Dual-Dominating Numbers in Neutrosophic Setting and Crisp Setting Obtained From Classes of Neutrosophic Graphs

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Abstract

New setting is introduced to study dual-dominating number and neutrosophic dual-dominating number arising from dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. Maximum number of dominated vertices, is a number which is representative based on those vertices. Maximum neutrosophic number of dominated vertices corresponded to dual-dominating set is called neutrosophic dual-dominating number. Forming sets from dominated vertices to figure out different types of number of vertices in the sets from dominated sets n in the terms of maximum number of vertices to get maximum number to assign to neutrosophic graphs is key type of approach to have these notions namely dual-dominating number and neutrosophic dual-dominating number arising from dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. Two numbers and one set are assigned to a neutrosophic graph, are obtained but now both settings lead to approach is on demand which is to compute and to find representatives of sets having largest number of dominated vertices from different types of sets in the terms of maximum number and maximum neutrosophic number forming it to get maximum number to assign to a neutrosophic graph. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then for given two vertices, s and n, if $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex s in S, there's at least one neutrosophic vertex n in $V \setminus S$ such that n dominates s, then the set of neutrosophic vertices, S is called dual-dominating set. The maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(NTG)$; for given two vertices, s and n, if $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex s in S, there's at least one neutrosophic vertex n in $V \setminus S$ such that n dominates s, then the set of neutrosophic vertices, S is called dual-dominating set. The maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(NTG)$. As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs, complete-t-partite-neutrosophic graphs, and wheel-neutrosophic graphs. The clarifications are also presented in both sections “Setting of dual-dominating number,” and “Setting of neutrosophic dual-dominating number,” for introduced results and used classes. This approach facilitates identifying sets which form dual-dominating number and neutrosophic dual-dominating number arising from dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. In both settings, some classes of well-known neutrosophic graphs are studied. Some clarifications for each result and each definition are provided. The cardinality of set of dominated vertices and neutrosophic cardinality of set of dominated vertices corresponded to dual-dominating set have eligibility to define dual-dominating number and neutrosophic dual-dominating number but different types of set of dominated vertices to define dual-dominating sets. Some results get more frameworks and perspective about these definitions. The way in that, different types of set of dominated vertices in the terms of maximum number to assign to neutrosophic graphs, opens the way to do some approaches. These notions are applied into neutrosophic graphs as individuals but not family of them as drawbacks for these notions. Finding special neutrosophic graphs which are well-known, is an open way to pursue this study. Neutrosophic dual-dominating notion is applied to different settings and classes of neutrosophic graphs. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.

Keywords: Dual-Dominating Number, Neutrosophic Dual-Dominating Number, Classes of Neutrosophic Graphs

AMS Subject Classification: 05C17, 05C22, 05E45

1 Background

Fuzzy set in **Ref.** [22] by Zadeh (1965), intuitionistic fuzzy sets in **Ref.** [3] by Atanassov (1986), a first step to a theory of the intuitionistic fuzzy graphs in **Ref.** [18] by Shannon and Atanassov (1994), a unifying field in logics neutrosophy: neutrosophic probability, set and logic, rehoboth in **Ref.** [19] by Smarandache (1998), single-valued neutrosophic sets in **Ref.** [20] by Wang et al. (2010), single-valued neutrosophic graphs in **Ref.** [5] by Broumi et al. (2016), operations on single-valued neutrosophic graphs in **Ref.** [1] by Akram and Shahzadi (2017), neutrosophic soft graphs in **Ref.** [17] by Shah and Hussain (2016), bounds on the average and minimum attendance in preference-based activity scheduling in **Ref.** [2] by Aronshtam and Ilani (2022), investigating the recoverable robust single machine scheduling problem under interval uncertainty in **Ref.** [4] by Bold and Goerigk (2022), new bounds for the b-chromatic number of vertex deleted graphs in **Ref.** [6] by Del-Vecchio and Kouider (2022), bipartite completion of colored graphs avoiding chordless cycles of given lengths in **Ref.** [7] by Elaine et al., infinite chromatic games in **Ref.** [12] by Janczewski et al. (2022), edge-disjoint rainbow triangles in edge-colored graphs in **Ref.** [13] by Li and Li (2022), rainbow triangles in arc-colored digraphs in **Ref.** [14] by Li et al. (2022), a sufficient condition for edge 6-colorable planar graphs with maximum degree 6 in **Ref.** [15] by Lu and Shi (2022), some comparative results concerning the Grundy and b-chromatic number of graphs in **Ref.** [16] by Masih and Zaker (2022), color neighborhood union conditions for proper edge-pancyclicity of edge-colored complete graphs in **Ref.** [21] by Wu et al. (2022), dimension and coloring alongside domination in neutrosophic hypergraphs in **Ref.** [9] by Henry Garrett (2022), three types of neutrosophic alliances based on connectedness and (strong) edges in **Ref.** [11] by Henry Garrett (2022), properties of SuperHyperGraph and neutrosophic SuperHyperGraph in **Ref.** [10] by Henry Garrett (2022), are studied. Also, some studies and researches about neutrosophic graphs, are proposed as a book in **Ref.** [8] by Henry Garrett (2022).

In this section, I use two subsections to illustrate a perspective about the background of this study.

1.1 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 1.1. *Is it possible to use mixed versions of ideas concerning "dual-dominating number", "neutrosophic dual-dominating number" and "Neutrosophic Graph" to define some notions which are applied to neutrosophic graphs?*

It's motivation to find notions to use in any classes of neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Having connection amid two paths have key roles to assign dual-dominating number and neutrosophic dual-dominating number arising from dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. Thus they're used to define new ideas which conclude to the structure of dual-dominating number and neutrosophic dual-dominating number arising from dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. The concept of having largest number of dominated vertices in the terms of crisp setting and in the terms of neutrosophic setting inspires us to study the behavior of all dominated vertices in the way that, some types of numbers, dual-dominating number and neutrosophic dual-dominating number arising from dominated vertices in neutrosophic graphs assigned to neutrosophic graphs, are the cases of study in the setting of individuals. In both settings, corresponded numbers conclude the discussion. Also, there are some avenues to extend these notions.

The framework of this study is as follows. In the beginning, I introduce basic definitions to clarify about preliminaries. In subsection "Preliminaries", new notions of dual-dominating number and neutrosophic dual-dominating number arising from dominated vertices in neutrosophic graphs assigned to neutrosophic graphs, are highlighted, are introduced and are clarified as individuals. In section "Preliminaries", Maximum number of dominated vertices, is a number which is representative based on those vertices, have the key role in this way. General results are obtained and also, the results about the basic notions of dual-dominating number and neutrosophic dual-dominating number arising from dominated vertices in neutrosophic graphs assigned to neutrosophic graphs, are elicited. Some classes of neutrosophic graphs are studied in the terms of dual-dominating number and neutrosophic dual-dominating number arising from dominated vertices in neutrosophic graphs assigned to neutrosophic graphs, in section "Setting of dual-dominating number," as individuals. In section "Setting of dual-dominating number," dual-dominating number is applied into individuals. As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs, complete-t-partite-neutrosophic graphs, and wheel-neutrosophic graphs. The clarifications are also presented in both sections "Setting of dual-dominating number," and "Setting of neutrosophic dual-dominating number," for introduced results and used classes. In section "Applications in Time Table and Scheduling", two applications are posed for quasi-complete and complete notions, namely complete-neutrosophic graphs and complete-t-partite-neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects and the mentioned models are considered as individual. In section "Open Problems", some problems and questions for further studies are proposed. In section "Conclusion and Closing Remarks", gentle discussion about results and applications is featured. In section "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions is formed.

1.2 Preliminaries

In this subsection, basic material which is used in this article, is presented. Also, new ideas and their clarifications are elicited.

Basic idea is about the model which is used. First definition introduces basic model.

Definition 1.2. (Graph).

$G = (V, E)$ is called a **graph** if V is a set of objects and E is a subset of $V \times V$ (E is a set of 2-subsets of V) where V is called **vertex set** and E is called **edge set**. Every two vertices have been corresponded to at most one edge.

Neutrosophic graph is the foundation of results in this paper which is defined as follows. Also, some related notions are demonstrated.

Definition 1.3. (Neutrosophic Graph And Its Special Case).

$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, and $\mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_i v_j \in E$,

$$\mu(v_i v_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

(i) : σ is called **neutrosophic vertex set**.

(ii) : μ is called **neutrosophic edge set**.

(iii) : $|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$.

(iv) : $\sum_{v \in V} \sum_{i=1}^{3} \sigma_i(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

(v) : $|E|$ is called **size** of NTG and it's denoted by $\mathcal{S}(NTG)$.

(vi) : $\sum_{e \in E} \sum_{i=1}^{3} \mu_i(e)$ is called **neutrosophic size** of NTG and it's denoted by $\mathcal{S}_n(NTG)$.

Some classes of well-known neutrosophic graphs are defined. These classes of neutrosophic graphs are used to form this study and the most results are about them.

Definition 1.4. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) : a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is called **path** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, \mathcal{O}(NTG) - 1$;

(ii) : **strength** of path $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is $\bigwedge_{i=0,\cdots,\mathcal{O}(NTG)-1} \mu(x_i x_{i+1})$;

(iii) : **connectedness** amid vertices x_0 and x_t is

$$\mu^{\infty}(x_0, x_t) = \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1});$$

(iv) : a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}, x_0$ is called **cycle** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, \mathcal{O}(NTG) - 1,\ x_{\mathcal{O}(NTG)} x_0 \in E$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$;

(v) : it's **t-partite** where V is partitioned to t parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge xy implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's complete, then it's denoted by $K_{\sigma_1,\sigma_2,\cdots,\sigma_t}$ where σ_i is σ on $V_i^{s_i}$ instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$;

(vi) : t-partite is **complete bipartite** if $t = 2$, and it's denoted by K_{σ_1,σ_2};

(vii) : complete bipartite is **star** if $|V_1| = 1$, and it's denoted by S_{1,σ_2};

$(viii)$: a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by W_{1,σ_2};

(ix) : it's **complete** where $\forall uv \in V,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$;

(x) : it's **strong** where $\forall uv \in E,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$.

To make them concrete, I bring preliminaries of this article in two upcoming definitions in other ways.

Definition 1.5. (Neutrosophic Graph And Its Special Case).

$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, and $\mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_iv_j \in E$,

$$\mu(v_iv_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

$|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$. $\Sigma_{v\in V}\sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

Definition 1.6. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then it's **complete** and denoted by CMT_σ if $\forall x, y \in V, xy \in E$ and $\mu(xy) = \sigma(x) \wedge \sigma(y)$; a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is called **path** and it's denoted by PTH where $x_ix_{i+1} \in E,\ i = 0, 1, \cdots, n-1$; a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}, x_0$ is called **cycle** and denoted by CYC where $x_ix_{i+1} \in E,\ i = 0, 1, \cdots, n-1,\ x_{\mathcal{O}(NTG)}x_0 \in E$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_iv_{i+1})$; it's **t-partite** where V is partitioned to t parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge xy implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's **complete**, then it's denoted by $CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}$ where σ_i is σ on $V_i^{s_i}$ instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$; t-partite is **complete bipartite** if $t = 2$, and it's denoted by CMT_{σ_1,σ_2}; complete bipartite is **star** if $|V_1| = 1$, and it's denoted by STR_{1,σ_2}; a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by WHL_{1,σ_2}.

Remark 1.7. Using notations which is mixed with literatures, are reviewed.

1. $NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3)),\ \mathcal{O}(NTG)$, and $\mathcal{O}_n(NTG)$;
2. $CMT_\sigma, PTH, CYC, STR_{1,\sigma_2}, CMT_{\sigma_1,\sigma_2}, CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}$, and WHL_{1,σ_2}.

Definition 1.8. (Dual-Dominating Numbers).

Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) for given two vertices, s and n, if $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex s in S, there's at least one neutrosophic vertex n in $V \setminus S$ such that n dominates s, then the set of neutrosophic vertices, S is called **dual-dominating set**. The maximum cardinality between all dual-dominating sets is called **dual-dominating number** and it's denoted by $\mathcal{D}(NTG)$;

(ii) for given two vertices, s and n, if $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let S be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex s in S, there's at least one neutrosophic vertex n in $V \setminus S$ such that n dominates s, then the set of neutrosophic vertices, S is called **dual-dominating set**. The maximum neutrosophic cardinality between all dual-dominating sets is called **neutrosophic dual-dominating number** and it's denoted by $\mathcal{D}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

Example 1.9. In Figure (1), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two vertices, s and n, $\mu(ns) = \sigma(n) \wedge \sigma(s)$. Thus s dominates n and n dominates s;

(ii) the existence of one vertex to do this function, dominating, is obvious thus this vertex form a set which is necessary and sufficient in the term of minimum dominating set and minimal dominating set;

(iii) for given two vertices, s and n, $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let $S = V \setminus \{n\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] If for every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = V \setminus \{n\})$ such that n dominates s, then the set of neutrosophic vertices, $S = V \setminus \{n\}$ is called dual-dominating set. The maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(NTG) = \mathcal{O}(NTG) - 1$;

(iv) the corresponded set doesn't have to be dominated by the set;

(v) V is exception when the set is considered in this notion;

(vi) for given two vertices, s and n, $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let $S = V \setminus \{n\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] If for every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = V \setminus \{n\})$ such that n dominates s, then the set of neutrosophic vertices, $S = V \setminus \{n\}$ is called dual-dominating set. The maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(NTG) = \mathcal{O}_n(NTG) - \sum_{i=1}^{3} \sigma_i(n_4) = 5$.

2 Setting of dual-dominating number

In this section, I provide some results in the setting of dual-dominating number. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, star-neutrosophic graph, bipartite-neutrosophic graph, t-partite-neutrosophic graph, and wheel-neutrosophic graph, are both of cases of study and classes which the results are about them.

Proposition 2.1. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{D}(CMT_\sigma) = \mathcal{O}(CMT_\sigma) - 1.$$

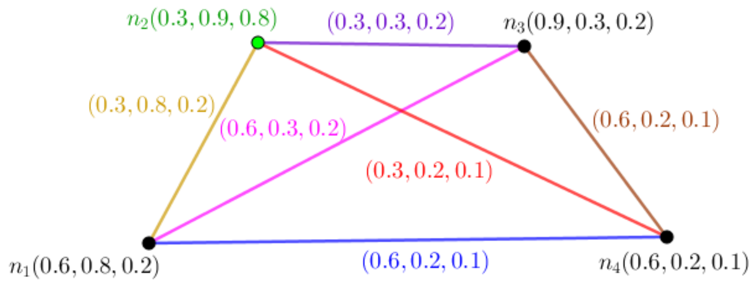

Figure 1. A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. For given two vertices, s and n, $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let $S = V \setminus \{n\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = V \setminus \{n\})$ such that n dominates s, then the set of neutrosophic vertices, $S = V \setminus \{n\}$ is called dual-dominating set. The maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(NTG) = \mathcal{O}(NTG) - 1$. Thus

$$\mathcal{D}(CMT_\sigma) = \mathcal{O}(CMT_\sigma) - 1.$$

□

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.2. In Figure (2), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two vertices, s and n, $\mu(ns) = \sigma(n) \wedge \sigma(s)$. Thus s dominates n and n dominates s;

(ii) the existence of one vertex to do this function, dominating, is obvious thus this vertex form a set which is necessary and sufficient in the term of minimum dominating set and minimal dominating set;

(iii) for given two vertices, s and n, $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let $S = V \setminus \{n\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] If for every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = V \setminus \{n\})$ such that n dominates s, then the set of neutrosophic vertices, $S = V \setminus \{n\}$ is called dual-dominating set. The maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMT_\sigma) = \mathcal{O}(CMT_\sigma) - 1$;

(iv) the corresponded set doesn't have to be dominated by the set;

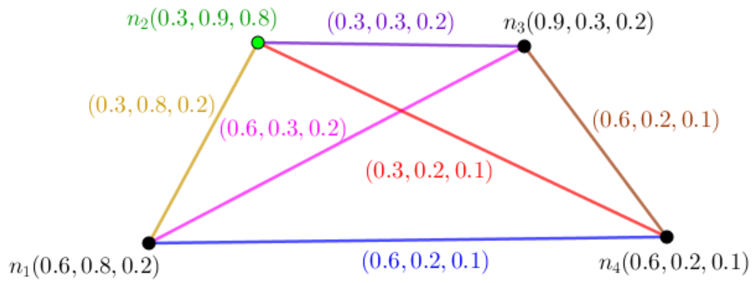

Figure 2. A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

(*v*) V is exception when the set is considered in this notion;

(*vi*) for given two vertices, s and n, $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let $S = V \setminus \{n\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] If for every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = V \setminus \{n\})$ such that n dominates s, then the set of neutrosophic vertices, $S = V \setminus \{n\}$ is called dual-dominating set. The maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(CMT_\sigma) = \mathcal{O}_n(CMT_\sigma) - \sum_{i=1}^{3} \sigma_i(n_4) = 5$.

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

Proposition 2.3. *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Then*

$$\mathcal{D}(PTH) = \lfloor \frac{2 \times \mathcal{O}(PTH)}{3} \rfloor.$$

Proof. Suppose $PTH : (V, E, \sigma, \mu)$ is a path-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(PTH)}$ be a path-neutrosophic graph. For given two vertices, x and y, there's one path from x to y. Let S be an intended set which is dual-dominating set. Despite leaves x_1, and $x_{\mathcal{O}(PTH)}$, two consecutive vertices belong to S. They could be dominated by previous vertex and upcoming vertex as if despite them so as maximal set S is constructed. Thus $S = \{x'_1, x'_2, \cdots, x'_{\lfloor \frac{2 \times \mathcal{O}(PTH)}{3} \rfloor}\}$ is the set S is a set of vertices from path-neutrosophic graph $PTH : (V, E, \sigma, \mu)$ with new arrangements of vertices in which there are two consecutive vertices which aren't neighbors. In this new arrangements, the notation of vertices from x_i is changed to x'_i. Leaves doesn't necessarily belong to S. Leaves are either belongs to S or doesn't belong to S. Adding only the vertices which aren't consecutive contradicts with maximality of S and maximum cardinality of S. It implies this construction is optimal. Thus, let

$$S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2 \times \mathcal{O}(PTH)}{3} \rfloor) - 1}, x_{\lfloor \frac{2 \times \mathcal{O}(PTH)}{3} \rfloor}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's at least one neutrosophic vertex n in $V \setminus (S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2 \times \mathcal{O}(PTH)}{3} \rfloor) - 1}, x_{\lfloor \frac{2 \times \mathcal{O}(PTH)}{3} \rfloor}\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2 \times \mathcal{O}(PTH)}{3} \rfloor) - 1}, x_{\lfloor \frac{2 \times \mathcal{O}(PTH)}{3} \rfloor}\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by

$$\mathcal{D}(PTH) = \lfloor \frac{2 \times \mathcal{O}(PTH)}{3} \rfloor.$$

Thus

$$\mathcal{D}(PTH) = \lfloor \frac{2 \times \mathcal{O}(PTH)}{3} \rfloor.$$

□

Example 2.4. There are two sections for clarifications.

(a) In Figure (3), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. New viewpoint implies different kinds of definitions to get more scrutiny and more discernment.

(i) Let $S = \{n_3, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_2\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_2\}$ is called dual-dominating set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(PTH) = 3$;

(ii) let $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren't consecutive vertices. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_4\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_4\}$ is called dual-dominating set and this set is maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(PTH) = 3$;

(iii) let $S = \{n_3, n_4, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_4, n_1\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(PTH) = 3$;

(iv) let $S = \{n_3, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_2, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_2, n_5\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(PTH) = 3$;

(v) let $S = \{n_3, n_4, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_4, n_1\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ is called dual-dominating set. As if it, 3.3, contradicts with the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(PTH) = 3.7$;

(vi) let $S = \{n_3, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_2, n_5\})$

such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_2, n_5\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(PTH) = 3.7$.

(*b*) In Figure (4), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. New definition is applied in this section.

(*i*) Let $S = \{n_3, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_2, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_2, n_5\}$ is called dual-dominating set and this set is maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(PTH) = 4$;

(*ii*) let $S = \{n_3, n_4, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren't consecutive vertices. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_4, n_1\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ is called dual-dominating set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(PTH) = 4$;

(*iii*) let $S = \{n_3, n_4, n_1, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_4, n_1, n_6\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1, n_6\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(PTH) = 4$;

(*iv*) let $S = \{n_3, n_2, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For a neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_2, n_6\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_2, n_6\}$ is called dual-dominating set and this set is maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(PTH) = 4$;

(*v*) every set containing three consecutive vertices isn't dual-dominating set. For instance, let $S = \{n_3, n_4, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For a neutrosophic vertex n_3 in S, there's no neutrosophic vertex n in $V \setminus (S = \{n_3, n_4, n_2\})$ such that n dominates n_3, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ isn't called dual-dominating set. So as maximum neutrosophic cardinality isn't related to the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(PTH) = 6.3$;

(*vi*) let $S = \{n_3, n_4, n_1, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_4, n_1, n_6\})$ such that n dominates s, then the set of

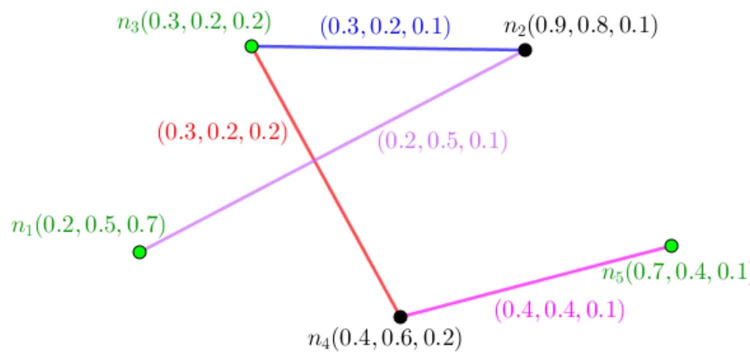

Figure 3. A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

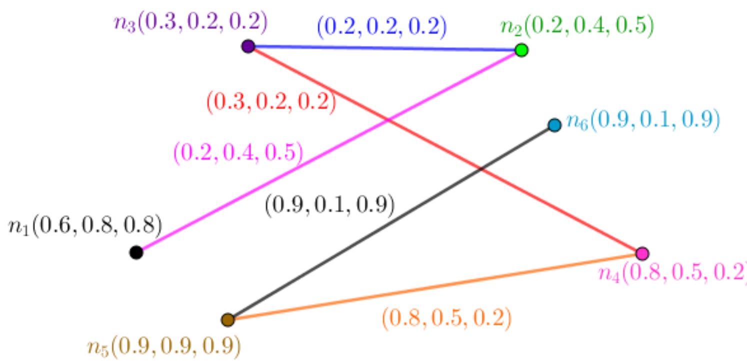

Figure 4. A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

neutrosophic vertices, $S = \{n_3, n_4, n_1, n_6\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(PTH) = 6.3$.

Proposition 2.5. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{D}(CYC) = \lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \rfloor.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

$$x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, x_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. Two consecutive vertices could belong to S which is dual-dominating set related to dual-dominating number. Since these two vertices could be dominated by previous vertex and upcoming vertex despite them. If there are no vertices which are consecutive, then it contradicts with maximality of set S and maximum cardinality of S. Thus, let

$$S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \rfloor)-1}, x_{\lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \rfloor}, x_1\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's at least one

neutrosophic vertex n in $V \setminus (S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2\times\mathcal{O}(CYC)}{3} \rfloor)-1}, x_{\lfloor \frac{2\times\mathcal{O}(CYC)}{3} \rfloor}, x_1\})$ such that n dominates s, then the set of neutrosophic vertices,
$S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2\times\mathcal{O}(CYC)}{3} \rfloor)-1}, x_{\lfloor \frac{2\times\mathcal{O}(CYC)}{3} \rfloor}, x_1\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by

$$\mathcal{D}(CYC) = \lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \rfloor.$$

Thus

$$\mathcal{D}(CYC) = \lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \rfloor.$$

□

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.6. There are two sections for clarifications.

(a) In Figure (5), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let $S = \{n_3, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_2, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_2, n_5\}$ is called dual-dominating set and this set is maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 4$;

(ii) let $S = \{n_3, n_4, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren't consecutive vertices. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_4, n_1\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ is called dual-dominating set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 4$;

(iii) let $S = \{n_3, n_4, n_1, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_4, n_1, n_6\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1, n_6\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 4$;

(iv) let $S = \{n_2, n_3, n_5, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_3, n_5, n_6\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_3, n_5, n_6\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 4$;

(v) let $S = \{n_1, n_2, n_4, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_1, n_2, n_4, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_1, n_2, n_4, n_5\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 4$;

(vi) let $S = \{n_2, n_3, n_5, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_3, n_5, n_6\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_3, n_5, n_6\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(CYC) = 5.9$.

(b) In Figure (6), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let $S = \{n_3, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_2\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_2\}$ is called dual-dominating set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 3$;

(ii) let $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren't consecutive vertices. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_4\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_4\}$ is called dual-dominating set and this set is maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 3$;

(iii) let $S = \{n_3, n_4, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_4, n_1\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 3$;

(iv) let $S = \{n_3, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_2, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_2, n_5\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 3$;

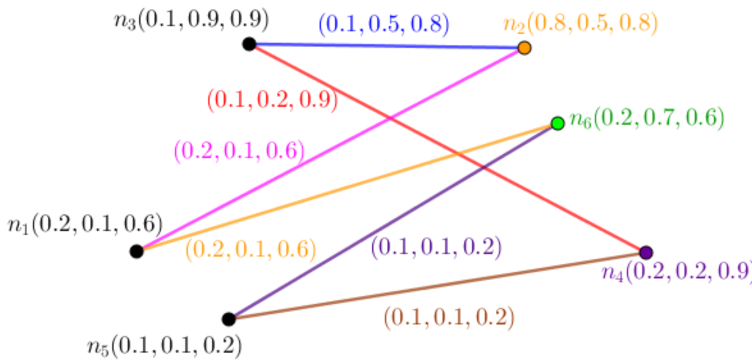

Figure 5. A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

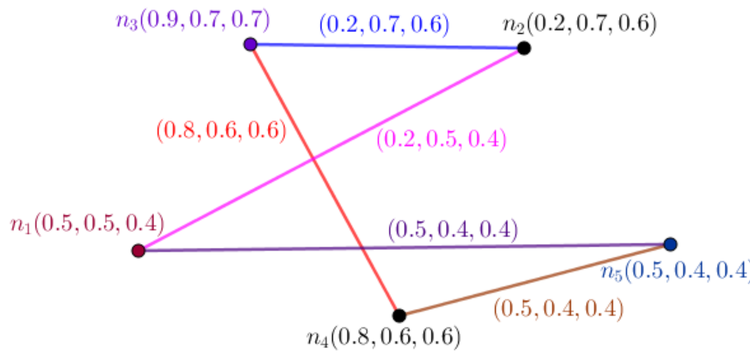

Figure 6. A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

(*v*) let $S = \{n_3, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_2, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_2, n_5\}$ is called dual-dominating set. As if it, 5.1, contradicts with the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(CYC) = 5.7$;

(*vi*) let $S = \{n_3, n_4, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_4, n_1\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(CYC) = 5.7$.

Proposition 2.7. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center c. Then*

$$\mathcal{D}(STR_{1,\sigma_2}) = \mathcal{O}(STR_{1,\sigma_2}) - 1.$$

Proof. Suppose $STR_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. An edge always has center, c, as one of its endpoints. All paths have one as their lengths, forever.

$S = V \setminus \{c\}$ is a dual-dominating set related dual-dominating number. Since, let

$$S = V \setminus \{c\} = \{x_1, x_2, \cdots, x_{\mathcal{O}(STR_{1,\sigma_2})-1}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex x_i in S, there's only one neutrosophic vertex c in $V \setminus (S = V \setminus \{c\} = \{x_1, x_2, \cdots, x_{\mathcal{O}(STR_{1,\sigma_2})-1}\})$ such that c dominates x_i, then the set of neutrosophic vertices, $S = V \setminus \{c\} = \{x_1, x_2, \cdots, x_{\mathcal{O}(STR_{1,\sigma_2})-1}\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by

$$\mathcal{D}(STR_{1,\sigma_2}) = \mathcal{O}(STR_{1,\sigma_2}) - 1.$$

Thus

$$\mathcal{D}(STR_{1,\sigma_2}) = \mathcal{O}(STR_{1,\sigma_2}) - 1.$$

□

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.8. There is one section for clarifications. In Figure (7), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let $S = \{n_1, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For a neutrosophic vertex n_2 in S, there's no neutrosophic vertex n in $V \setminus (S = \{n_1, n_2\})$ such that n dominates n_2, then the set of neutrosophic vertices, $S = \{n_1, n_2\}$ isn't called dual-dominating set and this set isn't maximal. So as it doesn't relate to maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(STR_{1,\sigma_2}) = 4$;

(ii) let $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren't consecutive vertices. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_4\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_4\}$ is called dual-dominating set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(STR_{1,\sigma_2}) = 4$;

(iii) let $S = \{n_2, n_3, n_4, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_3, n_4, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_3, n_4, n_5\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(STR_{1,\sigma_2}) = 4$;

(iv) let $S = \{n_1, n_3, n_4, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For a neutrosophic vertex n_3 in S, there's no neutrosophic vertex n in $V \setminus (S = \{n_1, n_3, n_4, n_5\})$ such that n dominates n_3, then the set of neutrosophic vertices, $S = \{n_1, n_3, n_4, n_5\}$ isn't called dual-dominating set. So as its cardinality doesn't relate to the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(STR_{1,\sigma_2}) = 4$;

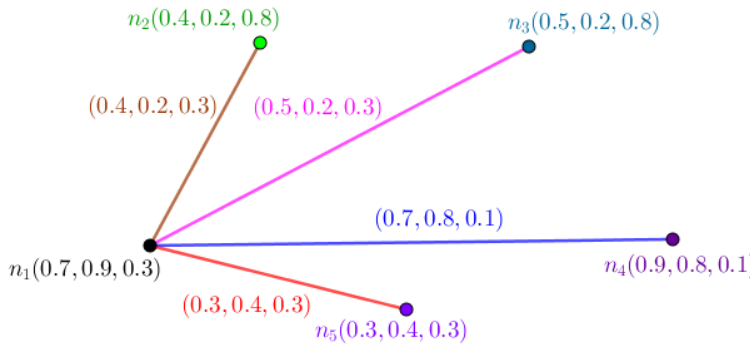

Figure 7. A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

(v) let $S = \{n_1, n_3, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For a neutrosophic vertex n_3 in S, there's no neutrosophic vertex n in $V \setminus (S = \{n_1, n_3, n_2, n_5\})$ such that n dominates n_3, then the set of neutrosophic vertices, $S = \{n_1, n_3, n_2, n_5\}$ isn't called dual-dominating set. So as its cardinality doesn't relate to the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(STR_{1,\sigma_2}) = 4$;

(vi) there's only one dual-dominating set thus let $S = \{n_2, n_3, n_4, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_3, n_4, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_3, n_4, n_5\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(STR_{1,\sigma_2}) = 5.7$.

Proposition 2.9. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-bipartite-neutrosophic graph. Then*

$$\mathcal{D}(CMC_{\sigma_1,\sigma_2}) = \mathcal{O}(CMC_{\sigma_1,\sigma_2}) - 2.$$

Proof. Suppose $CMC_{\sigma_1,\sigma_2} : (V, E, \sigma, \mu)$ is a complete-bipartite-neutrosophic graph. Every vertex in a part is dominated by another vertex in opposite part. Thus maximum cardinality implies excluding one vertex from each part. Let

$$S = V \setminus \{u, v\}_{u \in V_1, v \in V_2} = V_1 \setminus \{u\} \cup V_2 \setminus \{v\} = \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})-2}\}$$

be a dual-dominating set related to the dual-dominating number. This construction gives the proof. Since let

$$S = V \setminus \{u, v\}_{u \in V_1, v \in V_2} = V_1 \setminus \{u\} \cup V_2 \setminus \{v\} = \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})-2}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's a neutrosophic vertex n in

$$V \setminus (S = V \setminus \{u, v\}_{u \in V_1, v \in V_2} = V_1 \setminus \{u\} \cup V_2 \setminus \{v\} = \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})-2}\})$$

such that n dominates s, then the set of neutrosophic vertices,

$$S = V \setminus \{u, v\}_{u \in V_1, v \in V_2} = V_1 \setminus \{u\} \cup V_2 \setminus \{v\} = \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})-2}\}$$

is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by

$$\mathcal{D}(CMC_{\sigma_1,\sigma_2}) = \mathcal{O}(CMC_{\sigma_1,\sigma_2}) - 2.$$

Thus

$$\mathcal{D}(CMC_{\sigma_1,\sigma_2}) = \mathcal{O}(CMC_{\sigma_1,\sigma_2}) - 2.$$

□

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.10. There is one section for clarifications. In Figure (8), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) Let $S = \{n_1, n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For a neutrosophic vertex n_4 in S, there's no neutrosophic vertex n in $V \setminus (S = \{n_1, n_2, n_4\})$ such that n dominates n_4, then the set of neutrosophic vertices, $S = \{n_1, n_2, n_4\}$ isn't called dual-dominating set and this set isn't maximal. So as it doesn't relate to maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMC_{\sigma_1,\sigma_2}) = 2$;
- (ii) let $S = \{n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren't consecutive vertices. For every neutrosophic vertex s in S, there's a neutrosophic vertex n in $V \setminus (S = \{n_2\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2\}$ is called dual-dominating set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMC_{\sigma_1,\sigma_2}) = 2$;
- (iii) let $S = \{n_2, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_1\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_1\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMC_{\sigma_1,\sigma_2}) = 2$;
- (iv) let $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_4\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_4\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMC_{\sigma_1,\sigma_2}) = 2$;
- (v) let $S = \{n_4, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_4, n_3\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_4, n_3\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMC_{\sigma_1,\sigma_2}) = 2$;

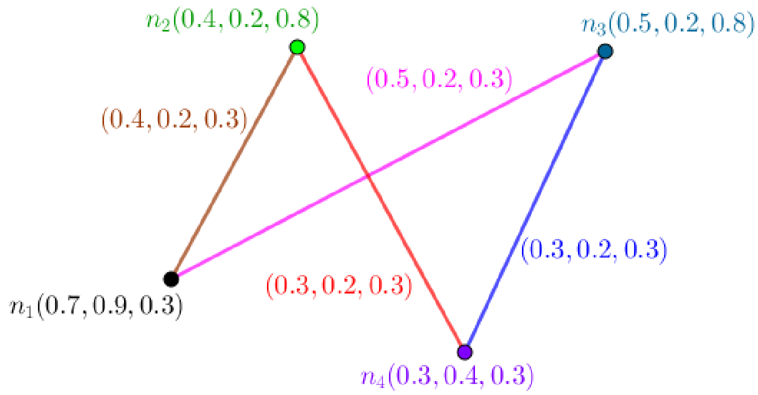

Figure 8. A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

(vi) let $S = \{n_1, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_1, n_3\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_1, n_3\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(CMC_{\sigma_1,\sigma_2}) = 3.4$.

Proposition 2.11. *Let* $NTG : (V, E, \sigma, \mu)$ *be a complete-t-partite-neutrosophic graph where* $t \geq 3$. *Then*

$$\mathcal{D}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - 2.$$

Proof. Suppose $CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t} : (V, E, \sigma, \mu)$ is a complete-t-partite-neutrosophic graph. Every vertex in a part is dominated by another vertex in opposite part. Thus maximum cardinality implies excluding two vertices from two different parts. Let

$$S = V \setminus \{u, v\}_{u \in V_1, v \in V_2} = V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t.$$

Thus

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t})-2}\}$$

be a dual-dominating set related to the dual-dominating number. This construction gives the proof. Since let

$$S = V \setminus \{u, v\}_{u \in V_1, v \in V_2} = V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t.$$

Thus

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t})-2}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's a neutrosophic vertex n in

$$V \setminus (S = V \setminus \{u, v\}_{u \in V_1, v \in V_2} = V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t.$$

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t})-2}\})$$

such that n dominates s, then the set of neutrosophic vertices,

$$S = V \setminus \{u, v\}_{u \in V_1, v \in V_2} = V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t.$$

Thus

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t})-2}\}$$

is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by

$$\mathcal{D}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - 2.$$

Thus

$$\mathcal{D}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - 2.$$

□

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.12. There is one section for clarifications. In Figure (9), a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let $S = \{n_1, n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For a neutrosophic vertex n_4 in S, there's no neutrosophic vertex n in $V \setminus (S = \{n_1, n_2, n_4\})$ such that n dominates n_4, then the set of neutrosophic vertices, $S = \{n_1, n_2, n_4\}$ isn't called dual-dominating set and this set isn't maximal. So as it doesn't relate to maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

(ii) let $S = \{n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren't consecutive vertices. For every neutrosophic vertex s in S, there's a neutrosophic vertex n in $V \setminus (S = \{n_2\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2\}$ is called dual-dominating set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

(iii) let $S = \{n_2, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_1\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_1\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

(iv) let $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_4\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_4\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

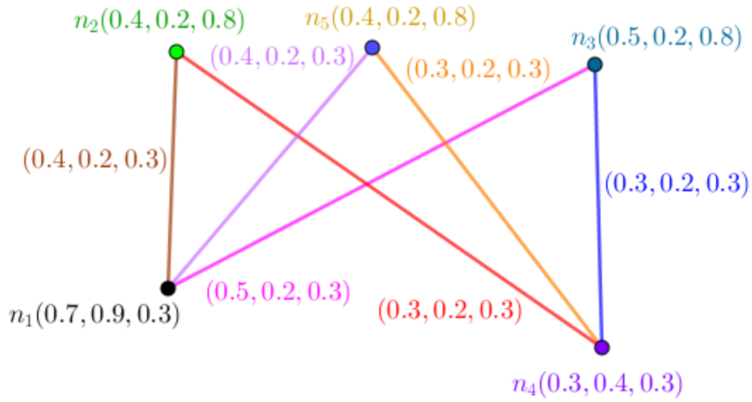

Figure 9. A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

(v) let $S = \{n_4, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_4, n_3\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_4, n_3\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

(vi) let $S = \{n_1, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_1, n_3\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_1, n_3\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 3.4$.

Proposition 2.13. *Let $NTG : (V, E, \sigma, \mu)$ be a wheel-neutrosophic graph. Then*

$$\mathcal{D}(WHL_{1,\sigma_2}) = \mathcal{O}(WHL_{1,\sigma_2}) - 1.$$

Proof. Suppose $WHL_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a wheel-neutrosophic graph. The argument is elementary. All vertices of a cycle join to one vertex, c. $S = V \setminus \{c\}$ is a dual-dominating set related dual-dominating number. Since, let

$$S = V \setminus \{c\} = \{x_1, x_2, \cdots, x_{\mathcal{O}(WHL_{1,\sigma_2})-1}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex x_i in S, there's a neutrosophic vertex c in $V \setminus (S = V \setminus \{c\} = \{x_1, x_2, \cdots, x_{\mathcal{O}(WHL_{1,\sigma_2})-1}\})$ such that c dominates x_i, then the set of neutrosophic vertices, $S = V \setminus \{c\} = \{x_1, x_2, \cdots, x_{\mathcal{O}(WHL_{1,\sigma_2})-1}\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by

$$\mathcal{D}(WHL_{1,\sigma_2}) = \mathcal{O}(WHL_{1,\sigma_2}) - 1.$$

Thus

$$\mathcal{D}(WHL_{1,\sigma_2}) = \mathcal{O}(WHL_{1,\sigma_2}) - 1.$$

□

The clarifications about results are in progress as follows. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.14. There is one section for clarifications. In Figure (10), a wheel-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let $S = \{n_1, n_2, n_3, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For a neutrosophic vertex n_2 in S, there's no neutrosophic vertex n in $V \setminus (S = \{n_1, n_2, n_3, n_5\})$ such that n dominates n_2, then the set of neutrosophic vertices, $S = \{n_1, n_2, n_3, n_5\}$ isn't called dual-dominating set and this set isn't maximal. So as it doesn't relate to maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(WHL_{1,\sigma_2}) = 4$;

(ii) let $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren't consecutive vertices. For every neutrosophic vertex s in S, there's a neutrosophic vertex n in $V \setminus (S = \{n_2, n_4\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_4\}$ is called dual-dominating set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(WHL_{1,\sigma_2}) = 4$;

(iii) let $S = \{n_2, n_3, n_4, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n_1 in $V \setminus (S = \{n_2, n_3, n_4, n_5\})$ such that n_1 dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_3, n_4, n_5\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(WHL_{1,\sigma_2}) = 4$;

(iv) let $S = \{n_2, n_3, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren't consecutive vertices. For every neutrosophic vertex s in S, there's a neutrosophic vertex n in $V \setminus (S = \{n_2, n_3, n_4\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_3, n_4\}$ is called dual-dominating set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(WHL_{1,\sigma_2}) = 4$;

(v) let $S = \{n_2, n_3, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren't consecutive vertices. For every neutrosophic vertex s in S, there's a neutrosophic vertex n in $V \setminus (S = \{n_2, n_3, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_3, n_5\}$ is called dual-dominating set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(WHL_{1,\sigma_2}) = 4$;

(vi) there's only one dual-dominating set thus let $S = \{n_2, n_3, n_4, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_3, n_4, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_3, n_4, n_5\}$ is called dual-dominating set.

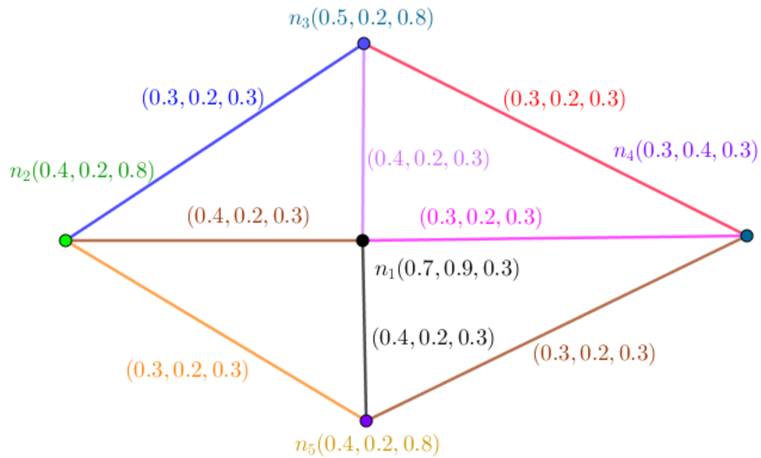

Figure 10. A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(WHL_{1,\sigma_2}) = 5.3$.

3 Setting of neutrosophic dual-dominating number

In this section, I provide some results in the setting of neutrosophic dual-dominating number. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, star-neutrosophic graph, bipartite-neutrosophic graph, t-partite-neutrosophic graph, and wheel-neutrosophic graph, are both of cases of study and classes which the results are about them.

Proposition 3.1. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{D}_n(CMT_\sigma) = \mathcal{O}_n(CMT_\sigma) - \min_{x \in V} \sum_{i=1}^{3} \sigma_i(x).$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. For given two vertices, s and n, $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let $S = V \setminus \{n\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = V \setminus \{n\})$ such that n dominates s, then the set of neutrosophic vertices, $S = V \setminus \{n\}$ is called dual-dominating set. The maximum neutrosophic cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(NTG) = \mathcal{O}(NTG) - 1$. Thus

$$\mathcal{D}_n(CMT_\sigma) = \mathcal{O}_n(CMT_\sigma) - \min_{x \in V} \sum_{i=1}^{3} \sigma_i(x).$$

□

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To

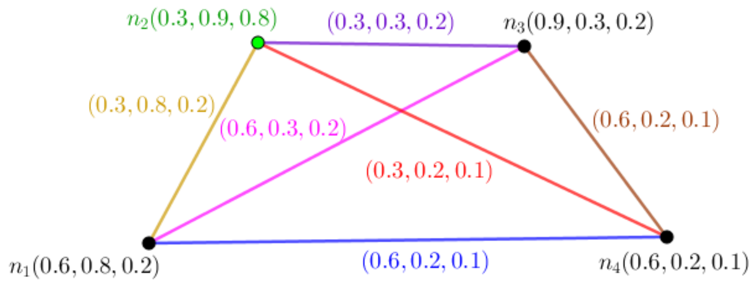

Figure 11. A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.2. In Figure (11), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two vertices, s and n, $\mu(ns) = \sigma(n) \wedge \sigma(s)$. Thus s dominates n and n dominates s;

(ii) the existence of one vertex to do this function, dominating, is obvious thus this vertex form a set which is necessary and sufficient in the term of minimum dominating set and minimal dominating set;

(iii) for given two vertices, s and n, $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let $S = V \setminus \{n\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] If for every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = V \setminus \{n\})$ such that n dominates s, then the set of neutrosophic vertices, $S = V \setminus \{n\}$ is called dual-dominating set. The maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMT_\sigma) = \mathcal{O}(CMT_\sigma) - 1$;

(iv) the corresponded set doesn't have to be dominated by the set;

(v) V is exception when the set is considered in this notion;

(vi) for given two vertices, s and n, $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let $S = V \setminus \{n\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] If for every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = V \setminus \{n\})$ such that n dominates s, then the set of neutrosophic vertices, $S = V \setminus \{n\}$ is called dual-dominating set. The maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(CMT_\sigma) = \mathcal{O}_n(CMT_\sigma) - \sum_{i=1}^{3} \sigma_i(n_4) = 5$.

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

Proposition 3.3. *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Then*

$$\mathcal{D}_n(PTH) = \max_{x \in S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2 \times \mathcal{O}(PTH)}{3} \rfloor) - 1}, x_{\lfloor \frac{2 \times \mathcal{O}(PTH)}{3} \rfloor}\}} \sum_{i=1}^{3} \sigma_i(x)$$

Proof. Suppose $PTH : (V, E, \sigma, \mu)$ is a path-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(PTH)}$ be a path-neutrosophic graph. For given two vertices, x and y, there's one path from x to y. Let S be an intended set which is dual-dominating set. Despite leaves x_1, and $x_{\mathcal{O}(PTH)}$, two consecutive vertices belong to S. They could be dominated by previous vertex and upcoming vertex as if despite them so as maximal set S is constructed. Thus $S = \{x'_1, x'_2, \cdots, x'_{\lfloor \frac{2\times\mathcal{O}(PTH)}{3} \rfloor}\}$ is the set S is a set of vertices from path-neutrosophic graph $PTH : (V, E, \sigma, \mu)$ with new arrangements of vertices in which there are two consecutive vertices which aren't neighbors. In this new arrangements, the notation of vertices from x_i is changed to x'_i. Leaves doesn't necessarily belong to S. Leaves are either belongs to S or doesn't belong to S. Adding only the vertices which aren't consecutive contradicts with maximality of S and maximum cardinality of S. It implies this construction is optimal. Thus, let

$$S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2\times\mathcal{O}(PTH)}{3} \rfloor)-1}, x_{\lfloor \frac{2\times\mathcal{O}(PTH)}{3} \rfloor}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's at least one neutrosophic vertex n in $V \setminus (S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2\times\mathcal{O}(PTH)}{3} \rfloor)-1}, x_{\lfloor \frac{2\times\mathcal{O}(PTH)}{3} \rfloor}\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2\times\mathcal{O}(PTH)}{3} \rfloor)-1}, x_{\lfloor \frac{2\times\mathcal{O}(PTH)}{3} \rfloor}\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by

$$\mathcal{D}_n(PTH) = \max_{x \in S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2\times\mathcal{O}(PTH)}{3} \rfloor)-1}, x_{\lfloor \frac{2\times\mathcal{O}(PTH)}{3} \rfloor}\}} \sum_{i=1}^{3} \sigma_i(x)$$

Thus

$$\mathcal{D}_n(PTH) = \max_{x \in S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2\times\mathcal{O}(PTH)}{3} \rfloor)-1}, x_{\lfloor \frac{2\times\mathcal{O}(PTH)}{3} \rfloor}\}} \sum_{i=1}^{3} \sigma_i(x)$$

□

Example 3.4. There are two sections for clarifications.

(a) In Figure (12), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. New viewpoint implies different kinds of definitions to get more scrutiny and more discernment.

- (i) Let $S = \{n_3, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_2\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_2\}$ is called dual-dominating set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(PTH) = 3$;
- (ii) let $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren't consecutive vertices. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_4\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_4\}$ is called dual-dominating set and this set is maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(PTH) = 3$;

(*iii*) let $S = \{n_3, n_4, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_4, n_1\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(PTH) = 3$;

(*iv*) let $S = \{n_3, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_2, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_2, n_5\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(PTH) = 3$;

(*v*) let $S = \{n_3, n_4, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_4, n_1\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ is called dual-dominating set. As if it, 3.3, contradicts with the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(PTH) = 3.7$;

(*vi*) let $S = \{n_3, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_2, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_2, n_5\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(PTH) = 3.7$.

(*b*) In Figure (13), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. New definition is applied in this section.

(*i*) Let $S = \{n_3, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_2, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_2, n_5\}$ is called dual-dominating set and this set is maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(PTH) = 4$;

(*ii*) let $S = \{n_3, n_4, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren't consecutive vertices. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_4, n_1\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ is called dual-dominating set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(PTH) = 4$;

(*iii*) let $S = \{n_3, n_4, n_1, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in

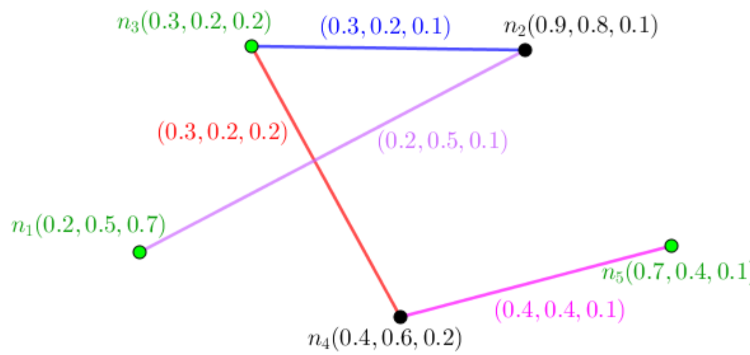

Figure 12. A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

$V \setminus (S = \{n_3, n_4, n_1, n_6\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1, n_6\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(PTH) = 4$;

(iv) let $S = \{n_3, n_2, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For a neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_2, n_6\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_2, n_6\}$ is called dual-dominating set and this set is maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(PTH) = 4$;

(v) every set containing three consecutive vertices isn't dual-dominating set. For instance, let $S = \{n_3, n_4, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For a neutrosophic vertex n_3 in S, there's no neutrosophic vertex n in $V \setminus (S = \{n_3, n_4, n_2\})$ such that n dominates n_3, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ isn't called dual-dominating set. So as maximum neutrosophic cardinality isn't related to the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(PTH) = 6.3$;

(vi) let $S = \{n_3, n_4, n_1, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_4, n_1, n_6\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1, n_6\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(PTH) = 6.3$.

Proposition 3.5. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{D}_n(CYC) = \max_{x \in S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \rfloor) - 1}, x_{\lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \rfloor}\}} \sum_{i=1}^{3} \sigma_i(x)$$

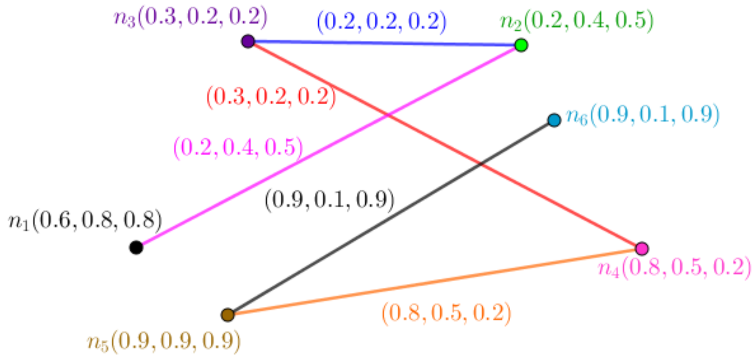

Figure 13. A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

$$x_1, x_2, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, x_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. Two consecutive vertices could belong to S which is dual-dominating set related to dual-dominating number. Since these two vertices could be dominated by previous vertex and upcoming vertex despite them. If there are no vertices which are consecutive, then it contradicts with maximality of set S and maximum cardinality of S. Thus, let

$$S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2\times\mathcal{O}(CYC)}{3} \rfloor)-1}, x_{\lfloor \frac{2\times\mathcal{O}(CYC)}{3} \rfloor}, x_1\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's at least one neutrosophic vertex n in $V \setminus (S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2\times\mathcal{O}(CYC)}{3} \rfloor)-1}, x_{\lfloor \frac{2\times\mathcal{O}(CYC)}{3} \rfloor}, x_1\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2\times\mathcal{O}(CYC)}{3} \rfloor)-1}, x_{\lfloor \frac{2\times\mathcal{O}(CYC)}{3} \rfloor}, x_1\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by

$$\mathcal{D}_n(CYC) = \max_{x \in S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2\times\mathcal{O}(CYC)}{3} \rfloor)-1}, x_{\lfloor \frac{2\times\mathcal{O}(CYC)}{3} \rfloor}\}} \sum_{i=1}^{3} \sigma_i(x)$$

Thus

$$\mathcal{D}_n(CYC) = \max_{x \in S = \{x_1, x_2, \cdots, x_{\lfloor \frac{2\times\mathcal{O}(CYC)}{3} \rfloor)-1}, x_{\lfloor \frac{2\times\mathcal{O}(CYC)}{3} \rfloor}\}} \sum_{i=1}^{3} \sigma_i(x)$$

□

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.6. There are two sections for clarifications.

(*a*) In Figure (14), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (*i*) Let $S = \{n_3, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_2, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_2, n_5\}$ is called dual-dominating set and this set is maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 4$;
- (*ii*) let $S = \{n_3, n_4, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren't consecutive vertices. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_4, n_1\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ is called dual-dominating set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 4$;
- (*iii*) let $S = \{n_3, n_4, n_1, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_4, n_1, n_6\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1, n_6\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 4$;
- (*iv*) let $S = \{n_2, n_3, n_5, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_3, n_5, n_6\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_3, n_5, n_6\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 4$;
- (*v*) let $S = \{n_1, n_2, n_4, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_1, n_2, n_4, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_1, n_2, n_4, n_5\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 4$;
- (*vi*) let $S = \{n_2, n_3, n_5, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_3, n_5, n_6\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_3, n_5, n_6\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(CYC) = 5.9$.

(*b*) In Figure (15), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let $S = \{n_3, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_2\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_2\}$ is called dual-dominating set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 3$;

(ii) let $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren't consecutive vertices. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_4\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_4\}$ is called dual-dominating set and this set is maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 3$;

(iii) let $S = \{n_3, n_4, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_4, n_1\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 3$;

(iv) let $S = \{n_3, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_2, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_2, n_5\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CYC) = 3$;

(v) let $S = \{n_3, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_2, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_2, n_5\}$ is called dual-dominating set. As if it, 5.1, contradicts with the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(CYC) = 5.7$;

(vi) let $S = \{n_3, n_4, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_3, n_4, n_1\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(CYC) = 5.7$.

Proposition 3.7. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center c. Then*

$$\mathcal{D}_n(STR_{1,\sigma_2}) = \mathcal{O}_n(STR_{1,\sigma_2}) - \sum_{i=1}^{3} \sigma_i(c).$$

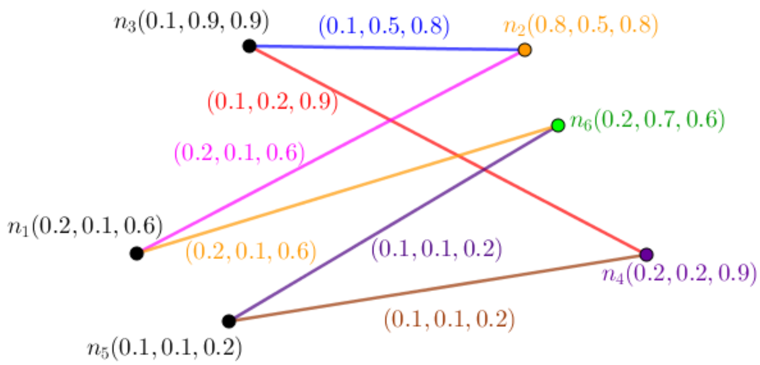

Figure 14. A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

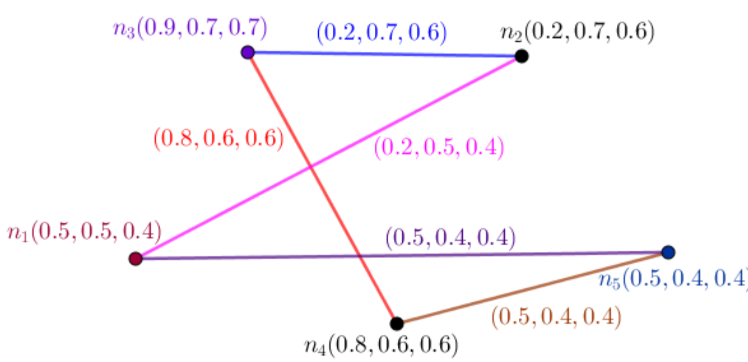

Figure 15. A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

Proof. Suppose $STR_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. An edge always has center, c, as one of its endpoints. All paths have one as their lengths, forever. $S = V \setminus \{c\}$ is a dual-dominating set related dual-dominating number. Since, let

$$S = V \setminus \{c\} = \{x_1, x_2, \cdots, x_{\mathcal{O}(STR_{1,\sigma_2})-1}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex x_i in S, there's only one neutrosophic vertex c in $V \setminus (S = V \setminus \{c\} = \{x_1, x_2, \cdots, x_{\mathcal{O}(STR_{1,\sigma_2})-1}\})$ such that c dominates x_i, then the set of neutrosophic vertices, $S = V \setminus \{c\} = \{x_1, x_2, \cdots, x_{\mathcal{O}(STR_{1,\sigma_2})-1}\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by

$$\mathcal{D}_n(STR_{1,\sigma_2}) = \mathcal{O}_n(STR_{1,\sigma_2}) - \sum_{i=1}^{3} \sigma_i(c).$$

Thus

$$\mathcal{D}_n(STR_{1,\sigma_2}) = \mathcal{O}_n(STR_{1,\sigma_2}) - \sum_{i=1}^{3} \sigma_i(c).$$

□

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.8. There is one section for clarifications. In Figure (16), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) Let $S = \{n_1, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For a neutrosophic vertex n_2 in S, there's no neutrosophic vertex n in $V \setminus (S = \{n_1, n_2\})$ such that n dominates n_2, then the set of neutrosophic vertices, $S = \{n_1, n_2\}$ isn't called dual-dominating set and this set isn't maximal. So as it doesn't relate to maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(STR_{1,\sigma_2}) = 4$;

(*ii*) let $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren't consecutive vertices. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_4\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_4\}$ is called dual-dominating set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(STR_{1,\sigma_2}) = 4$;

(*iii*) let $S = \{n_2, n_3, n_4, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_3, n_4, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_3, n_4, n_5\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(STR_{1,\sigma_2}) = 4$;

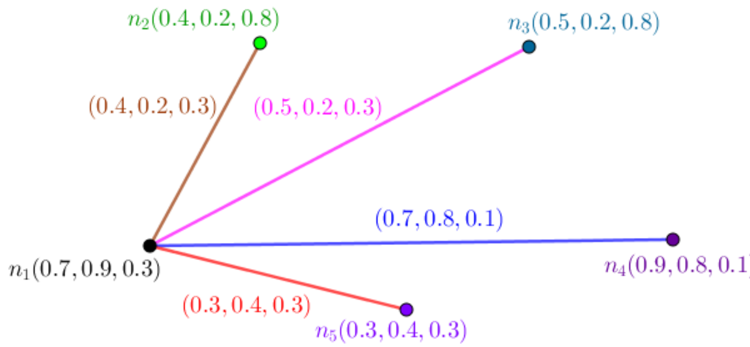

Figure 16. A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

(iv) let $S = \{n_1, n_3, n_4, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For a neutrosophic vertex n_3 in S, there's no neutrosophic vertex n in $V \setminus (S = \{n_1, n_3, n_4, n_5\})$ such that n dominates n_3, then the set of neutrosophic vertices, $S = \{n_1, n_3, n_4, n_5\}$ isn't called dual-dominating set. So as its cardinality doesn't relate to the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(STR_{1,\sigma_2}) = 4$;

(v) let $S = \{n_1, n_3, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For a neutrosophic vertex n_3 in S, there's no neutrosophic vertex n in $V \setminus (S = \{n_1, n_3, n_2, n_5\})$ such that n dominates n_3, then the set of neutrosophic vertices, $S = \{n_1, n_3, n_2, n_5\}$ isn't called dual-dominating set. So as its cardinality doesn't relate to the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(STR_{1,\sigma_2}) = 4$;

(vi) there's only one dual-dominating set thus let $S = \{n_2, n_3, n_4, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_3, n_4, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_3, n_4, n_5\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(STR_{1,\sigma_2}) = 5.7$.

Proposition 3.9. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-bipartite-neutrosophic graph. Then*

$$\mathcal{D}_n(CMC_{\sigma_1,\sigma_2}) = \mathcal{O}_n(CMC_{\sigma_1,\sigma_2}) - \min_{x \in V_1, y \in V_2} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)).$$

Proof. Suppose $CMC_{\sigma_1,\sigma_2} : (V, E, \sigma, \mu)$ is a complete-bipartite-neutrosophic graph. Every vertex in a part is dominated by another vertex in opposite part. Thus maximum cardinality implies excluding one vertex from each part. Let

$$S = V \setminus \{u, v\}_{u \in V_1, v \in V_2} = V_1 \setminus \{u\} \cup V_2 \setminus \{v\} = \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})-2}\}$$

be a dual-dominating set related to the dual-dominating number. This construction gives the proof. Since let

$$S = V \setminus \{u, v\}_{u \in V_1, v \in V_2} = V_1 \setminus \{u\} \cup V_2 \setminus \{v\} = \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})-2}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's a neutrosophic vertex n in

$$V \setminus (S = V \setminus \{u, v\}_{u\in V_1, v\in V_2} = V_1 \setminus \{u\} \cup V_2 \setminus \{v\} = \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})-2}\})$$

such that n dominates s, then the set of neutrosophic vertices,

$$S = V \setminus \{u, v\}_{u\in V_1, v\in V_2} = V_1 \setminus \{u\} \cup V_2 \setminus \{v\} = \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})-2}\}$$

is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by

$$\mathcal{D}_n(CMC_{\sigma_1,\sigma_2}) = \mathcal{O}_n(CMC_{\sigma_1,\sigma_2}) - \min_{x\in V_1, y\in V_2} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)).$$

Thus

$$\mathcal{D}_n(CMC_{\sigma_1,\sigma_2}) = \mathcal{O}_n(CMC_{\sigma_1,\sigma_2}) - \min_{x\in V_1, y\in V_2} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)).$$

□

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.10. There is one section for clarifications. In Figure (17), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) Let $S = \{n_1, n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For a neutrosophic vertex n_4 in S, there's no neutrosophic vertex n in $V \setminus (S = \{n_1, n_2, n_4\})$ such that n dominates n_4, then the set of neutrosophic vertices, $S = \{n_1, n_2, n_4\}$ isn't called dual-dominating set and this set isn't maximal. So as it doesn't relate to maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMC_{\sigma_1,\sigma_2}) = 2$;
- (ii) let $S = \{n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren't consecutive vertices. For every neutrosophic vertex s in S, there's a neutrosophic vertex n in $V \setminus (S = \{n_2\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2\}$ is called dual-dominating set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMC_{\sigma_1,\sigma_2}) = 2$;
- (iii) let $S = \{n_2, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_1\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_1\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMC_{\sigma_1,\sigma_2}) = 2$;

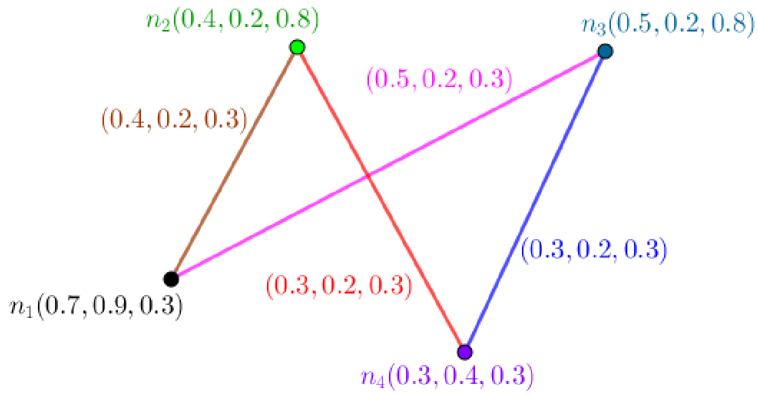

Figure 17. A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

(iv) let $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_4\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_4\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMC_{\sigma_1,\sigma_2}) = 2$;

(v) let $S = \{n_4, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_4, n_3\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_4, n_3\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMC_{\sigma_1,\sigma_2}) = 2$;

(vi) let $S = \{n_1, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_1, n_3\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_1, n_3\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(CMC_{\sigma_1,\sigma_2}) = 3.4$.

Proposition 3.11. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-t-partite-neutrosophic graph where $t \geq 3$. Then*

$$\mathcal{D}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - \min_{x \in V_i, y \in V_j, i \neq j} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)).$$

Proof. Suppose $CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t} : (V, E, \sigma, \mu)$ is a complete-t-partite-neutrosophic graph. Every vertex in a part is dominated by another vertex in opposite part. Thus maximum cardinality implies excluding two vertices from two different parts. Let

$$S = V \setminus \{u, v\}_{u \in V_1, v \in V_2} = V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t.$$

Thus

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t})-2}\}$$

be a dual-dominating set related to the dual-dominating number. This construction gives the proof. Since let

$$S = V \setminus \{u, v\}_{u \in V_1, v \in V_2} = V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t.$$

Thus

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t})-2}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's a neutrosophic vertex n in

$$V \setminus (S = V \setminus \{u, v\}_{u \in V_1, v \in V_2} = V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t.$$

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t})-2}\})$$

such that n dominates s, then the set of neutrosophic vertices,

$$S = V \setminus \{u, v\}_{u \in V_1, v \in V_2} = V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t.$$

Thus

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t})-2}\}$$

is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by

$$\mathcal{D}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - \min_{x \in V_i, y \in V_j, i \neq j} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)).$$

Thus

$$\mathcal{D}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - \min_{x \in V_i, y \in V_j, i \neq j} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)).$$

□

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.12. There is one section for clarifications. In Figure (18), a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) Let $S = \{n_1, n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For a neutrosophic vertex n_4 in S, there's no neutrosophic vertex n in $V \setminus (S = \{n_1, n_2, n_4\})$ such that n dominates n_4, then the set of neutrosophic vertices, $S = \{n_1, n_2, n_4\}$ isn't called dual-dominating set and this set isn't maximal. So as it doesn't relate to maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

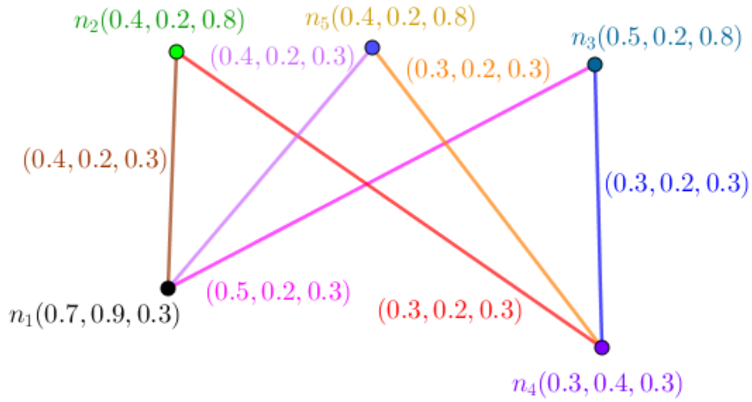

Figure 18. A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

(ii) let $S = \{n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren't consecutive vertices. For every neutrosophic vertex s in S, there's a neutrosophic vertex n in $V \setminus (S = \{n_2\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2\}$ is called dual-dominating set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

(iii) let $S = \{n_2, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_1\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_1\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

(iv) let $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_4\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_4\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

(v) let $S = \{n_4, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_4, n_3\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_4, n_3\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

(vi) let $S = \{n_1, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_1, n_3\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_1, n_3\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 3.4$.

Proposition 3.13. *Let $NTG : (V, E, \sigma, \mu)$ be a wheel-neutrosophic graph. Then*

$$\mathcal{D}_n(WHL_{1,\sigma_2}) = \mathcal{O}_n(WHL_{1,\sigma_2}) - \sum_{i=1}^{3} \sigma_i(c).$$

Proof. Suppose $WHL_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a wheel-neutrosophic graph. The argument is elementary. All vertices of a cycle join to one vertex, c. $S = V \setminus \{c\}$ is a dual-dominating set related dual-dominating number. Since, let

$$S = V \setminus \{c\} = \{x_1, x_2, \cdots, x_{\mathcal{O}(WHL_{1,\sigma_2})-1}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex x_i in S, there's a neutrosophic vertex c in $V \setminus (S = V \setminus \{c\} = \{x_1, x_2, \cdots, x_{\mathcal{O}(WHL_{1,\sigma_2})-1}\})$ such that c dominates x_i, then the set of neutrosophic vertices, $S = V \setminus \{c\} = \{x_1, x_2, \cdots, x_{\mathcal{O}(WHL_{1,\sigma_2})-1}\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by

$$\mathcal{D}_n(WHL_{1,\sigma_2}) = \mathcal{O}_n(WHL_{1,\sigma_2}) - \sum_{i=1}^{3} \sigma_i(c).$$

Thus

$$\mathcal{D}_n(WHL_{1,\sigma_2}) = \mathcal{O}_n(WHL_{1,\sigma_2}) - \sum_{i=1}^{3} \sigma_i(c).$$

□

The clarifications about results are in progress as follows. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.14. There is one section for clarifications. In Figure (19), a wheel-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let $S = \{n_1, n_2, n_3, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For a neutrosophic vertex n_2 in S, there's no neutrosophic vertex n in $V \setminus (S = \{n_1, n_2, n_3, n_5\})$ such that n dominates n_2, then the set of neutrosophic vertices, $S = \{n_1, n_2, n_3, n_5\}$ isn't called dual-dominating set and this set isn't maximal. So as it doesn't relate to maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(WHL_{1,\sigma_2}) = 4$;

(ii) let $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren't consecutive vertices. For every neutrosophic vertex s in S, there's a neutrosophic vertex n in $V \setminus (S = \{n_2, n_4\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_4\}$ is called dual-dominating set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(WHL_{1,\sigma_2}) = 4$;

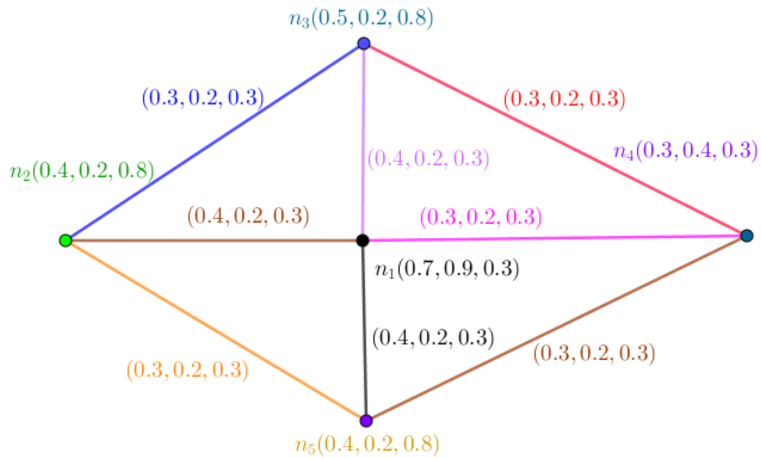

Figure 19. A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

(iii) let $S = \{n_2, n_3, n_4, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n_1 in $V \setminus (S = \{n_2, n_3, n_4, n_5\})$ such that n_1 dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_3, n_4, n_5\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(WHL_{1,\sigma_2}) = 4$;

(iv) let $S = \{n_2, n_3, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren't consecutive vertices. For every neutrosophic vertex s in S, there's a neutrosophic vertex n in $V \setminus (S = \{n_2, n_3, n_4\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_3, n_4\}$ is called dual-dominating set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(WHL_{1,\sigma_2}) = 4$;

(v) let $S = \{n_2, n_3, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren't consecutive vertices. For every neutrosophic vertex s in S, there's a neutrosophic vertex n in $V \setminus (S = \{n_2, n_3, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_3, n_5\}$ is called dual-dominating set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(WHL_{1,\sigma_2}) = 4$;

(vi) there's only one dual-dominating set thus let $S = \{n_2, n_3, n_4, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_3, n_4, n_5\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_3, n_4, n_5\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(WHL_{1,\sigma_2}) = 5.3$.

4 Applications in Time Table and Scheduling

In this section, two applications for time table and scheduling are provided where the models are either complete models which mean complete connections are formed as individual and family of complete models with common neutrosophic vertex set or quasi-complete models which mean quasi-complete connections are formed as individual and family of quasi-complete models with common neutrosophic vertex set.

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has importance to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive sections. Beyond that, sometimes sections are not the same.

Step 3. (Model) The situation is designed as a model. The model uses data to assign every section and to assign to relation amid sections, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relations amid them. Table (1), clarifies about the assigned numbers to these situations.

Table 1. Scheduling concerns its Subjects and its Connections as a neutrosophic graph in a Model.

Sections of NTG	n_1	$n_2 \cdots$	n_5
Values	$(0.7, 0.9, 0.3)$	$(0.4, 0.2, 0.8) \cdots$	$(0.4, 0.2, 0.8)$
Connections of NTG	E_1	$E_2 \cdots$	E_6
Values	$(0.4, 0.2, 0.3)$	$(0.5, 0.2, 0.3) \cdots$	$(0.3, 0.2, 0.3)$

4.1 Case 1: Complete-t-partite Model alongside its dual-dominating number and its neutrosophic dual-dominating number

Step 4. (Solution) The neutrosophic graph alongside its dual-dominating number and its neutrosophic dual-dominating number as model, propose to use specific number. Every subject has connection with some subjects. Thus the connection is applied as possible and the model demonstrates quasi-full connections as quasi-possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is star, the number is different. Also, it holds for other types such that complete, wheel, path, and cycle. The collection of situations is another application of its dual-dominating number and its neutrosophic dual-dominating number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are five subjects which are represented as Figure (20). This model is strong and even more it's quasi-complete. And the study proposes using specific number which is called its dual-dominating number and its neutrosophic dual-dominating number. There are

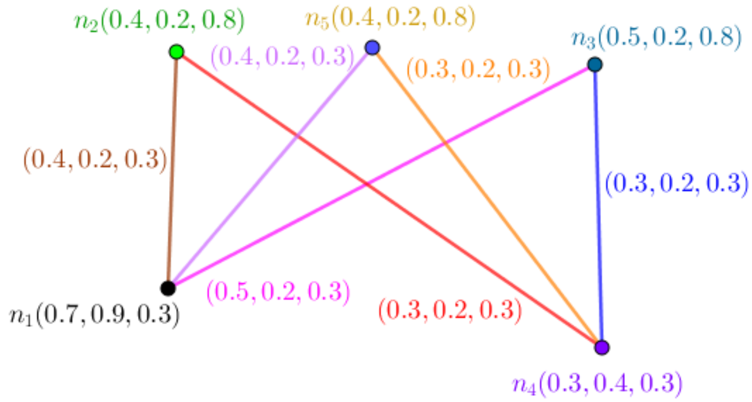

Figure 20. A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number

also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to this model and situation to compare them with same situations to get more precise. Consider Figure (20). In Figure (20), an complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let $S = \{n_1, n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For a neutrosophic vertex n_4 in S, there's no neutrosophic vertex n in $V \setminus (S = \{n_1, n_2, n_4\})$ such that n dominates n_4, then the set of neutrosophic vertices, $S = \{n_1, n_2, n_4\}$ isn't called dual-dominating set and this set isn't maximal. So as it doesn't relate to maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

(ii) let $S = \{n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren't consecutive vertices. For every neutrosophic vertex s in S, there's a neutrosophic vertex n in $V \setminus (S = \{n_2\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2\}$ is called dual-dominating set and this set isn't maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

(iii) let $S = \{n_2, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_1\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_1\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

(iv) let $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_2, n_4\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_2, n_4\}$ is called dual-dominating set. So as the maximum cardinality between all

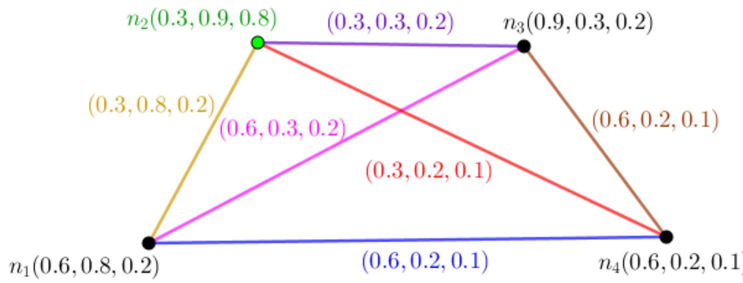

Figure 21. A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number

dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

(v) let $S = \{n_4, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_4, n_3\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_4, n_3\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

(vi) let $S = \{n_1, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = \{n_1, n_3\})$ such that n dominates s, then the set of neutrosophic vertices, $S = \{n_1, n_3\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 3.4$.

4.2 Case 2: Complete Model alongside its A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number

Step 4. (Solution) The neutrosophic graph alongside its dual-dominating number and its neutrosophic dual-dominating number as model, propose to use specific number. Every subject has connection with every given subject in deemed way. Thus the connection applied as possible and the model demonstrates full connections as possible between parts but with different view where symmetry amid vertices and edges are the matters. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete multipartite, the number is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application of its dual-dominating number and its neutrosophic dual-dominating number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are four subjects which are represented in the formation of one model as Figure (21). This model is neutrosophic strong as individual and even more it's complete. And the study proposes using specific number which is called its dual-dominating number and its neutrosophic dual-dominating number for this model. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to these models as individual. A model as a collection of situations to compare them with another model as a collection of situations to get more precise. Consider Figure (21). There is one section for clarifications.

- (i) For given two vertices, s and n, $\mu(ns) = \sigma(n) \wedge \sigma(s)$. Thus s dominates n and n dominates s;
- (ii) the existence of one vertex to do this function, dominating, is obvious thus this vertex form a set which is necessary and sufficient in the term of minimum dominating set and minimal dominating set;
- (iii) for given two vertices, s and n, $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let $S = V \setminus \{n\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] If for every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = V \setminus \{n\})$ such that n dominates s, then the set of neutrosophic vertices, $S = V \setminus \{n\}$ is called dual-dominating set. The maximum cardinality between all dual-dominating sets is called dual-dominating number and it's denoted by $\mathcal{D}(CMT_{\sigma}) = \mathcal{O}(CMT_{\sigma}) - 1$;
- (iv) the corresponded set doesn't have to be dominated by the set;
- (v) V is exception when the set is considered in this notion;
- (vi) for given two vertices, s and n, $\mu(ns) = \sigma(n) \wedge \sigma(s)$, then s dominates n and n dominates s. Let $S = V \setminus \{n\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] If for every neutrosophic vertex s in S, there's only one neutrosophic vertex n in $V \setminus (S = V \setminus \{n\})$ such that n dominates s, then the set of neutrosophic vertices, $S = V \setminus \{n\}$ is called dual-dominating set. The maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it's denoted by $\mathcal{D}_n(CMT_{\sigma}) = \mathcal{O}_n(CMT_{\sigma}) - \sum_{i=1}^{3} \sigma_i(n_4) = 5$.

5 Open Problems

In this section, some questions and problems are proposed to give some avenues to pursue this study. The structures of the definitions and results give some ideas to make new settings which are eligible to extend and to create new study.

Notion concerning its dual-dominating number and its neutrosophic dual-dominating number are defined in neutrosophic graphs. Thus,

Question 5.1. *Is it possible to use other types of its dual-dominating number and its neutrosophic dual-dominating number?*

Question 5.2. *Are existed some connections amid different types of its dual-dominating number and its neutrosophic dual-dominating number in neutrosophic graphs?*

Question 5.3. *Is it possible to construct some classes of neutrosophic graphs which have "nice" behavior?*

Question 5.4. *Which mathematical notions do make an independent study to apply these types in neutrosophic graphs?*

Problem 5.5. *Which parameters are related to this parameter?*

Problem 5.6. *Which approaches do work to construct applications to create independent study?*

Problem 5.7. *Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?*

6 Conclusion and Closing Remarks

In this section, concluding remarks and closing remarks are represented. The drawbacks of this article are illustrated. Some benefits and advantages of this study are highlighted.

This study uses two definitions concerning dual-dominating number and neutrosophic dual-dominating number arising from dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. Maximum number of dominated vertices, is a number which is representative based on those vertices. Maximum neutrosophic number of dominated vertices corresponded to dual-dominating set is called neutrosophic dual-dominating number. The connections of vertices which aren't clarified by strong edges differ them from each other and put them in different categories to represent a

Table 2. A Brief Overview about Advantages and Limitations of this Study

Advantages	Limitations
1. Dual-Dominating Number of Model	1. Connections amid Classes
2. Neutrosophic Dual-Dominating Number of Model	
3. Maximal Dual-Dominating Sets	2. Study on Families
4. Dominated Vertices amid all Vertices	
5. Acting on All Vertices	3. Same Models in Family

number which is called dual-dominating number and neutrosophic dual-dominating number arising from dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. Further studies could be about changes in the settings to compare these notions amid different settings of neutrosophic graphs theory. One way is finding some relations amid all definitions of notions to make sensible definitions. In Table (2), some limitations and advantages of this study are pointed out.

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