4, \end{aligned}$$

corresponded to k-number-dominating number as if there's one k-number-dominating set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is the determiner;

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(vi) all k-number-dominating sets corresponded to k-number-dominating number are

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$$\begin{aligned} &\{n_1\}^1, \{n_2\}^1, \{n_3\}^1, \\ &\{n_4\}^1, \{n_1, n_2\}^2, \{n_1, n_3\}^2, \\ &\{n_1, n_4\}^2, \{n_2, n_3\}^2, \{n_2, n_4\}^2, \\ &\{n_3, n_4\}^2, \{n_1, n_2, n_3\}^3, \{n_1, n_2, n_4\}^3, \\ &\{n_2, n_3, n_4\}^3, \{n_1, n_2, n_3, n_4\}^4. \end{aligned}$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum neutrosophic cardinality between all k-number-dominating sets is called neutrosophic k-number-dominating number and it's denoted by  $\mathcal{N}_n^k(CMT_\sigma) = 0.9^1, 2.3^2, 3.9^3, 5.9^4$ ; and corresponded to k-number-dominating sets are

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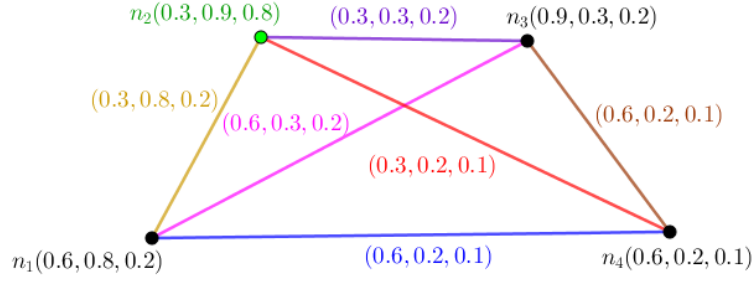
$$\begin{aligned} &\{n_4\}^1, \{n_4, n_3\}^2, \{n_4, n_3, n_1\}^3, \\ &\{n_1, n_2, n_3, n_4\}^4. \end{aligned}$$

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

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**Proposition 3.6.** Let  $NTG : (V, E, \sigma, \mu)$  be a path-neutrosophic graph. Then

$$\mathcal{N}_n^1(PTH) = \min_{x_j \in \{x_1, x_2, \dots, x_{\lfloor \frac{\mathcal{O}(PTH)}{3} \rfloor}\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^{\lfloor \frac{\mathcal{O}(PTH)}{3} \rfloor} \sigma_i(x_j).$$



**Figure 11.** A Neutrosophic Graph in the Viewpoint of its k-number-dominating number and its neutrosophic k-number-dominating number.

$$\mathcal{N}_n^2(PTH) = \min_{x_j \in \{x_1, x_2, \dots, x_{\lfloor \frac{\mathcal{O}(PTH)}{2} \rfloor}\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^{\lfloor \frac{\mathcal{O}(PTH)}{2} \rfloor} \sigma_i(x_j).$$

$$k \leq 2.$$

*Proof.* Suppose  $PTH : (V, E, \sigma, \mu)$  is a path-neutrosophic graph. Let  $n_1, n_2, \dots, n_{\mathcal{O}(PTH)}$  be a path-neutrosophic graph. For given two vertices,  $x$  and  $y$ , there's one path from  $x$  to  $y$ . In the setting of path, a vertex of dominating set corresponded to dominating number dominates as if it doesn't k-number-dominate in the setting of dominating. All minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\{n_1, n_4, \dots\}_{\lfloor \frac{\mathcal{O}(PTH)}{3} \rfloor}^1, \{n_2, n_5, \dots\}_{\lfloor \frac{\mathcal{O}(PTH)}{3} \rfloor}^1, \{n_2, n_4, \dots\}_{\lfloor \frac{\mathcal{O}(PTH)}{3} \rfloor}^1, \dots$$

$$\{n_1, n_{\mathcal{O}(PTH)}, n_3, \dots\}_{\lfloor \frac{\mathcal{O}(PTH)}{2} \rfloor}^{1,2}, \{n_1, n_{\mathcal{O}(PTH)}, n_2, n_4, \dots\}_{\lfloor \frac{\mathcal{O}(PTH)}{2} \rfloor}^{1,2}, \dots$$

For given vertex  $n$ , if  $s_1 n, s_2 n, \dots, s_k n \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum neutrosophic cardinality between all k-number-dominating sets is called neutrosophic k-number-dominating number and it's denoted by

$$\mathcal{N}_n^1(PTH) = \min_{x_j \in \{x_1, x_2, \dots, x_{\lfloor \frac{\mathcal{O}(PTH)}{3} \rfloor}\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^{\lfloor \frac{\mathcal{O}(PTH)}{3} \rfloor} \sigma_i(x_j).$$

$$\mathcal{N}_n^2(PTH) = \min_{x_j \in \{x_1, x_2, \dots, x_{\lfloor \frac{\mathcal{O}(PTH)}{2} \rfloor}\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^{\lfloor \frac{\mathcal{O}(PTH)}{2} \rfloor} \sigma_i(x_j).$$

$$k \leq 2;$$

and corresponded to k-number-dominating sets are

$$\{n_1, n_4, \dots\}_{\lfloor \frac{\mathcal{O}(PTH)}{3} \rfloor}^1, \{n_2, n_5, \dots\}_{\lfloor \frac{\mathcal{O}(PTH)}{3} \rfloor}^1, \{n_2, n_4, \dots\}_{\lfloor \frac{\mathcal{O}(PTH)}{3} \rfloor}^1, \dots$$

$$\{n_1, n_{\mathcal{O}(PTH)}, n_3, \dots\}_{\lfloor \frac{\mathcal{O}(PTH)}{2} \rfloor}^{1,2}, \{n_1, n_{\mathcal{O}(PTH)}, n_2, n_4, \dots\}_{\lfloor \frac{\mathcal{O}(PTH)}{2} \rfloor}^{1,2}, \dots$$

Thus

$$\mathcal{N}_n^1(PTH) = \min_{x_j \in \{x_1, x_2, \dots, x_{\lfloor \frac{\mathcal{O}(PTH)}{3} \rfloor}\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^{\lfloor \frac{\mathcal{O}(PTH)}{3} \rfloor} \sigma_i(x_j).$$

$$\mathcal{N}_n^2(PTH) = \min_{x_j \in \{x_1, x_2, \dots, x_{\lfloor \frac{\mathcal{O}(PTH)}{2} \rfloor}\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^{\lfloor \frac{\mathcal{O}(PTH)}{2} \rfloor} \sigma_i(x_j).$$

$$k \leq 2.$$

□ 755

**Proposition 3.7.** Let  $NTG : (V, E, \sigma, \mu)$  be a path-neutrosophic graph. If  $k$  isn't equal to one, then all leaves belong  $k$ -number-dominating sets corresponded to  $k$ -number-dominating number. 756  
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**Proposition 3.8.** Let  $NTG : (V, E, \sigma, \mu)$  be a path-neutrosophic graph. If at least one leaf doesn't belong  $k$ -number-dominating sets corresponded to  $k$ -number-dominating number, then  $k$  is equal to one. 759  
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**Example 3.9.** There are two sections for clarifications. 762

- (a) In Figure (12), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. 763  
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- (i) For given neutrosophic vertex,  $s$ , there's only one path with other vertices; 765
  - (ii) in the setting of path, a vertex of dominating set corresponded to dominating number dominates as if it doesn't  $k$ -number-dominate in the setting of dominating; 766  
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  - (iii) all minimal  $k$ -number-dominating sets corresponded to  $k$ -number-dominating number are 769  
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$$\{n_1, n_4\}^1, \{n_2, n_5\}^1, \{n_2, n_4\}^1, \\ \{n_1, n_5, n_3\}^{1,2}.$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$   $k$ -number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$   $k$ -number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called  $k$ -number-dominating set. The minimum cardinality between all  $k$ -number-dominating sets is called  $k$ -number-dominating number and it's denoted by  $\mathcal{N}^k(PTH) = k + 1$ ,  $k = 1, 2$ ; and corresponded to  $k$ -number-dominating sets are 771  
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$$\{n_1, n_4\}^1, \{n_2, n_5\}^1, \{n_2, n_4\}^1, \\ \{n_1, n_5, n_3\}^{1,2};$$

- (iv) there are thirteen  $k$ -number-dominating sets 781

$$\{n_1, n_4\}^1, \{n_2, n_5\}^1, \{n_2, n_4\}^1, \\ \{n_1, n_4, n_2\}^1, \{n_1, n_4, n_3\}^1, \{n_1, n_4, n_5\}^1, \\ \{n_2, n_4, n_3\}^1, \{n_2, n_4, n_5\}^1, \{n_2, n_5, n_1\}^1, \\ \{n_2, n_5, n_3\}^1, \{n_1, n_5, n_3, n_2\}^{1,2}, \{n_1, n_5, n_3, n_4\}^{1,2}, \\ \{n_1, n_5, n_3, n_4, n_2\}^{1,2},$$

so as it's possible to have one of them as a set corresponded to neutrosophic  
k-number-dominating number so as neutrosophic cardinality is characteristic;

(v) there are four k-number-dominating sets

$$\{n_1, n_4\}^1, \{n_2, n_5\}^1, \{n_2, n_4\}^1, \\ \{n_1, n_5, n_3\}^{1,2},$$

corresponded to k-number-dominating number as if there's one  
k-number-dominating set corresponded to neutrosophic k-number-dominating  
number so as neutrosophic cardinality is the determiner;

(vi) all minimal k-number-dominating sets corresponded to k-number-dominating  
number are

$$\{n_1, n_4\}^1, \{n_2, n_5\}^1, \{n_2, n_4\}^1, \\ \{n_1, n_5, n_3\}^{1,2}, \{n_1, n_5, n_4\}^{1,2}, \{n_1, n_5, n_2\}^{1,2}.$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$   
k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex  
alongside triple pair of its values is called neutrosophic vertex.]. If for every  
neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  
 $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set  
of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum  
neutrosophic cardinality between all k-number-dominating sets is called  
neutrosophic k-number-dominating number and it's denoted by  
 $\mathcal{N}_n^1(PTH) = 2.6$ ,  $\mathcal{N}_n^2(PTH) = 3.3$ ; and corresponded to  
k-number-dominating sets are

$$\{n_1, n_4\}^1, \{n_1, n_5, n_3\}^{1,2}.$$

(b) In Figure (13), an even-path-neutrosophic graph is illustrated. Some points are  
represented in follow-up items as follows.

- (i) For given neutrosophic vertex,  $s$ , there's only one path with other vertices;
- (ii) in the setting of path, a vertex of dominating set corresponded to dominating  
number dominates as if it doesn't k-number-dominate in the setting of  
dominating;
- (iii) all minimal k-number-dominating sets corresponded to k-number-dominating  
number are

$$\{n_2, n_5\}^1, \{n_1, n_6, n_3, n_5\}^{1,2}, \{n_1, n_6, n_3, n_4\}^{1,2}, \{n_1, n_6, n_2, n_4\}^{1,2}.$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$   
k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex  
alongside triple pair of its values is called neutrosophic vertex.]. If for every  
neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  
 $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set  
of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum  
cardinality between all k-number-dominating sets is called  
k-number-dominating number and it's denoted by  
 $\mathcal{N}^1(PTH) = 2$ ,  $\mathcal{N}^2(PTH) = 4$ ; and corresponded to k-number-dominating  
sets are

$$\{n_2, n_5\}^1, \{n_1, n_6, n_3, n_5\}^{1,2}, \{n_1, n_6, n_3, n_4\}^{1,2}, \{n_1, n_6, n_2, n_4\}^{1,2};$$

(iv) there are twenty k-number-dominating sets

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$$\begin{aligned} & \{n_2, n_5\}^1, \{n_2, n_5, n_1\}^1, \{n_2, n_5, n_3\}^1, \\ & \{n_2, n_5, n_4\}^1, \{n_2, n_5, n_6\}^1, \{n_2, n_5, n_1, n_3\}^1, \\ & \{n_2, n_5, n_1, n_4\}^1, \{n_2, n_5, n_1, n_6\}^{1,2}, \{n_2, n_5, n_3, n_4\}^1, \\ & \{n_2, n_5, n_3, n_6\}^1, \{n_2, n_5, n_4, n_6\}^1, \{n_2, n_5, n_1, n_3, n_4\}^1, \\ & \{n_2, n_5, n_1, n_3, n_6\}^{1,2}, \{n_2, n_5, n_1, n_3, n_4\}^1, \{n_2, n_5, n_3, n_4, n_6\}^1, \\ & \{n_2, n_5, n_4, n_6, n_3, n_1\}^{1,2}, \{n_1, n_6, n_3, n_5, n_4\}^{1,2}, \{n_1, n_6, n_3, n_5\}^{1,2}, \\ & \{n_1, n_6, n_3, n_4\}^{1,2}, \{n_1, n_6, n_2, n_4\}^{1,2}, \end{aligned}$$

so as it's possible to have one of them as a set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is characteristic;

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(v) there are four k-number-dominating sets

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$$\begin{aligned} & \{n_2, n_5\}^1, \{n_1, n_6, n_3, n_5\}^{1,2}, \{n_1, n_6, n_3, n_4\}^{1,2}, \\ & \{n_1, n_6, n_2, n_4\}^{1,2}, \end{aligned}$$

corresponded to k-number-dominating number as if there's one k-number-dominating set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is the determiner;

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(vi) all minimal k-number-dominating sets corresponded to k-number-dominating number are

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$$\{n_2, n_5\}^1, \{n_1, n_6, n_3, n_5\}^{1,2}, \{n_1, n_6, n_3, n_4\}^{1,2}, \{n_1, n_6, n_2, n_4\}^{1,2}.$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum neutrosophic cardinality between all k-number-dominating sets is called neutrosophic k-number-dominating number and it's denoted by  $\mathcal{N}_n^1(PTH) = 3.8, \mathcal{N}_n^2(PTH) = 2.2$ ; and corresponded to k-number-dominating sets are

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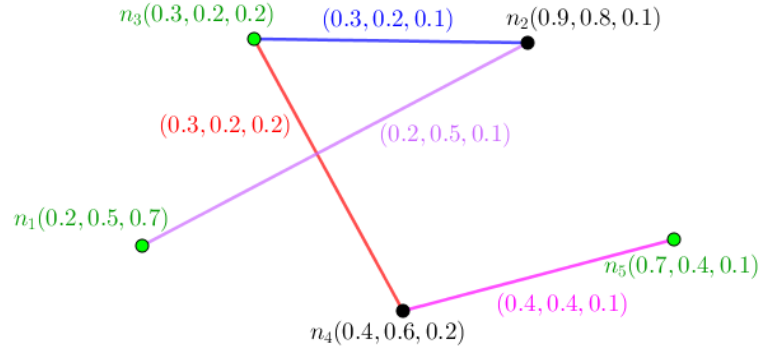
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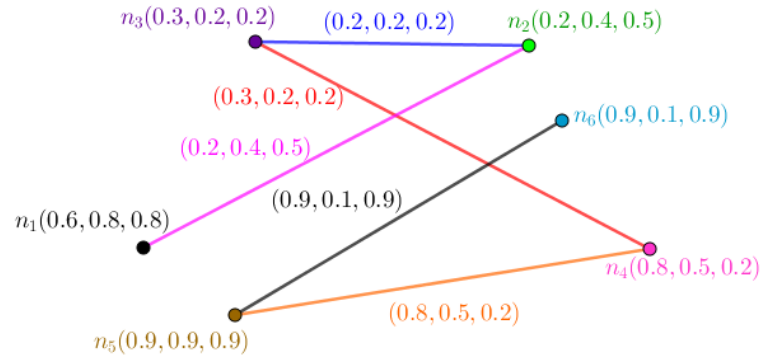
$$\{n_2, n_5\}^1, \{n_1, n_6, n_3, n_4\}^{1,2}.$$

**Proposition 3.10.** Let  $NTG : (V, E, \sigma, \mu)$  be a cycle-neutrosophic graph where  $O(CYC) \geq 3$ . Then

$$\begin{aligned} \mathcal{N}_n^1(CYC) &= \min_{x_j \in \{x_1, x_2, \dots, x_{\lfloor \frac{O(CYC)}{3} \rfloor}\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^{\lfloor \frac{O(CYC)}{3} \rfloor} \sigma_i(x_j). \\ \mathcal{N}_n^2(CYC) &= \min_{x_j \in \{x_1, x_2, \dots, x_{\lfloor \frac{O(CYC)}{2} \rfloor}\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^{\lfloor \frac{O(CYC)}{2} \rfloor} \sigma_i(x_j). \\ k &\leq 2. \end{aligned}$$



**Figure 12.** A Neutrosophic Graph in the Viewpoint of its k-number-dominating number and its neutrosophic k-number-dominating number.



**Figure 13.** A Neutrosophic Graph in the Viewpoint of its k-number-dominating number and its neutrosophic k-number-dominating number.

*Proof.* Suppose  $CYC : (V, E, \sigma, \mu)$  is a cycle-neutrosophic graph. For given two vertices,  $x$  and  $y$ , there are only two paths with distinct edges from  $x$  to  $y$ . Let

$$n_1, n_2, \dots, n_{\mathcal{O}(CYC)-1}, n_{\mathcal{O}(CYC)}, n_1$$

be a cycle-neutrosophic graph  $CYC : (V, E, \sigma, \mu)$ . In the setting of cycle, a vertex of dominating set corresponded to dominating number dominates as if it doesn't k-number-dominate in the setting of dominating. All minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\begin{aligned} & \{n_1, n_4, \dots\}_{\lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor}^1, \{n_2, n_5, \dots\}_{\lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor}^1, \{n_2, n_4, \dots\}_{\lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor}^1, \dots \\ & \{n_1, n_{\mathcal{O}(CYC)}, n_3, \dots\}_{\lfloor \frac{\mathcal{O}(CYC)}{2} \rfloor}^{1,2}, \{n_1, n_{\mathcal{O}(CYC)}, n_2, n_4, \dots\}_{\lfloor \frac{\mathcal{O}(CYC)}{2} \rfloor}^{1,2}, \dots \end{aligned}$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum neutrosophic cardinality between all k-number-dominating sets is called neutrosophic k-number-dominating number and it's denoted by

$$\begin{aligned} \mathcal{N}_n^1(CYC) &= \min_{x_j \in \{x_1, x_2, \dots, x_{\lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor}\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^{\lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor} \sigma_i(x_j). \\ \mathcal{N}_n^2(CYC) &= \min_{x_j \in \{x_1, x_2, \dots, x_{\lfloor \frac{\mathcal{O}(CYC)}{2} \rfloor}\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^{\lfloor \frac{\mathcal{O}(CYC)}{2} \rfloor} \sigma_i(x_j). \\ k &\leq 2; \end{aligned}$$

and corresponded to k-number-dominating sets are

$$\begin{aligned} & \{n_1, n_4, \dots\}_{\lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor}^1, \{n_2, n_5, \dots\}_{\lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor}^1, \{n_2, n_4, \dots\}_{\lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor}^1, \dots \\ & \{n_1, n_{\mathcal{O}(CYC)}, n_3, \dots\}_{\lfloor \frac{\mathcal{O}(CYC)}{2} \rfloor}^{1,2}, \{n_1, n_{\mathcal{O}(CYC)}, n_2, n_4, \dots\}_{\lfloor \frac{\mathcal{O}(CYC)}{2} \rfloor}^{1,2}, \dots \end{aligned}$$

Thus

$$\begin{aligned} \mathcal{N}_n^1(CYC) &= \min_{x_j \in \{x_1, x_2, \dots, x_{\lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor}\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^{\lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor} \sigma_i(x_j). \\ \mathcal{N}_n^2(CYC) &= \min_{x_j \in \{x_1, x_2, \dots, x_{\lfloor \frac{\mathcal{O}(CYC)}{2} \rfloor}\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^{\lfloor \frac{\mathcal{O}(CYC)}{2} \rfloor} \sigma_i(x_j). \\ k &\leq 2. \end{aligned}$$

□ 842

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

**Example 3.11.** There are two sections for clarifications. 849

- (a) In Figure (14), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. 850
- (i) For given neutrosophic vertex,  $s$ , there's only one path with other vertices; 852
- (ii) in the setting of cycle, a vertex of dominating set corresponded to dominating number dominates as if it doesn't  $k$ -number-dominate in the setting of dominating; 853
- (iii) all minimal  $k$ -number-dominating sets corresponded to  $k$ -number-dominating number are 854
- 855

$$\{n_2, n_5\}^1, \{n_1, n_4\}^1, \{n_3, n_6\}^1, \\ \{n_1, n_3, n_5\}^{1,2}, \{n_2, n_4, n_6\}^{1,2}.$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$   $k$ -number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$   $k$ -number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called  $k$ -number-dominating set. The minimum cardinality between all  $k$ -number-dominating sets is called  $k$ -number-dominating number and it's denoted by  $\mathcal{N}^k(CYC) = k + 1$ ,  $k = 1, 2$ ; and corresponded to  $k$ -number-dominating sets are 856

857

$$\{n_2, n_5\}^1, \{n_1, n_4\}^1, \{n_3, n_6\}^1, \\ \{n_1, n_3, n_5\}^{1,2}, \{n_2, n_4, n_6\}^{1,2};$$

- (iv) there are some  $k$ -number-dominating sets 868

$$\{n_2, n_5\}^1, \{n_2, n_5, n_1\}^1, \{n_2, n_5, n_3\}^1, \\ \{n_2, n_5, n_4\}^1, \{n_2, n_5, n_6\}^1, \{n_2, n_5, n_1, n_3\}^{1,2}, \\ \{n_2, n_5, n_1, n_4\}^{1,2}, \{n_2, n_5, n_1, n_6\}^1, \{n_2, n_5, n_3, n_4\}^1, \\ \{n_2, n_5, n_3, n_6\}^{1,2}, \{n_2, n_5, n_4, n_6\}^{1,2}, \{n_2, n_5, n_1, n_3, n_4\}^{1,2}, \\ \{n_2, n_5, n_1, n_3, n_6\}^{1,2}, \{n_2, n_5, n_1, n_3, n_4\}^{1,2}, \{n_2, n_5, n_3, n_4, n_6\}^{1,2}, \\ \{n_2, n_5, n_4, n_6, n_3, n_1\}^{1,2}, \dots,$$

so as it's possible to have one of them as a set corresponded to neutrosophic  $k$ -number-dominating number so as neutrosophic cardinality is characteristic; 869

870

- (v) there are five  $k$ -number-dominating sets 871

$$\{n_2, n_5\}^1, \{n_1, n_4\}^1, \{n_3, n_6\}^1, \\ \{n_1, n_3, n_5\}^{1,2}, \{n_2, n_4, n_6\}^{1,2},$$

corresponded to  $k$ -number-dominating number as if there's one  $k$ -number-dominating set corresponded to neutrosophic  $k$ -number-dominating number so as neutrosophic cardinality is the determiner; 872

873

- (vi) all minimal  $k$ -number-dominating sets corresponded to  $k$ -number-dominating number are 874
- 875
- 876

$$\{n_2, n_5\}^1, \{n_1, n_4\}^1, \{n_3, n_6\}^1, \\ \{n_1, n_3, n_5\}^{1,2}, \{n_2, n_4, n_6\}^{1,2}.$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum neutrosophic cardinality between all k-number-dominating sets is called neutrosophic k-number-dominating number and it's denoted by  $\mathcal{N}_n^1(CYC) = 2.2$ ,  $\mathcal{N}_n^2(CYC) = 3.2$ ; and corresponded to k-number-dominating sets are

$$\{n_1, n_4\}^1, \{n_1, n_3, n_5\}^{1,2}.$$

(b) In Figure (15), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given neutrosophic vertex,  $s$ , there's only one path with other vertices;
- (ii) in the setting of cycle, a vertex of dominating set corresponded to dominating number dominates as if it doesn't k-number-dominate in the setting of dominating;
- (iii) all minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\begin{aligned} &\{n_1, n_4\}^1, \{n_1, n_3\}^1, \{n_2, n_4\}^1, \\ &\{n_2, n_5\}^1, \{n_3, n_5\}^1, \{n_1, n_3, n_5\}^{1,2}, \\ &\{n_1, n_2, n_4\}^{1,2}, \{n_2, n_3, n_5\}^{1,2}, \{n_3, n_4, n_1\}^{1,2}, \\ &\{n_2, n_4, n_5\}^{1,2}. \end{aligned}$$

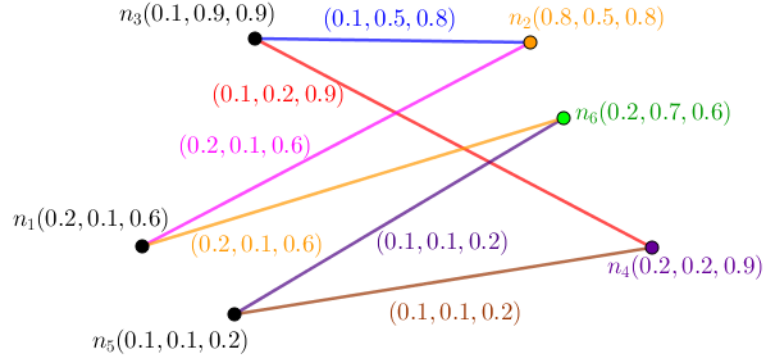
For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum cardinality between all k-number-dominating sets is called k-number-dominating number and it's denoted by  $\mathcal{N}^k(CYC) = k + 1$ ,  $k = 1, 2$ ; and corresponded to k-number-dominating sets are

$$\begin{aligned} &\{n_1, n_4\}^1, \{n_1, n_3\}^1, \{n_2, n_4\}^1, \\ &\{n_2, n_5\}^1, \{n_3, n_5\}^1, \{n_1, n_3, n_5\}^{1,2}, \\ &\{n_1, n_2, n_4\}^{1,2}, \{n_2, n_3, n_5\}^{1,2}, \{n_3, n_4, n_1\}^{1,2}, \\ &\{n_2, n_4, n_5\}^{1,2}; \end{aligned}$$

(iv) there are thirteen k-number-dominating sets

$$\begin{aligned} &\{n_1, n_4\}^1, \{n_2, n_5\}^1, \{n_2, n_4\}^1, \\ &\{n_1, n_4, n_2\}^1, \{n_1, n_4, n_3\}^1, \{n_1, n_4, n_5\}^1, \\ &\{n_2, n_4, n_3\}^1, \{n_2, n_4, n_5\}^1, \{n_2, n_5, n_1\}^1, \\ &\{n_2, n_5, n_3\}^1, \{n_1, n_5, n_3, n_2\}^{1,2}, \{n_1, n_5, n_3, n_4\}^{1,2}, \\ &\{n_1, n_5, n_3, n_4, n_2\}^{1,2}, \end{aligned}$$

so as it's possible to have one of them as a set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is characteristic;



**Figure 14.** A Neutrosophic Graph in the Viewpoint of its k-number-dominating number and its neutrosophic k-number-dominating number.

(v) there are twenty-three k-number-dominating sets

$$\begin{aligned} & \{n_1, n_4\}^1, \{n_2, n_5\}^1, \{n_2, n_4\}^1, \\ & \{n_1, n_4, n_2\}^1, \{n_1, n_4, n_3\}^1, \{n_1, n_4, n_5\}^1, \\ & \{n_2, n_4, n_3\}^1, \{n_2, n_4, n_5\}^1, \{n_2, n_5, n_1\}^1, \\ & \{n_2, n_5, n_3\}^1, \{n_1, n_5, n_3, n_2\}^{1,2}, \{n_1, n_5, n_3, n_4\}^{1,2}, \\ & \{n_1, n_5, n_3, n_4, n_2\}^{1,2}, \{n_1, n_2, n_3\}^1, \{n_2, n_3, n_4\}^1, \\ & \{n_3, n_4, n_5\}^1, \{n_5, n_1, n_2\}^1, \{n_1, n_2, n_3, n_4\}^{1,2}, \\ & \{n_2, n_3, n_4, n_5\}^{1,2}, \{n_3, n_4, n_5, n_1\}^{1,2}, \{n_4, n_5, n_1, n_2\}^{1,2}, \\ & \{n_5, n_1, n_2, n_3\}^{1,2}, \{n_1, n_2, n_3, n_4, n_5\}^{1,2}, \end{aligned}$$

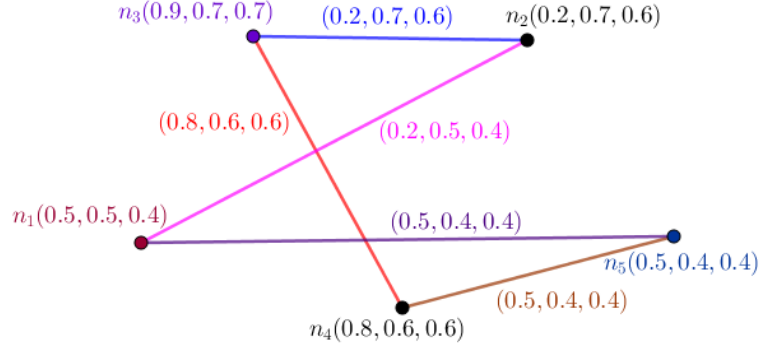
corresponded to k-number-dominating number as if there's one  
k-number-dominating set corresponded to neutrosophic k-number-dominating  
number so as neutrosophic cardinality is the determiner;

(vi) all minimal k-number-dominating sets corresponded to k-number-dominating  
number are

$$\begin{aligned} & \{n_1, n_4\}^1, \{n_1, n_3\}^1, \{n_2, n_4\}^1, \\ & \{n_2, n_5\}^1, \{n_3, n_5\}^1, \{n_1, n_3, n_5\}^{1,2}, \\ & \{n_1, n_2, n_4\}^{1,2}, \{n_2, n_3, n_5\}^{1,2}, \{n_3, n_4, n_1\}^{1,2}, \\ & \{n_2, n_4, n_5\}^{1,2}. \end{aligned}$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$   
k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex  
alongside triple pair of its values is called neutrosophic vertex.]. If for every  
neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  
 $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set  
of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum  
neutrosophic cardinality between all k-number-dominating sets is called  
neutrosophic k-number-dominating number and it's denoted by  
 $\mathcal{N}_n^1(CYC) = 2.8$ ,  $\mathcal{N}_n^2(CYC) = 4.8$ ; and corresponded to  
k-number-dominating sets are

$$\{n_2, n_5\}^1, \{n_2, n_4, n_5\}^{1,2}.$$



**Figure 15.** A Neutrosophic Graph in the Viewpoint of its k-number-dominating number and its neutrosophic k-number-dominating number.

**Proposition 3.12.** Let  $NTG : (V, E, \sigma, \mu)$  be a star-neutrosophic graph with center  $c$ . Then

$$\mathcal{N}_n^1(STR_{1,\sigma_2}) = \sum_{i=1}^3 \sigma_i(c).$$

$$\mathcal{N}_n^{\mathcal{O}(STR_{1,\sigma_2})-1}(STR_{1,\sigma_2}) = \min_{x_j \in \{x_1, x_2, \dots, x_{\mathcal{O}(STR_{1,\sigma_2})-1}\} \subseteq V \setminus \{c\}} \sum_{i=1}^3 \sum_{j=1}^{\mathcal{O}(STR_{1,\sigma_2})-1} \sigma_i(x_j).$$

$$k = 1, \mathcal{O}(STR_{1,\sigma_2}) - 1.$$

*Proof.* Suppose  $STR_{1,\sigma_2} : (V, E, \sigma, \mu)$  is a star-neutrosophic graph. An edge always has center,  $c$ , as one of its endpoints where  $n_{\mathcal{O}(STR_{1,\sigma_2})} = c$ . All paths have one as their lengths, forever. In the setting of star, a vertex of dominating set corresponded to dominating number dominates as if it doesn't k-number-dominate in the setting of dominating. All minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\{n_{\mathcal{O}(STR_{1,\sigma_2})}\}^1, \{n_1, n_2, n_3, \dots, n_{\mathcal{O}(STR_{1,\sigma_2})-1}\}^{1, \mathcal{O}(STR_{1,\sigma_2})-1}.$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum neutrosophic cardinality between all k-number-dominating sets is called neutrosophic k-number-dominating number and it's denoted by

$$\mathcal{N}_n^1(STR_{1,\sigma_2}) = \sum_{i=1}^3 \sigma_i(c).$$

$$\mathcal{N}_n^{\mathcal{O}(STR_{1,\sigma_2})-1}(STR_{1,\sigma_2}) = \min_{x_j \in \{x_1, x_2, \dots, x_{\mathcal{O}(STR_{1,\sigma_2})-1}\} \subseteq V \setminus \{c\}} \sum_{i=1}^3 \sum_{j=1}^{\mathcal{O}(STR_{1,\sigma_2})-1} \sigma_i(x_j).$$

$$k = 1, \mathcal{O}(STR_{1,\sigma_2}) - 1;$$

and corresponded to k-number-dominating sets are

$$\{n_{\mathcal{O}(STR_{1,\sigma_2})}\}^1, \{n_1, n_2, n_3, \dots, n_{\mathcal{O}(STR_{1,\sigma_2})-1}\}^{1, \mathcal{O}(STR_{1,\sigma_2})-1}.$$

Thus

$$\mathcal{N}_n^1(STR_{1,\sigma_2}) = \sum_{i=1}^3 \sigma_i(c).$$

$$\mathcal{N}_n^{\mathcal{O}(STR_{1,\sigma_2})-1}(STR_{1,\sigma_2}) = \min_{x_j \in \{x_1, x_2, \dots, x_{\mathcal{O}(STR_{1,\sigma_2})-1}\} \subseteq V \setminus \{c\}} \sum_{i=1}^3 \sum_{j=1}^{\mathcal{O}(STR_{1,\sigma_2})-1} \sigma_i(x_j).$$

$$k = 1, \mathcal{O}(STR_{1,\sigma_2}) - 1.$$

□ 931

**Proposition 3.13.** *Let  $NTG : (V, E, \sigma, \mu)$  be a star-neutrosophic graph. Then  $k$ -number-dominating number isn't equal to dominating number where  $k \neq 1$ .*

**Proposition 3.14.** *Let  $NTG : (V, E, \sigma, \mu)$  be a star-neutrosophic graph with center  $c$ . Then*

(i) *the number of  $k$ -number-dominating sets is  $2^{\mathcal{O}(STR_{1,\sigma_2})-1}$  where  $k = 1$ ;*

(ii) *the number of  $k$ -number-dominating sets is one where  $k \neq 1$ .*

**Proposition 3.15.** *Let  $NTG : (V, E, \sigma, \mu)$  be a star-neutrosophic graph with center  $c$ . Then*

(i) *the number of  $k$ -number-dominating sets corresponded to  $k$ -number-dominating number is one where  $k = 1$ ;*

(ii) *the number of  $k$ -number-dominating sets corresponded to  $k$ -number-dominating number is one where  $k \neq 1$ .*

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

**Example 3.16.** There is one section for clarifications. In Figure (16), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given two neutrosophic vertices,  $s$  and  $n_1$ , there's only one path, precisely one edge between them and there's no path despite them;
- (ii) in the setting of star, a vertex of dominating set corresponded to dominating number dominates as if it doesn't  $k$ -number-dominate in the setting of dominating;
- (iii) all minimal  $k$ -number-dominating sets corresponded to  $k$ -number-dominating number are

$$\{n_1\}^1, \{n_2, n_3, n_4, n_5\}^{1,4}.$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$   $k$ -number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$   $k$ -number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called  $k$ -number-dominating set. The minimum cardinality between all  $k$ -number-dominating sets is called  $k$ -number-dominating number and it's denoted

by  $\mathcal{N}^1(STR_{1,\sigma_2}) = 1$ ,  $\mathcal{N}^4(STR_{1,\sigma_2}) = 4$ ; and corresponded to k-number-dominating sets are

$$\{n_1\}^1, \{n_2, n_3, n_4, n_5\}^{1,4};$$

(iv) there are seventeen k-number-dominating sets

$$\begin{aligned} &\{n_1\}^1, \{n_1, n_2\}^1, \{n_1, n_3\}^1, \\ &\{n_1, n_4\}^1, \{n_1, n_5\}^1, \{n_1, n_2, n_3\}^1, \\ &\{n_1, n_2, n_4\}^1, \{n_1, n_2, n_5\}^1, \{n_2, n_3, n_4\}^1, \\ &\{n_2, n_3, n_5\}^1, \{n_3, n_4, n_5\}^1, \{n_2, n_3, n_4, n_5\}^1, \\ &\{n_1, n_3, n_4, n_5\}^1, \{n_1, n_2, n_4, n_5\}^1, \{n_1, n_2, n_3, n_5\}^1, \\ &\{n_1, n_2, n_3, n_4\}^1, \{n_2, n_3, n_4, n_5\}^{1,4}, \end{aligned}$$

so as it's possible to have one of them as a set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is characteristic;

(v) there are two k-number-dominating sets

$$\{n_1\}^1, \{n_2, n_3, n_4, n_5\}^{1,4},$$

corresponded to k-number-dominating number as if there's one k-number-dominating set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is the determiner;

(vi) all minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\{n_1\}^1, \{n_2, n_3, n_4, n_5\}^{1,4}.$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum neutrosophic cardinality between all k-number-dominating sets is called neutrosophic k-number-dominating number and it's denoted by  $\mathcal{N}_n^1(STR_{1,\sigma_2}) = 1.9$ ,  $\mathcal{N}_n^4(STR_{1,\sigma_2}) = 5.7$ ; and corresponded to k-number-dominating sets are

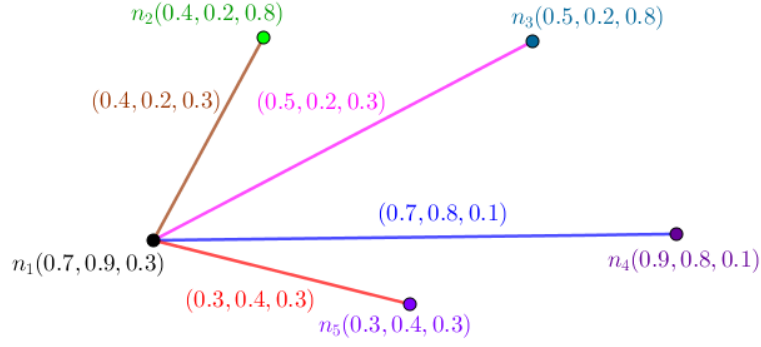
$$\{n_1\}^1, \{n_2, n_3, n_4, n_5\}^{1,4}.$$

**Proposition 3.17.** Let  $NTG : (V, E, \sigma, \mu)$  be a complete-bipartite-neutrosophic graph which isn't star-neutrosophic graph which means  $|V_1|, |V_2| \geq 2$ . Then

$$\mathcal{N}_n^k(CMC_{\sigma_1, \sigma_2}) = \min_{x_j \in \{x_1, x_2, \dots, x_{2k}\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^{2k} \sigma_i(x_j)$$

where  $k = 1, 2, \dots, \min\{|V_1|, |V_2|\}$ .

*Proof.* Suppose  $CMC_{\sigma_1, \sigma_2} : (V, E, \sigma, \mu)$  is a complete-bipartite-neutrosophic graph. Every vertex in a part and another vertex in opposite part k-number-dominates any given vertex. Assume same parity for same partition of vertex set which means  $V_1$  has



**Figure 16.** A Neutrosophic Graph in the Viewpoint of its k-number-dominating number and its neutrosophic k-number-dominating number.

odd indexes and  $V_2$  has even indexes. In the setting of complete-bipartite, a vertex of dominating set corresponded to dominating number dominates as if it doesn't k-number-dominate so as dominating is different from k-number-dominating. All minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\begin{aligned}
& \{n_1, n_2\}^1, \{n_1, n_4\}^1, \{n_1, n_6\}^1, \dots, \{n_1, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})} (n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1})\}^1, \\
& \{n_2, n_3\}^1, \{n_2, n_5\}^1, \{n_2, n_7\}^1, \dots, \{n_2, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1} (n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})})\}^1, \\
& \{n_3, n_4\}^1, \{n_3, n_6\}^1, \{n_3, n_8\}^1, \dots, \{n_3, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})} (n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1})\}^1, \\
& \dots \\
& \{n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})}\}^1, \\
& \dots, \\
& \{n_1, n_3, n_2, n_4\}^{1,2}, \dots, \{n_1, n_3, n_2, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})} (n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1})\}^{1,2}, \\
& \{n_1, n_3, n_6, n_8\}^{1,2}, \dots, \{n_1, n_3, n_6, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})} (n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1})\}^{1,2}, \\
& \{n_1, n_3, n_8, n_{10}\}^{1,2}, \dots, \{n_1, n_3, n_8, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})} (n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1})\}^{1,2}, \\
& \dots, \\
& \{n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-3}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-2}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})}\}^{1,2}, \\
& \dots, \\
& \{n_{i+1}, n_{i+2}, n_{i+3}, \dots, n_{i+2 \min\{|V_1|, |V_2|\}}\}^{1,2, \dots, \min\{|V_1|, |V_2|\}}.
\end{aligned}$$

For given vertex  $n$ , if  $s_1 n, s_2 n, \dots, s_k n \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum neutrosophic cardinality between all k-number-dominating sets is called neutrosophic k-number-dominating number and it's denoted by

$$\mathcal{N}_n^k(CMC_{\sigma_1, \sigma_2}) = \min_{x_j \in \{x_1, x_2, \dots, x_{2k}\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^{2k} \sigma_i(x_j)$$

where  $k = 1, 2, \dots, \min\{|V_1|, |V_2|\}$ ; and corresponded to  $k$ -number-dominating sets are

$$\begin{aligned} & \{n_1, n_2\}^1, \{n_1, n_4\}^1, \{n_1, n_6\}^1, \dots, \{n_1, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})} (n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1})\}^1, \\ & \{n_2, n_3\}^1, \{n_2, n_5\}^1, \{n_2, n_7\}^1, \dots, \{n_2, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1} (n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})})\}^1, \\ & \{n_3, n_4\}^1, \{n_3, n_6\}^1, \{n_3, n_8\}^1, \dots, \{n_3, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})} (n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1})\}^1, \\ & \dots \\ & \{n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})}\}^1, \\ & \dots, \\ & \{n_1, n_3, n_2, n_4\}^{1,2}, \dots, \{n_1, n_3, n_2, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})} (n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1})\}^{1,2}, \\ & \{n_1, n_3, n_6, n_8\}^{1,2}, \dots, \{n_1, n_3, n_6, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})} (n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1})\}^{1,2}, \\ & \{n_1, n_3, n_8, n_{10}\}^{1,2}, \dots, \{n_1, n_3, n_8, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})} (n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1})\}^{1,2}, \\ & \dots, \\ & \{n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-3}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-2}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})-1}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2})}\}^{1,2}, \\ & \dots, \\ & \{n_{i+1}, n_{i+2}, n_{i+3}, \dots, n_{i+2 \min\{|V_1|, |V_2|\}}\}^{1,2, \dots, \min\{|V_1|, |V_2|\}}. \end{aligned}$$

Thus

$$\mathcal{N}_n^k(CMC_{\sigma_1, \sigma_2}) = \min_{x_j \in \{x_1, x_2, \dots, x_{2k}\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^{2k} \sigma_i(x_j)$$

where  $k = 1, 2, \dots, \min\{|V_1|, |V_2|\}$ . □

**Proposition 3.18.** *Let  $NTG : (V, E, \sigma, \mu)$  be a complete-bipartite-neutrosophic graph. Then  $k$ -number-dominating number isn't equal to dominating number where  $k \neq 1$ .*

**Proposition 3.19.** *Let  $NTG : (V, E, \sigma, \mu)$  be a complete-bipartite-neutrosophic graph. Then the number of  $k$ -number-dominating sets is multiplying  $2^{|V_1|+|V_2|-2k}$  by multiplying  $|V_1|$  choose  $k$  by  $|V_2|$  choose  $k$ .*

**Proposition 3.20.** *Let  $NTG : (V, E, \sigma, \mu)$  be a complete-bipartite-neutrosophic graph. Then the number of  $k$ -number-dominating sets corresponded to  $k$ -number-dominating number is multiplying  $|V_1|$  choose  $k$  by  $|V_2|$  choose  $k$ .*

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

**Example 3.21.** There is one section for clarifications. In Figure (17), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given two neutrosophic vertices,  $n$  and  $n'$ , there is either one path with length one or one path with length two between them;
- (ii) in the setting of complete-bipartite, a vertex of dominating set corresponded to dominating number dominates as if it doesn't  $k$ -number-dominate so as dominating is different from  $k$ -number-dominating;

(iii) all minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\{n_1, n_2\}^1, \{n_1, n_3\}^1, \{n_4, n_2\}^1, \\ \{n_4, n_3\}^1, \{n_1, n_2, n_3, n_4\}^{1,2}.$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum cardinality between all k-number-dominating sets is called k-number-dominating number and it's denoted by  $\mathcal{N}^k(CMC_{\sigma_1, \sigma_2}) = 2k$ ,  $k = 1, 2$ ; and corresponded to k-number-dominating sets are

$$\{n_1, n_2\}^1, \{n_1, n_3\}^1, \{n_4, n_2\}^1, \\ \{n_4, n_3\}^1, \{n_1, n_2, n_3, n_4\}^{1,2};$$

(iv) there are nine k-number-dominating sets

$$\{n_1, n_2\}^1, \{n_1, n_3\}^1, \{n_4, n_2\}^1, \\ \{n_4, n_3\}^1, \{n_1, n_2, n_3, n_4\}^{1,2}, \{n_1, n_2, n_3\}^1, \\ \{n_1, n_2, n_4\}^1, \{n_1, n_3, n_4\}^1, \{n_4, n_2, n_3\}^1,$$

so as it's possible to have one of them as a set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is characteristic;

(v) there are five k-number-dominating sets

$$\{n_1, n_2\}^1, \{n_1, n_3\}^1, \{n_4, n_2\}^1, \\ \{n_4, n_3\}^1, \{n_1, n_2, n_3, n_4\}^{1,2},$$

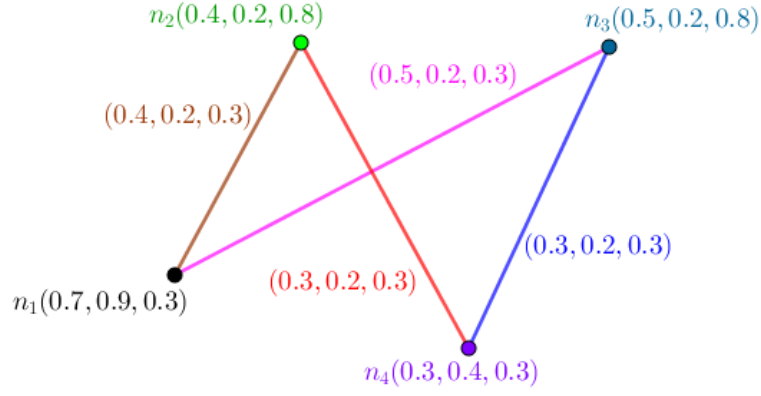
corresponded to k-number-dominating number as if there's one k-number-dominating set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is the determiner;

(vi) all minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\{n_1, n_2\}^1, \{n_1, n_3\}^1, \{n_4, n_2\}^1, \\ \{n_4, n_3\}^1, \{n_1, n_2, n_3, n_4\}^{1,2}.$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum neutrosophic cardinality between all k-number-dominating sets is called neutrosophic k-number-dominating number and it's denoted by  $\mathcal{N}_n^1(CMC_{\sigma_1, \sigma_2}) = 2.4$ ,  $\mathcal{N}_n^2(CMC_{\sigma_1, \sigma_2}) = 5.8$ ; and corresponded to k-number-dominating sets are

$$\{n_4, n_2\}^1, \{n_1, n_2, n_3, n_4\}^{1,2}.$$



**Figure 17.** A Neutrosophic Graph in the Viewpoint of its k-number-dominating number and its neutrosophic k-number-dominating number.

**Proposition 3.22.** Let  $NTG : (V, E, \sigma, \mu)$  be a complete- $t$ -partite-neutrosophic graph where  $t \geq 3$ . Then

$$\mathcal{N}_n^k(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t}) = \min_{x_j \in \{x_1, x_2, \dots, x_{2k}\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^{2k} \sigma_i(x_j)$$

where  $k = 1, 2, \dots, \min\{|V_1|, |V_2|, \dots, |V_t|\}$ .

*Proof.* Suppose  $CMC_{\sigma_1, \sigma_2, \dots, \sigma_t} : (V, E, \sigma, \mu)$  is a complete- $t$ -partite-neutrosophic graph. Every vertex in a part is k-number-dominated by another vertex in another part.

Assume same parity for same partition of vertex set which means  $V_i$  has odd indexes and  $V_j$  has even indexes. In the setting of complete- $t$ -partite, a vertex of dominating set corresponded to dominating number dominates as if it doesn't k-number-dominate so as dominating is different from k-number-dominating. All minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\begin{aligned} & \{n_1, n_2\}^1, \{n_1, n_4\}^1, \{n_1, n_6\}^1, \dots, \{n_1, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})}(\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1)\}^1, \\ & \{n_2, n_3\}^1, \{n_2, n_5\}^1, \{n_2, n_7\}^1, \dots, \{n_2, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1}(\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t}))\}^1, \\ & \{n_3, n_4\}^1, \{n_3, n_6\}^1, \{n_3, n_8\}^1, \dots, \{n_3, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})}(\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1)\}^1, \\ & \dots \\ & \{n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})}\}^1, \\ & \dots, \\ & \{n_1, n_3, n_2, n_4\}^{1,2}, \dots, \{n_1, n_3, n_2, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})}(\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1)\}^{1,2}, \\ & \{n_1, n_3, n_6, n_8\}^{1,2}, \dots, \{n_1, n_3, n_6, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})}(\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1)\}^{1,2}, \\ & \{n_1, n_3, n_8, n_{10}\}^{1,2}, \dots, \{n_1, n_3, n_8, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})}(\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1)\}^{1,2}, \\ & \dots, \\ & \{n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-3}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-2}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})}\}^{1,2}, \\ & \dots, \\ & \{n_{i+1}, n_{i+2}, n_{i+3}, \dots, n_{i+2 \min\{|V_1|, |V_2|, \dots, |V_t|\}}\}^{1,2, \dots, \min\{|V_1|, |V_2|, \dots, |V_t|\}}. \end{aligned}$$

For given vertex  $n$ , if  $s_1 n, s_2 n, \dots, s_k n \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$

k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum neutrosophic cardinality between all k-number-dominating sets is called neutrosophic k-number-dominating number and it's denoted by

$$\mathcal{N}_n^k(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t}) = \min_{x_j \in \{x_1, x_2, \dots, x_{2k}\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^{2k} \sigma_i(x_j)$$

where  $k = 1, 2, \dots, \min\{|V_1|, |V_2|, \dots, |V_t|\}$ ; and corresponded to k-number-dominating sets are

$$\begin{aligned} & \{n_1, n_2\}^1, \{n_1, n_4\}^1, \{n_1, n_6\}^1, \dots, \{n_1, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})}(\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1)\}^1, \\ & \{n_2, n_3\}^1, \{n_2, n_5\}^1, \{n_2, n_7\}^1, \dots, \{n_2, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1}(\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t}))\}^1, \\ & \{n_3, n_4\}^1, \{n_3, n_6\}^1, \{n_3, n_8\}^1, \dots, \{n_3, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})}(\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1)\}^1, \\ & \dots \\ & \{n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})}\}^1, \\ & \dots, \\ & \{n_1, n_3, n_2, n_4\}^{1,2}, \dots, \{n_1, n_3, n_2, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})}(\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1)\}^{1,2}, \\ & \{n_1, n_3, n_6, n_8\}^{1,2}, \dots, \{n_1, n_3, n_6, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})}(\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1)\}^{1,2}, \\ & \{n_1, n_3, n_8, n_{10}\}^{1,2}, \dots, \{n_1, n_3, n_8, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})}(\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1)\}^{1,2}, \\ & \dots, \\ & \{n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-3}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-2}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})-1}, n_{\mathcal{O}(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t})}\}^{1,2}, \\ & \dots, \\ & \{n_{i+1}, n_{i+2}, n_{i+3}, \dots, n_{i+2 \min\{|V_1|, |V_2|, \dots, |V_t|\}}\}^{1,2, \dots, \min\{|V_1|, |V_2|, \dots, |V_t|\}}. \end{aligned}$$

Thus

$$\mathcal{N}_n^k(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t}) = \min_{x_j \in \{x_1, x_2, \dots, x_{2k}\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^{2k} \sigma_i(x_j)$$

where  $k = 1, 2, \dots, \min\{|V_1|, |V_2|, \dots, |V_t|\}$ . □

**Proposition 3.23.** Let  $NTG : (V, E, \sigma, \mu)$  be a complete-t-partite-neutrosophic graph. Then k-number-dominating number isn't equal to dominating number where  $k \neq 1$ .

**Proposition 3.24.** Let  $NTG : (V, E, \sigma, \mu)$  be a complete-t-partite-neutrosophic graph. Then the number of k-number-dominating sets is multiplying  $2^{|V_i|+|V_j|-2k}$  by the summation of multiplying  $|V_i|$  choose  $k$  by  $|V_j|$  choose  $k$  on  $i$  and  $j$ .

**Proposition 3.25.** Let  $NTG : (V, E, \sigma, \mu)$  be a complete-t-partite-neutrosophic graph. Then the number of k-number-dominating sets corresponded to k-number-dominating number is the summation of multiplying  $|V_i|$  choose  $k$  by  $|V_j|$  choose  $k$  on  $i$  and  $j$ .

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

**Example 3.26.** There is one section for clarifications. In Figure (18), a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given two neutrosophic vertices,  $n$  and  $n'$ , there is either one path with length one or one path with length two between them; 1074  
1075
- (ii) in the setting of complete-t-partite, a vertex of dominating set corresponded to dominating number dominates as if it doesn't k-number-dominate so as dominating is different from k-number-dominating; 1076  
1077  
1078
- (iii) all minimal k-number-dominating sets corresponded to k-number-dominating number are 1079  
1080

$$\begin{aligned} &\{n_1, n_2\}^1, \{n_1, n_3\}^1, \{n_1, n_5\}^1, \\ &\{n_4, n_2\}^1, \{n_4, n_3\}^1, \{n_4, n_5\}^1, \\ &\{n_1, n_4, n_2, n_3\}^{1,2}, \{n_1, n_4, n_2, n_5\}^{1,2}, \{n_1, n_4, n_3, n_5\}^{1,2}. \end{aligned}$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum cardinality between all k-number-dominating sets is called k-number-dominating number and it's denoted by  $\mathcal{N}^k(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t}) = 2k$ ,  $k = 1, 2$ ; and corresponded to k-number-dominating sets are 1081  
1082  
1083  
1084  
1085  
1086  
1087  
1088  
1089

$$\begin{aligned} &\{n_1, n_2\}^1, \{n_1, n_3\}^1, \{n_1, n_5\}^1, \\ &\{n_4, n_2\}^1, \{n_4, n_3\}^1, \{n_4, n_5\}^1, \\ &\{n_1, n_4, n_2, n_3\}^{1,2}, \{n_1, n_4, n_2, n_5\}^{1,2}, \{n_1, n_4, n_3, n_5\}^{1,2}; \end{aligned}$$

- (iv) there are eighteen k-number-dominating sets 1090

$$\begin{aligned} &\{n_1, n_2\}^1, \{n_1, n_3\}^1, \{n_1, n_5\}^1, \\ &\{n_4, n_2\}^1, \{n_4, n_3\}^1, \{n_4, n_5\}^1, \\ &\{n_1, n_2, n_3, n_4\}^{1,2}, \{n_1, n_2, n_3, n_5\}^{1,2}, \{n_1, n_2, n_4, n_5\}^{1,2}, \\ &\{n_1, n_2, n_3\}^1, \{n_1, n_2, n_4\}^1, \{n_1, n_2, n_5\}^1, \\ &\{n_1, n_3, n_4\}^1, \{n_1, n_3, n_5\}^1, \{n_4, n_2, n_3\}^1, \\ &\{n_4, n_2, n_5\}^1, \{n_4, n_3, n_5\}^1, \{n_1, n_2, n_3, n_4, n_5\}^{1,2}, \end{aligned}$$

so as it's possible to have one of them as a set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is characteristic; 1091  
1092

- (v) there are nine k-number-dominating sets 1093

$$\begin{aligned} &\{n_1, n_2\}^1, \{n_1, n_3\}^1, \{n_1, n_5\}^1, \\ &\{n_4, n_2\}^1, \{n_4, n_3\}^1, \{n_4, n_5\}^1, \\ &\{n_1, n_4, n_2, n_3\}^{1,2}, \{n_1, n_4, n_2, n_5\}^{1,2}, \{n_1, n_4, n_3, n_5\}^{1,2}, \end{aligned}$$

corresponded to k-number-dominating number as if there's one k-number-dominating set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is the determiner; 1094  
1095  
1096

- (vi) all minimal k-number-dominating sets corresponded to k-number-dominating number are 1097  
1098

$$\begin{aligned} &\{n_1, n_2\}^1, \{n_1, n_3\}^1, \{n_1, n_5\}^1, \\ &\{n_4, n_2\}^1, \{n_4, n_3\}^1, \{n_4, n_5\}^1, \\ &\{n_1, n_4, n_2, n_3\}^{1,2}, \{n_1, n_4, n_2, n_5\}^{1,2}, \{n_1, n_4, n_3, n_5\}^{1,2}. \end{aligned}$$



setting of wheel, a vertex of dominating set corresponded to dominating number dominates as if it doesn't k-number-dominate so as dominating is different from k-number-dominating. All minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\begin{aligned} & \{n_{\mathcal{O}(WHL_{1,\sigma_2})}\}^1, \{n_2, n_4, \dots, n_{\mathcal{O}(WHL_{1,\sigma_2})-1}(n_{\mathcal{O}(WHL_{1,\sigma_2})-2})\}^{1,2}, \\ & \{n_3, n_5, \dots, n_{\mathcal{O}(WHL_{1,\sigma_2})-2}(n_{\mathcal{O}(WHL_{1,\sigma_2})-1})\}^{1,2}, \\ & \{n_1, n_2, n_4, \dots, n_{\mathcal{O}(WHL_{1,\sigma_2})-1}(n_{\mathcal{O}(WHL_{1,\sigma_2})-2})\}^{1,2,3}, \\ & \{n_1, n_3, n_5, \dots, n_{\mathcal{O}(WHL_{1,\sigma_2})-2}(n_{\mathcal{O}(WHL_{1,\sigma_2})-1})\}^{1,2,3}, \\ & \{n_1, n_2, n_3, \dots, n_{\mathcal{O}(WHL_{1,\sigma_2})-3}, n_{\mathcal{O}(WHL_{1,\sigma_2})-2}, n_{\mathcal{O}(WHL_{1,\sigma_2})-1}\}^{\mathcal{O}(WHL_{1,\sigma_2})-1}. \end{aligned}$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum neutrosophic cardinality between all k-number-dominating sets is called neutrosophic k-number-dominating number and it's denoted by

$$\begin{aligned} \mathcal{N}_n^1(WHL_{1,\sigma_2}) &= \sum_{i=1}^3 \sigma_i(c). \\ \mathcal{N}_n^2(WHL_{1,\sigma_2}) &= \min_{x_j \in \{x_1, x_2, \dots, x_{\lfloor \frac{\mathcal{O}(WHL_{1,\sigma_2})}{2} \rfloor}\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^{\lfloor \frac{\mathcal{O}(WHL_{1,\sigma_2})}{2} \rfloor} \sigma_i(x_j). \\ \mathcal{N}_n^3(WHL_{1,\sigma_2}) &= \min_{x_j \in \{x_1, x_2, \dots, x_{\lfloor \frac{\mathcal{O}(WHL_{1,\sigma_2})}{2} \rfloor}\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^{\lfloor \frac{\mathcal{O}(WHL_{1,\sigma_2})}{2} \rfloor} \sigma_i(x_j) + \sum_{i=1}^3 \sigma_i(c). \\ \mathcal{N}_n^{\mathcal{O}(WHL_{1,\sigma_2})-1}(WHL_{1,\sigma_2}) &= \min_{x_j \in \{x_1, x_2, \dots, x_{\mathcal{O}(WHL_{1,\sigma_2})-1}\} \subseteq V \setminus \{c\}} \sum_{i=1}^3 \sum_{j=1}^{\mathcal{O}(WHL_{1,\sigma_2})-1} \sigma_i(x_j). \\ k &= 1, 2, 3, \mathcal{O}(WHL_{1,\sigma_2}) - 1; \end{aligned}$$

and corresponded to k-number-dominating sets are

$$\begin{aligned} & \{n_{\mathcal{O}(WHL_{1,\sigma_2})}\}^1, \{n_2, n_4, \dots, n_{\mathcal{O}(WHL_{1,\sigma_2})-1}(n_{\mathcal{O}(WHL_{1,\sigma_2})-2})\}^{1,2}, \\ & \{n_3, n_5, \dots, n_{\mathcal{O}(WHL_{1,\sigma_2})-2}(n_{\mathcal{O}(WHL_{1,\sigma_2})-1})\}^{1,2}, \\ & \{n_1, n_2, n_4, \dots, n_{\mathcal{O}(WHL_{1,\sigma_2})-1}(n_{\mathcal{O}(WHL_{1,\sigma_2})-2})\}^{1,2,3}, \\ & \{n_1, n_3, n_5, \dots, n_{\mathcal{O}(WHL_{1,\sigma_2})-2}(n_{\mathcal{O}(WHL_{1,\sigma_2})-1})\}^{1,2,3}, \\ & \{n_1, n_2, n_3, \dots, n_{\mathcal{O}(WHL_{1,\sigma_2})-3}, n_{\mathcal{O}(WHL_{1,\sigma_2})-2}, n_{\mathcal{O}(WHL_{1,\sigma_2})-1}\}^{\mathcal{O}(WHL_{1,\sigma_2})-1}. \end{aligned}$$

Thus

$$\begin{aligned} \mathcal{N}_n^1(WHL_{1,\sigma_2}) &= \sum_{i=1}^3 \sigma_i(c). \\ \mathcal{N}_n^2(WHL_{1,\sigma_2}) &= \min_{x_j \in \{x_1, x_2, \dots, x_{\lfloor \frac{\mathcal{O}(WHL_{1,\sigma_2})}{2} \rfloor}\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^{\lfloor \frac{\mathcal{O}(WHL_{1,\sigma_2})}{2} \rfloor} \sigma_i(x_j). \end{aligned}$$

$$\mathcal{N}_n^3(WHL_{1,\sigma_2}) = \min_{x_j \in \{x_1, x_2, \dots, x_{\lfloor \frac{\mathcal{O}(WHL_{1,\sigma_2})}{2} \rfloor}\} \subseteq V} \sum_{i=1}^3 \sum_{j=1}^{\lfloor \frac{\mathcal{O}(WHL_{1,\sigma_2})}{2} \rfloor} \sigma_i(x_j) + \sum_{i=1}^3 \sigma_i(c).$$

$$\mathcal{N}_n^{\mathcal{O}(WHL_{1,\sigma_2})-1}(WHL_{1,\sigma_2}) = \min_{x_j \in \{x_1, x_2, \dots, x_{\mathcal{O}(WHL_{1,\sigma_2})-1}\} \subseteq V \setminus \{c\}} \sum_{i=1}^3 \sum_{j=1}^{\mathcal{O}(WHL_{1,\sigma_2})-1} \sigma_i(x_j).$$

$$k = 1, 2, 3, \mathcal{O}(WHL_{1,\sigma_2}) - 1.$$

□ 1115

**Proposition 3.28.** Let  $NTG : (V, E, \sigma, \mu)$  be a wheel-neutrosophic graph. Then  $k$ -number-dominating number isn't equal to dominating number where  $k > 1$ .

**Proposition 3.29.** Let  $NTG : (V, E, \sigma, \mu)$  be a wheel-partite-neutrosophic graph. Then the number of  $k$ -number-dominating sets corresponded to  $k$ -number-dominating number is one where  $k = 1$ .

**Proposition 3.30.** Let  $NTG : (V, E, \sigma, \mu)$  be a wheel-partite-neutrosophic graph. Then the number of  $k$ -number-dominating sets corresponded to  $k$ -number-dominating number is one where  $k = \mathcal{O}(WHL_{1,\sigma_2}) - 1$ .

The clarifications about results are in progress as follows. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

**Example 3.31.** There is one section for clarifications. In Figure (19), a wheel-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given two neutrosophic vertices,  $s$  and  $n_1$ , there's only one edge between them;
- (ii) in the setting of wheel, a vertex of dominating set corresponded to dominating number dominates as if it doesn't  $k$ -number-dominate so as dominating is different from  $k$ -number-dominating;
- (iii) all minimal  $k$ -number-dominating sets corresponded to  $k$ -number-dominating number are

$$\{n_1\}^1, \{n_2, n_4\}^{1,2}, \{n_3, n_5\}^{1,2},$$

$$\{n_2, n_4, n_1\}^{1,2,3}, \{n_3, n_5, n_1\}^{1,2,3}, \{n_2, n_3, n_4, n_5\}^{1,2,3,4}.$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$   $k$ -number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$   $k$ -number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called  $k$ -number-dominating set. The minimum cardinality between all  $k$ -number-dominating sets is called  $k$ -number-dominating number and it's denoted by  $\mathcal{N}^k(WHL_{1,\sigma_2}) = k$ ,  $k = 1, 2, 3, 4$ ; and corresponded to  $k$ -number-dominating sets are

$$\{n_1\}^1, \{n_2, n_4\}^{1,2}, \{n_3, n_5\}^{1,2},$$

$$\{n_2, n_4, n_1\}^{1,2,3}, \{n_3, n_5, n_1\}^{1,2,3}, \{n_2, n_3, n_4, n_5\}^{1,2,3,4};$$

(iv) there are twenty k-number-dominating sets

$$\begin{aligned} &\{n_1\}^1, \{n_1, n_2\}^1, \{n_1, n_3\}^1, \\ &\{n_1, n_4\}^1, \{n_1, n_5\}^1, \{n_2, n_3\}^1, \\ &\{n_2, n_4\}^{1,2}, \{n_2, n_5\}^1, \{n_3, n_4\}^1, \\ &\{n_3, n_5\}^{1,2}, \{n_4, n_5\}^1, \{n_1, n_2, n_3\}^{1,2}, \\ &\{n_1, n_2, n_4\}^{1,2,3}, \{n_1, n_2, n_5\}^{1,2}, \{n_2, n_3, n_4\}^{1,2}, \\ &\{n_2, n_3, n_5\}^{1,2}, \{n_3, n_4, n_5\}^{1,2}, \{n_1, n_2, n_3, n_4\}^{1,2,3}, \\ &\{n_1, n_2, n_3, n_5\}^{1,2,3}, \{n_2, n_3, n_4, n_5\}^{1,2,3,4}; \end{aligned}$$

so as it's possible to have one of them as a set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is characteristic;

(v) there are six k-number-dominating sets

$$\begin{aligned} &\{n_1\}^1, \{n_2, n_4\}^{1,2}, \{n_3, n_5\}^{1,2}, \\ &\{n_2, n_4, n_1\}^{1,2,3}, \{n_3, n_5, n_1\}^{1,2,3}, \{n_2, n_3, n_4, n_5\}^{1,2,3,4}; \end{aligned}$$

corresponded to k-number-dominating number as if there's one k-number-dominating set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is the determiner;

(vi) all minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\begin{aligned} &\{n_1\}^1, \{n_2, n_4\}^{1,2}, \{n_3, n_5\}^{1,2}, \\ &\{n_2, n_4, n_1\}^{1,2,3}, \{n_3, n_5, n_1\}^{1,2,3}, \{n_2, n_3, n_4, n_5\}^{1,2,3,4}. \end{aligned}$$

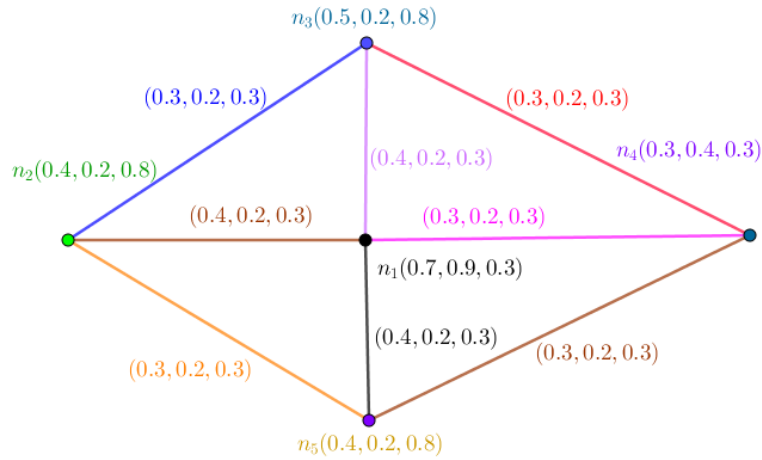
For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum neutrosophic cardinality between all k-number-dominating sets is called neutrosophic k-number-dominating number and it's denoted by  $\mathcal{N}_n^1(WHL_{1,\sigma_2}) = 1.9, \mathcal{N}_n^2(WHL_{1,\sigma_2}) = 2.4, \mathcal{N}_n^3(WHL_{1,\sigma_2}) = 4.3, \mathcal{N}_n^4(WHL_{1,\sigma_2}) = 5.3$ ; and corresponded to k-number-dominating sets are

$$\begin{aligned} &\{n_1\}^1, \{n_2, n_4\}^{1,2}, \{n_2, n_4, n_1\}^{1,2,3}, \\ &\{n_2, n_3, n_4, n_5\}^{1,2,3,4}. \end{aligned}$$

## 4 Applications in Time Table and Scheduling

In this section, two applications for time table and scheduling are provided where the models are either complete models which mean complete connections are formed as individual and family of complete models with common neutrosophic vertex set or quasi-complete models which mean quasi-complete connections are formed as individual and family of quasi-complete models with common neutrosophic vertex set.

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has importance to avoid mixing up.



**Figure 19.** A Neutrosophic Graph in the Viewpoint of its k-number-dominating number and its neutrosophic k-number-dominating number.

**Step 1. (Definition)** Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

**Step 2. (Issue)** Scheduling of program has faced with difficulties to differ amid consecutive sections. Beyond that, sometimes sections are not the same.

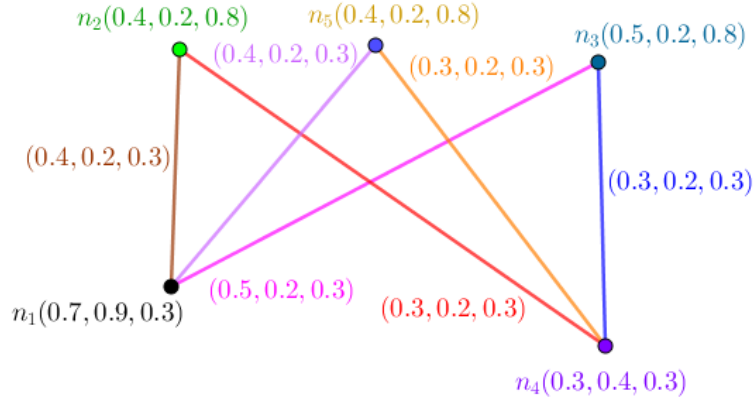
**Step 3. (Model)** The situation is designed as a model. The model uses data to assign every section and to assign to relation amid sections, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relations amid them. Table (1), clarifies about the assigned numbers to these situations.

**Table 1.** Scheduling concerns its Subjects and its Connections as a neutrosophic graph in a Model.

Sections of $NTG$	$n_1$	$n_2 \cdots$	$n_5$
Values	$(0.7, 0.9, 0.3)$	$(0.4, 0.2, 0.8) \cdots$	$(0.4, 0.2, 0.8)$
Connections of $NTG$	$E_1$	$E_2 \cdots$	$E_6$
Values	$(0.4, 0.2, 0.3)$	$(0.5, 0.2, 0.3) \cdots$	$(0.3, 0.2, 0.3)$

#### 4.1 Case 1: Complete-t-partite Model alongside its k-number-dominating number and its neutrosophic k-number-dominating number

**Step 4. (Solution)** The neutrosophic graph alongside its k-number-dominating number and its neutrosophic k-number-dominating number as model, propose to use specific number. Every subject has connection with some subjects. Thus the connection is applied as possible and the model demonstrates quasi-full connections as quasi-possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is star, the number is different. Also, it holds for other types such



**Figure 20.** A Neutrosophic Graph in the Viewpoint of its k-number-dominating number and its neutrosophic k-number-dominating number

that complete, wheel, path, and cycle. The collection of situations is another application of its k-number-dominating number and its neutrosophic k-number-dominating number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, there are five subjects which are represented as Figure (20). This model is strong and even more it's quasi-complete. And the study proposes using specific number which is called its k-number-dominating number and its neutrosophic k-number-dominating number. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to this model and situation to compare them with same situations to get more precise. Consider Figure (20). In Figure (20), an complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) For given two neutrosophic vertices,  $n$  and  $n'$ , there is either one path with length one or one path with length two between them;
- (ii) in the setting of complete-t-partite, a vertex of dominating set corresponded to dominating number dominates as if it doesn't k-number-dominate so as dominating is different from k-number-dominating;
- (iii) all minimal k-number-dominating sets corresponded to k-number-dominating number are

$$\begin{aligned} &\{n_1, n_2\}^1, \{n_1, n_3\}^1, \{n_1, n_5\}^1, \\ &\{n_4, n_2\}^1, \{n_4, n_3\}^1, \{n_4, n_5\}^1, \\ &\{n_1, n_4, n_2, n_3\}^{1,2}, \{n_1, n_4, n_2, n_5\}^{1,2}, \{n_1, n_4, n_3, n_5\}^{1,2}. \end{aligned}$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum cardinality between all k-number-dominating sets is called k-number-dominating number and it's denoted by  $\mathcal{N}^k(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t}) = 2k$ ,  $k = 1, 2$ ; and corresponded to

k-number-dominating sets are

$$\begin{aligned} &\{n_1, n_2\}^1, \{n_1, n_3\}^1, \{n_1, n_5\}^1, \\ &\{n_4, n_2\}^1, \{n_4, n_3\}^1, \{n_4, n_5\}^1, \\ &\{n_1, n_4, n_2, n_3\}^{1,2}, \{n_1, n_4, n_2, n_5\}^{1,2}, \{n_1, n_4, n_3, n_5\}^{1,2}; \end{aligned}$$

(iv) there are eighteen k-number-dominating sets

$$\begin{aligned} &\{n_1, n_2\}^1, \{n_1, n_3\}^1, \{n_1, n_5\}^1, \\ &\{n_4, n_2\}^1, \{n_4, n_3\}^1, \{n_4, n_5\}^1, \\ &\{n_1, n_2, n_3, n_4\}^{1,2}, \{n_1, n_2, n_3, n_5\}^{1,2}, \{n_1, n_2, n_4, n_5\}^{1,2}, \\ &\{n_1, n_2, n_3\}^1, \{n_1, n_2, n_4\}^1, \{n_1, n_2, n_5\}^1, \\ &\{n_1, n_3, n_4\}^1, \{n_1, n_3, n_5\}^1, \{n_4, n_2, n_3\}^1, \\ &\{n_4, n_2, n_5\}^1, \{n_4, n_3, n_5\}^1, \{n_1, n_2, n_3, n_4, n_5\}^{1,2}, \end{aligned}$$

so as it's possible to have one of them as a set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is characteristic;

(v) there are nine k-number-dominating sets

$$\begin{aligned} &\{n_1, n_2\}^1, \{n_1, n_3\}^1, \{n_1, n_5\}^1, \\ &\{n_4, n_2\}^1, \{n_4, n_3\}^1, \{n_4, n_5\}^1, \\ &\{n_1, n_4, n_2, n_3\}^{1,2}, \{n_1, n_4, n_2, n_5\}^{1,2}, \{n_1, n_4, n_3, n_5\}^{1,2}, \end{aligned}$$

corresponded to k-number-dominating number as if there's one k-number-dominating set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is the determiner;

(vi) all minimal k-number-dominating sets corresponded to k-number-dominating number are

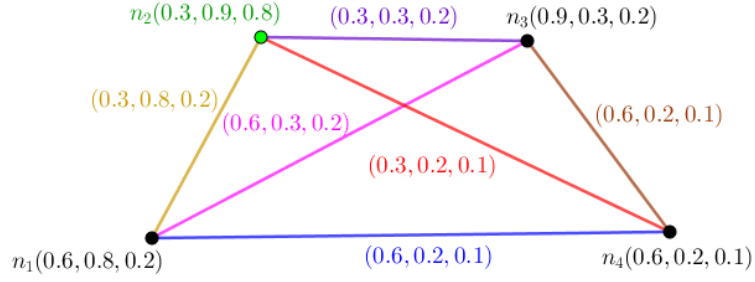
$$\begin{aligned} &\{n_1, n_2\}^1, \{n_1, n_3\}^1, \{n_1, n_5\}^1, \\ &\{n_4, n_2\}^1, \{n_4, n_3\}^1, \{n_4, n_5\}^1, \\ &\{n_1, n_4, n_2, n_3\}^{1,2}, \{n_1, n_4, n_2, n_5\}^{1,2}, \{n_1, n_4, n_3, n_5\}^{1,2}. \end{aligned}$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least a neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum neutrosophic cardinality between all k-number-dominating sets is called neutrosophic k-number-dominating number and it's denoted by  $\mathcal{N}_n^1(CMC_{\sigma_1, \sigma_2, \dots, \sigma_t}) = 2.4$ ,  $\mathcal{N}_n^2(CMC_{\sigma_1, \sigma_2}) = 5.7$ ; and corresponded to k-number-dominating sets are

$$\{n_4, n_2\}^1, \{n_4, n_5\}^1, \{n_1, n_4, n_2, n_5\}^{1,2}.$$

## 4.2 Case 2: Complete Model alongside its Neutrosophic Graph in the Viewpoint of its k-number-dominating number and its neutrosophic k-number-dominating number

**Step 4. (Solution)** The neutrosophic graph alongside its k-number-dominating number and its neutrosophic k-number-dominating number as model, propose to



**Figure 21.** A Neutrosophic Graph in the Viewpoint of its k-number-dominating number and its neutrosophic k-number-dominating number

use specific number. Every subject has connection with every given subject in deemed way. Thus the connection applied as possible and the model demonstrates full connections as possible between parts but with different view where symmetry amid vertices and edges are the matters. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete multipartite, the number is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application of its k-number-dominating number and its neutrosophic k-number-dominating number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, there are four subjects which are represented in the formation of one model as Figure (21). This model is neutrosophic strong as individual and even more it's complete. And the study proposes using specific number which is called its k-number-dominating number and its neutrosophic k-number-dominating number for this model. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to these models as individual. A model as a collection of situations to compare them with another model as a collection of situations to get more precise. Consider Figure (21). There is one section for clarifications.

- (i) For given neutrosophic vertex,  $s$ , there's an edge with other vertices;
- (ii) in the setting of complete, a vertex of dominating set corresponded to dominating number dominates as if it doesn't k-number-dominate so as dominating is different from k-number-dominating. Dominating number and k-number-dominating number are the same if  $k = 1$ ;
- (iii) all k-number-dominating sets corresponded to k-number-dominating number are

$$\begin{aligned} &\{n_1\}^1, \{n_2\}^1, \{n_3\}^1, \\ &\{n_4\}^1, \{n_1, n_2\}^2, \{n_1, n_3\}^2, \\ &\{n_1, n_4\}^2, \{n_2, n_3\}^2, \{n_2, n_4\}^2, \\ &\{n_3, n_4\}^2, \{n_1, n_2, n_3\}^3, \{n_1, n_2, n_4\}^3, \\ &\{n_2, n_3, n_4\}^3, \{n_1, n_2, n_3, n_4\}^4. \end{aligned}$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every

neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum cardinality between all k-number-dominating sets is called k-number-dominating number and it's denoted by  $\mathcal{N}^k(CMT_\sigma) = k$ ,  $k = 1, 2, \dots, \mathcal{O}(CMT_\sigma)$ ; and corresponded to k-number-dominating sets are

$$\begin{aligned} &\{n_1\}^1, \{n_2\}^1, \{n_3\}^1, \\ &\{n_4\}^1, \{n_1, n_2\}^2, \{n_1, n_3\}^2, \\ &\{n_1, n_4\}^2, \{n_2, n_3\}^2, \{n_2, n_4\}^2, \\ &\{n_3, n_4\}^2, \{n_1, n_2, n_3\}^3, \{n_1, n_2, n_4\}^3, \\ &\{n_2, n_3, n_4\}^3, \{n_1, n_2, n_3, n_4\}^4; \end{aligned}$$

(iv) there are some k-number-dominating sets

$$\begin{aligned} &\{n_1\}^{1,2,3,4}, \{n_2\}^{1,2,3,4}, \{n_3\}^{1,2,3,4}, \\ &\{n_4\}^{1,2,3,4}, \{n_1, n_2\}^{2,3,4}, \{n_1, n_3\}^{2,3,4}, \\ &\{n_1, n_4\}^{2,3,4}, \{n_2, n_3\}^{2,3,4}, \{n_2, n_4\}^{2,3,4}, \\ &\{n_3, n_4\}^{2,3,4}, \{n_1, n_2, n_3\}^{3,4}, \{n_1, n_2, n_4\}^{3,4}, \\ &\{n_2, n_3, n_4\}^{3,4}, \{n_1, n_2, n_3, n_4\}^4. \end{aligned}$$

so as it's possible to have one of them as a set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is characteristic;

(v) there are some k-number-dominating sets

$$\begin{aligned} &\{n_1\}^1, \{n_2\}^1, \{n_3\}^1, \\ &\{n_4\}^1, \{n_1, n_2\}^2, \{n_1, n_3\}^2, \\ &\{n_1, n_4\}^2, \{n_2, n_3\}^2, \{n_2, n_4\}^2, \\ &\{n_3, n_4\}^2, \{n_1, n_2, n_3\}^3, \{n_1, n_2, n_4\}^3, \\ &\{n_2, n_3, n_4\}^3, \{n_1, n_2, n_3, n_4\}^4, \end{aligned}$$

corresponded to k-number-dominating number as if there's one k-number-dominating set corresponded to neutrosophic k-number-dominating number so as neutrosophic cardinality is the determiner;

(vi) all k-number-dominating sets corresponded to k-number-dominating number are

$$\begin{aligned} &\{n_1\}^1, \{n_2\}^1, \{n_3\}^1, \\ &\{n_4\}^1, \{n_1, n_2\}^2, \{n_1, n_3\}^2, \\ &\{n_1, n_4\}^2, \{n_2, n_3\}^2, \{n_2, n_4\}^2, \\ &\{n_3, n_4\}^2, \{n_1, n_2, n_3\}^3, \{n_1, n_2, n_4\}^3, \\ &\{n_2, n_3, n_4\}^3, \{n_1, n_2, n_3, n_4\}^4. \end{aligned}$$

For given vertex  $n$ , if  $s_1n, s_2n, \dots, s_kn \in E$ , then  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ . Let  $S$  be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex  $n$  in  $V \setminus S$ , there are at least neutrosophic vertices  $s_1, s_2, \dots, s_k$  in  $S$  such that  $s_1, s_2, \dots, s_k$  k-number-dominate  $n$ , then the set of neutrosophic vertices,  $S$  is called k-number-dominating set. The minimum

neutrosophic cardinality between all k-number-dominating sets is called  
neutrosophic k-number-dominating number and it's denoted by  
 $\mathcal{N}_n^k(CMT_\sigma) = 0.9^1, 2.3^2, 3.9^3, 5.9^4$ ; and corresponded to  
k-number-dominating sets are

$$\{n_4\}^1, \{n_4, n_3\}^2, \{n_4, n_3, n_1\}^3, \\ \{n_1, n_2, n_3, n_4\}^4.$$

## 5 Open Problems

In this section, some questions and problems are proposed to give some avenues to pursue this study. The structures of the definitions and results give some ideas to make new settings which are eligible to extend and to create new study.

Notion concerning its k-number-dominating number and its neutrosophic k-number-dominating number are defined in neutrosophic graphs. Thus,

**Question 5.1.** *Is it possible to use other types of its k-number-dominating number and its neutrosophic k-number-dominating number?*

**Question 5.2.** *Are existed some connections amid different types of its k-number-dominating number and its neutrosophic k-number-dominating number in neutrosophic graphs?*

**Question 5.3.** *Is it possible to construct some classes of neutrosophic graphs which have “nice” behavior?*

**Question 5.4.** *Which mathematical notions do make an independent study to apply these types in neutrosophic graphs?*

**Problem 5.5.** *Which parameters are related to this parameter?*

**Problem 5.6.** *Which approaches do work to construct applications to create independent study?*

**Problem 5.7.** *Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?*

## 6 Conclusion and Closing Remarks

In this section, concluding remarks and closing remarks are represented. The drawbacks of this article are illustrated. Some benefits and advantages of this study are highlighted.

This study uses two definitions concerning k-number-dominating number and neutrosophic k-number-dominating number arising from k-number-dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. Minimum number of k-number-dominated vertices, is a number which is representative based on those vertices. Minimum neutrosophic number of k-number-dominated vertices corresponded to k-number-dominating set is called neutrosophic k-number-dominating number. The connections of vertices which aren't clarified by minimum number of edges amid them differ them from each other and put them in different categories to represent a number which is called k-number-dominating number and neutrosophic k-number-dominating number arising from k-number-dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. Further studies could be about changes in the settings to compare these notions amid different settings of neutrosophic graphs theory. One way is finding some relations amid all definitions of notions to make sensible definitions. In Table (2), some limitations and advantages of this study are pointed out.

**Table 2.** A Brief Overview about Advantages and Limitations of this Study

Advantages	Limitations
1. <b>k-number-dominating Number of Model</b>	1. <b>Connections amid Classes</b>
2. <b>Neutrosophic k-number-dominating Number of Model</b>	
3. <b>Minimal k-number-dominating Sets</b>	2. <b>Study on Families</b>
4. <b>k-number-dominated Vertices amid all Vertices</b>	
5. <b>Acting on All Vertices</b>	3. <b>Same Models in Family</b>

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