Dense Numbers and Minimal Dense Sets of Neutrosophic Graphs

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Abstract

New setting is introduced to study dense number and neutrosophic dense number arising from neighborhood and the number of neighbors in neutrosophic graphs assigned to neutrosophic graphs. Consider a vertex. Maximum number of neighbors based on that vertex to compare to its neighbors, is a number which is representative based on that vertex. Minimum neutrosophic number of vertices amid neutrosophic cardinality of all sets of vertices is called neutrosophic dense number. Forming sets from special vertices to figure out different types of number of neighborhoods in the terms of largest number of edges to get minimum number to assign to neutrosophic graphs is key type of approach to have these notions namely dense number and neutrosophic dense number arising from neighborhood and the number of neighbors in neutrosophic graphs assigned to neutrosophic graphs. Two numbers and one set are assigned to a neutrosophic graph, are obtained but now both settings lead to approach is on demand which is to compute and to find representatives of neighborhoods forming different types of sets of vertices in the terms of minimum number and minimum neutrosophic number forming it to get minimum number to assign to a neutrosophic graph. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then a set of vertices is called dense set if for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is greater than the number of neighbors of y. The minimum cardinality between all dense sets is called dense number and it's denoted by $\mathcal{D}(NTG)$; a set of vertices S is called dense set if for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is greater than the number of neighbors of y. The minimum neutrosophic cardinality $\sum_{s\in S}\sum_{i=1}^{3}\sigma_i(s)$ between all dense sets is called neutrosophic dense number and it's denoted by $\mathcal{D}_n(NTG)$. As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs, complete-t-partite-neutrosophic graphs and wheel-neutrosophic graphs. The clarifications are also presented in both sections "Setting of dense number," and "Setting of neutrosophic dense number," for introduced results and used classes. This approach facilitates identifying vertices which form dense number and neutrosophic dense number arising from neighborhood and the number of neighbors in neutrosophic graphs assigned to neutrosophic graphs. In both settings, some classes of well-known neutrosophic graphs are studied. Some clarifications for each result and each definition are provided. The cardinality of set of vertices and neutrosophic cardinality of set of vertices have eligibility to define dense number and neutrosophic dense number but different types of vertices has eligibility to define dense sets. Some results get more frameworks and perspective about these definitions. The way in that, different types of vertices forming different types of sets in the terms of largest number of neighbors and smallest number of vertices forming it to get minimum number to assign to neutrosophic graphs or in other words, the way in that, consider a vertex. Maximum number of neighbors based on that vertex is a number which is representative based on that vertex. Minimum neutrosophic number amid all neutrosophic cardinalities of sets containing representative numbers is called neutrosophic dense number, opens the way to do some approaches. These notions are applied into neutrosophic graphs as individuals but not family of them as drawbacks for these notions. Finding special neutrosophic graphs which are well-known, is an open way to pursue this study. Neutrosophic dense notion is applied to different settings and classes of neutrosophic graphs. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.

Keywords: Dense Number, Neutrosophic Dense Number, Classes of Neutrosophic Graphs

AMS Subject Classification: 05C17, 05C22, 05E45

1 Background

Chromatic number and neutrosophic chromatic number in **Ref.** [2], closing numbers and super-closing numbers as (dual)resolving and (dual)coloring alongside (dual)dominating in (neutrosophic)n-SuperHyperGraph in **Ref.** [3], co-degree and degree of classes of neutrosophic hypergraphs in **Ref.** [4], different types of neutrosophic chromatic number in **Ref.** [5], dimension and coloring alongside domination in neutrosophic hypergraphs in **Ref.** [6], independent set in neutrosophic graphs in **Ref.** [7], neutrosophic chromatic number based on connectedness in **Ref.** [8], properties of SuperHyperGraph and neutrosophic SuperHyperGraph in **Ref.** [9], some polynomials related to numbers in classes of neutrosophic graphs in **Ref.** [10], three types of neutrosophic alliances based on connectedness and edges in **Ref.** [11], are studied. Also, some studies and researches about neutrosophic graphs, are proposed as a book in **Ref.** [1].

In this section, I use two subsections to illustrate a perspective about the background of this study.

1.1 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 1.1. *Is it possible to use mixed versions of ideas concerning "Dense Number", "Neutrosophic Dense Number" and "Neutrosophic Graph" to define some notions which are applied to neutrosophic graphs?*

It's motivation to find notions to use in any classes of neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Having connection amid two vertices have key roles to assign dense number and neutrosophic dense number arising from length and strength of cycles in neutrosophic graphs assigned to neutrosophic graphs. Thus they're used to define new ideas which conclude to the structure of dense number and neutrosophic dense number arising from length and strength of cycles in neutrosophic graphs assigned to neutrosophic graphs. The concept of having large number of neighbors inspires us to study the behavior of edges in the way that, some types of numbers, dense number and neutrosophic dense number arising from length and strength of cycles in neutrosophic graphs assigned to neutrosophic graphs, are the cases of study in the setting of individuals. In both settings, a corresponded numbers concludes the discussion. Also, there are some avenues to extend these notions.

The framework of this study is as follows. In the beginning, I introduce basic definitions to clarify about preliminaries. In subsection "Preliminaries", new notions of dense number and neutrosophic dense number arising from neighborhood and the number of neighbors in neutrosophic graphs assigned to neutrosophic graphs, are highlighted, are introduced and are clarified as individuals. In section "Preliminaries", different numbers of neighbors forming different types of neighborhoods in the terms of large numbers and minimal sets forming it to get minimum number to assign to neutrosophic graphs, have the key role in this way. General results are obtained and also, the results about the basic notions of dense number and neutrosophic dense number arising from neighborhood and the number of neighbors in neutrosophic graphs assigned to neutrosophic graphs, are elicited. Some classes of neutrosophic graphs are studied in the terms of dense number and neutrosophic dense number arising from neighborhood and the number of neighbors in neutrosophic graphs assigned to neutrosophic graphs, in section "Setting of dense number," as individuals. In section "Setting of dense number," dense number is applied into individuals. As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs, complete-t-partite-neutrosophic graphs and wheel-neutrosophic graphs. The clarifications are also presented in both sections "Setting of dense number," and "Setting of neutrosophic dense number," for introduced results and used classes. In section "Applications in Time Table and Scheduling", two applications are posed for quasi-complete and complete notions, namely complete-neutrosophic graphs and complete-t-partite-neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects and the mentioned models are considered as individual. In section "Open Problems", some problems and questions for further studies are proposed. In section "Conclusion and Closing Remarks", gentle discussion about results and applications is featured. In section "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions is formed.

1.2 Preliminaries

In this subsection, basic material which is used in this article, is presented. Also, new ideas and their clarifications are elicited.

Basic idea is about the model which is used. First definition introduces basic model.

Definition 1.2. (Graph).

$G = (V, E)$ is called a **graph** if V is a set of objects and E is a subset of $V \times V$ (E is a set of 2-subsets of V) where V is called **vertex set** and E is called **edge set**. Every two vertices have been corresponded to at most one edge.

Neutrosophic graph is the foundation of results in this paper which is defined as follows. Also, some related notions are demonstrated.

Definition 1.3. (Neutrosophic Graph And Its Special Case).

$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, and $\mu_i : E \to [0, 1]$. We add one condition on it and we use

special case of neutrosophic graph but with same name. The added condition is as follows, for every $v_iv_j \in E$,
$$\mu(v_iv_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

(i) : σ is called **neutrosophic vertex set**.

(ii) : μ is called **neutrosophic edge set**.

(iii) : $|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$.

(iv) : $\sum_{v\in V}\sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

(v) : $|E|$ is called **size** of NTG and it's denoted by $\mathcal{D}(NTG)$.

(vi) : $\sum_{e\in E}\sum_{i=1}^{3}\mu_i(e)$ is called **neutrosophic size** of NTG and it's denoted by $\mathcal{S}_n(NTG)$.

Some classes of well-known neutrosophic graphs are defined. These classes of neutrosophic graphs are used to form this study and the most results are about them.

Definition 1.4. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) : a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is called **path** where $x_ix_{i+1} \in E,\ i = 0, 1, \cdots, \mathcal{O}(NTG) - 1$;

(ii) : **strength** of path $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is $\bigwedge_{i=0,\cdots,\mathcal{O}(NTG)-1}\mu(x_ix_{i+1})$;

(iii) : **connectedness** amid vertices x_0 and x_t is
$$\mu^{\infty}(x_0, x_t) = \bigvee_{P:x_0,x_1,\cdots,x_t}\ \bigwedge_{i=0,\cdots,t-1}\mu(x_ix_{i+1});$$

(iv) : a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}, x_0$ is called **cycle** where $x_ix_{i+1} \in E,\ i = 0, 1, \cdots, \mathcal{O}(NTG) - 1,\ x_{\mathcal{O}(NTG)}x_0 \in E$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1}\mu(v_iv_{i+1})$;

(v) : it's **t-partite** where V is partitioned to t parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge xy implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's complete, then it's denoted by $K_{\sigma_1,\sigma_2,\cdots,\sigma_t}$ where σ_i is σ on $V_i^{s_i}$ instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$;

(vi) : t-partite is **complete bipartite** if $t = 2$, and it's denoted by K_{σ_1,σ_2};

(vii) : complete bipartite is **star** if $|V_1| = 1$, and it's denoted by S_{1,σ_2};

$(viii)$: a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by W_{1,σ_2};

(ix) : it's **complete** where $\forall uv \in V,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$;

(x) : it's **strong** where $\forall uv \in E,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$.

To make them concrete, I bring preliminaries of this article in two upcoming definitions in other ways.

Definition 1.5. (Neutrosophic Graph And Its Special Case).

$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, and $\mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_i v_j \in E$,

$$\mu(v_i v_j) \le \sigma(v_i) \wedge \sigma(v_j).$$

$|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$. $\Sigma_{v \in V} \sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

Definition 1.6. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then it's **complete** and denoted by CMT_σ if $\forall x, y \in V, xy \in E$ and $\mu(xy) = \sigma(x) \wedge \sigma(y)$; a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is called **path** and it's denoted by PTH where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1$; a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}, x_0$ is called **cycle** and denoted by CYC where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1,\ x_{\mathcal{O}(NTG)} x_0 \in E$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$; it's **t-partite** where V is partitioned to t parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge xy implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's **complete**, then it's denoted by $CMT_{\sigma_1, \sigma_2, \cdots, \sigma_t}$ where σ_i is σ on $V_i^{s_i}$ instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$; t-partite is **complete bipartite** if $t = 2$, and it's denoted by CMT_{σ_1, σ_2}; complete bipartite is **star** if $|V_1| = 1$, and it's denoted by STR_{1, σ_2}; a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by WHL_{1, σ_2}.

Remark 1.7. Using notations which is mixed with literatures, are reviewed.

1. $NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3)),\ \mathcal{O}(NTG)$, and $\mathcal{O}_n(NTG)$;

2. $CMT_\sigma, PTH, CYC, STR_{1,\sigma_2}, CMT_{\sigma_1,\sigma_2}, CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}$, and WHL_{1,σ_2}.

Definition 1.8. (Dense Numbers).

Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) a set of vertices is called **dense set** if for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is greater than the number of neighbors of y. The minimum cardinality between all dense sets is called **dense number** and it's denoted by $\mathcal{D}(NTG)$;

(ii) a set of vertices S is called **dense set** if for every vertex y outside, there's at least one vertex x inside such that they're endpoints $xy \in E$ and the number of neighbors of x is greater than the number of neighbors of y. The minimum neutrosophic cardinality $\sum_{s \in S} \sum_{i=1}^{3} \sigma_i(s)$ between all dense sets is called **neutrosophic dense number** and it's denoted by $\mathcal{D}_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

Example 1.9. In Figure (1), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since every vertex inside has one neighbor inside and two neighbors outside. Hence the number of neighbors inside isn't greater than the number of neighbors outside;

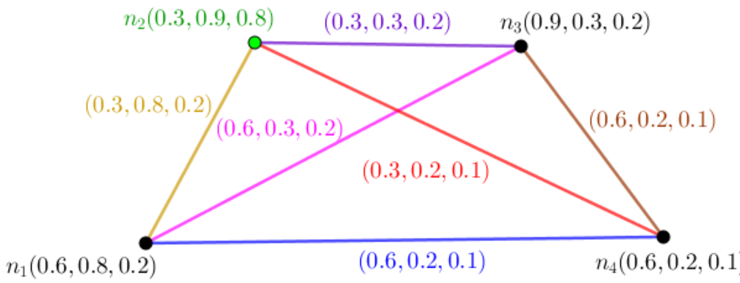

Figure 1. A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number.

(*ii*) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since every vertex inside has no neighbor inside and three neighbors outside. Hence the number of neighbors inside isn't greater than the number of neighbors outside;

(*iii*) if $S_1 = \{n_1, n_2, n_3\}, S_2 = \{n_1, n_2, n_4\}, S_3 = \{n_2, n_3, n_4\}$ are sets of vertices, then they're dense sets since every vertex inside has two neighbors inside and one neighbor outside. Hence the number of neighbors inside is greater than the number of neighbors outside. The minimum cardinality between all dense sets is 3. Thus $\mathcal{D}(CMT_\sigma) = 3$;

(*iv*) if $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since every vertex inside has one neighbor inside and two neighbors outside. Hence the number of neighbors inside isn't greater than the number of neighbors outside;

(*v*) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since every vertex inside has no neighbor inside and three neighbors outside. Hence the number of neighbors inside isn't greater than the number of neighbors outside;

(*vi*) if $S_1 = \{n_1, n_2, n_3\}, S_2 = \{n_1, n_2, n_4\}, S_3 = \{n_2, n_3, n_4\}$ are sets of vertices, then they're dense sets since every vertex inside has two neighbors inside and one neighbor outside. Hence the number of neighbors inside is greater than the number of neighbors outside. The minimum neutrosophic cardinality $\sum_{s \in S} \sum_{i=1}^{3} \sigma_i(s)$ between all dense sets is 3.9. Thus $\mathcal{D}_n(CMT_\sigma) = 3.9$.

2 Setting of dense number

In this section, I provide some results in the setting of dense number. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, star-neutrosophic graph, bipartite-neutrosophic graph, t-partite-neutrosophic graph, and wheel-neutrosophic graph, are both of cases of study and classes which the results are about them.

Proposition 2.1. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{D}(CMT_\sigma) = \lfloor \frac{\mathcal{O}(CMT_\sigma)}{2} \rfloor + 1.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. The number of vertices is $\mathcal{O}(CMT_\sigma)$. Sets of vertices with cardinality $\lfloor \frac{\mathcal{O}(CMT_\sigma)}{2} \rfloor + 1$ are dense sets since every vertex inside has $\lfloor\frac{\mathcal{O}(CMT_\sigma)}{2}\rfloor$ neighbors inside and $\lfloor\frac{\mathcal{O}(CMT_\sigma)}{2}\rfloor - 1$ neighbors outside. Hence the number of neighbors inside is greater than the number of neighbors outside. The minimum cardinality between all dense sets is $\lfloor\frac{\mathcal{O}(CMT_\sigma)}{2}\rfloor + 1$. Thus

$$\mathcal{D}(CMT_\sigma) = \lfloor\frac{\mathcal{O}(CMT_\sigma)}{2}\rfloor + 1.$$

□

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.2. In Figure (2), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since every vertex inside has one neighbor inside and two neighbors outside. Hence the number of neighbors inside isn't greater than the number of neighbors outside;

(ii) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since every vertex inside has no neighbor inside and three neighbors outside. Hence the number of neighbors inside isn't greater than the number of neighbors outside;

(iii) if $S_1 = \{n_1, n_2, n_3\}, S_2 = \{n_1, n_2, n_4\}, S_3 = \{n_2, n_3, n_4\}$ are sets of vertices, then they're dense sets since every vertex inside has two neighbors inside and one neighbor outside. Hence the number of neighbors inside is greater than the number of neighbors outside. The minimum cardinality between all dense sets is 3. Thus $\mathcal{D}(CMT_\sigma) = 3$;

(iv) if $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since every vertex inside has one neighbor inside and two neighbors outside. Hence the number of neighbors inside isn't greater than the number of neighbors outside;

(v) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since every vertex inside has no neighbor inside and three neighbors outside. Hence the number of neighbors inside isn't greater than the number of neighbors outside;

(vi) if $S_1 = \{n_1, n_2, n_3\}, S_2 = \{n_1, n_2, n_4\}, S_3 = \{n_2, n_3, n_4\}$ are sets of vertices, then they're dense sets since every vertex inside has two neighbors inside and one neighbor outside. Hence the number of neighbors inside is greater than the number of neighbors outside. The minimum neutrosophic cardinality $\sum_{s\in S}\sum_{i=1}^{3}\sigma_i(s)$ between all dense sets is 3.9. Thus $\mathcal{D}_n(CMT_\sigma) = 3.9$.

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

Proposition 2.3. *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Then*

$$\mathcal{D}(PTH) = \lfloor\frac{\mathcal{O}(PTH)}{3}\rfloor.$$

Proof. Suppose $PTH : (V, E, \sigma, \mu)$ is a path-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(PTH)}$ be a path-neutrosophic graph. Every vertex which isn't leaf, has

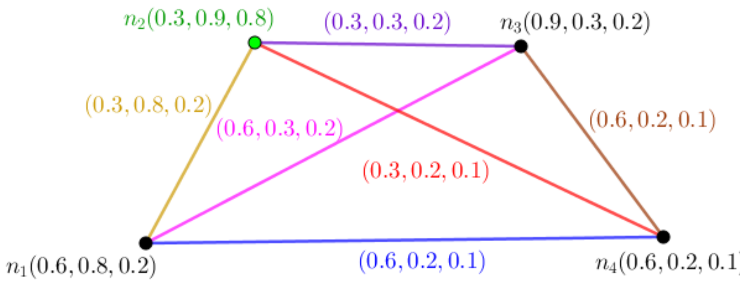

Figure 2. A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number.

two neighbors. So these vertices have same positions and by the minimum number of vertices is on demand, the result is obtained. Thus

$$\mathcal{D}(PTH) = \lfloor \frac{\mathcal{O}(PTH)}{3} \rfloor.$$

□

Example 2.4. There are two sections for clarifications.

(a) In Figure (3), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since there are some vertices n_4 and n_5 such that have no neighbor in S. Consider the vertex n_3. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_3 which is two;

(ii) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since there are some vertices n_3, n_4, and n_5 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is one which isn't [greater than] equal to the number of neighbors for n_2 which is two;

(iii) $S = \{n_2, n_4\}$ is only set of vertices which is minimal set such that it's dense set since either the vertex n_2 or the vertex n_1 has to be chosen but it's impossible to have vertex n_1 in S. Hence only choice is the vertex n_2. After that, either the vertex n_3, n_5 or the vertex n_4 has to be chosen but it's impossible to have vertex n_3, n_5 in S. Hence only choice is the vertex n_4. Every vertex inside has two neighbors and every vertex outside has either one or two neighbors. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There's only one dense set. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}(PTH) = 2$;

(iv) if $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since there are some vertices n_4 and n_5 such that have no neighbor in S. Consider the vertex n_3. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_3 which is two;

(v) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since there are some vertices n_3, n_4, and n_5 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is one which isn't [greater than] equal to the number of neighbors for n_2 which is two;

(vi) $S = \{n_2, n_4\}$ is only set of vertices which is minimal set such that it's dense set since either the vertex n_2 or the vertex n_1 has to be chosen but it's impossible to have vertex n_1 in S. Hence only choice is the vertex n_2. After that, either the vertex n_3, n_5 or the vertex n_4 has to be chosen but it's impossible to have vertex n_3, n_5 in S. Hence only choice is the vertex n_4. Every vertex inside has two neighbors and every vertex outside has either one or two neighbors. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There's only one dense set. So the minimum cardinality between all dense sets is 2. Also, $\mathcal{D}_n(PTH) = 3$.

(b) In Figure (4), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since there are two vertices n_4, n_5 and n_6 such that have no neighbor in S. Consider the vertex n_3. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_3 which is two;

(ii) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since there are two vertices n_3, n_4, n_5 and n_6 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is one which isn't [greater than] equal to the number of neighbors for n_2 which is two;

(iii) $S = \{n_2, n_5\}$ is only set of vertices which is minimal set such that it's dense set since either the vertex n_2 or the vertex n_1 has to be chosen but it's impossible to have vertex n_1 in S. Hence only choice is the vertex n_2. After that, either the vertex n_3, n_5, n_6 or the vertex n_4 has to be chosen but it's impossible to have vertex n_3, n_4, n_6 in S. Hence only choice is the vertex n_5. Every vertex inside has two neighbors and every vertex outside has either one or two neighbors. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There's only one dense set. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}(PTH) = 2$;

(iv) If $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since there are some vertices n_4, n_5 and n_6 such that have no neighbor in S. Consider the vertex n_3. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_3 which is two;

(v) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since there are some vertices n_3, n_4, n_5 and n_6 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is one which isn't [greater than] equal to the number of neighbors for n_2 which is two;

(vi) $S = \{n_2, n_5\}$ is only set of vertices which is minimal set such that it's dense set since either the vertex n_2 or the vertex n_1 has to be chosen but it's impossible to have vertex n_1 in S. Hence only choice is the vertex n_2. After that, either the vertex n_3, n_5, n_6 or the vertex n_4 has to be chosen but it's impossible to have vertex n_3, n_4, n_6 in S. Hence only choice is the vertex n_5. Every vertex inside has two neighbors and every vertex outside has either one or two neighbors. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There's only one dense set. So the minimum cardinality between all dense sets is 2. Also, $\mathcal{D}_n(PTH) = 3.8$.

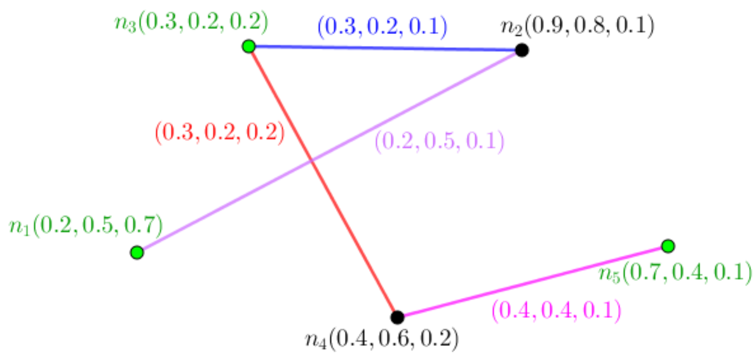

Figure 3. A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number.

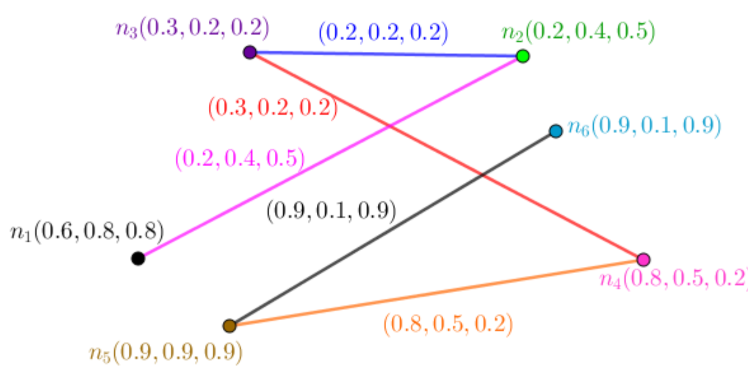

Figure 4. A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number.

Proposition 2.5. *Let* $NTG : (V, E, \sigma, \mu)$ *be a cycle-neutrosophic graph where* $\mathcal{O}(CYC) \geq 3$. *Then*

$$\mathcal{D}(CYC) = \lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor.$$

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(CYC)}, x_1$ be a cycle-neutrosophic graph. Every vertex has two neighbors. So these vertices have same positions and by the minimum number of vertices is on demand, the result is obtained. Thus

$$\mathcal{D}(CYC) = \lfloor \frac{\mathcal{O}(CYC)}{3} \rfloor.$$

□

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.6. There are two sections for clarifications.

(a) In Figure (5), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) If $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since there are some vertices n_4 and n_5 such that have no neighbor in S. Consider the vertex n_3. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_3 which is two;
- (ii) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since there are some vertices n_3, n_4 and n_5 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is two which is [greater than] equal to the number of neighbors for n_2 which is two;
- (iii) $S_1 = \{n_1, n_4\}, S_2 = \{n_2, n_5\}, S_3 = \{n_3, n_6\}$ are only sets of vertices which are minimal sets such that they're dense sets. Since every vertex inside has two neighbors and every vertex outside has two neighbors. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There're only three dense sets. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}(CYC) = 2$;
- (iv) if $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since there are some vertices n_4 and n_5 such that have no neighbor in S. Consider the vertex n_3. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_3 which is two;
- (v) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since there are some vertices n_3, n_4 and n_5 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is two which is [greater than] equal to the number of neighbors for n_2 which is two;
- (vi) $S_1 = \{n_1, n_4\}, S_2 = \{n_2, n_5\}, S_3 = \{n_3, n_6\}$ are only sets of vertices which are minimal sets such that they're dense sets. Since every vertex inside has two neighbors and every vertex outside has two neighbors. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There're only three dense sets. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}_n(CYC) = 2.2$ corresponded to S_1;

(*b*) In Figure (6), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (*i*) If $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since there's one vertex n_4 such that have no neighbor in S. Consider the vertex n_3. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_3 which is two;
- (*ii*) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since there are some vertices n_3, and n_4 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is two which is [greater than] equal to the number of neighbors for n_2 which is two;
- (*iii*) $S_1 = \{n_1, n_3\}, S_2 = \{n_1, n_4\}, S_3 = \{n_2, n_4\}, S_4 = \{n_2, n_5\}, S_5 = \{n_3, n_5\}$ are only sets of vertices which are minimal sets such that they're dense sets. Since every vertex inside has two neighbors and every vertex outside has two neighbors. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There're only five dense sets. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}(CYC) = 2$;
- (*iv*) if $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since there's one vertex n_4 such that have no neighbor in S. Consider the vertex n_3. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_3 which is two;
- (*v*) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since there are some vertices n_3, and n_4 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is two which is [greater than] equal to the number of neighbors for n_2 which is two;
- (*vi*)

$$\begin{aligned} S_1 &= \{n_1, n_3\} \to 2.8 \\ S_2 &= \{n_1, n_4\} \to 2.2 \\ S_3 &= \{n_2, n_4\} \to 3.4 \\ S_4 &= \{n_2, n_5\} \to 2.5 \\ S_5 &= \{n_3, n_5\} \to 2.3 \\ &\text{Minimum number is } 2.2 \end{aligned}$$

  are only sets of vertices which are minimal sets such that they're dense sets. Since every vertex inside has two neighbors and every vertex outside has two neighbors. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There're only five dense sets. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}_n(CYC) = 2$ corresponded to S_2.

Proposition 2.7. *Let* $NTG : (V, E, \sigma, \mu)$ *be a star-neutrosophic graph with center* c*. Then*

$$\mathcal{D}(STR_{1,\sigma_2}) = 1.$$

Proof. Suppose $STR_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. For non-center vertices, the number of neighbors is 1 and center has $\mathcal{O}(STR_{1,\sigma_2}) - 1$ neighbors. If $S = \{c\}$ is a set of vertices, then every vertex inside has $\mathcal{O}(STR_{1,\sigma_2}) - 1$ neighbors and every vertex outside has one neighbor. So the number of neighbors for every vertex

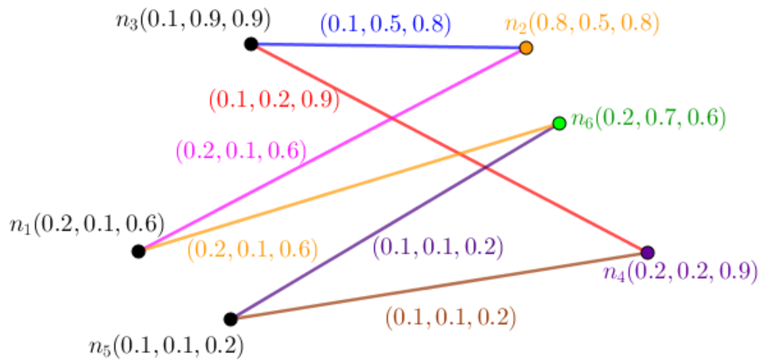

Figure 5. A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number.

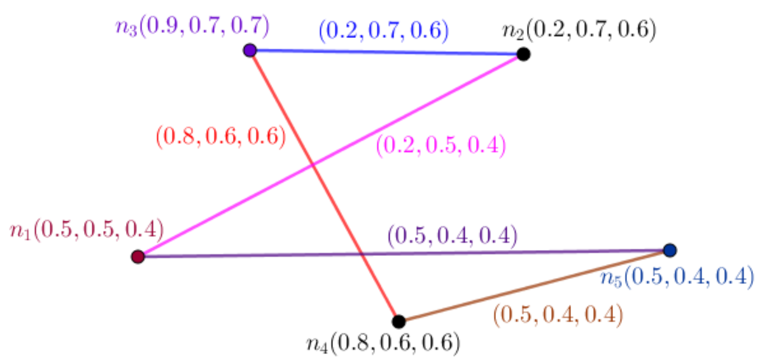

Figure 6. A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number.

inside is greater than the number of neighbors for every vertex outside and also every vertex inside is a neighbor for every vertex outside. Thus

$$\mathcal{D}(STR_{1,\sigma_2}) = 1.$$

□

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.8. There is one section for clarifications. In Figure (7), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_4, n_2\}$ is a set of vertices, then it isn't dense set since there are some vertices n_3 and n_5 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is four which isn't [greater than] equal to the number of neighbors for n_2 which is one;

(ii) if $S = \{n_2, n_3, n_4, n_5\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified. Consider the vertex n_1. The number of neighbors for $n_i,\ i = 2, 3, 4, 5$ is one which isn't [greater than] equal to the number of neighbors for n_1 which is four;

(iii) $S = \{n_1\}$ is only set of vertices which is minimal set such that it's dense set. Since every vertex inside has four neighbors and every vertex outside has one neighbor. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There's only one dense set in the term of minimal set. So the minimum cardinality between all dense sets is 1. Thus $\mathcal{D}(STR_{1,\sigma_2}) = 1$;

(iv) If $S = \{n_4, n_2\}$ is a set of vertices, then it isn't dense set since there are some vertices n_3 and n_5 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is four which isn't [greater than] equal to the number of neighbors for n_2 which is one;

(v) if $S = \{n_2, n_3, n_4, n_5\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified. Consider the vertex n_1. The number of neighbors for $n_i,\ i = 2, 3, 4, 5$ is one which isn't [greater than] equal to the number of neighbors for n_1 which is four;

(vi) $S = \{n_1\}$ is only set of vertices which is minimal set such that it's dense set. Since every vertex inside has four neighbors and every vertex outside has one neighbor. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There's only one dense set in the term of minimal set. So the minimum cardinality between all dense sets is 1. Thus $\mathcal{D}_n(STR_{1,\sigma_2}) = 1.9$

Proposition 2.9. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-bipartite-neutrosophic graph. Then*

$$\mathcal{D}(CMC_{\sigma_1,\sigma_2}) = \min\{|V_1|, |V_2|\}.$$

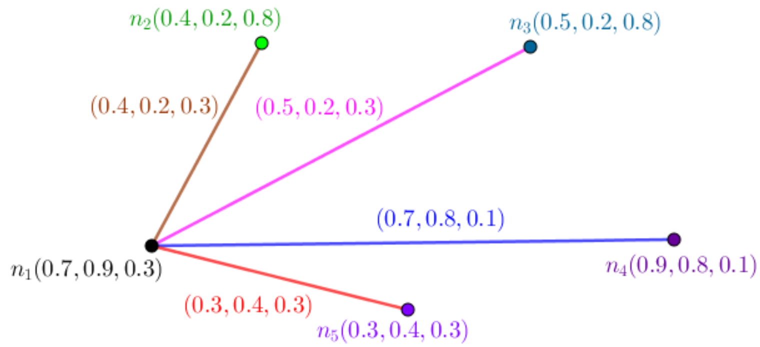

Figure 7. A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number.

Proof. Suppose $CMC_{\sigma_1,\sigma_2} : (V, E, \sigma, \mu)$ is a complete-bipartite-neutrosophic graph. Consider $V_1 = \min\{|V_1|, |V_2|\}$. For vertices in V_2, the number of neighbors is $|V_1|$ and for vertices in V_1, the number of neighbors is $|V_2|$ where $|V_1| \leq |V_2|$. If $S = V_1$ is a set of vertices, then every vertex inside has $|V_2|$ neighbors and every vertex outside has $|V_1|$ neighbors. So the number of neighbors for every vertex inside is greater than the number of neighbors for every vertex outside and also every vertex inside is a neighbor for every vertex outside. Thus

$$\mathcal{D}(CMC_{\sigma_1,\sigma_2}) = \min\{|V_1|, |V_2|\}.$$

□

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.10. There is one section for clarifications. In Figure (8), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If $S = \{n_4, n_2\}$ is a set of vertices, then it's dense set since there's no vertex such that have neighbor in S. Consider the vertex n_1. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_1 which is two;

(*ii*) if $S = \{n_4\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified;

(*iii*) $S_1 = \{n_1, n_2\}, S_2 = \{n_1, n_3\}, S_3 = \{n_1, n_4\}, S_4 = \{n_2, n_3\}, S_5 = \{n_2, n_4\}, S_6 = \{n_3, n_4\}$ are only sets of vertices which are minimal sets such that they're dense sets. Since every vertex inside has two neighbors and every vertex outside has two neighbors. Hence the number of neighbors for vertices in either of $S_i,\ i = 1, 2, \cdots, 6$ is equal to [greater than] the number of neighbors for vertices in $V \setminus S_i,\ i = 1, 2, \cdots, 6$. There're only four dense sets in the term of minimal set. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}(CMC_{\sigma_1,\sigma_2}) = 2$;

(*iv*) If $S = \{n_4, n_2\}$ is a set of vertices, then it's dense set since there's no vertex such that have neighbor in S. Consider the vertex n_1. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_1 which is two;

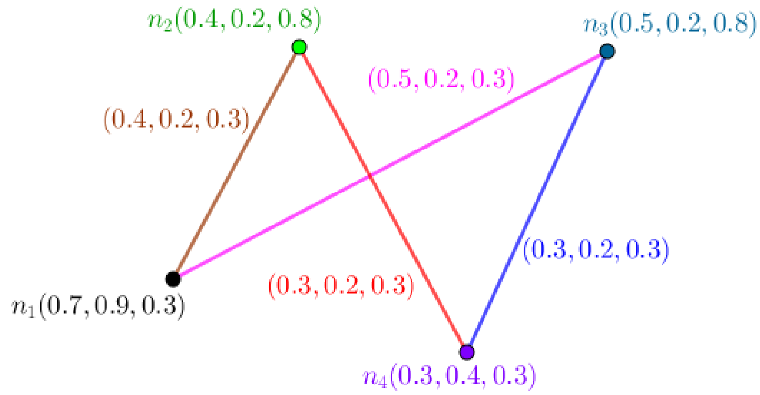

Figure 8. A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number.

(*v*) if $S = \{n_4\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified;

(*vi*) $S_1 = \{n_1, n_2\}, S_2 = \{n_1, n_3\}, S_3 = \{n_1, n_4\}, S_4 = \{n_2, n_3\}, S_5 = \{n_2, n_4\}, S_6 = \{n_3, n_4\}$ are only sets of vertices which are minimal sets such that they're dense sets. Since every vertex inside has two neighbors and every vertex outside has two neighbors. Hence the number of neighbors for vertices in either of $S_i,\ i = 1, 2, \cdots, 6$ is equal to [greater than] the number of neighbors for vertices in $V \setminus S_i,\ i = 1, 2, \cdots, 6$. There're only four dense sets in the term of minimal set. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}_n(CMC_{\sigma_1,\sigma_2}) = 2.4$ corresponded to $S_5 = \{n_2, n_4\}$.

Proposition 2.11. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-t-partite-neutrosophic graph where $t \geq 3$. Then*

$$\mathcal{D}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \min\{|V_1|, |V_2|, \cdots, |V_t|\}.$$

Proof. Suppose $CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t} : (V, E, \sigma, \mu)$ is a complete-t-partite-neutrosophic graph. Consider $|V_1| = \min\{|V_1|, |V_2|, \cdots, |V_t|\}$. For vertices in $V_i,\ i = 2, 3, \cdots, t$, the number of neighbors is $|V_1| + |V_2| + \cdots + |V_{i-1}| + |V_{i+1}| + \cdots + |V_t|$ and for vertices in V_1, the number of neighbors is $|V_2| + |V_3| + \cdots + |V_t|$ where $|V_1| \leq |V_i|,\ i = 2, 3, \cdots, t$. If $S = V_1$ is a set of vertices, then every vertex inside has $|V_2| + |V_3| + \cdots + |V_t|$ neighbors and every vertex outside has $|V_1| + |V_2| + \cdots + |V_{i-1}| + |V_{i+1}| + \cdots + |V_t|$ neighbors where $|V_1| \leq |V_i|,\ i = 2, 3, \cdots, t$. So the number of neighbors for every vertex inside is greater than the number of neighbors for every vertex outside and also every vertex inside is a neighbor for every vertex outside. Thus

$$\mathcal{D}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \min\{|V_1|, |V_2|, \cdots, |V_t|\}.$$

□

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

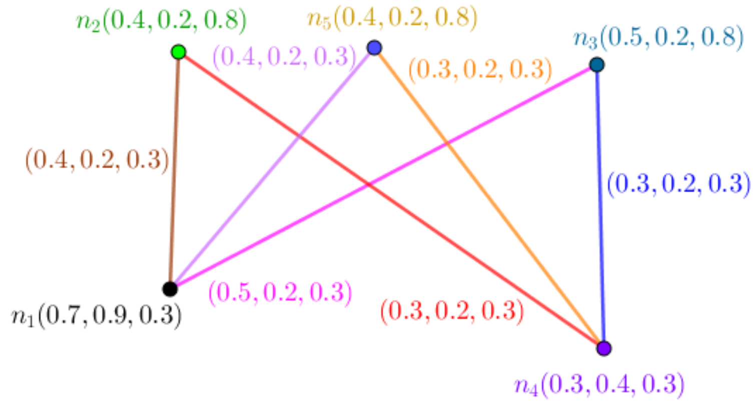

Figure 9. A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number.

Example 2.12. There is one section for clarifications. In Figure (9), a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If $S = \{n_4, n_2\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified in the terms of S. Consider the vertex n_1. The number of neighbors for n_2 is two which isn't greater than [equal to] the number of neighbors for n_1 which is three;

(*ii*) if $S = \{n_4\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified in the terms of having neighbor from a member of S;

(*iii*) $S = \{n_1, n_4\}$, is only set of vertices which is minimal set such that it's dense set. Since every vertex inside has three neighbors and every vertex outside has two neighbors. Hence the number of neighbors for vertices in S is [equal to] greater than the number of neighbors for vertices in $V \setminus S$. There's only one dense set in the term of minimal set. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

(*iv*) If $S = \{n_4, n_2\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified in the terms of S. Consider the vertex n_1. The number of neighbors for n_2 is two which isn't greater than [equal to] the number of neighbors for n_1 which is three;

(*v*) if $S = \{n_4\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified in the terms of having neighbor from a member of S;

(*vi*) $S = \{n_1, n_4\}$, is only set of vertices which is minimal set such that it's dense set. Since every vertex inside has three neighbors and every vertex outside has two neighbors. Hence the number of neighbors for vertices in S is [equal to] greater than the number of neighbors for vertices in $V \setminus S$. There's only one dense set in the term of minimal set. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2.9$.

Proposition 2.13. *Let $NTG : (V, E, \sigma, \mu)$ be a wheel-neutrosophic graph. Then*

$$\mathcal{D}(WHL_{1,\sigma_2}) = 1.$$

Proof. Suppose $WHL_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a wheel-neutrosophic graph. The argument is elementary. Since all vertices of a cycle join to one vertex, consider c as a center. For non-center vertices, the number of neighbors is three and for center, the number of neighbors is $\mathcal{O}(WHL_{1,\sigma_2}) - 1$. If $S = \{c\}$ is a set of vertices, then every vertex inside has $\mathcal{O}(WHL_{1,\sigma_2}) - 1$ neighbors and every vertex outside has three neighbors. So the number of neighbors for every vertex inside is greater than the number of neighbors for every vertex outside and also every vertex inside is a neighbor for every vertex outside. Thus

$$\mathcal{D}(WHL_{1,\sigma_2}) = 1.$$

□

The clarifications about results are in progress as follows. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.14. There is one section for clarifications. In Figure (10), a wheel-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) If $S = \{n_4, n_2\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified in the terms of S. Consider the vertex n_1. The number of neighbors for n_2 is three which isn't greater than [equal to] the number of neighbors for n_1 which is four;
- (ii) if $S = \{n_4\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified in the terms of having neighbor from a member of S;
- (iii) $S = \{n_1\}$ is only set of vertices which is minimal set such that it's dense set. Since every vertex inside has four neighbors and every vertex outside has three neighbors. Hence the number of neighbors for vertices in S is [equal to] greater than the number of neighbors for vertices in $V \setminus S$. There's only one dense set in the term of minimal set. So the minimum cardinality between all dense sets is 1. Thus $\mathcal{D}(WHL_{1,\sigma_2}) = 1$;
- (iv) If $S = \{n_4, n_2\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified in the terms of S. Consider the vertex n_1. The number of neighbors for n_2 is three which isn't greater than [equal to] the number of neighbors for n_1 which is four;
- (v) if $S = \{n_4\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified in the terms of having neighbor from a member of S;
- (vi) $S = \{n_1\}$ is only set of vertices which is minimal set such that it's dense set. Since every vertex inside has four neighbors and every vertex outside has three neighbors. Hence the number of neighbors for vertices in S is [equal to] greater than the number of neighbors for vertices in $V \setminus S$. There's only one dense set in the term of minimal set. So the minimum cardinality between all dense sets is 1. Thus $\mathcal{D}_n(WHL_{1,\sigma_2}) = 1.9$.

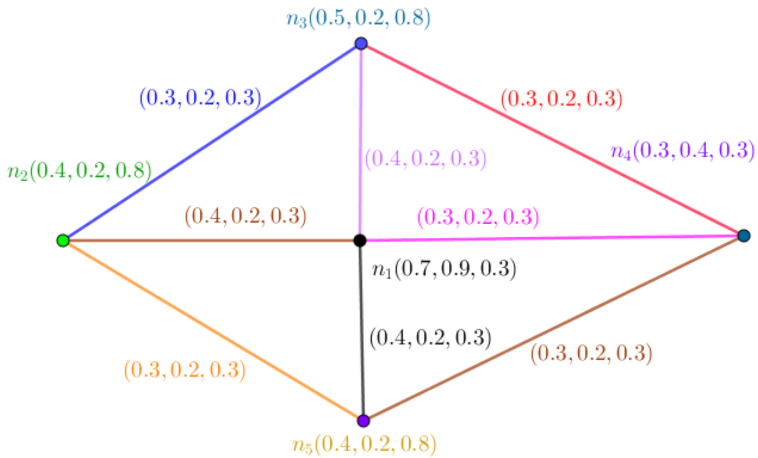

Figure 10. A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number.

3 Setting of neutrosophic dense number

In this section, I provide some results in the setting of neutrosophic dense number. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, star-neutrosophic graph, bipartite-neutrosophic graph, t-partite-neutrosophic graph, and wheel-neutrosophic graph, are both of cases of study and classes which the results are about them.

Proposition 3.1. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{D}_n(CMT_\sigma) = \min\{\sum_{i=1}^{\lfloor\frac{\mathcal{O}(CMT_\sigma)}{2}\rfloor+1} \sigma(x_i)\}.$$

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there's one edge between two vertices. The number of vertices is $\mathcal{O}(CMT_\sigma)$. Sets of vertices with cardinality $\lfloor\frac{\mathcal{O}(CMT_\sigma)}{2}\rfloor + 1$ are dense sets since every vertex inside has $\lfloor\frac{\mathcal{O}(CMT_\sigma)}{2}\rfloor$ neighbors inside and $\lfloor\frac{\mathcal{O}(CMT_\sigma)}{2}\rfloor - 1$ neighbors outside. Hence the number of neighbors inside is greater than the number of neighbors outside. The minimum cardinality between all dense sets is $\lfloor\frac{\mathcal{O}(CMT_\sigma)}{2}\rfloor + 1$. Thus

$$\mathcal{D}_n(CMT_\sigma) = \min\{\sum_{i=1}^{\lfloor\frac{\mathcal{O}(CMT_\sigma)}{2}\rfloor+1} \sigma(x_i)\}.$$

□

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.2. In Figure (11), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

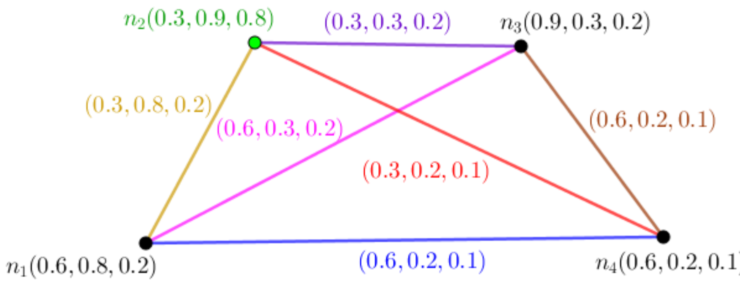

Figure 11. A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number.

(*i*) If $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since every vertex inside has one neighbor inside and two neighbors outside. Hence the number of neighbors inside isn't greater than the number of neighbors outside;

(*ii*) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since every vertex inside has no neighbor inside and three neighbors outside. Hence the number of neighbors inside isn't greater than the number of neighbors outside;

(*iii*) if $S_1 = \{n_1, n_2, n_3\}, S_2 = \{n_1, n_2, n_4\}, S_3 = \{n_2, n_3, n_4\}$ are sets of vertices, then they're dense sets since every vertex inside has two neighbors inside and one neighbor outside. Hence the number of neighbors inside is greater than the number of neighbors outside. The minimum cardinality between all dense sets is 3. Thus $\mathcal{D}(CMT_\sigma) = 3$;

(*iv*) if $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since every vertex inside has one neighbor inside and two neighbors outside. Hence the number of neighbors inside isn't greater than the number of neighbors outside;

(*v*) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since every vertex inside has no neighbor inside and three neighbors outside. Hence the number of neighbors inside isn't greater than the number of neighbors outside;

(*vi*) if $S_1 = \{n_1, n_2, n_3\}, S_2 = \{n_1, n_2, n_4\}, S_3 = \{n_2, n_3, n_4\}$ are sets of vertices, then they're dense sets since every vertex inside has two neighbors inside and one neighbor outside. Hence the number of neighbors inside is greater than the number of neighbors outside. The minimum neutrosophic cardinality $\sum_{s \in S} \sum_{i=1}^{3} \sigma_i(s)$ between all dense sets is 3.9. Thus $\mathcal{D}_n(CMT_\sigma) = 3.9$.

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

Proposition 3.3. *Let* $NTG : (V, E, \sigma, \mu)$ *be a path-neutrosophic graph* $x_1, x_2, \cdots, x_{\mathcal{O}(PTH)}$. *Then*

$$\mathcal{D}_n(PTH) = \min\{\sum_{i=-1}^{\mathcal{O}(PTH)-4} \sigma(x_{i+3}), \sum_{i=0}^{\mathcal{O}(PTH)-4} \sigma(x_{i+3})\} + c.$$

Proof. Suppose $PTH : (V, E, \sigma, \mu)$ is a path-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(PTH)}$ be a path-neutrosophic graph. Every vertex which isn't leaf, has two neighbors. So these vertices have same positions and by the minimum number of vertices is on demand, the result is obtained. Thus

$$\mathcal{D}_n(PTH) = \min\{\sum_{i=-1}^{\mathcal{O}(PTH)-4} \sigma(x_{i+3}), \sum_{i=0}^{\mathcal{O}(PTH)-4} \sigma(x_{i+3})\} + c.$$

$\square$

Example 3.4. There are two sections for clarifications.

(a) In Figure (12), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since there are some vertices n_4 and n_5 such that have no neighbor in S. Consider the vertex n_3. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_3 which is two;

(ii) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since there are some vertices n_3, n_4, and n_5 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is one which isn't [greater than] equal to the number of neighbors for n_2 which is two;

(iii) $S = \{n_2, n_4\}$ is only set of vertices which is minimal set such that it's dense set since either the vertex n_2 or the vertex n_1 has to be chosen but it's impossible to have vertex n_1 in S. Hence only choice is the vertex n_2. After that, either the vertex n_3, n_5 or the vertex n_4 has to be chosen but it's impossible to have vertex n_3, n_5 in S. Hence only choice is the vertex n_4. Every vertex inside has two neighbors and every vertex outside has either one or two neighbors. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There's only one dense set. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}(PTH) = 2$;

(iv) if $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since there are some vertices n_4 and n_5 such that have no neighbor in S. Consider the vertex n_3. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_3 which is two;

(v) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since there are some vertices n_3, n_4, and n_5 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is one which isn't [greater than] equal to the number of neighbors for n_2 which is two;

(vi) $S = \{n_2, n_4\}$ is only set of vertices which is minimal set such that it's dense set since either the vertex n_2 or the vertex n_1 has to be chosen but it's impossible to have vertex n_1 in S. Hence only choice is the vertex n_2. After that, either the vertex n_3, n_5 or the vertex n_4 has to be chosen but it's impossible to have vertex n_3, n_5 in S. Hence only choice is the vertex n_4. Every vertex inside has two neighbors and every vertex outside has either one or two neighbors. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There's only one dense set. So the minimum cardinality between all dense sets is 2. Also, $\mathcal{D}_n(PTH) = 3$.

(b) In Figure (13), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since there are two vertices n_4, n_5 and n_6 such that have no neighbor in S. Consider the vertex n_3. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_3 which is two;

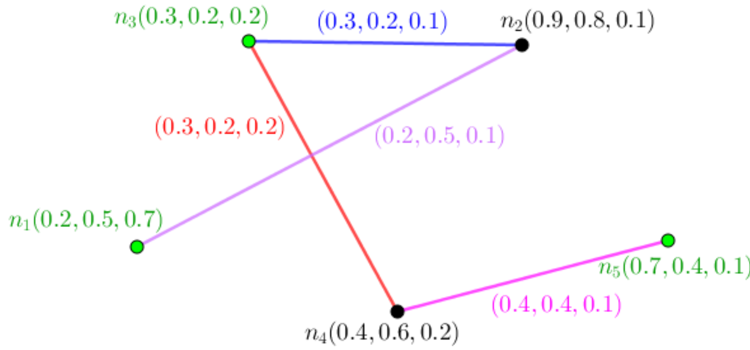

Figure 12. A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number.

(ii) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since there are two vertices n_3, n_4, n_5 and n_6 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is one which isn't [greater than] equal to the number of neighbors for n_2 which is two;

(iii) $S = \{n_2, n_5\}$ is only set of vertices which is minimal set such that it's dense set since either the vertex n_2 or the vertex n_1 has to be chosen but it's impossible to have vertex n_1 in S. Hence only choice is the vertex n_2. After that, either the vertex n_3, n_5, n_6 or the vertex n_4 has to be chosen but it's impossible to have vertex n_3, n_4, n_6 in S. Hence only choice is the vertex n_5. Every vertex inside has two neighbors and every vertex outside has either one or two neighbors. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There's only one dense set. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}(PTH) = 2$;

(iv) If $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since there are some vertices n_4, n_5 and n_6 such that have no neighbor in S. Consider the vertex n_3. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_3 which is two;

(v) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since there are some vertices n_3, n_4, n_5 and n_6 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is one which isn't [greater than] equal to the number of neighbors for n_2 which is two;

(vi) $S = \{n_2, n_5\}$ is only set of vertices which is minimal set such that it's dense set since either the vertex n_2 or the vertex n_1 has to be chosen but it's impossible to have vertex n_1 in S. Hence only choice is the vertex n_2. After that, either the vertex n_3, n_5, n_6 or the vertex n_4 has to be chosen but it's impossible to have vertex n_3, n_4, n_6 in S. Hence only choice is the vertex n_5. Every vertex inside has two neighbors and every vertex outside has either one or two neighbors. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There's only one dense set. So the minimum cardinality between all dense sets is 2. Also, $\mathcal{D}_n(PTH) = 3.8$.

Proposition 3.5. *Let* $NTG : (V, E, \sigma, \mu)$ *be a cycle-neutrosophic graph where* $\mathcal{O}(CYC) \geq 3$. *Then*

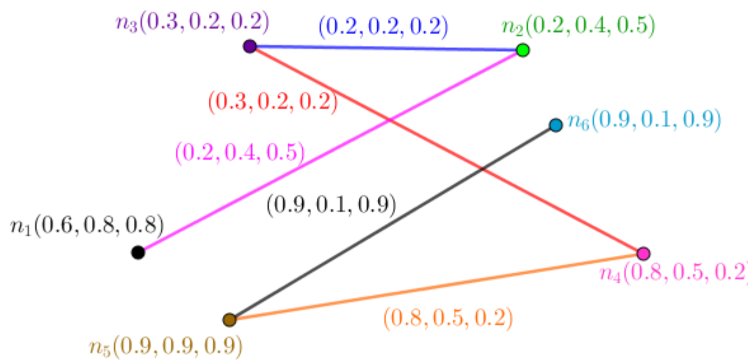

Figure 13. A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number.

$$\mathcal{D}_n(CYC) = \min\{\sum_{i=-2}^{\mathcal{O}(CYC)-3} \sigma(x_{i+3})\}$$

where rearrangements of indexes are possible in any arbitrary ways.

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(CYC)}, x_1$ be a cycle-neutrosophic graph. Every vertex has two neighbors. So these vertices have same positions and by the minimum number of vertices is on demand, the result is obtained. Thus

$$\mathcal{D}_n(CYC) = \min\{\sum_{i=-2}^{\mathcal{O}(CYC)-3} \sigma(x_{i+3})\}$$

where rearrangements of indexes are possible in any arbitrary ways. □

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.6. There are two sections for clarifications.

(a) In Figure (14), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

  (i) If $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since there are some vertices n_4 and n_5 such that have no neighbor in S. Consider the vertex n_3. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_3 which is two;

  (ii) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since there are some vertices n_3, n_4 and n_5 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is two which is [greater than] equal to the number of neighbors for n_2 which is two;

  (iii) $S_1 = \{n_1, n_4\}, S_2 = \{n_2, n_5\}, S_3 = \{n_3, n_6\}$ are only sets of vertices which are minimal sets such that they're dense sets. Since every vertex inside has two neighbors and every vertex outside has two neighbors. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There're only three dense sets. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}(CYC) = 2$;

(iv) if $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since there are some vertices n_4 and n_5 such that have no neighbor in S. Consider the vertex n_3. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_3 which is two;

(v) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since there are some vertices n_3, n_4 and n_5 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is two which is [greater than] equal to the number of neighbors for n_2 which is two;

(vi) $S_1 = \{n_1, n_4\}, S_2 = \{n_2, n_5\}, S_3 = \{n_3, n_6\}$ are only sets of vertices which are minimal sets such that they're dense sets. Since every vertex inside has two neighbors and every vertex outside has two neighbors. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There're only three dense sets. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}_n(CYC) = 2.2$ corresponded to S_1;

(b) In Figure (15), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since there's one vertex n_4 such that have no neighbor in S. Consider the vertex n_3. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_3 which is two;

(ii) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since there are some vertices n_3, and n_4 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is two which is [greater than] equal to the number of neighbors for n_2 which is two;

(iii) $S_1 = \{n_1, n_3\}, S_2 = \{n_1, n_4\}, S_3 = \{n_2, n_4\}, S_4 = \{n_2, n_5\}, S_5 = \{n_3, n_5\}$ are only sets of vertices which are minimal sets such that they're dense sets. Since every vertex inside has two neighbors and every vertex outside has two neighbors. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There're only five dense sets. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}(CYC) = 2$;

(iv) if $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since there's one vertex n_4 such that have no neighbor in S. Consider the vertex n_3. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_3 which is two;

(v) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since there are some vertices n_3, and n_4 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is two which is [greater than] equal to the number of neighbors for n_2 which is two;

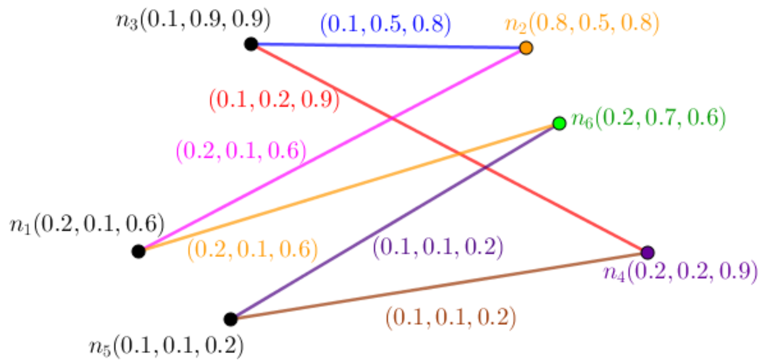

Figure 14. A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number.

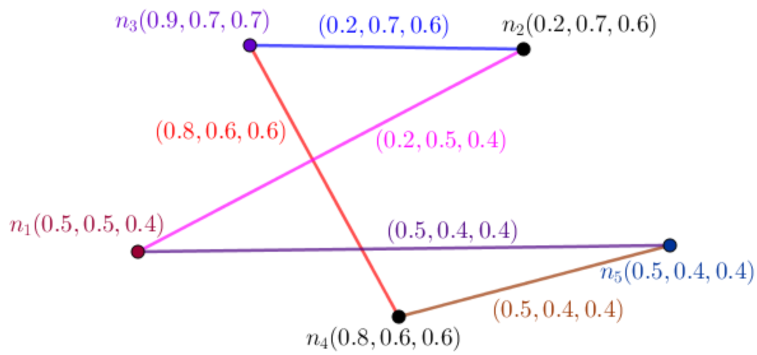

Figure 15. A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number.

(vi)

$$
\begin{aligned}
S_1 &= \{n_1, n_3\} \to 2.8 \\
S_2 &= \{n_1, n_4\} \to 2.2 \\
S_3 &= \{n_2, n_4\} \to 3.4 \\
S_4 &= \{n_2, n_5\} \to 2.5 \\
S_5 &= \{n_3, n_5\} \to 2.3 \\
&\text{Minimum number is } 2.2
\end{aligned}
$$

are only sets of vertices which are minimal sets such that they're dense sets. Since every vertex inside has two neighbors and every vertex outside has two neighbors. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There're only five dense sets. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}_n(CYC) = 2$ corresponded to S_2.

Proposition 3.7. *Let* $NTG : (V, E, \sigma, \mu)$ *be a star-neutrosophic graph with center* c*. Then*

$$\mathcal{D}_n(STR_{1,\sigma_2}) = \sigma(c).$$

Proof. Suppose $STR_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. For non-center vertices, the number of neighbors is 1 and center has $\mathcal{O}(STR_{1,\sigma_2}) - 1$ neighbors. If $S = \{c\}$ is a set of vertices, then every vertex inside has $\mathcal{O}(STR_{1,\sigma_2}) - 1$ neighbors and every vertex outside has one neighbor. So the number of neighbors for every vertex inside is greater than the number of neighbors for every vertex outside and also every vertex inside is a neighbor for every vertex outside. Thus

$$\mathcal{D}_n(STR_{1,\sigma_2}) = \sigma(c).$$

□

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.8. There is one section for clarifications. In Figure (16), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_4, n_2\}$ is a set of vertices, then it isn't dense set since there are some vertices n_3 and n_5 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is four which isn't [greater than] equal to the number of neighbors for n_2 which is one;

(ii) if $S = \{n_2, n_3, n_4, n_5\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified. Consider the vertex n_1. The number of neighbors for $n_i,\ i = 2, 3, 4, 5$ is one which isn't [greater than] equal to the number of neighbors for n_1 which is four;

(iii) $S = \{n_1\}$ is only set of vertices which is minimal set such that it's dense set. Since every vertex inside has four neighbors and every vertex outside has one neighbor. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There's only one dense set in the term of minimal set. So the minimum cardinality between all dense sets is 1. Thus $\mathcal{D}(STR_{1,\sigma_2}) = 1$;

(iv) If $S = \{n_4, n_2\}$ is a set of vertices, then it isn't dense set since there are some vertices n_3 and n_5 such that have no neighbor in S. Consider the vertex n_2. The number of neighbors for n_1 is four which isn't [greater than] equal to the number of neighbors for n_2 which is one;

(v) if $S = \{n_2, n_3, n_4, n_5\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified. Consider the vertex n_1. The number of neighbors for $n_i,\ i = 2, 3, 4, 5$ is one which isn't [greater than] equal to the number of neighbors for n_1 which is four;

(vi) $S = \{n_1\}$ is only set of vertices which is minimal set such that it's dense set. Since every vertex inside has four neighbors and every vertex outside has one neighbor. Hence the number of neighbors for vertices in S is greater than [equal to] the number of neighbors for vertices in $V \setminus S$. There's only one dense set in the term of minimal set. So the minimum cardinality between all dense sets is 1. Thus $\mathcal{D}_n(STR_{1,\sigma_2}) = 1.9$

Proposition 3.9. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-bipartite-neutrosophic graph. Then*

$$\mathcal{D}_n(CMC_{\sigma_1,\sigma_2}) = \sum_{x \in V_1} \sigma(x).$$

where $|V_1| \leq |V_2|\}$.

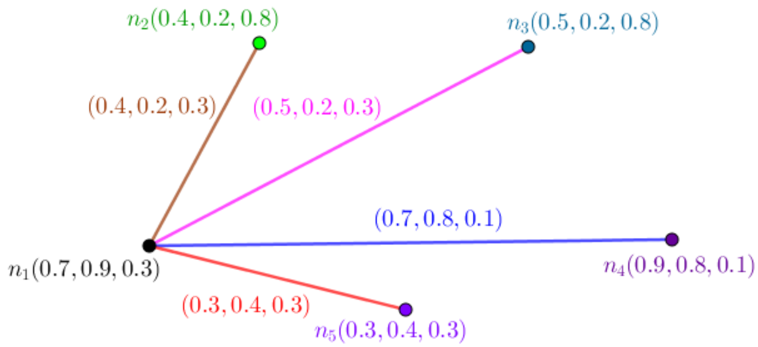

Figure 16. A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number.

Proof. Suppose $CMC_{\sigma_1,\sigma_2} : (V, E, \sigma, \mu)$ is a complete-bipartite-neutrosophic graph. Consider $V_1 = \min\{|V_1|, |V_2|\}$. For vertices in V_2, the number of neighbors is $|V_1|$ and for vertices in V_1, the number of neighbors is $|V_2|$ where $|V_1| \leq |V_2|$. If $S = V_1$ is a set of vertices, then every vertex inside has $|V_2|$ neighbors and every vertex outside has $|V_1|$ neighbors. So the number of neighbors for every vertex inside is greater than the number of neighbors for every vertex outside and also every vertex inside is a neighbor for every vertex outside. Thus

$$\mathcal{D}_n(CMC_{\sigma_1,\sigma_2}) = \sum_{x \in V_1} \sigma(x).$$

where $|V_1| \leq |V_2|\}$. □

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.10. There is one section for clarifications. In Figure (17), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If $S = \{n_4, n_2\}$ is a set of vertices, then it's dense set since there's no vertex such that have neighbor in S. Consider the vertex n_1. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_1 which is two;

(*ii*) if $S = \{n_4\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified;

(*iii*) $S_1 = \{n_1, n_2\}, S_2 = \{n_1, n_3\}, S_3 = \{n_1, n_4\}, S_4 = \{n_2, n_3\}, S_5 = \{n_2, n_4\}, S_6 = \{n_3, n_4\}$ are only sets of vertices which are minimal sets such that they're dense sets. Since every vertex inside has two neighbors and every vertex outside has two neighbors. Hence the number of neighbors for vertices in either of $S_i,\ i = 1, 2, \cdots, 6$ is equal to [greater than] the number of neighbors for vertices in $V \setminus S_i,\ i = 1, 2, \cdots, 6$. There're only four dense sets in the term of minimal set. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}(CMC_{\sigma_1,\sigma_2}) = 2$;

(*iv*) If $S = \{n_4, n_2\}$ is a set of vertices, then it's dense set since there's no vertex such that have neighbor in S. Consider the vertex n_1. The number of neighbors for n_2 is two which is [greater than] equal to the number of neighbors for n_1 which is two;

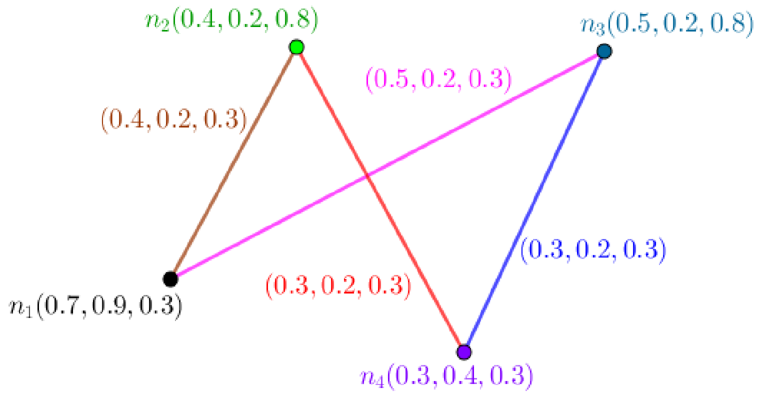

Figure 17. A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number.

(*v*) if $S = \{n_4\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified;

(*vi*) $S_1 = \{n_1, n_2\}, S_2 = \{n_1, n_3\}, S_3 = \{n_1, n_4\}, S_4 = \{n_2, n_3\}, S_5 = \{n_2, n_4\}, S_6 = \{n_3, n_4\}$ are only sets of vertices which are minimal sets such that they're dense sets. Since every vertex inside has two neighbors and every vertex outside has two neighbors. Hence the number of neighbors for vertices in either of $S_i, \ i = 1, 2, \cdots, 6$ is equal to [greater than] the number of neighbors for vertices in $V \setminus S_i, \ i = 1, 2, \cdots, 6$. There're only four dense sets in the term of minimal set. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}_n(CMC_{\sigma_1,\sigma_2}) = 2.4$ corresponded to $S_5 = \{n_2, n_4\}$.

Proposition 3.11. *Let* $NTG : (V, E, \sigma, \mu)$ *be a complete-t-partite-neutrosophic graph where* $t \geq 3$. *Then*

$$\mathcal{D}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \sum_{x \in V_1} \sigma(x).$$

where $|V_1| = \min\{|V_1|, |V_2|, \cdots, |V_t|\}$.

Proof. Suppose $CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t} : (V, E, \sigma, \mu)$ is a complete-t-partite-neutrosophic graph. Consider $|V_1| = \min\{|V_1|, |V_2|, \cdots, |V_t|\}$. For vertices in $V_i, \ i = 2, 3, \cdots, t$, the number of neighbors is $|V_1| + |V_2| + \cdots + |V_{i-1}| + |V_{i+1}| + \cdots + |V_t|$ and for vertices in V_1, the number of neighbors is $|V_2| + |V_3| + \cdots + |V_t|$ where $|V_1| \leq |V_i|, \ i = 2, 3, \cdots, t$. If $S = V_1$ is a set of vertices, then every vertex inside has $|V_2| + |V_3| + \cdots + |V_t|$ neighbors and every vertex outside has $|V_1| + |V_2| + \cdots + |V_{i-1}| + |V_{i+1}| + \cdots + |V_t|$ neighbors where $|V_1| \leq |V_i|, \ i = 2, 3, \cdots, t$. So the number of neighbors for every vertex inside is greater than the number of neighbors for every vertex outside and also every vertex inside is a neighbor for every vertex outside. Thus

$$\mathcal{D}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \sum_{x \in V_1} \sigma(x).$$

where $|V_1| = \min\{|V_1|, |V_2|, \cdots, |V_t|\}$. □

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about

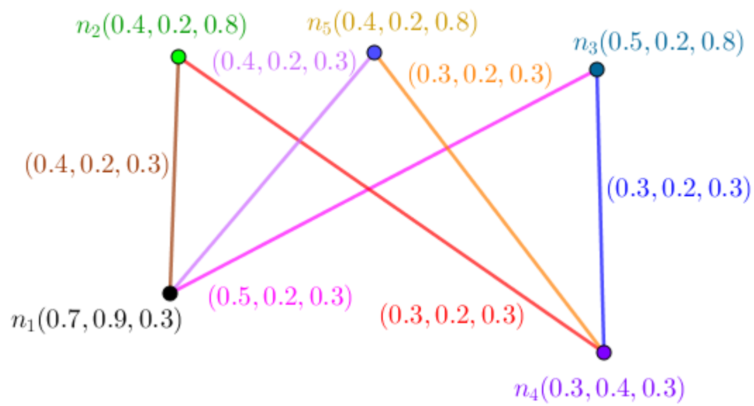

Figure 18. A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number.

new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.12. There is one section for clarifications. In Figure (18), a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_4, n_2\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified in the terms of S. Consider the vertex n_1. The number of neighbors for n_2 is two which isn't greater than [equal to] the number of neighbors for n_1 which is three;

(ii) if $S = \{n_4\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified in the terms of having neighbor from a member of S;

(iii) $S = \{n_1, n_4\}$, is only set of vertices which is minimal set such that it's dense set. Since every vertex inside has three neighbors and every vertex outside has two neighbors. Hence the number of neighbors for vertices in S is [equal to] greater than the number of neighbors for vertices in $V \setminus S$. There's only one dense set in the term of minimal set. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

(iv) If $S = \{n_4, n_2\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified in the terms of S. Consider the vertex n_1. The number of neighbors for n_2 is two which isn't greater than [equal to] the number of neighbors for n_1 which is three;

(v) if $S = \{n_4\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified in the terms of having neighbor from a member of S;

(vi) $S = \{n_1, n_4\}$, is only set of vertices which is minimal set such that it's dense set. Since every vertex inside has three neighbors and every vertex outside has two neighbors. Hence the number of neighbors for vertices in S is [equal to] greater than the number of neighbors for vertices in $V \setminus S$. There's only one dense set in the term of minimal set. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2.9$.

Proposition 3.13. *Let* $NTG : (V, E, \sigma, \mu)$ *be a wheel-neutrosophic graph. Then*

$$\mathcal{D}_n(WHL_{1,\sigma_2}) = \sigma(c).$$

Proof. Suppose $WHL_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a wheel-neutrosophic graph. The argument is elementary. Since all vertices of a cycle join to one vertex, consider c as a center. For non-center vertices, the number of neighbors is three and for center, the number of neighbors is $\mathcal{O}(WHL_{1,\sigma_2}) - 1$. If $S = \{c\}$ is a set of vertices, then every vertex inside has $\mathcal{O}(WHL_{1,\sigma_2}) - 1$ neighbors and every vertex outside has three neighbors. So the number of neighbors for every vertex inside is greater than the number of neighbors for every vertex outside and also every vertex inside is a neighbor for every vertex outside. Thus

$$\mathcal{D}_n(WHL_{1,\sigma_2}) = \sigma(c).$$

□

The clarifications about results are in progress as follows. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.14. There is one section for clarifications. In Figure (19), a wheel-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_4, n_2\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified in the terms of S. Consider the vertex n_1. The number of neighbors for n_2 is three which isn't greater than [equal to] the number of neighbors for n_1 which is four;

(ii) if $S = \{n_4\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified in the terms of having neighbor from a member of S;

(iii) $S = \{n_1\}$ is only set of vertices which is minimal set such that it's dense set. Since every vertex inside has four neighbors and every vertex outside has three neighbors. Hence the number of neighbors for vertices in S is [equal to] greater than the number of neighbors for vertices in $V \setminus S$. There's only one dense set in the term of minimal set. So the minimum cardinality between all dense sets is 1. Thus $\mathcal{D}(WHL_{1,\sigma_2}) = 1$;

(iv) If $S = \{n_4, n_2\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified in the terms of S. Consider the vertex n_1. The number of neighbors for n_2 is three which isn't greater than [equal to] the number of neighbors for n_1 which is four;

(v) if $S = \{n_4\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified in the terms of having neighbor from a member of S;

(vi) $S = \{n_1\}$ is only set of vertices which is minimal set such that it's dense set. Since every vertex inside has four neighbors and every vertex outside has three neighbors. Hence the number of neighbors for vertices in S is [equal to] greater than the number of neighbors for vertices in $V \setminus S$. There's only one dense set in the term of minimal set. So the minimum cardinality between all dense sets is 1. Thus $\mathcal{D}_n(WHL_{1,\sigma_2}) = 1.9$.

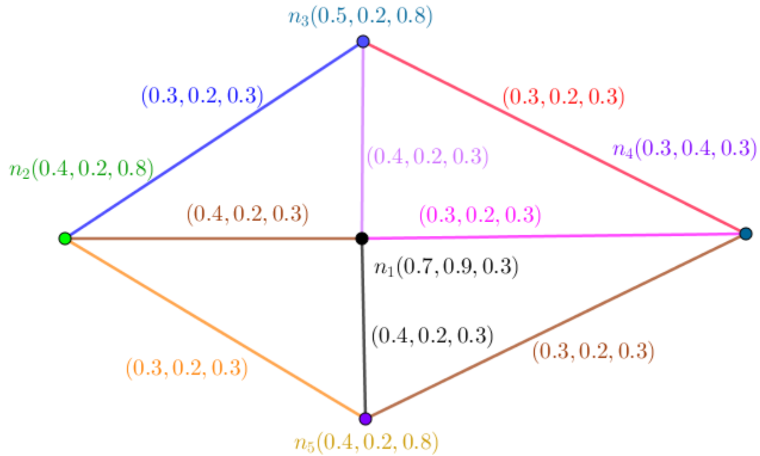

Figure 19. A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number.

4 Applications in Time Table and Scheduling

In this section, two applications for time table and scheduling are provided where the models are either complete models which mean complete connections are formed as individual and family of complete models with common neutrosophic vertex set or quasi-complete models which mean quasi-complete connections are formed as individual and family of quasi-complete models with common neutrosophic vertex set.

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has importance to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive sections. Beyond that, sometimes sections are not the same.

Step 3. (Model) The situation is designed as a model. The model uses data to assign every section and to assign to relation amid sections, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relations amid them. Table (1), clarifies about the assigned numbers to these situations.

Table 1. Scheduling concerns its Subjects and its Connections as a neutrosophic graph in a Model.

Sections of NTG	n_1	$n_2 \cdots$	n_5
Values	$(0.7, 0.9, 0.3)$	$(0.4, 0.2, 0.8) \cdots$	$(0.4, 0.2, 0.8)$
Connections of NTG	E_1	$E_2 \cdots$	E_6
Values	$(0.4, 0.2, 0.3)$	$(0.5, 0.2, 0.3) \cdots$	$(0.3, 0.2, 0.3)$

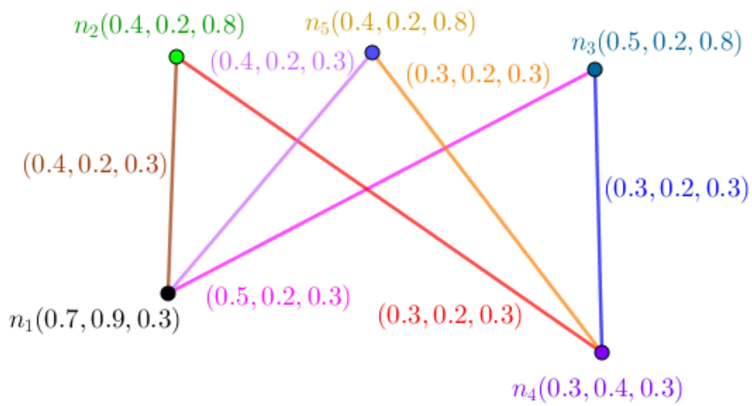

Figure 20. A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number

4.1 Case 1: Complete-t-partite Model alongside its dense number and its neutrosophic dense number

Step 4. (Solution) The neutrosophic graph alongside its dense number and its neutrosophic dense number as model, propose to use specific number. Every subject has connection with some subjects. Thus the connection is applied as possible and the model demonstrates quasi-full connections as quasi-possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is star, the number is different. Also, it holds for other types such that complete, wheel, path, and cycle. The collection of situations is another application of its dense number and its neutrosophic dense number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are five subjects which are represented as Figure (20). This model is strong and even more it's quasi-complete. And the study proposes using specific number which is called its dense number and its neutrosophic dense number. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to this model and situation to compare them with same situations to get more precise. Consider Figure (20). In Figure (20), an complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If $S = \{n_4, n_2\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified in the terms of S. Consider the vertex n_1. The number of neighbors for n_2 is two which isn't greater than [equal to] the number of neighbors for n_1 which is three;

(ii) if $S = \{n_4\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified in the terms of having neighbor from a member of S;

(iii) $S = \{n_1, n_4\}$, is only set of vertices which is minimal set such that it's dense set. Since every vertex inside has three neighbors and every vertex outside has two neighbors. Hence the number of neighbors for vertices in S is [equal to] greater than the number of neighbors for vertices in $V \setminus S$. There's only

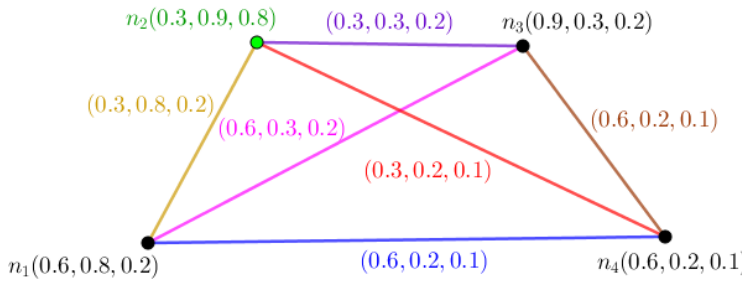

Figure 21. A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number

one dense set in the term of minimal set. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$;

(iv) If $S = \{n_4, n_2\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified in the terms of S. Consider the vertex n_1. The number of neighbors for n_2 is two which isn't greater than [equal to] the number of neighbors for n_1 which is three;

(v) if $S = \{n_4\}$ is a set of vertices, then it isn't dense set since there's one vertex n_1 such that doesn't qualified in the terms of having neighbor from a member of S;

(vi) $S = \{n_1, n_4\}$, is only set of vertices which is minimal set such that it's dense set. Since every vertex inside has three neighbors and every vertex outside has two neighbors. Hence the number of neighbors for vertices in S is [equal to] greater than the number of neighbors for vertices in $V \setminus S$. There's only one dense set in the term of minimal set. So the minimum cardinality between all dense sets is 2. Thus $\mathcal{D}_n(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2.9$.

4.2 Case 2: Complete Model alongside its A Neutrosophic Graph in the Viewpoint of its dense number and its neutrosophic dense number

Step 4. (Solution) The neutrosophic graph alongside its dense number and its neutrosophic dense number as model, propose to use specific number. Every subject has connection with every given subject in deemed way. Thus the connection applied as possible and the model demonstrates full connections as possible between parts but with different view where symmetry amid vertices and edges are the matters. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete multipartite, the number is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application of its dense number and its neutrosophic dense number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are four subjects which are represented in the formation of one model as Figure (21). This model is neutrosophic strong as individual and even more it's complete. And the study proposes using specific number which is called its dense number and its neutrosophic dense number for this model. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to these models as individual. A model as a collection of situations to compare them with another model as a collection of situations to get more precise. Consider Figure (21). There is one section for clarifications.

(i) If $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since every vertex inside has one neighbor inside and two neighbors outside. Hence the number of neighbors inside isn't greater than the number of neighbors outside;

(ii) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since every vertex inside has no neighbor inside and three neighbors outside. Hence the number of neighbors inside isn't greater than the number of neighbors outside;

(iii) if $S_1 = \{n_1, n_2, n_3\}, S_2 = \{n_1, n_2, n_4\}, S_3 = \{n_2, n_3, n_4\}$ are sets of vertices, then they're dense sets since every vertex inside has two neighbors inside and one neighbor outside. Hence the number of neighbors inside is greater than the number of neighbors outside. The minimum cardinality between all dense sets is 3. Thus $\mathcal{D}(CMT_\sigma) = 3$;

(iv) if $S = \{n_1, n_2\}$ is a set of vertices, then it isn't dense set since every vertex inside has one neighbor inside and two neighbors outside. Hence the number of neighbors inside isn't greater than the number of neighbors outside;

(v) if $S = \{n_1\}$ is a set of vertices, then it isn't dense set since every vertex inside has no neighbor inside and three neighbors outside. Hence the number of neighbors inside isn't greater than the number of neighbors outside;

(vi) if $S_1 = \{n_1, n_2, n_3\}, S_2 = \{n_1, n_2, n_4\}, S_3 = \{n_2, n_3, n_4\}$ are sets of vertices, then they're dense sets since every vertex inside has two neighbors inside and one neighbor outside. Hence the number of neighbors inside is greater than the number of neighbors outside. The minimum neutrosophic cardinality $\sum_{s \in S} \sum_{i=1}^{3} \sigma_i(s)$ between all dense sets is 3.9. Thus $\mathcal{D}_n(CMT_\sigma) = 3.9$.

5 Open Problems

In this section, some questions and problems are proposed to give some avenues to pursue this study. The structures of the definitions and results give some ideas to make new settings which are eligible to extend and to create new study.

Notion concerning its dense number and its neutrosophic dense number are defined in neutrosophic graphs. Thus,

Question 5.1. *Is it possible to use other types of its dense number and its neutrosophic dense number?*

Question 5.2. *Are existed some connections amid different types of its dense number and its neutrosophic dense number in neutrosophic graphs?*

Question 5.3. *Is it possible to construct some classes of neutrosophic graphs which have "nice" behavior?*

Question 5.4. *Which mathematical notions do make an independent study to apply these types in neutrosophic graphs?*

Problem 5.5. *Which parameters are related to this parameter?*

Problem 5.6. *Which approaches do work to construct applications to create independent study?*

Problem 5.7. *Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?*

6 Conclusion and Closing Remarks

In this section, concluding remarks and closing remarks are represented. The drawbacks of this article are illustrated. Some benefits and advantages of this study are highlighted.

This study uses two definitions concerning dense number and neutrosophic dense number arising from neighborhood and the number of neighbors in neutrosophic graphs assigned to neutrosophic graphs. The connections of consecutive vertices which aren't clarified by a neighborhood differ them from each other and put them in different categories to represent a number which is called dense number and neutrosophic dense number arising from neighborhood and the number of neighbors in neutrosophic graphs assigned to neutrosophic graphs. Further studies could be about changes in the settings to compare these notions amid different settings of neutrosophic graphs theory. One way is finding some relations amid all definitions of notions to make sensible definitions. In Table (2), some limitations and advantages of this study are pointed out.

Table 2. A Brief Overview about Advantages and Limitations of this Study

Advantages	Limitations
1. Dense Number of Model	1. Connections amid Classes
2. Neutrosophic Dense Number of Model	
3. Minimal Dense Sets	2. Study on Families
4. Number of Neighbors	
5. Neighborhood	3. Same Models in Family

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